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Some Fixed Point Theorems in Complete Neutrosophic Metric Spaces for Neutrosophic ψ-Quasi Contractions

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Abstract. Fixed point theory is a fundamental tool in mathematics and numerous scientific fields. This study employs the concept of neutrosophic metric spaces as defined by Kirişci and Şimşek to introduce fixed point theorems related to neutrosophic ψ-quasi contractions. We demonstrate that each ψ-quasi contractions possesses a unique fixed point, provided the fuzzy sets are continuous. Additionally, we provide some examples to illustrate our primary theorem. Moreover, we derive several fixed point results based on the main theorem.

Keywords: Fixed point; ψ-Quasi-contractions; Banach contraction principle; Chatterjea contractions; Kannan contractions; Neutrosophic metric; Quasi-contraction; Non-linear contractions.

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1. **Introduction**

In 1965, Zadeh [1] introduced fuzzy sets, enabling elements to have varying degrees of membership. Despite initial skepticism, this concept became influential across many fields. However, fuzzy sets faced limitations in solving certain problems. In 1986, Atanassov [2] proposed Intuitionistic Fuzzy Sets, incorporating both membership and non-membership. Later, Smarandache [3] advanced the theory further with Neutrosophic Sets, adding indeterminacy to membership and non-membership, providing a more robust framework for handling uncertainty. These developments have significantly expanded the scope of applications in decision-making and optimization.

In recent years, the study of fuzzy sets and their generalizations has garnered significant attention due to their wide-ranging applications in decision-making and optimization. For instance, [4] explored bipolar complex fuzzy soft sets and their practical applications, while [5] investigated the normality and cosets of (γ, ∂)-fuzzy HX-subgroups. The concept of time fuzzy soft sets and their role in decision-making was introduced by [6], and the influence of time on fuzzy soft expert sets was further examined by [7]. By integrating time elements and neutrosophic logic, [8] introduced the idea of time-effective fuzzy soft sets and examined some of their applications, both with and without neutrosophic components, adding a new dimension to the theory of fuzzy soft sets. In a different study, [9] introduced the idea of time-shadow soft sets and examined their applications, further broadening the field of soft set theory and its application in dynamic systems and decision-making processes. A thorough investigation of complicated hesitant fuzzy graphs was presented by [10], which broadened the theory and offered fresh perspectives on its uses in graph theory and decision-making. A numerical solution to the fuzzy heat equation with complicated Dirichlet boundary conditions was described in a later study by [11], providing a useful technique for dealing with fuzzy parameters in heat conduction issues. Last but not least, [12] presented the idea of homomorphism of tripolar fuzzy soft Γ-semirings, a noteworthy advancement in the algebraic structure of fuzzy systems that facilitates comprehension of their characteristics and their applications in a variety of domains.

Neutrosophic sets, as a powerful extension of fuzzy sets, have demonstrated remarkable utility across diverse domains. For example, [13] provided a detailed characterization of effective and optimal solutions for scalar optimization problems, including the derivation of Kuhn-Tucker conditions for efficiency and proper efficiency. [14] initiated a new approach known as possibility interval-valued neutrosophic soft sets (PIVNSSs) as a new development in a fuzzy soft computing environment. Further, [15] explored applications of interval-valued Q-neutrosophic soft matrices, while [16] introduced the concept of a mapping on classes of complex multi-fuzzy soft expert sets, and studied the images and inverse images of complex multi-fuzzy soft expert sets. [17] Introduced a more flexible and extended approach to the previously established concept of Interval-Valued Q-Neutrosophic Soft Matrix (IV-Q-NSM), offering a broader generalization of existing mathematical frameworks.

Additionally, neutrosophic logic has been applied to address real-world challenges. For instance, [18] utilized neutrosophic logic to navigate uncertainty in security events in Mexico, and in [19], combined Neutrosophic Set Theory (NST) with machine learning (ML) to develop a novel hybrid methodology (NS-ML) aimed at enhancing breast cancer diagnosis. In the agricultural domain, [20] introduced a multi-attribute neutrosophic optimization technique for optimal crop selection in the Ariyalur District. Meanwhile, [21] presented an innovative telemedical framework by integrating vague Type-2 Neutrosophic Sets (T2NSs) with the Ordered Weighted Continuous Multiplicative Ratio Assessment Method (OWCM-RAM) for intelligent Medical 4.0 evaluations. These studies underscore the versatility and applicability of neutrosophic sets in addressing complex, real-world problems.

Fixed point theory plays a crucial role in both applied and theoretical mathematics. At its core is the Banach contraction principle [22], introduced by Banach in 1922. Since then, numerous researchers have extended and generalized this principle in various ways. Notably, common fixed point theorems have been established in [23–25]. In [26,27], the authors explored fixed point results in neutrosophic metric spaces. Additionally, [28,29] investigated fixed points by introducing new types of distance spaces. Additional research on fixed point theory in generalized metric spaces can be found in the following studies: [30] introduced some results on b-metric spaces and related it with results in metric and G-metric spaces, [31] established fixed point results and investigated (α, β)-triangular admissibility within the framework of complete extended b-metric spaces, and [32] introduced new type of contractions and proved fixed point results based on extended quasi b-metric spaces, broadening the field of fixed point theory. [33] contributed to the study of distance functions in nonlinear spaces by presenting fixed point findings for nonlinear contractions with generalized Ω-distance mappings. Further insights into G-metric spaces and their associated fixed point theorems were provided by [34]. Finally, [35] advanced the subject of fixed point theory by introducing common fixed point theorems in G-metric spaces with Ω-distance.

The following studies are cited for further research on fixed points in various distance spaces and their uses in order to better grasp fixed point theory in partly ordered metric spaces, [36] introduced tripled coincidence point theorems for weak Φ-contractions. Important findings in ordered b-metric spaces and graphic b-metric spaces were obtained by [37], who proposed coincidence and common fixed point theorems for four mappings fulfilling $(\alpha(s), F)$-contractions. In a different study, [38] addressed asymptotic regularity and (α, p)-convex contractions, providing a fresh perspective on regularity and convergence in fixed point theory. Fixed point results for Geraghty-type contractions with comparable distance were introduced by [39] through a new type of distance spaces. [40] contributed to the study of approximating fixed point theory by presenting a numerical scheme to approximate common fixed points for a wide class of operators.

2. Preliminary

In this document, we define the sets as follows: $\mathbb{R}^+ = [0, \infty)$ and $I = [0, 1]$.

The following section revisits the definitions of triangular norms and t-norms, concepts initially proposed by Menger [41] (see also [42]). These definitions are essential for the characterization of neutrosophic metric spaces.

Definition 2.1. An operation $\bullet : I \times I \to I$ is called as continuous t-norm (CTN) if for each $\kappa, \kappa', \tau, \tau' \in I$ it satisfies the following:

(1) $\kappa \bullet 1 = \kappa$,
(2) If $\kappa \leq \kappa'$ and $\tau \leq \tau'$, than $\kappa \bullet \tau \leq \kappa' \bullet \tau'$,
(3) $\bullet$ is continuous,
(4) $\bullet$ is commutative and associate.

Definition 2.2. An operation $\diamond : I \times I \to I$ is called as continuous t-conorm (CTCN) if for each $\kappa, \kappa', \tau, \tau' \in I$ it satisfies the following:

(1) $\kappa \diamond 0 = \kappa$,
(2) If $\kappa \leq \kappa'$ and $\tau \leq \tau'$, than $\kappa \diamond \tau \leq \kappa' \diamond \tau'$,
(3) $\diamond$ is continuous,
(4) $\diamond$ is commutative and associate.

The concept of neutrosophic metric spaces was proposed by Kirişci and Şimşek in the following manner.

Definition 2.3. [42] A 6-tuple $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ is defined as a Neutrophic Metric Space (NMS) when the set $\mathcal{Z}$ constitutes a non-empty arbitrary collection. In this context, $\bullet$ represents a continuous t-norm, while $\diamond$ denotes a continuous t-conorm. Additionally, the elements $\mathcal{T}, \mathcal{F}$, and $\mathcal{I}$ are three fuzzy sets defined on the Cartesian product $\mathcal{Z}^2 \times (0, \infty)$. These components are required to fulfill specific conditions applicable to all elements $\zeta, \varrho, c \in \mathcal{Z}$ and for all positive real numbers γ, ρ.

(1) $0 \leq \mathcal{T}(\zeta, \varrho, \gamma) \leq 1$, $0 \leq \mathcal{F}(\zeta, \varrho, \gamma) \leq 1$, $0 \leq \mathcal{I}(\zeta, \varrho, \gamma) \leq 1$,
(2) $0 \leq \mathcal{T}(\zeta, \varrho, \gamma) + \mathcal{F}(\zeta, \varrho, \gamma) + \mathcal{I}(\zeta, \varrho, \gamma) \leq 3$,
(3) $\mathcal{T}(\zeta, \varrho, \gamma) = 1$, for $\gamma > 0$ iff $\zeta = \varrho$
(4) $\mathcal{T}(\zeta, \varrho, \gamma) = H(\varrho, \zeta, \gamma)$, for $\gamma > 0$
(5) $\mathcal{T}(\zeta, \varrho, \gamma) \bullet \mathcal{T}(\varrho, c, \rho) \leq \mathcal{T}(\zeta, c, \gamma + \rho)$
(6) $\mathcal{T}(\zeta, \varrho, \cdot) : \mathbb{R}^+ \to I$ is continuous
(7) $\lim_{\gamma \to \infty} \mathcal{T}(\zeta, \varrho, \gamma) = 1$
(8) $\mathcal{F}(\zeta, \varrho, \gamma) = 0$ iff $\zeta = \varrho$
(9) $\mathcal{F}(\zeta, \varrho, \gamma) = \mathcal{F}(\varrho, \zeta, \gamma)$,

(10) $\mathcal{F}(\zeta, \varrho, \gamma) \diamond \mathcal{F}(\varrho, c, \rho) \geq \mathcal{F}(\zeta, c, \gamma + \rho)$,

(11) $\mathcal{F}(\zeta, \varrho, \cdot) : \mathbb{R}^+ \to I$ is continuous

(12) $\lim_{\gamma \to \infty} \mathcal{F}(\zeta, \varrho, \gamma) = 0$

(13) $\mathcal{I}(\zeta, \varrho, \gamma) = 0$, for $\gamma > 0$ iff $\zeta = \varrho$

(14) $\mathcal{I}(\zeta, \varrho, \gamma) = \mathcal{I}(\varrho, \zeta, \gamma)$,

(15) $\mathcal{I}(\zeta, \varrho, \gamma) \diamond \mathcal{I}(\varrho, c, \rho) \geq S(\zeta, c, \gamma + \rho)$,

(16) $\mathcal{I}(\zeta, \varrho, \cdot) : \mathbb{R} \to I$ is continuous

(17) $\lim_{\gamma \to \infty} \mathcal{I}(\zeta, \varrho, \gamma) = 0$

(18) If $\gamma \leq 0$, then $\mathcal{T}(\zeta, \varrho, \gamma) = 0$, $\mathcal{F}(\zeta, \varrho, \gamma) = \mathcal{I}(\zeta, \varrho, \gamma) = 1$

The functions $\mathcal{T}(\zeta, \varrho, \gamma)$, $\mathcal{F}(\zeta, \varrho, \gamma)$, and $\mathcal{I}(\zeta, \varrho, \gamma)$ represent the degrees of nearness, neutralness, and non-nearness between the elements ζ and ϱ in relation to the parameter γ, respectively.

The topologies concepts including convergence, Cauchy sequences, and completeness in NMS, are outlined as follows.

Definition 2.4. [42] Let (ζ_n) be a sequence in a NMS $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$. Then

(1) (ζ_n) converges to $\zeta \in \mathcal{Z}$ iff for a given $\epsilon \in (0, 1)$, $\gamma > 0$ there is $n_0 \in \mathbb{N}$ such that for each $n \geq n_0$

$$\mathcal{T}(\zeta_n, \zeta, \gamma) > 1 - \epsilon,\ \mathcal{F}(\zeta_n, \zeta, \gamma) < \epsilon,\ \mathcal{I}(\zeta_n, \zeta, \gamma) < \epsilon$$

i.e.,

$$\lim_{n \to \infty} \mathcal{T}(\zeta_n, \zeta, \gamma) = 1,\ \lim_{n \to \infty} \mathcal{F}(\zeta_n, \zeta, \gamma) = 0,\ \lim_{n \to \infty} \mathcal{I}(\zeta_n, \zeta, \gamma) = 0$$

(2) (ζ_n) is called Cauchy iff for a given $\epsilon \in (0, 1)$, $\gamma > 0$ there is $n_0 \in \mathbb{N}$ such that for each $n, m \geq n_0$

$$\mathcal{T}(\zeta_n, \zeta_m, \gamma) > 1 - \epsilon,\ \mathcal{F}(\zeta_n, \zeta_m, \gamma) < \epsilon,\ \mathcal{I}(\zeta_n, \zeta_m, \gamma) < \epsilon$$

i.e.,

$$\lim_{n.m \to \infty} \mathcal{T}(\zeta_n, \zeta_m, \gamma) = 1,\ \lim_{n,m \to \infty} \mathcal{F}(\zeta_n, \zeta_m, \gamma) = 0,\ \lim_{n,m \to \infty} \mathcal{I}(\zeta_n, \zeta_m, \gamma) = 0$$

(3) $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ is called complete if each Cauchy sequence is convergent to an element in $\mathcal{Z}$.

In the research conducted by Simsek and Kirisci [43], the concept of NC-contractions was presented in the setting of neutrosophic metric spaces, establishing that every NC-contraction in a complete neutrosophic metric space has a distinct fixed point when certain conditions are met.

The following useful lemma presented in [44].

Lemma 2.5. *[44] Let $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ be an NMS. Then*

(1) $\mathcal{T}(\zeta, \varrho, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-decreasing*
(2) $\mathcal{F}(\zeta, \varrho, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-increasing*
(3) $\mathcal{I}(\zeta, \varrho, \cdot) : \mathbb{R} \to \mathbb{R}$ *is non-increasing*

Bataihah and Hazaymeh [44] established fixed point results for quasi-contractions within the framework of neutrosophic fuzzy metric spaces. Building upon this foundation, we introduce the concept of neutrosophic ψ-quasi contractions, a broader class of mappings that generalizes and unifies various contraction conditions. We prove that such contractions, under suitable assumptions, guarantee the existence of a unique fixed point. Our results not only extend several well-known types of contractions but also contribute to the ongoing development of fixed point theory in neutrosophic settings, offering potential applications in diverse mathematical and applied sciences.

3. Main Result

In this section, we introduce the concept of neutrosophic ψ-quasi contractions within the framework of complete neutrosophic metric spaces (NMS). We then establish a fixed point theorem, proving the existence and uniqueness of fixed points for such mappings.

To facilitate our analysis, we first present the following necessary notations and definitions: If $\mathcal{G} : \mathcal{Z}^2 \times \mathbb{R} \to \mathbb{R}$, and f is a self-map on $\mathcal{Z}$, then

(1) $\mathscr{M}_{\mathcal{G},f}(\zeta, \varrho, \gamma) =$
$$\max\left\{\frac{1}{\mathcal{G}(\zeta,\varrho,\gamma)} - 1, \frac{1}{\mathcal{G}(\zeta,f\zeta,\gamma)} - 1, \frac{1}{\mathcal{G}(\varrho,f\varrho,\gamma)} - 1, \frac{1}{\mathcal{G}(\zeta,f\varrho,\gamma)} - 1, \frac{1}{\mathcal{G}(f\zeta,\varrho,\gamma)} - 1\right\},$$

(2) $\mathscr{N}_{\mathcal{G},f}(\zeta, \varrho, \gamma) = \max\left\{\mathcal{G}(\zeta,\varrho,\gamma), \mathcal{G}(\zeta,f\zeta,\gamma), \mathcal{G}(\varrho,f\varrho,\gamma), \mathcal{G}(\zeta,f\varrho,\gamma), \mathcal{G}(f\zeta,\varrho,\gamma)\right\}.$

And

(1) $\delta_{\mathcal{G}}(\zeta, f, \gamma) = \max\limits_{i,j\in\mathbb{N}}\left\{\frac{1}{\mathcal{G}(f^i\zeta, f^j\zeta, \gamma)} - 1\right\},$

(2) $\sigma_{\mathcal{G}}(\zeta, f, \gamma) = \max\limits_{i,j\in\mathbb{N}}\left\{\mathcal{G}(f^i\zeta, f^j\zeta, \gamma)\right\}.$

Definition 3.1. [45] A function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is called comparison function if it fullfiled the following:

(1) Where $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is monotone increasing
(2) $\lim_{n\to\infty} \psi^n(a) = 0$ for each $a > 0$.

Remark 3.2. [45] If ψ is comparison function, then

(1) $\psi(0) = 0$

(2) $\psi(a) < a$ for each $a > 0$

We will now present the concept of neutrosophic ψ-quasi contractions.

Definition 3.3. Let $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ be a NMS. A mapping $f : \mathcal{Z} \to \mathcal{Z}$ is called neutrosophic ψ-quasi-contraction if for each $\zeta, \varrho \in \mathcal{Z}$ and each $\gamma > 0$, we have

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 \leq \psi(\mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)),$$

$$\mathcal{F}(f\zeta, f\varrho, \gamma) \leq \psi(\mathscr{N}_{\mathcal{F},f}(\zeta, \varrho, \gamma)),$$

and

$$\mathcal{I}(f\zeta, f\varrho, \gamma) \leq \psi(\mathscr{N}_{\mathcal{I},f}(\zeta, \varrho, \gamma)).$$

Theorem 3.4. *Let $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ be a complete NMS, Suppose that $f : \mathcal{Z} \to \mathcal{Z}$ is neutrosophic ψ-quasi-contraction. Also, assume that one of the following holds true:*

(a) *f is sequential continuous i.e if $\zeta_n \to \zeta$, then $f\zeta_n \to f\zeta$,*

(b) *the fuzzy sets $\mathcal{T}, \mathcal{F}$ and $\mathcal{I}$ are continuous in the first two coordinates.*

Consequently, the function f possesses a unique fixed point.

Proof. Let $\zeta_0 \in \mathcal{Z}$ represent an arbitrary point. We examine the Picard sequence (ζ_n) characterized by the relation $\zeta_{n+1} = f^n\zeta_0$ for all $n \geq 0$. By Definition 3.3 we have for all $i, j \in \mathbb{N}$

$$\begin{aligned}\frac{1}{\mathcal{T}(f^{n+i}\zeta_0, f^{n+j}\zeta_0, \gamma)} - 1 \quad &\leq \psi \;\max\Big\{\frac{1}{\mathcal{T}(f^{n+i-1}\zeta_0, f^{n+j-1}\zeta_0, \gamma)} - 1, \frac{1}{\mathcal{T}(f^{n+i-1}\zeta_0, f^{n+i}\zeta_0, \gamma)} - 1,\\ &\qquad \frac{1}{\mathcal{T}(f^{n+j-1}\zeta_0, f^{n+j}\zeta_0, \gamma)} - 1, \frac{1}{\mathcal{T}(f^{n+i-1}\zeta_0, f^{n+j}\zeta_0, \gamma)} - 1,\\ &\qquad \frac{1}{\mathcal{T}(f^{n+j-1}\zeta_0, f^{n+i}\zeta_0, \gamma)} - 1\Big\},\end{aligned}$$

$$\begin{aligned}\mathcal{F}(f^{n+i}\zeta_0, f^{n+j}\zeta_0, \gamma) \quad &\leq \psi \;\max\{\mathcal{F}(f^{n+i-1}\zeta_0, f^{n+j-1}\zeta_0, \gamma), \mathcal{F}(f^{n+i-1}\zeta_0, f^{n+i}\zeta_0, \gamma),\\ &\qquad \mathcal{F}(f^{n+j-1}\zeta_0, f^{n+j}\zeta_0, \gamma), \mathcal{F}(f^{n+i-1}\zeta_0, f^{n+j}\zeta_0, \gamma),\\ &\qquad \mathcal{F}(f^{n+j-1}\zeta_0, f^{n+i}\zeta_0, \gamma)\},\end{aligned}$$

and

$$\begin{aligned}\mathcal{I}(f^{n+i}\zeta_0, f^{n+j}\zeta_0, \gamma) \quad &\leq \psi \;\max\{\mathcal{I}(f^{n+i-1}\zeta_0, f^{n+j-1}\zeta_0, \gamma), \mathcal{I}(f^{n+i-1}\zeta_0, f^{n+i}\zeta_0, \gamma),\\ &\qquad \mathcal{I}(f^{n+j-1}\zeta_0, f^{n+j}\zeta_0, \gamma), \mathcal{I}(f^{n+i-1}\zeta_0, f^{n+j}\zeta_0, \gamma),\\ &\qquad \mathcal{I}(f^{n+j-1}\zeta_0, f^{n+i}\zeta_0, \gamma)\}.\end{aligned}$$

Thus, we conclude

$$\delta_{\mathcal{T}}(f^n\zeta_0, f, \gamma) \leq \psi\; \delta_{\mathcal{T}}(f^{n-1}\zeta_0, f, \gamma),$$

$$\sigma_{\mathcal{F}}(f^n\zeta_0, f, \gamma) \leq \psi\; \sigma_{\mathcal{F}}(f^{n-1}\zeta_0, f, \gamma),$$

and

$$\sigma_{\mathcal{I}}(f^n\zeta_0, f, \gamma) \leq \psi\ \sigma_{\mathcal{I}}(f^{n-1}\zeta_0, f, \gamma).$$

Hence, we deduce that for all $n \geq 1$

$$\delta_{\mathcal{T}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \delta_{\mathcal{T}}(\zeta_0, f, \gamma),$$
$$\sigma_{\mathcal{F}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \sigma_{\mathcal{F}}(\zeta_0, f, \gamma),$$

and

$$\sigma_{\mathcal{I}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \sigma_{\mathcal{I}}(\zeta_0, f, \gamma).$$

From the above inequalities, we have

$$\frac{1}{\mathcal{T}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma)} - 1 \leq \delta_{\mathcal{T}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \delta_{\mathcal{T}}(\zeta_0, f, \gamma),$$
$$\mathcal{F}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma) \leq \sigma_{\mathcal{F}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \sigma_{\mathcal{F}}(\zeta_0, f, \gamma),$$

and

$$\mathcal{I}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma) \leq \sigma_{\mathcal{I}}(f^n\zeta_0, f, \gamma) \leq \psi^n\ \sigma_{\mathcal{I}}(\zeta_0, f, \gamma).$$

By taking the limit when both n, m approach ∞, we get

$$\lim_{n,m\to\infty} \mathcal{T}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma) = 1,$$
$$\lim_{n,m\to\infty} \mathcal{F}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma) = 0,$$
$$\lim_{n,m\to\infty} \mathcal{I}(f^n\zeta_0, f^{n+m}\zeta_0, \gamma) = 0.$$

Hence $(f^n\zeta_0)$ is a Cauchy sequence, thus, there is $u \in \mathcal{Z}$ such that $f^n\zeta_0 \to u$.
mathcalIf (a) holds, then $f^{n+1}\zeta_0 = ff^n\zeta_0 \to fu$ and so, $u = fu$.
mathcalIf (b) holds, then Definition 3.3 implies that

$$\begin{aligned}\frac{1}{\mathcal{T}(fu, f^{n+1}\zeta_0, \gamma)} - 1 \quad &\leq \psi\ \max\Big\{\frac{1}{\mathcal{T}(u, f^n\zeta_0, \gamma)} - 1, \frac{1}{\mathcal{T}(u, fu, \gamma)} - 1, \frac{1}{\mathcal{T}(f^n\zeta_0, f^{n+1}\zeta_0, \gamma)} - 1, \\ &\qquad \frac{1}{\mathcal{T}(u, f^{n+1}\zeta_0, \gamma)} - 1, \frac{1}{\mathcal{T}(f^n\zeta_0, fu, \gamma)} - 1\ \Big\},\end{aligned}$$

$$\begin{aligned}\mathcal{F}(fu, f^{n+1}\zeta_0, \gamma) \quad &\leq \psi\ \max\big\{\mathcal{F}(u, f^n\zeta_0, \gamma), \mathcal{F}(u, fu, \gamma), \mathcal{F}(f^n\zeta_0, f^{n+1}\zeta_0, \gamma), \\ &\qquad \mathcal{F}(u, f^{n+1}\zeta_0, \gamma), \mathcal{F}(f^n\zeta_0, fu, \gamma)\ \big\},\end{aligned}$$

and

$$\mathcal{I}(fu, f^{n+1}\zeta_0, \gamma) \leq \psi \ \max\left\{\mathcal{I}(u, f^n\zeta_0, \gamma), \mathcal{I}(u, fu, \gamma), \mathcal{I}(f^n\zeta_0, f^{n+1}\zeta_0, \gamma), \mathcal{I}(u, f^{n+1}\zeta_0, \gamma), \mathcal{I}(f^n\zeta_0, fu, \gamma) \right\}.$$

By taking the limit, we get

$$\frac{1}{\mathcal{T}(fu,u,\gamma)} - 1 \leq \psi(\frac{1}{\mathcal{T}(fu,u,\gamma)} - 1),$$

$$\mathcal{F}(fu,u,\gamma) \leq \psi(\mathcal{F}(fu,u,\gamma)),$$

$$\mathcal{I}(fu,u,\gamma) \leq \psi(\mathcal{I}(fu,u,\gamma)).$$

If one of $\frac{1}{\mathcal{T}(fu,u,\gamma)} - 1, \mathcal{F}(fu,u,\gamma), \mathcal{I}(fu,u,\gamma)$ not equal 0, then

$$\frac{1}{\mathcal{T}(fu,u,\gamma)} - 1 < \frac{1}{\mathcal{T}(fu,u,\gamma)} - 1,$$

or

$$\mathcal{F}(fu,u,\gamma) < \mathcal{F}(fu,u,\gamma),$$

or

$$\mathcal{I}(fu,u,\gamma) < \mathcal{I}(fu,u,\gamma).$$

Which leads to a contradiction in each case. Hence $u = fu$.

Now, let $w \in \mathcal{Z}$ with $w = fw$. If $u \neq w$, then from Definition 3.3, it follows that

$$\begin{aligned}\frac{1}{\mathcal{T}(u,w,\gamma)} - 1 = \frac{1}{\mathcal{T}(fu,fw,\gamma)} - 1 &\leq \psi \ \max\left\{\frac{1}{\mathcal{T}(u,w,\gamma)} - 1, \frac{1}{\mathcal{T}(u,u,\gamma)} - 1, \frac{1}{\mathcal{T}(w,w,\gamma)} - 1, \frac{1}{\mathcal{T}(u,w,\gamma)} - 1, \frac{1}{\mathcal{T}(w,u,\gamma)} - 1 \right\} \\ &= \psi \ \left(\frac{1}{\mathcal{T}(u,w,\gamma)} - 1\right),\end{aligned}$$

$$\begin{aligned}\mathcal{F}(u,w,\gamma) = \mathcal{F}(fu,fw,\gamma) &\leq \psi \ \max\{\mathcal{F}(u,w,\gamma), \mathcal{F}(u,u,\gamma), \mathcal{F}(w,w,\gamma), \mathcal{F}(u,w,\gamma), \mathcal{F}(w,u,\gamma)\} \\ &= \psi \ \mathcal{F}(w,u,\gamma),\end{aligned}$$

and

$$\begin{aligned}\mathcal{I}(u,w,\gamma) = \mathcal{I}(fu,fw,\gamma) &\leq \psi \ \max\{\mathcal{I}(u,w,\gamma), \mathcal{I}(u,u,\gamma), \mathcal{I}(w,w,\gamma), \mathcal{I}(u,w,\gamma), \mathcal{I}(w,u,\gamma)\} \\ &= \psi \ \mathcal{I}(w,u,\gamma).\end{aligned}$$

Therefore, by the same argument as in the previous step, we conclude that $u = w$. □

4. Examples and Consequences

In this section, we provide some of illustrative examples to demonstrate the applicability of our primary theorem in various contexts. These examples will highlight the significance of the conditions imposed in the theorem and showcase different scenarios where the results hold. Following this, we derive several interesting consequences and related findings, further emphasizing the theoretical and practical implications of our main result.

Example 4.1. Let $\mathcal{Z} = [0, 1]$ and define the neutrosophic metric $\mathcal{T}, \mathcal{F}, \mathcal{I}$ as follows:

$$\mathcal{T}(\zeta, \varrho, \gamma) = \frac{\gamma}{\gamma + |\zeta - \varrho|},$$

$$\mathcal{F}(\zeta, \varrho, \gamma) = \frac{|\zeta - \varrho|}{\gamma + |\zeta - \varrho|},$$

$$\mathcal{I}(\zeta, \varrho, \gamma) = \frac{|\zeta - \varrho|}{\gamma + |\zeta - \varrho|}.$$

Then, $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ is a complete NMS where $\bullet$ and $\diamond$ are as follows: $\bullet(a, b) = \min(a, b)$ and $\diamond(a, b) = \max(a, b)$.

Define the mapping $f : \mathcal{Z} \to \mathcal{Z}$ as: $f(\zeta) = \frac{\zeta}{2}$.

Let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ be defined as $\psi(a) = \frac{a}{2}$. Then, ψ is a comparison function.

Proof. Now, we verify the conditions for f being a neutrosophic ψ-quasi-contraction.

For any $\zeta, \varrho \in \mathcal{Z}$ and any $\gamma > 0$, we have

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 = \frac{|\zeta/2 - \varrho/2|}{\gamma} = \frac{|\zeta - \varrho|}{2\gamma}.$$

On the other hand:

$$\mathcal{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma) = \max\left\{\frac{1}{\mathcal{T}(\zeta, \varrho, \gamma)} - 1, \frac{1}{\mathcal{T}(\zeta, f\zeta, \gamma)} - 1, \frac{1}{\mathcal{T}(\varrho, f\varrho, \gamma)} - 1, \frac{1}{\mathcal{T}(\zeta, f\varrho, \gamma)} - 1, \frac{1}{\mathcal{T}(f\zeta, \varrho, \gamma)} - 1 \right\}.$$

Since f is a contraction, it can be shown that:

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 \leq \psi\left(\mathcal{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)\right).$$

Similarly, we can verify:

$$\mathcal{F}(f\zeta, f\varrho, \gamma) \leq \psi\left(\mathcal{N}_{\mathcal{F},f}(\zeta, \varrho, \gamma)\right),$$

$$\mathcal{I}(f\zeta, f\varrho, \gamma) \leq \psi\left(\mathcal{N}_{\mathcal{I},f}(\zeta, \varrho, \gamma)\right).$$

Moreover, the mapping f is sequential continuous because if $\zeta_n \to \zeta$, then $f\zeta_n = \frac{\zeta_n}{2} \to \frac{\zeta}{2} = f\zeta$. Additionally, the fuzzy sets $\mathcal{T}, \mathcal{F}, \mathcal{I}$ are continuous in the first two coordinates.

Hence, by Theorem 3.4, the mapping f has a unique fixed point. In this case, the fixed point is $\zeta = 0$, since $f(0) = 0$. □

Example 4.2. Let $\mathcal{Z} = \left\{\frac{1}{3^n} : n \in \mathbb{N}\right\} \cup \{0\}$ with the standard metric by $d(\zeta, \rho) = |\zeta - \rho|$. Also, Let the t-norm and t-conorm be defined as follows $\zeta \diamond \rho = \min\{\zeta, \rho\}$, $\zeta \bullet \rho = \max\{\zeta, \rho\}$. Additionally, let the fuzzy sets be defined as follows:

$$\mathcal{T}(\zeta, \rho, \gamma) = \frac{\gamma}{\gamma + d(\zeta, \rho)},\ \mathcal{F}(\zeta, \rho, \gamma) = \frac{d(\zeta, \rho)}{\gamma + d(\zeta, \rho)},\ \mathcal{I}(\zeta, \rho, \gamma) = \frac{d(\zeta, \rho)}{d(\zeta, \rho) + \gamma}.$$

Let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ be defined as $\psi(a) = \frac{1}{2}a$.

Define $f : \mho \to \mho$ by

$$f(\zeta) = \begin{cases} 0 & , \ \zeta = 0 \\ \frac{1}{3^{n+1}} & , \ \zeta = \frac{1}{3^n} \end{cases}$$

Then, f, has a unique fixed point.

Proof. From [42], the six-tuple $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ represents a complete NMS. Now, for any elements ζ and ρ in $\mathcal{Z}$ and for any $\gamma > 0$, we have.

$$\begin{aligned} \mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma) &= \max\left\{\frac{1}{\mathcal{T}(\zeta, \varrho, \gamma)} - 1, \frac{1}{\mathcal{T}(\zeta, f\zeta, \gamma)} - 1, \frac{1}{\mathcal{T}(\varrho, f\varrho, \gamma)} - 1, \right. \\ &\qquad \left. \frac{1}{\mathcal{T}(\zeta, f\varrho, \gamma)} - 1, \frac{1}{\mathcal{T}(f\zeta, \varrho, \gamma)} - 1 \right\} \\ &= \max\left\{\frac{|\zeta - \varrho|}{\gamma}, \frac{|\zeta - f\zeta|}{\gamma}, \frac{|\varrho - f\varrho|}{\gamma}, \frac{|\zeta - f\varrho|}{\gamma}, \frac{|f\zeta - \varrho|}{\gamma}\right\}. \end{aligned}$$

If $\zeta = \varrho = 0$, then $0 = \left(\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1\right) \leq \psi(\mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)) = 0$.

If $\zeta = 0,\ \varrho = \frac{1}{3^m}$, then

$$\frac{|\frac{1}{3^{m+1}}|}{\gamma} = \left(\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1\right) \leq \psi(\mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)) = \frac{1}{2}\max\left\{\frac{|\frac{1}{3^m}|}{\gamma}, \frac{|\frac{2}{3^{m+1}}|}{\gamma}, \frac{|\frac{1}{3^{m+1}}|}{\gamma}\right\}.$$

If $\zeta = \frac{1}{3^n},\ \varrho = \frac{1}{3^m}$, then

$$\left(\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1\right) = \frac{1}{3}\frac{|\frac{1}{3^m} - \frac{1}{3^n}|}{\gamma}.$$

Also,

$$\psi(\mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)) = \frac{1}{2}\max\left\{\frac{|\frac{1}{3^m} - \frac{1}{3^n}|}{\gamma}, \frac{|\frac{2}{3^{n+1}}|}{\gamma}, \frac{|\frac{2}{3^{m+1}}|}{\gamma}, \frac{|\frac{1}{3^n} - \frac{1}{3^{m+1}}|}{\gamma}, \frac{|\frac{1}{3^m} - \frac{1}{3^{n+1}}|}{\gamma}\right\}.$$

So,

$$\left(\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1\right) \leq \psi(\mathscr{M}_{\mathcal{T},f}(\zeta, \varrho, \gamma)).$$

Also, we have

$$\begin{aligned}\psi(\mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma)) &= \frac{1}{2}\max\left\{\mathcal{F}(\zeta,\varrho,\gamma),\mathcal{F}(\zeta,f\zeta,\gamma),\mathcal{F}(\varrho,f\varrho,\gamma),\mathcal{F}(\zeta,f\varrho,\gamma),\mathcal{F}(f\zeta,\varrho,\gamma)\right\}\\ &= \frac{1}{2}\max\left\{\frac{|\zeta-\varrho|}{\gamma+|\zeta-\varrho|},\frac{|\zeta-f\zeta|}{\gamma+|\zeta-f\zeta|},\frac{|\varrho-f\varrho|}{\gamma+|\varrho-f\varrho|},\frac{|\zeta-f\varrho|}{\gamma+|\zeta-f\varrho|},\frac{|f\zeta-\varrho|}{\gamma+|f\zeta-\varrho|}\right\}.\end{aligned}$$

If $\zeta=\varrho=0$, then $0=\mathcal{F}(f\zeta,f\varrho,\gamma)\le\psi(\mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma))=0$.

If $\zeta=0,\ \varrho=\frac{1}{3^m}$, then

$$\frac{|\frac{1}{3^{m+1}}|}{\gamma+|\frac{1}{3^{m+1}}|}=\mathcal{F}(f\zeta,f\varrho,\gamma)\le\psi(\mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma))=\frac{1}{2}\max\left\{\frac{|\frac{1}{3^m}|}{\gamma+|\frac{1}{3^m}|},\frac{|\frac{2}{3^{m+1}}|}{\gamma+|\frac{2}{3^{m+1}}|},\frac{|\frac{1}{3^{m+1}}|}{\gamma+|\frac{1}{3^{m+1}}|}\right\}.$$

If $\zeta=\frac{1}{3^n},\ \varrho=\frac{1}{3^m}$, then

$$\mathcal{F}(f\zeta,f\varrho,\gamma)=\frac{|\frac{1}{3^{m+1}}-\frac{1}{3^{n+1}}|}{\gamma+|\frac{1}{3^{m+1}}-\frac{1}{3^{n+1}}|}.$$

Also,

$$\psi(\mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma))=\frac{1}{2}\max\left\{\frac{|\frac{1}{3^m}-\frac{1}{3^n}|}{\gamma+|\frac{1}{3^m}-\frac{1}{3^n}|},\frac{|\frac{2}{3^{n+1}}|}{\gamma+|\frac{2}{3^{n+1}}|},\frac{|\frac{2}{3^{m+1}}|}{\gamma+|\frac{2}{3^{m+1}}|},\frac{|\frac{1}{3^n}-\frac{1}{3^{m+1}}|}{\gamma+|\frac{1}{3^n}-\frac{1}{3^{m+1}}|},\frac{|\frac{1}{3^m}-\frac{1}{3^{n+1}}|}{\gamma+|\frac{1}{3^m}-\frac{1}{3^{n+1}}|}\right\}.$$

So,

$$\mathcal{F}(f\zeta,f\varrho,\gamma)\le\psi(\mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma)).$$

The remaining portion can be illustrated in a similar fashion. □

The following corollaries are direct consequences of the main theorem, extending its applicability to specific cases and providing further insights into its implications.

Corollary 4.3 (Fixed Point for Quasi-Contractions)**.** *Let $(\mathcal{Z},\mathcal{T},\mathcal{F},\mathcal{I},\bullet,\diamond)$ be a complete NMS, Suppose that there is $k\in[0,1)$ such that for each $\zeta,\varrho\in\mathcal{Z}$ and each $\gamma>0$ the self map $f:\mathcal{Z}\to\mathcal{Z}$ satisfies the following:*

$$\frac{1}{\mathcal{T}(f\zeta,f\varrho,\gamma)}-1\le k\ \mathscr{M}_{\mathcal{T},f}(\zeta,\varrho,\gamma),$$

$$\mathcal{F}(f\zeta,f\varrho,\gamma)\le k\ \mathscr{N}_{\mathcal{F},f}(\zeta,\varrho,\gamma),$$

and

$$\mathcal{I}(f\zeta,f\varrho,\gamma)\le k\ \mathscr{N}_{\mathcal{I},f}(\zeta,\varrho,\gamma).$$

Also, assume that one of the following holds true:

(1) *f is sequential continuous,*
(2) *the fuzzy sets $\mathcal{T},\mathcal{F}$ and $\mathcal{I}$ are continuous in the first two coordinates.*

Therefore, the function f has a unique fixed point.

Corollary 4.4 (Fixed Point for Contraction Mappings). *Let* $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ *be a complete NMS, Suppose that there is* $k \in [0,1)$ *such that for each* $\zeta, \varrho \in \mathcal{Z}$ *and each* $\gamma > 0$ *the self map* $f : \mathcal{Z} \to \mathcal{Z}$ *satisfies the following:*

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 \leq k \left(\frac{1}{\mathcal{T}(\zeta, \varrho, \gamma)} - 1 \right),$$
$$\mathcal{F}(f\zeta, f\varrho, \gamma) \leq k\ \mathcal{F}(\zeta, \varrho, \gamma),$$

and

$$\mathcal{I}(f\zeta, f\varrho, \gamma) \leq k\ \mathcal{I}(\zeta, \varrho, \gamma).$$

Also, assume that one of the following holds true:

(1) f *is sequential continuous,*

(2) *the fuzzy sets* $\mathcal{T}, \mathcal{F}$ *and* $\mathcal{I}$ *are continuous in the first two coordinates.*

Therefore, the function f *has a unique fixed point.*

Corollary 4.5 (Fixed Point for Kannan-Type Contractions). *Let* $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ *be a complete NMS, Suppose that there is* $k \in [0,1)$ *such that for each* $\zeta, \varrho \in \mathcal{Z}$ *and each* $\gamma > 0$ *the self map* $f : \mathcal{Z} \to \mathcal{Z}$ *satisfies the following:*

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 \leq \frac{k}{2} \left(\frac{1}{\mathcal{T}(\zeta, f\zeta, \gamma)} - 1 + \frac{1}{\mathcal{T}(\varrho, f\varrho, \gamma)} - 1 \right),$$
$$\mathcal{F}(f\zeta, f\varrho, \gamma) \leq \frac{k}{2}\ (\mathcal{F}(\zeta, f\zeta, \gamma) + \mathcal{F}(\varrho, f\varrho, \gamma)),$$

and

$$\mathcal{I}(f\zeta, f\varrho, \gamma) \leq \frac{k}{2}\ (\mathcal{I}(\zeta, f\zeta, \gamma) + \mathcal{I}(\varrho, f\varrho, \gamma)).$$

Also, assume that one of the following holds true:

(1) f *is sequential continuous,*

(2) *the fuzzy sets* $\mathcal{T}, \mathcal{F}$ *and* $\mathcal{I}$ *are continuous in the first two coordinates.*

Therefore, the function f *has a unique fixed point.*

Corollary 4.6 (Fixed Point for Chatterjea type contraction). *Let* $(\mathcal{Z}, \mathcal{T}, \mathcal{F}, \mathcal{I}, \bullet, \diamond)$ *be a complete NMS, Suppose that there is* $k \in [0,1)$ *such that for each* $\zeta, \varrho \in \mathcal{Z}$ *and each* $\gamma > 0$ *the self map* $f : \mathcal{Z} \to \mathcal{Z}$ *satisfies the following:*

$$\frac{1}{\mathcal{T}(f\zeta, f\varrho, \gamma)} - 1 \leq \frac{k}{2} \left(\frac{1}{\mathcal{T}(\zeta, f\varrho, \gamma)} - 1 + \frac{1}{\mathcal{T}(\zeta, f\varrho, \gamma)} - 1 \right),$$
$$\mathcal{F}(f\zeta, f\varrho, \gamma) \leq \frac{k}{2}\ (\mathcal{F}(\zeta, f\varrho, \gamma) + \mathcal{F}(\varrho, f\zeta, \gamma)),$$

and

$$\mathcal{I}(f\zeta, f\varrho, \gamma) \leq \frac{k}{2}\ (\mathcal{I}(\zeta, f\varrho, \gamma) + \mathcal{I}(\varrho, f\zeta, \gamma)).$$

Also, assume that one of the following holds true:

(1) *f is sequential continuous,*

(2) *the fuzzy sets $\mathcal{T}, \mathcal{F}$ and $\mathcal{I}$ are continuous in the first two coordinates.*

Therefore, the function f has a unique fixed point.

5. Conclusions

The fixed point theory is a crucial tool in mathematics and various scientific fields. This paper discusses fixed point results in the context of complete neutrosophic metric spaces, focusing on the concept of neutrosophic ψ-quasi contractions, with an example provided to demonstrate the main theorem. Additionally, we build upon previous findings documented in the literature. Future research may aim to generalize the primary results to encompass broader neutrosophic metric spaces and other abstract spaces. Another avenue for future investigation could involve exploring applications to fractional differential equations through the lens of neutrosophic sets, as referenced in [46–48].

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353.
2. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96.
3. Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. *Int. J. Pure Appl. Math.* **2005**, *24*, 287–297.
4. Alqaraleh, S.M.; Alkouri, A.U.M.J.S.; Massa'deh, M.O.; Talafha, A.G.; Bataihah, A. Bipolar complex fuzzy soft sets and their application. *Int. J. Fuzzy Syst. Appl.* **2022**, *11*(1), 1–23.
5. Falltah, A.; Massa'deh, M.O.; Alkouri, A.U. Normal and Cosets of (γ, ∂)-Fuzzy HX-Subgroups. *J. Appl. Math. Inform.* **2022**, *40*(3–4), 719–727.
6. Hazaymeh, A. Time Fuzzy Soft Sets and its application in decision-making. *Int. J. Neutrosophic Sci.* **2025**, *3*, 37–50.
7. Hazaymeh, A. Time Factor's Impact On Fuzzy Soft Expert Sets. *Int. J. Neutrosophic Sci.* **2025**, *3*, 155–176.
8. Hazaymeh, A. Time Effective Fuzzy Soft Set and Its Some Applications with and Without a Neutrosophic. *Int. J. Neutrosophic Sci.* **2024**, *2*, 129–129.
9. Hazaymeh, A. Time-Shadow Soft Set: Concepts and Applications. *Int. J. Fuzzy Log. Intell. Syst.* **2024**, *24*(4), 387–398.
10. Alkouri, A.; AbuHijleh, E.A.; Alafifi, G.; Almuhur, E.; Al-Zubi, F.M. More on complex hesitant fuzzy graphs. *AIMS Math.* **2023**, *8*(12), 30429–30444.
11. Zureigat, H.; Alkouri, A.A.U.; Abuteen, E.; Abu-Ghurra, S. Numerical Solution of Fuzzy Heat Equation with Complex Dirichlet Conditions. *Int. J. Fuzzy Log. Intell. Syst.* **2023**, *23*(1), 11–19.
12. Fallatah, A.; Massa'deh, M.O.; Alkouri, A. Homomorphism Of Tripolar Fuzzy Soft Γ-Semiring. *WSEAS Trans. Math.* **2020**, *19*, 239–246.
13. Abd Elwahed, H.; Alburaikan, A.; Smarandache, F. On characterizing efficient and properly efficient solutions for multi-objective programming problems in a complex space. *J. Optim. Ind. Eng.* **2024**, *16*(2), 369–375.

14. Al-Qudah, Y.; Al-Sharqi, F. Algorithm for decision-making based on similarity measures of possibility interval-valued neutrosophic soft setting settings. *Int. J. Neutrosophic Sci.* **2023**, *22*(3), 69–83. DOI:10.54216/IJNS.220305.
15. Hazaymeh, A.; Al-Qudah, Y.; Al-Sharqi, F.; Bataihah, A. A novel Q-neutrosophic soft under interval matrix setting and its applications. *Int. J. Neutrosophic Sci.* **2025**, *25*(4), 156–168. DOI:10.54216/IJNS.250413.
16. Al-Qudah, Y.; Hassan, N. Mapping on complex multi-fuzzy soft expert classes. *J. Phys. Conf. Ser.* **2019**, *1212*(1), 012019. DOI:10.1088/1742-6596/1212/1/012019.
17. Al-Qudah, Y.; Hamadameen, A.O.; Abdalla, N.; Al-Sharqi, F. A new generalization of interval-valued Q-neutrosophic soft matrix and its applications. *Int. J. Neutrosophic Sci.* **2025**, *25*(3), 242–257. DOI:10.54216/IJNS.250322.
18. Wajid, M.S.; Terashima-Marin, H.; Wajid, M.A.; Smarandache, F.; Verma, S.B.; Wajid, M.K. Neutrosophic Logic to Navigate Uncertainty of Security Events in Mexico. *Neutrosophic Sets Syst.* **2024**, *73*, 120–130.
19. Ashika, T.; Grace, H.; Martin, N.; Smarandache, F. Enhanced Neutrosophic Set and Machine Learning Approach for Breast Cancer Prediction. *Neutrosophic Sets Syst.* **2024**, *73*, 206–217.
20. Angammal, S.; Martin, N.; Smarandache, F. Multi Attribute Neutrosophic Optimization Technique for Optimal Crop Selection in Ariyalur District. *Neutrosophic Sets Syst.* **2024**, *73*, 153–168.
21. Salem, A.; Mohamed, M.; Smarandache, F. IM4.0EF: Tele-Medical Realization via Integrating Vague T2NSs with OWCM-RAM Toward Intelligent Medical 4.0 Evaluator Framework. *Sustain. Mach. Intell. J.* **2024**, *9*, 79–88.
22. Banach, S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund. Math.* **1922**, *3*, 133–181.
23. Shatanawi, W.; Maniu, G.; Bataihah, A.; Ahmad, F.B. Common fixed points for mappings of cyclic form satisfying linear contractive conditions with Omega-distance. *UPB Sci. Bull. Ser. A* **2017**, *79*, 11–20.
24. Batiha, I.M.; Aoua, L.B.; Oussaeif, T.E.; Ouannas, A.; Alshorman, S.; Jebril, I.H.; Momani, S. Common fixed point theorem in non-Archimedean Menger PM-spaces using CLR property with application to functional equations. *IAENG Int. J. Appl. Math.* **2023**, *53*(1), 1–9.
25. Laid, G.H.; Batiha, I.M.; Benaoua, L.; Oussaeif, T.E.; Laouadi, B.; Jebril, I.H. On a common fixed point theorem in intuitionistic Menger space via C class and inverse C class functions with CLR property. *Nonlinear Funct. Anal. Appl.* **2024**, 899–912.
26. Bataihah, A.; Hazaymeh, A. Neutrosophic fuzzy metric spaces and fixed points results with integral contraction type. *Int. J. Neutrosophic Sci.* **2025**, *25*(3), 561–572. DOI:10.54216/IJNS.250344.
27. Hazaymeh, A.; Bataihah, A. Neutrosophic fuzzy metric spaces and fixed points for contractions of nonlinear type. *Neutrosophic Sets Syst.* **2024**, *77*, 96–112. https://fs.unm.edu/nss8/index.php/111/article/view/5270.
28. Bataihah, A. Some fixed point results with application to fractional differential equation via new type of distance spaces. *Results Nonlinear Anal.* **2024**, *7*, 202–208.
29. Bataihah, A.; Qawasmeh, T. A new type of distance spaces and fixed point results. *J. Math. Anal.* **2024**, *15*(4), 81–90.
30. Bataihah, A.; Qawasmeh, T.; Shatnawi, M. Discussion on b-metric spaces and related results in metric and G-metric spaces. *Nonlinear Funct. Anal. Appl.* **2022**, *27*(2), 233–247.
31. Qawasmeh, T.; Shatanawi, W.; Bataihah, A.; Tallafha, A. Fixed point results and (α, β)-triangular admissibility in the frame of complete extended b-metric spaces and application. *UPB Sci. Bull. Ser. A* **2021**, *83*(1), 113–124.
32. Shatanawi, W.; Qawasmeh, T.; Bataihah, A.; Tallafha, A. New contractions and some fixed point results with application based on extended quasi b-metric spaces. *UPB Sci. Bull. Ser. A* **2021**, *83*(2), 1223–7027.
33. Abu-Irwaq, I.; Shatanawi, W.; Bataihah, A.; Nuseir, I. Fixed point results for nonlinear contractions with generalized Ω-distance mappings. *UPB Sci. Bull. Ser. A* **2019**, *81*(1), 57–64.

34. Shatanawi, W.; Bataihah, A. Remarks on G-Metric Spaces and Related Fixed Point Theorems. *Thai J. Math.* **2021**, *19*(2), 445–455. https://thaijmath2.in.cmu.ac.th/index.php/thaijmath/article/view/1168.
35. Abu-Irwaq, I.; Nuseir, I.; Bataihah, A. Common fixed point theorems in G-metric spaces with Ω-distance. *J. Math. Anal.* **2017**, *8*(1), 120–129.
36. Aydi, H.; Karapinar, E.; Postolache, M. Tripled coincidence point theorems for weak Φ-contractions in partially ordered metric spaces. *Fixed Point Theory Appl.* **2012**, *2012*, 44. DOI:10.1186/1687-1812-2012-44.
37. Nazam, M.; Arshad, M.; Postolache, M. Coincidence and common fixed point theorems for four mappings satisfying (alpha (s), F)-contraction. *Nonlinear Anal. Model. Control* **2018**, *23*(5), 664–690.
38. Khan, M.S.; Singh, Y.M.; Maniu, G.; Postolache, M. On (α, p)-convex contraction and asymptotic regularity. *J. Math. Comput. Sci.* **2018**, *18*, 132–145.
39. Bataihah, A. Fixed point results of Geraghty type contractions with equivalent distance. *Int. J. Neutrosophic Sci.* **2025**, *25*(3), 177–186. DOI:10.54216/IJNS.250316.
40. Latif, A.; Postolache, M.; Alansari, M.O. Numerical reckoning common fixed point in CAT (0) spaces for a general class of operators. *UPB Sci. Bull.* **2022**, *84*, 3–12.
41. Menger, K. Statistical Metrics. *Proc. Natl. Acad. Sci. USA* **1942**, *28*(12), 535–537.
42. Kirişci, M.; Şimşek, N. Neutrosophic metric spaces. *Math. Sci.* **2020**, *14*(3), 241–248.
43. Şimşek, N.; Kirişci, M. Fixed point theorems in Neutrosophic metric spaces. *Infinite Study* **2019**.
44. Bataihah, A.; Hazaymeh, A. Quasi Contractions and Fixed Point Theorems in the Context of Neutrosophic Fuzzy Metric Spaces. *Eur. J. Pure Appl. Math.* **2025**, *18*(1), 5785–5785.
45. Matkowski, J. Integrable solutions of functional equations. *Diss. Math.* **1975**, *127*, 1–68.
46. Batiha, I.M.; Ouannas, A.; Albadarneh, R.; Al-Nana, A.A.; Momani, S. Existence and uniqueness of solutions for generalized Sturm–Liouville and Langevin equations via Caputo–Hadamard fractional-order operator. *Eng. Comput.* **2022**, *39*(7), 2581–2603.
47. Alshanti, W.G.; Batiha, I.M.; Hammad, M.A.; Khalil, R. A novel analytical approach for solving partial differential equations via a tensor product theory of Banach spaces. *Partial Differ. Equ. Appl. Math.* **2023**, *8*, 100531.
48. Bouchenak, A.; Batiha, I.M.; Aljazzazi, M.; Jebril, I.H.; Al-Horani, M.; Khalil, R. Atomic exact solution for some fractional partial differential equations in Banach spaces. *Partial Differ. Equ. Appl. Math.* **2024**, *9*, 100626.

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