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Complex Fermatean Neutrosophic Sets and their Applications in Decision Making

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Abstract: Complex Neutrosophic Sets have emerged as a groundbreaking paradigm for handling uncertainty in complex-valued variables. By defining truth, indeterminacy, and falsity membership functions within the unit circle of the complex plane, this innovative framework provides a robust foundation for uncertainty management. This paper introduces a novel hesitance distance measure for Complex Fermatean Neutrosophic Sets, enabling the quantification of uncertainty in complex decision-making scenarios. The proposed distance measure is applied to solve a Multi-Criteria Decision Making Problem, demonstrating its effectiveness in handling uncertainty and ambiguity. This research has far-reaching implications for various fields, including decision theory, artificial intelligence, and operations research. The proposed hesitance distance measure provides a valuable tool for decision-makers to quantify uncertainty and make informed decisions in complex environments.

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Keywords: Neutrosophic Sets, Complex Neutrosophic Sets, Compex Fermatean Neutrosophic Sets

1. Introduction

Fuzzy Sets introduced by Zadeh[1] paved way to deal with problems with uncertainty. Atanassov[2] added a non-membership function to Fuzzy set to coin Intuitionistic fuzzy sets, which helped to solve problems in real life. The concept of neutrosophic sets (NS) and neutrosophic logic was introduced by Smarandache [3] as a generalization of fuzzy sets [1] and intuitionistic fuzzy sets [2].

The concept of Complex Fuzzy Sets was introduced by Ramot et al[4]. This was extended to Complex Intuitionistic Fuzzy Set[5], Complex Pythagorean Fuzzy Set[6] and Complex Neutrosophic Sets[7]. The idea of Fermatean Neutrosophic Fuzzy Sets was proposed by Antony and Jansi[8]. Later, Broumi et al[6], defined Complex Fermatean Neutrosophic sets.

Thangaraj Beaula and Vijaya[9-11] have adopted different fuzzy numbers and ranking methods to solve critical path problems. Vijaya et al[13,14] have used Pythagorean Fuzzy numbers and Neutrosophic Fuzzy numbers to find the solution of Decision making problem and critical path problems. Madad Khan[12] proposed a new distance measure in Complex Decision making problems. Ali, Smarandache, Khan & Ye[15-17] have discussed the applications of Complex Neutrosophic sets. Ranulfo Paiva Barbosa (Sobrinho), & Smarandache, F.[18] introduced Pura Vida Neutrosophic Algebra. Jayasudha and Raghavi[19] studied the properties of Neutrosophic Hypersoft Matrices.

Complex Fermatean Neutrosophic Sets(CFNS) provides a robust framework for handling uncertainty, ambiguity, and imprecision in decision-making problems. CFNS is particularly useful in complex decision scenarios where multiple criteria, constraints, and stakeholders are involved. CFNS can capture non-linear relationships between variables, which is essential in many real-world decision-making problems. CFNS can incorporate human judgment and expertise into the decision-making process, making it more realistic and effective.

CFNS lacks a comprehensive axiomatization, which is essential for establishing a solid theoretical foundation. The properties of CFNS, such as commutativity, associativity, and distributivity, require further investigation. Research on operators, such as union, intersection, and complement, is limited, and more work is needed to develop a comprehensive operator theory. Efficient algorithms for computing CFNS operations, such as membership functions and weighted aggregation, are needed. Research on distance measures for CFNS is limited, and more work is needed to develop effective distance measures. CFNS has limited applications in real-world problems, such as decision-making, optimization, and risk assessment. CFNS-based models, such as decision-making models and optimization models, require further development.

In this paper, we have used Hesitance degree of Neutrosophic Sets to coin the distance measure of Complex Neutrosphic Fermatean Sets. Using the distance measure and Score function, a Multicriteria Decision Making Problem is solved whose parameters are Complex Neutrosophic Fermatean Sets.

2. Preliminaries

Definition 2.1[1]

Let X be a Universal Set. Then a Fuzzy Set $\tilde{A}$ is usually characterized by the membership function $\mu_{\tilde{A}}(x)$, where $\mu_{\tilde{A}}(x): X \to [0,1]$ and it is denoted by $\tilde{A} = \{x, \mu_{\tilde{A}}(x); x \in X\}$

Definition 2.2[3]

Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A is an object having the form

$$A=\{< x : T_A(x), I_A(x), F_A(x) > x \in X\}$$

where the functions $T, I, F: X \to]0^-, 1^+[$ define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with, condition $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$ The functions $T_A(x), I_A(x) \,\&\, F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$

Definition 2.3[4]

Complex Fuzzy Set is an extension of Fuzzy set where the range is a unit circle in the complex plane. The membership function is complex valued which is given by $\mu_{\tilde{A}}(x) = r(x)e^{i\theta(x)}$, where $r(x) \in [0,1], \theta(x) \in [0, 2\pi]$

Definition 2.4[7]

A Neutrosophic set can be extended to Complex Neutrosophic Fuzzy Set where the Truth membership, Indeterminacy membership and Falsity membership functions are all complex valued functions and is given by $A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$ where $T_A(x) = r_A(x)e^{i\theta_1(u)}$, $I_A(x) = s_A(x)e^{i\theta_2(u)}$, $F_A(x) = t_A(x)e^{i\theta_3(u)}$, $0^- \le r_A + s_A + t_A \le 3^+$, $0 \le \theta_1 + \theta_2 + \theta_3 \le 2\pi$

Definition 2.5[8]

A Fermatean Neutrosophic Set A defined on X is represented as

$$A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$$

with the condition $0^- \le T_A^3 + I_A^3 + F_A^3 \le 2^+$ and $0 \le T_A^3 + F_A^3 \le 1, 0 \le I_A^3 \le 1$

3. Complex Fermatean Neutrosophic Sets(CFNS)

Definition 3.1[6]

A Complex Fermatean Neutrosophic Set A on X is denoted by $A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$, where $T_A(x) = r_A(x)e^{2\pi i\theta_1(u)}$, $I_A(x) = s_A(x)e^{2\pi i\theta_2(u)}$, $F_A(x) = t_A(x)e^{2\pi i\theta_3(u)}$, represents the truth, indeterminacy and falsity membership functions with the condition $0^- \le r_A^3 + s_A^3 + t_A^3 \le 3^+$, $0^- \le \theta_1^3 + \theta_2^3 + \theta_3^3 \le 3^+$. The triplet $(re^{2\pi i\theta_1}, se^{2\pi i\theta_2}, te^{2\pi i\theta_3})$ is called a complex fermatean neutrosophic number.

3.2 Operations on Complex Fermatean Neutrosophic Sets

If $A = (x, r_1e^{2\pi i\theta_1}, s_1e^{2\pi i\theta_2}, t_1e^{2\pi i\theta_3})$ & $B = (x, r_2e^{2\pi i\phi_1}, s_2e^{2\pi i\phi_2}, t_2e^{2\pi i\phi_3})$ are any two complex fermatean neutrosophic sets, then their union, intersection and complement are defined as follows

Union: $A \cup B = \{x, \max(r_1, r_2)e^{2\pi i \max(\theta_1, \phi_1)}, \max(s_1, s_2)e^{2\pi i \max(\theta_2, \phi_2)}, \max(t_1, t_2)e^{2\pi i \max(\theta_3, \phi_3)})\}$

Intersection: $A \cap B = \{x, \min(r_1, r_2)e^{2\pi i \min(\theta_1, \phi_1)}, \min(s_1, s_2)e^{2\pi i \min(\theta_2, \phi_2)}, \min(t_1, t_2)e^{2\pi i \min(\theta_3, \phi_3)})\}$

Complement: $A^C = \{x, (1-r_1)e^{2\pi i(1-\theta_1)}, (1-s_1)e^{2\pi i(1-\theta_2)}, (1-t_1)e^{2\pi i(1-\theta_3)})\}$

Theorem 3.3

Operations on Complex Fermatean Neutrtosophic Sets satisfies Commutative, Associative, Distributive and Demorgan's properties

Proof:

Let $A = (x, r_1e^{2\pi i\theta_1}, s_1e^{2\pi i\theta_2}, t_1e^{2\pi i\theta_3})$, $B = (x, r_2e^{2\pi i\phi_1}, s_2e^{2\pi i\phi_2}, t_2e^{2\pi i\phi_3})$ &

$C = (x, r_3e^{2\pi i\omega_1}, s_3e^{2\pi i\omega_2}, t_3e^{2\pi i\omega_3})$ be complex fermatean neutrosophic sets.

Commutative property:

i. $A \cup B = B \cup A$

ii. $A \cap B = B \cap A$

$$A \cup B = \{x, \max(r_1, r_2)e^{2\pi i \max(\theta_1, \phi_1)}, \max(s_1, s_2)e^{2\pi i \max(\theta_2, \phi_2)}, \max(t_1, t_2)e^{2\pi i \max(\theta_3, \phi_3)})\}$$

$$= \{x, \max(r_2, r_1)e^{2\pi i \max(\phi_1, \theta_1)}, \max(s_2, s_1)e^{2\pi i \max(\phi_2, \theta_2)}, \max(t_2, t_1)e^{2\pi i \max(\phi_3, \theta_3)})\}$$

$$= B \cup A$$

$$A \cap B = \{x, \min(r_1, r_2)e^{2\pi i \min(\theta_1, \phi_1)}, \min(s_1, s_2)e^{2\pi i \min(\theta_2, \phi_2)}, \min(t_1, t_2)e^{2\pi i \min(\theta_3, \phi_3)})\}$$

$$= \{x, \min(r_2, r_1)e^{2\pi i \min(\phi_1, \theta_1)}, \min(s_2, s_1)e^{2\pi i \min(\phi_2, \theta_2)}, \min(t_2, t_1)e^{2\pi i \min(\phi_3, \theta_3)})\}$$

$$= B \cap A$$

Associative property

i. $A \cup (B \cup C) = (A \cup B) \cup C$

ii. $A \cap (B \cap C) = (A \cap B) \cap C$

$$A \cup (B \cup C) = \{x, \max(r_1, \max(r_2, r_3)e^{2\pi i \max(\theta_1, \max(\phi_1, \omega_1)}, \max(s_1, \max(s_2, s_3)e^{2\pi i \max(\theta_2, \max(\phi_2, \omega_3)},$$

$$\max(t_1, \max(t_2, t_3)e^{2\pi i \max(\theta_3, \max(\phi_3, \omega_3)})\}$$

$$= \{x, \max(r_1, r_2, r_3)e^{2\pi i \max(\theta_1, \phi_1, \omega_1)}, \max(s_1, s_2, s_3)e^{2\pi i \max(\theta_2, \phi_2, \omega_3)}, \max(t_1, t_2, t_3)e^{2\pi i \max(\theta_3, \phi_3, \omega_3)})\}$$

$$= (A \cup B) \cup C$$

Similarly $A \cap (B \cap C) = (A \cap B) \cap C$ can be proved.

Distributive property:

i. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$A \cup B = \{x, \max(r_1, r_2)e^{2\pi i \max(\theta_1,\phi_1)}, \max(s_1, s_2)e^{2\pi i \max(\theta_2,\phi_2)}, \max(t_1, t_2)e^{2\pi i \max(\theta_3,\phi_3)})\}$$

$$A \cup C = \{x, \max(r_1, r_3)e^{2\pi i \max(\theta_1,\omega_1)}, \max(s_1, s_3)e^{2\pi i \max(\theta_2,\omega_2)}, \max(t_1, t_3)e^{2\pi i \max(\theta_3,\omega_3)})\}$$

$$(A \cup B) \cap (A \cup C) = \{x, \min(\max(r_1, r_2), \max(r_1, r_3))e^{2\pi i \min(\max(\theta_1,\phi_1),\max(\theta_1,\omega_1))},$$

$$\min(\max(s_1, s_2), \max(s_1, s_3))e^{2\pi i \min(\max(\theta_2,\phi_2),\max(\theta_2,\omega_2))},$$

$$\min(\max(t_1, t_2), \max(t_1, t_3))e^{2\pi i \min(\max(\theta_3,\phi_3),\max(\theta_3,\omega_3))}\}$$

$$= \{x, \max(r_1, \min(r_2, r_3))e^{2\pi i \max(\theta_1,\min(\phi_1,\omega_1))}, \max(s_1, \min(s_2, s_3))e^{2\pi i \max(\theta_2,\min(\phi_2,\omega_{12}))},$$

$$\max(t_1, \min(t_2, t_3))e^{2\pi i \max(\theta_3,\min(\phi_3,\omega_3))}\}$$

$$= A \cup (B \cap C)$$

Similarly $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ can be proved.

Demorgan's property

i. $(A \cup B)^c = A^c \cap B^c$

ii. $(A \cap B)^c = A^c \cup B^c$

$$A \cup B = \{x, \max(r_1, r_2)e^{2\pi i \max(\theta_1,\phi_1)}, \max(s_1, s_2)e^{2\pi i \max(\theta_2,\phi_2)}, \max(t_1, t_2)e^{2\pi i \max(\theta_3,\phi_3)})\}$$

$$(A \cup B)^c = \{x, 1 - \max(r_1, r_2)e^{2\pi i(1-\max(\theta_1,\phi_1))}, 1 - \max(s_1, s_2)e^{2\pi i(1-\max(\theta_2,\phi_2))}, 1 - \max(t_1, t_2)e^{2\pi i(1-\max(\theta_3,\phi_3))})\}$$

$$A^C = \{x, (1 - r_1)e^{2\pi i(1-\theta_1)}, (1 - s_1)e^{2\pi i(1-\theta_2)}, (1 - t_1)e^{2\pi i(1-\theta_3)})\}$$

$$B^C = \{x, (1 - r_2)e^{2\pi i(1-\phi_1)}, (1 - s_2)e^{2\pi i(1-\phi_2)}, (1 - t_2)e^{2\pi i(1-\phi_3)})\}$$

$$A^c \cap B^c = \{x, \min(1 - r_1, 1 - r_2)e^{2\pi i \min(1-\theta_1,1-\phi_1)}, \min(1 - s_1, 1 - s_2)e^{2\pi i \min(1-\theta_2,1-\phi_2)},$$

$$\min(1 - t_1, 1 - t_2)e^{2\pi i \min(1-\theta_3,1-\phi_3)}\}$$

$$=$$

$$\{x, 1 - \max(r_1, r_2)e^{2\pi i(1-\max(\theta_1,\phi_1))}, 1 - \max(s_1, s_2)e^{2\pi i(1-\max(\theta_2,\phi_2))}, 1 - \max(t_1, t_2)e^{2\pi i(1-\max(\theta_3,\phi_3))})\}$$

$$=(A\cup B)^c$$

Similarly $(A\cap B)^c = A^c \cup B^c$ can be proved.

4. Distance measures of Complex Fermatean Neutrosophic set

Definition 4.1

Let $A=(x, r_1 e^{2\pi i\theta_1}, s_1 e^{2\pi i\theta_2}, t_1 e^{2\pi i\theta_3})$, $B=(x, r_2 e^{2\pi i\phi_1}, s_2 e^{2\pi i\phi_2}, t_2 e^{2\pi i\phi_3})$ & be any two complex fermatean neutrosophic sets. Then the Hamming distance between A and B is defined as

$$d_H(A,B)=\frac{1}{3}\left[\left|T_A - T_B\right| + \left|I_A - I_B\right| + \left|F_A - F_B\right|\right]$$

Definition 4.2

Let $A=(x, r_1 e^{2\pi i\theta_1}, s_1 e^{2\pi i\theta_2}, t_1 e^{2\pi i\theta_3})$, $B=(x, r_2 e^{2\pi i\phi_1}, s_2 e^{2\pi i\phi_2}, t_2 e^{2\pi i\phi_3})$ & be any two complex fermatean neutrosophic sets. Then the Normalised Hamming distance between A and B is defined as

$$d_{NH}(A,B)=\frac{1}{3n}\sum_{1}^{n}\left|T_{A_i} - T_{B_i}\right| + \left|I_{A_i} - I_{B_i}\right| + \left|F_{A_i} - F_{B_i}\right|$$

Definition 4.3

If $A=(re^{2\pi i\theta_1}, se^{2\pi i\theta_2}, te^{2\pi i\theta_3})$ is any complex Fermatean Neutrosophic number, then the hesitance degree of A is given by

$$H_A = \frac{s}{r+s+t}$$

Definition 4.4

Let $A=(x, r_1 e^{2\pi i\theta_1}, s_1 e^{2\pi i\theta_2}, t_1 e^{2\pi i\theta_3})$, $B=(x, r_2 e^{2\pi i\phi_1}, s_2 e^{2\pi i\phi_2}, t_2 e^{2\pi i\phi_3})$ & be any two complex fermatean neutrosophic sets. The distance measure incorporating hesitance degree is defined as

$$D_{HD}(A,B)=\frac{1}{3n}\sum_{i=1}^{n}\left|T_{A_i} - T_{B_i}\right| + \left|I_{A_i} - I_{B_i}\right| + \left|F_{A_i} - F_{B_i}\right| + \left|\theta_{A_i} - \phi_{B_i}\right| + \left|H_{A_i} - H_{B_i}\right|$$

Theorem 4.5

The function defined above is a metric function

Proof:

i. It is obvious that

$$D_{HD}(A,B)\geq 0$$

ii. $$D_{HD}(A,B)=\frac{1}{3n}\sum_{i=1}^{n}\left|T_{A_i} - T_{B_i}\right| + \left|I_{A_i} - I_{B_i}\right| + \left|F_{A_i} - F_{B_i}\right| + \left|\theta_{A_i} - \phi_{B_i}\right| + \left|H_{A_i} - H_{B_i}\right|$$

$$=\frac{1}{3n}\sum_{i=1}^{n}\left|T_{B_i}-T_{A_i}\right|+\left|I_{B_i}-I_{A_i}\right|+\left|F_{B_i}-F_{A_i}\right|+\left|\theta_{B_i}-\phi_{A_i}\right|+\left|H_{B_i}-H_{A_i}\right|$$

$$=D_{HD}(B,A)$$

iii. $$D_{HD}(A,C)=\frac{1}{3n}\sum_{i=1}^{n}\left|T_{A_i}-T_{C_i}\right|+\left|I_{A_i}-I_{C_i}\right|+\left|F_{A_i}-F_{C_i}\right|+\left|\theta_{A_i}-\omega_{C_i}\right|+\left|H_{A_i}-H_{C_i}\right|$$

$$=\frac{1}{3n}\sum_{i=1}^{n}\left|T_{A_i}+T_{B_i}-T_{B_i}-T_{C_i}\right|+\left|I_{A_i}+I_{B_i}-I_{B_i}-I_{C_i}\right|+\left|F_{A_i}+F_{B_i}-F_{B_i}-F_{C_i}\right|$$

$$+\left|\theta_{A_i}+\phi_{B_i}-\phi_{B_i}-\omega_{C_i}\right|+\left|H_{A_i}+H_{B_i}-H_{B_i}-H_{C_i}\right|$$

$$\leq\frac{1}{3n}\sum_{i=1}^{n}\left|T_{A_i}-T_{B_i}\right|+\left|T_{B_i}-T_{C_i}\right|+\left|I_{A_i}-I_{B_i}\right|+\left|I_{B_i}-I_{C_i}\right|+\left|F_{A_i}-F_{B_i}\right|+\left|F_{B_i}-F_{C_i}\right|$$

$$+\left|\theta_{A_i}-\phi_{B_i}\right|+\left|\phi_{B_i}-\omega_{C_i}\right|+\left|H_{A_i}-H_{B_i}\right|+\left|H_{B_i}-H_{C_i}\right|$$

$$=D_{HD}(A,B)+D_{HD}(B,C)$$

Hence $D_{HD}(A,B)\leq D_{HD}(A,B)+D_{HD}(B,C)$

From i, ii and iii, we can see that $D_{HD}(A,B)$ is a metric function.

5. Complex Fermatean Neutrosophic Sets in Multi criteria decision making problems

Different algorithms for making decisions are made for particular situations and issues. A novel approach to decision-making based on CFNSs and distance measure is suggested in this section. By adding complex fermatean neutrosophic numbers, CNFS theory expands on the potential of conventional fuzzy set theory and provides a more sophisticated way to represent and manipulate uncertainty in decision-making scenarios. It provides decision-makers with a strong foundation for handling challenging and unpredictable situations and coming to wise decisions. CFNSs can be used to more accurately model conflicting criteria, preferences, and imprecise information in decision-making situations. Decision-makers are able to take into account a wider range of options and make better decisions.

Consider a Multi Criteria Decision Making(MCDM) problem with i alternatives $a_1,a_2,...a_i$, j attributes $b_1,b_2,...b_j$ and k experts $E_1,E_2,...E_k$

Defintion 5.5

The Score function of an alternative $a_i;i=1,2,....n$ is defined as

$$S(a_i) = \frac{\sum_{i=1}^{n} D_{HD}(a_i^{E_j}, a_i^{E_k})}{n}$$

5.6 Algorithm to solve a Multi Criteria Decision Making Problem based on Complex Fermatean Neutrosphic Sets:

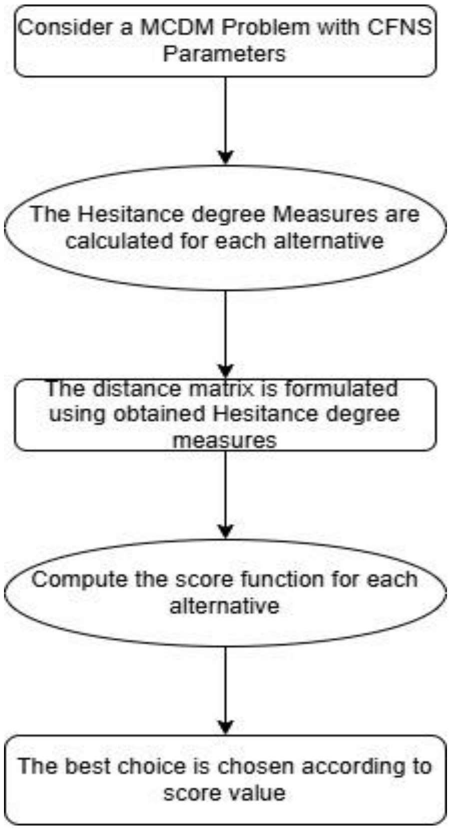
Step 1: Construct the decision making table where the entries are CFNS.

Step 2: Calculate the Hesitance distance measure between the values assessed by various experts for each alternative.

Step 3: Formulate the distance matrix utilizing the values of hesitance distance measure

Step 4: Compute the Score function for each alternative

Step 5: The Best Option can be chosen by ranking the alternatives using the Score function.



Structured Outline of the Proposed algorithm

6 Illustrative example

Consider the following example of multi criteria decision making problem.

A university wants to select the best candidate for a faculty position. There are three candidates (A,B,C), five attributes Research Experience(RE), Teaching Experience(TE), Publication Record(PR), Communication Skills(CS) and Teamwork Ability(TA). Three experts Department Head(E1), Senior Faculty Member(E2), and External Reviewer(E3) have to analyse and submit the reports. The evaluated values of the candidates are given in Table I. Choose the best candidate.

Experts	Candidates	Attributes

		RE	TE	PR	CS	TA
E1	A	$(0.8e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.5)}, 0.7e^{i2\pi(0.2)})$	$(0.6e^{i2\pi(0.4)}, 0.7e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.5)})$	$(0.7e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)})$	$(0.9e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.2)})$	$(0.4e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.5)})$
	B	$(0.4e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.5)})$	$(0.8e^{i2\pi(0.6)}, 0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.2)})$	$(0.6e^{i2\pi(0.5)}, 0.8e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.4)})$	$(0.7e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.6)}, 0.9e^{i2\pi(0.5)})$	$(0.5e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.5)}, 0.8e^{i2\pi(0.4)})$
	C	$(0.2e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.7)})$	$(0.4e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.4)})$	$(0.6e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.3)})$	$(0.5e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)}, 0.7e^{i2\pi(0.2)})$	$(0.3e^{i2\pi(0.6)}, 0.7e^{i2\pi(0..7)}, 0.9e^{i2\pi(0.5)})$
E2	A	$(0.9e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.4)}, 0.7e^{i2\pi(0.6)})$	$(0.8e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.3)})$	$(0.6e^{i2\pi(0.4)}, 0.9e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.7)})$	$(0.7e^{i2\pi(0.8)}, 0.9e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.2)})$	$(0.5e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.9)})$
	B	$(0.5e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)})$	$(0.7e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.3)})$	$(0.4e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.6)}, 0.8e^{i2\pi(0.7)})$	$(0.8e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.5)}, 0.9e^{i2\pi(0.7)})$	$(0.3e^{i2\pi(0.9)}, 0.4e^{i2\pi(0..7)}, 0.5e^{i2\pi(0.2)})$
	C	$(0.4e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.5)}, 0.7e^{i2\pi(0.9)})$	$(0.6e^{i2\pi(0.6)}, 0.7e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.2)})$	$(0.7e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.6)}, 0.9e^{i2\pi(0.7)})$	$(0.8e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.3)})$	$(0.5e^{i2\pi(0.9)}, 0.4e^{i2\pi(0.3)}, 0.6e^{i2\pi(0.4)})$
E3	A	$(0.7e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.5)})$	$(0.5e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.6)})$	$(0.9e^{i2\pi(0.4)}, 0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)})$	$(0.8e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.4)})$	$(0.6e^{i2\pi(0.5)}, 0.8e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.2)})$
	B	$(0.6e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.2)})$	$(0.9e^{i2\pi(0.6)}, 0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.6)})$	$(0.8e^{i2\pi(0.7)}, 0.9e^{i2\pi(0.4)}, 0.7e^{i2\pi(0.5)})$	$(0.7e^{i2\pi(0.9)}, 0.5e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.8)})$	$(0.5e^{i2\pi(0.8)}, 0.4e^{i2\pi(0.6)}, 0.9e^{i2\pi(0.3)})$
	C	$(0.3e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.6)})$	$(0.5e^{i2\pi(0.8)}, 0.7e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.3})$	$(0.4e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.4)})$	$(0.6e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.2)})$	$(0.8e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.5)})$

Table 1- Evaluated Values of Alternatives

The hesitance degree is calculated and represented in the following table

Experts	Candidates	Attributes				
		RE	TE	PR	CS	TA

E1	A	0.32	0.35	0.38	0.22	0.28
	B	0.33	0.33	0.38	0.24	0.35
	C	0.57	0.33	0.27	0.33	0.37
E2	A	0.27	0.26	0.39	0.43	0.4
	B	0.39	0.35	0.29	0.26	0.33
	C	0.31	0.33	0.24	0.33	0.27
E3	A	0.29	0.5	0.32	0.25	0.35
	B	0.3	0.32	0.38	0.31	0.22
	C	0.33	0.39	0.31	0.47	0.33

Table 2- Hesitance degree value

The Hesitance distance measure of each candidate corresponding to the evaluation of experts is calculated and represented in a matrix

$$D_{HD}(A^{E1}, A^{E2}) = 0.40$$

$$D_{HD}(A^{E1}, A^{E3}) = 0.30$$

$$D_{HD}(A^{E2}, A^{E3}) = 0.42$$

$$D_{HD}(B^{E1}, B^{E2}) = 0.334$$

$$D_{HD}(B^{E1}, B^{E3}) = 0.283$$

$$D_{HD}(B^{E2}, B^{E3}) = 0.356$$

$$D_{HD}(C^{E1}, C^{E2}) = 0.459$$

$$D_{HD}(C^{E1}, C^{E3}) = 0.368$$

$$D_{HD}(C^{E2}, C^{E3}) = 0.363$$

Distance measure matrix is given by

$$M = \begin{array}{c} A \\ B \\ C \end{array} \begin{pmatrix} 0.40 & 0.30 & 0.42 \\ 0.334 & 0.283 & 0.356 \\ 0.459 & 0.368 & 0.363 \end{pmatrix}$$

Now the Score function is calculated for each candidate

$$S(A) = \frac{0.40 + 0.30 + 0.42}{3} = 0.37$$

$$S(B) = \frac{0.334 + 0.283 + 0.356}{3} = 0.324$$

$$S(C) = \frac{0.459 + 0.368 + 0.363}{3} = 0.396$$

We have $S(C) > S(A) > S(B)$

Hence C can be chosen the best among the three candidates.

6.1 Sensitivity Analysis

The study of Complex Fermatean Neutrosophic Sets is analysed for various values of truth, indeterminacy and falsity membership functions. Different truth, indeterminacy and falsity degree values are generated and each is used in the calculation of hesitance degree and the score function. It is found that, if the indeterminacy is of degree 1, then there will be no impact on final result. On the other hand if either truth or falsity is of degree 1, then the decision will be favourable in case of truth value 1 and unfavourable in case of falsity degree.

If hesitance degree increases then the corresponding attribute will also increase considerably thereby resulting in an increase in the distance measure. This impact on the score function and the corresponding choice will be chosen by the decision maker.

7 Discussion:

7.1 Need for study of CFNS

CFNS can handle complex uncertainty, ambiguity, and imprecision in real-world problems. CFNS can capture non-linear relationships between variables, which is common in real-world problems. CFNS can help assess risks in complex systems. CFNS provides opportunities for collaboration with other disciplines, such as mathematics, computer science, and engineering.

7.2 Impacts of CFNS in Decision Making

CFNS can lead to improved decision quality by incorporating uncertainty, ambiguity, and imprecision into the decision-making process. CFNS provides a transparent and auditable decision-making process, which can increase stakeholder trust and confidence. CFNS can help identify and manage risks more effectively by capturing uncertainty and ambiguity. CFNS can lead to innovative solutions by incorporating human judgment and expertise into the decision-making process.

7.3 Limitations of CFNS in Decision Making

CFNS requires complex computations, which can be time-consuming and challenging, especially for large-scale problems. The results obtained from CFNS can be difficult to interpret, especially for non-technical stakeholders.Currently, there is limited software support for CFNS, making it challenging to implement and apply.

Overall, CFNS has the potential to revolutionize decision-making by providing a robust framework for handling uncertainty, ambiguity, and imprecision. However, its limitations, such as computational complexity and data requirements, need to be addressed to fully realize its potential.

7.4 Future Scope of CFNS

CFNS-based models can be developed for solving real-world problems, such as medical diagnosis, financial forecasting, and environmental modeling. The application of CFNS in big data analytics can be investigated, such as data mining, data warehousing, and business intelligence.The application of CFNS in cognitive computing can be studied, such as natural language processing, machine learning, and expert systems. The use of Complex Fermatean Neutrosophic Sets can be investigated in image and signal processing, such as in image segmentation, denoising, and feature extraction. Efficient computational algorithms and tools can be developed to support the widespread adoption of Complex Fermatean Neutrosophic Sets in MCDM problems.

8 Conclusion:

The integration of Complex Fermatean Neutrosophic Sets with Multi-Criteria Decision Making (MCDM) problems has far-reaching implications. By leveraging the hesitance degree distance measure, decision-makers can navigate complex decision-making landscapes with unprecedented ease. The proposed approach reduces computational complexity, enabling faster and more efficient decision-making. Complex Fermatean Neutrosophic Sets effectively model uncertainty and ambiguity, providing a more comprehensive understanding of decision-making scenarios. By capturing the nuances of uncertainty and ambiguity, decision-makers can make more informed choices, leading to improved outcomes. The proposed approach has the potential to revolutionize decision-making in complex environments, enabling organizations to make more informed, efficient, and effective decisions.

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