The integral of 2- refined irrational neutrosophic functions

Yaser Ahmad Alhasan[1,*], Raja Abdullah Abdulfatah[2] and Issam Mohammed Abdallah Abdalaziz[3]

[1]Deanship of the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj, Saudi Arabia.; y.alhasan@psau.edu.sa
[2]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj Saudi Arabia.; r.abdulfatah@psau.edu.sa
[3]Deanship the Preparatory Year, Prince Sattam bin Abdulaziz University, Alkharj Saudi Arabia.; i.abdalaziz@psau.edu.sa

*Corresponding author: y.alhasan@psau.edu.sa

Abstract: In mathematics, we often come across several ways to solve the same problem, so we should find direct rules that make it easier for us to find the direct solution to these problems. This is what prompted us to present this paper, which dealt with the integral of 2- refined irrational neutrosophic functions. Where we presented seven rules to find the integral of 2- refined irrational neutrosophic functions directly, in addition to proving each case.

Keywords: irrational neutrosophic functions; 2- refined irrational; neutrosophic integrals; standard irrational.

1. Introduction and Preliminaries

As an alternative to the existing logics, Smarandache proposed the neutrosophic logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1I_1, b_2I_2, \dots, b_nI_n)$ where $a, b_1, b_2, \dots, b_n \in R \text{ or } C$ [1]. Agboola introduced the concept of refined neutrosophic algebraic structures [2]. In addition, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. Abobala presented the papers on some special substructures of refined neutrosophic rings and a study of ah-substructures in n-refined neutrosophic vector spaces [6-7].

Alhasan.Y and Abdulfatah. R also presented a study on the division of refined neutrosophic number [8].

There are papers presented in n-Valued Refined Neutrosophic Logic and Its Applications in Physics, Neutrosophic Rings I [4-5] and studying the integral calculus according to the logic of neutrosophic by presenting a set of papers on that [9-10].

In addition, the AH-Isometry was extended to n-Refined AH-Isometry by Smarandache & Abobala in 2024 [11] https://fs.unm.edu/NSS/RefinedLiteral21.pdf.

2. Integral of standard 2- refined irrational neutrosophic functions

- **Standard 2- refined irrational neutrosophic integral I:**

$$\int \frac{a_1+b_1I_1+c_1I_2}{\sqrt{(a_2+b_2I_1+c_2I_2)^2-x^2}}dx$$

$$=(a_1+b_1I_1+c_1I_2)sin^{-1}\left(\left(\frac{1}{a_2}+\left[\frac{-b_2}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1-\left[\frac{c_2}{a_2(a_2+c_2)}\right]I_2\right)x\right)+C$$

Whereas $a_2\neq 0,\ a_2\neq -c_2,\ a_2\neq -b_2-c_2, C=a_0+b_0I_1+c_0I_2\ and\ a_0,\ b_0,\ c_0$ are real numbers.

Proof:
Let's put: $x=(a_2+b_2I_1+c_2I_2)\sin\vartheta \quad \Rightarrow dx=(a_2+b_2I_1+c_2I_2)\cos\vartheta\, d\vartheta$

Then:

$$(a_2+b_2I_1+c_2I_2)^2-x^2=(a_2+b_2I_1+c_2I_2)^2-(a_2+b_2I_1+c_2I_2)^2sin^2\vartheta$$

$$=(a_2+b_2I_1+c_2I_2)^2(1-sin^2\vartheta)$$

$$=(a_2+b_2I_1+c_2I_2)^2cos^2\vartheta$$

$$\Rightarrow \int \frac{a_1+b_1I_1+c_1I_2}{\sqrt{(a_2+b_2I_1+c_2I_2)^2-x^2}}dx=\int \frac{a_1+b_1I_1+c_1I_2}{\sqrt{(a_2+b_2I_1+c_2I_2)^2cos^2\vartheta}}(a_2+b_2I_1+c_2I_2)\cos\vartheta\ d\vartheta$$

$$=\int (a_1+b_1I_1+c_1I_2)d\vartheta=(a_1+b_1I_1+c_1I_2)\int d\vartheta$$

$$=(a_1+b_1I_1+c_1I_2)\vartheta=(a_1+b_1I_1+c_1I_2)sin^{-1}\left(\frac{x}{a_2+b_2I_1+c_2I_2}\right)+C$$

where

$$\vartheta=sin^{-1}\left(\frac{x}{a_2+b_2I_1+c_2I_2}\right)$$

hence:

$$\int \frac{a_1+b_1I_1+c_1I_2}{\sqrt{(a_2+b_2I_1+c_2I_2)^2-x^2}}dx$$

$$=(a_1+b_1I_1+c_1I_2)sin^{-1}\left(\left(\frac{1}{a_2}+\left[\frac{-b_2}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1-\left[\frac{c_2}{a_2(a_2+c_2)}\right]I_2\right)x\right)+C$$

Whereas $a_2\neq 0,\ a_2\neq -c_2,\ a_2\neq -b_2-c_2$

Example 1

$$\int \frac{1+I_1+I_2}{\sqrt{100+75I_1+21I_2-x^2}}dx$$

Solution:

$$100+75I_1+21I_2-x^2=\left(\sqrt{100+75I_1+21I_2}\right)^2-x^2$$

Whereas: $\sqrt{100+75I_1+21I_2}=\sqrt{100}+\left[\sqrt{100+75+21}-\sqrt{100+21}\right]I_1+\left[\sqrt{100+21}-\sqrt{100}\right]I_2$

$$\Longrightarrow \quad \sqrt{100+60I_1+36I_2}=10+3I_1+I_2$$

$$\int \frac{1+I_1+I_2}{\sqrt{100+75I_1+21I_2-x^2}}dx=\int \frac{1+I_1+I_2}{\sqrt{(10+3I_1+I_2)^2-x^2}}dx$$

$$=(1+I_1+I_2)sin^{-1}\left(\left(\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2\right)x\right)+C$$

Let's check the answer:

$$\frac{d}{dx}\left[(1+I_1+I_2)sin^{-1}\left(\left(\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2\right)x\right)+C\right]$$

$$=(1+I_1+I_2)\frac{\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2}{\sqrt{1-\left(\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2\right)^2x^2}}$$

$$=(1+I_1+I_2)\frac{\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2}{\left(\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2\right)\left(\sqrt{\frac{1}{\left(\frac{1}{10}-\frac{3}{154}I_1-\frac{1}{110}I_2\right)^2}-x^2}\right)}$$

$$=(1+I_1+I_2)\frac{1}{\left(\sqrt{\frac{1}{\frac{1}{100}-\frac{75}{23716}I_1-\frac{21}{12100}I_2}-x^2}\right)}$$

$$=\frac{1+I_1+I_2}{\sqrt{100+75I_1+21I_2-x^2}}\quad (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral II:**

$$\int \frac{a_1+b_1I_1+c_1I_2}{x\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}}dx$$

$$=\left(\frac{a_1}{a_2}+\left[\frac{a_2b_1+b_1c_2-a_1b_2-b_2c_1}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1+\left[\frac{a_2c_1-a_1c_2}{a_2(a_2+c_2)}\right]I_2\right)sec^{-1}\left(\left(\frac{1}{a_2}+\left[\frac{-b_2}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1-\left[\frac{c_2}{a_2(a_2+c_2)}\right]I_2\right)x\right)+C$$

Whereas $a_2\neq 0,\ a_2\neq -c_2,\ a_2\neq -b_2-c_2, C=a_0+b_0I_1+c_0I_2\ and\ a_0,\ b_0,\ c_0$ are real numbers.

Proof:

Let's put: $x=(a_2+b_2I_1+c_2I_2)\sec\vartheta \quad \Rightarrow dx=(a_2+b_2I_1+c_2I_2)\sec\vartheta\tan\vartheta\ d\vartheta$

Then:

$$x\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}=(a_2+b_2I_1+c_2I_2)\sec\vartheta\sqrt{(a_2+b_2I_1+c_2I_2)^2sec^2\vartheta-(a_2+b_2I_1+c_2I_2)^2}$$

$$= (a_2 + b_2I_1 + c_2I_2)\, sec\,\vartheta \sqrt{(a_2 + b_2I_1 + c_2I_2)^2(sec^2\vartheta - 1)}$$

$$= (a_2 + b_2I_1 + c_2I_2)^2\, sec\,\vartheta\, tan\,\vartheta$$

$$\Rightarrow \int \frac{a_1 + b_1I_1 + c_1I_2}{x\sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2}}dx = \int \frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2)^2\, sec\,\vartheta\, tan\,\vartheta}(a_2 + b_2I_1 + c_2I_2)\, sec\,\vartheta\, tan\,\vartheta\, d\vartheta$$

$$= \int \frac{a_1 + b_1I_1 + c_1I_2}{a_2 + b_2I_1 + c_2I_2}d\vartheta = \frac{a_1 + b_1I_1 + c_1I_2}{a_2 + b_2I_1 + c_2I_2}\int d\vartheta$$

$$= \frac{a_1 + b_1I_1 + c_1I_2}{a_2 + b_2I_1 + c_2I_2}\vartheta = \frac{a_1 + b_1I_1 + c_1I_2}{a_2 + b_2I_1 + c_2I_2} sec^{-1}\left(\frac{x}{a_2 + b_2I_1 + c_2I_2}\right) + C$$

Whereas:

$$\vartheta = sec^{-1}\left(\frac{x}{a_2 + b_2I_1 + c_2I_2}\right)$$

Hence:

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{x\sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2}}dx$$

$$= \left(\frac{a_1}{a_2} + \left[\frac{a_2b_1 + b_1c_2 - a_1b_2 - b_2c_1}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right]I_1 + \left[\frac{a_2c_1 - a_1c_2}{a_2(a_2 + c_2)}\right]I_2\right) sec^{-1}\left(\left(\frac{1}{a_2} + \left[\frac{-b_2}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right]I_1 - \left[\frac{c_2}{a_2(a_2 + c_2)}\right]I_2\right)x\right) + C$$

Whereas $a_2 \neq 0,\ a_2 \neq -c_2,\ a_2 \neq -b_2 - c_2$

Example 2

Evaluate:

$$\int \frac{3 + 7I_2}{x\sqrt{x^2 - 144 - 56I_1 - 25I_2}}dx$$

Solution:

$$x^2 - 144 - 56I_1 - 25I_2 = x^2 - \left(\sqrt{144 + 56I_1 + 25I_2}\right)^2$$

$$x^2 - 144 - 56I_1 - 25I_2 = x^2 - (12 + 2I_1 + I_2)^2$$

Whereas: $\sqrt{144 + 56I_1 + 25I_2} = \sqrt{144} + \left[\sqrt{144 + 56 + 25} - \sqrt{144 + 25}\right]I_1 + \left[\sqrt{144 + 25} - \sqrt{144}\right]I_2$

$$\Rightarrow \sqrt{1 + 72I_1 + 8I_2} = 12 + 2I_1 + I_2$$

$$\int \frac{3 + 7I_2}{x\sqrt{x^2 - 144 - 56I_1 - 25I_2}}dx = \int \frac{3 + 7I_2}{x\sqrt{x^2 - (12 + 2I_1 + I_2)^2}}dx$$

$$= \left(\frac{3 + 7I_2}{12 + 2I_1 + I_2}\right) sec^{-1}\left(\left(\frac{1}{12 + 2I_1 + I_2}\right)x\right)$$

$$= \left(\frac{1}{4} - \frac{4}{39}I_1 + \frac{27}{52}I_2\right) sec^{-1}\left(\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)x\right) + C$$

Let's check the answer:

$$\frac{d}{dx}\left[\left(\frac{1}{4} - \frac{4}{39}I_1 + \frac{27}{52}I_2\right) sec^{-1}\left(\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)x\right) + C\right]$$

$$= \left(\frac{1}{4} - \frac{4}{39}I_1 + \frac{27}{52}I_2\right)\frac{\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2}{\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)x\sqrt{\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)^2 x^2 - 1}}$$

$$= \left(\frac{1}{4} - \frac{4}{39}I_1 + \frac{27}{52}I_2\right)\frac{\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2}{\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)^2\left(x\sqrt{x^2 - \frac{1}{\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)^2}}\right)}$$

$$= \left(\frac{\frac{1}{4} - \frac{4}{39}I_1 + \frac{27}{52}I_2}{\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2}\right)\frac{1}{\left(x\sqrt{x^2 - \frac{1}{\left(\frac{1}{12} - \frac{2}{195}I_1 - \frac{1}{156}I_2\right)^2}}\right)}$$

$$= (3 + 7I_2)\frac{1}{\left(x\sqrt{x^2 - \frac{1}{\frac{1}{144} - \frac{56}{38025}I_1 - \frac{25}{24336}I_2}}\right)}$$

$$= \frac{3 + 7I_2}{x\sqrt{x^2 - 144 - 56I_1 - 25I_2}} \quad (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral III:**

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{\sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2}}dx = (a_1 + b_1I_1 + c_1I_2)ln\left|x + \sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2}\right| + C$$

Proof:

Let's put: $x = (a_2 + b_2I_1 + c_2I_2)\sec\vartheta \quad \Rightarrow \quad dx = (a_2 + b_2I_1 + c_2I_2)\sec\vartheta\ \tan\vartheta\, d\vartheta$

Then:

$$\sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2} = \sqrt{(a_2 + b_2I_1 + c_2I_2)^2 sec^2\vartheta - (a_2 + b_2I_1 + c_2I_2)^2}$$

$$= \sqrt{(a_2 + b_2I_1 + c_2I_2)^2(sec^2\vartheta - 1)}$$

$$= (a_2 + b_2I_1 + c_2I_2)\tan\vartheta$$

$$\Rightarrow \int \frac{a_1 + b_1I_1 + c_1I_2}{\sqrt{x^2 - (a_2 + b_2I_1 + c_2I_2)^2}}dx = \int \frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2)\tan\vartheta}(a_2 + b_2I_1 + c_2I_2)\sec\vartheta\tan\vartheta\ d\vartheta$$

$$= \int (a_1 + b_1 I_1 + c_1 I_2)\sec\vartheta \; d\vartheta = (a_1 + b_1 I_1 + c_1 I_2)\int \sec\vartheta \; d\vartheta$$

$$= (a_1 + b_1 I_1 + c_1 I_2)ln|\sec\vartheta + \tan\vartheta| + C_1 = (a_1 + b_1 I_1 + c_1 I_2)ln\left|\sec\vartheta + \sqrt{\sec^2\vartheta - 1}\right| + C_1$$

$$= (a_1 + b_1 I_1 + c_1 I_2)ln\left|\frac{x}{a_2 + b_2 I_1 + c_2 I_2} + \sqrt{\left(\frac{x}{a_2 + b_2 I_1 + c_2 I_2}\right)^2 - 1}\right| + C_1$$

$$= (a_1 + b_1 I_1 + c_1 I_2)ln\left|\frac{x}{a_2 + b_2 I_1 + c_2 I_2} + \frac{1}{a_2 + b_2 I_1 + c_2 I_2}\sqrt{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2}\right| + C_1$$

$$= (a_1 + b_1 I_1 + c_1 I_2)ln\left|\frac{1}{a_2 + b_2 I_1 + c_2 I_2}\left(x + \sqrt{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2}\right)\right| + C_1$$

$$= (a_1 + b_1 I_1 + c_1 I_2)\left[ln\left|\left(x + \sqrt{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2}\right)\right| - ln|a_2 + b_2 I_1 + c_2 I_2| + C_1\right]$$

hence:

$$\int \frac{a_1 + b_1 I_1 + c_1 I_2}{\sqrt{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2}}dx = (a_1 + b_1 I_1 + c_1 I_2)ln\left|x + \sqrt{x^2 - (a_2 + b_2 I_1 + c_2 I_2)^2}\right| + C$$

Whereas:

$$\sec\vartheta = \frac{x}{a_2 + b_2 I_1 + c_2 I_2} \; and \; C = -ln|a_2 + b_2 I_1 + c_2 I_2| + C_1$$

Example 3

Evaluate:

$$\int \frac{1 - I_1 + 2I_2}{\sqrt{x^2 - 1 - 32I_1 - 3I_2}}dx$$

Solution:

$$x^2 - 1 - 32I_1 - 3I_2 = x^2 - \left(\sqrt{1 + 32I_1 + 3I_2}\right)^2$$

$$x^2 - 1 - 32I_1 - 3I_2 = x^2 - (1 + 4I_1 + 3I_2)^2$$

Whereas: $\sqrt{1 + 32I_1 + 3I_2} = 1 + \left[\sqrt{1 + 32 + 3} - \sqrt{4}\right]I_1 + \left[\sqrt{4} - 1\right]I_2$

$\Rightarrow \quad \sqrt{1 + 32I_1 + 3I_2} = 1 + 4I_1 + 3I_2$

$$\int \frac{1 - I_1 + 2I_2}{\sqrt{x^2 - 1 - 32I_1 - 3I_2}}dx = \int \frac{1 - I_1 + 2I_2}{\sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}}dx$$

$$= (1 - I_1 + 2I_2)ln\left|x + \sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}\right| + C$$

Let's check the answer:

$$\frac{d}{dx}\left[(1 - I_1 + 2I_2)ln\left|x + \sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}\right| + C\right]$$

$$= (1 - I_1 + 2I_2)\frac{1 + \frac{2x}{2\sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}}}{x + \sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}} = (1 - I_1 + 2I_2)\frac{\frac{\sqrt{x^2 - (1 + 4I_1 + 3I_2)^2} + x}{\sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}}}{x + \sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}}$$

$$= (1 - I_1 + 2I_2)\frac{1}{\sqrt{x^2 - (1 + 4I_1 + 3I_2)^2}} = \frac{1 - I_1 + 2I_2}{\sqrt{x^2 - 1 - 32I_1 - 3I_2}} \ (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral IV:**

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}}dx = (a_1 + b_1I_1 + c_1I_2)ln\left|x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right| + C$$

Proof:
Let's put: $x = (a_2 + b_2I_1 + c_2I_2)\,tan\,\vartheta \ \Rightarrow\ dx = (a_2 + b_2I_1 + c_2I_2)sec^2\vartheta\ d\vartheta$

Then:

$$\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2} = \sqrt{(a_2 + b_2I_1 + c_2I_2)^2tan^2\vartheta + (a_2 + b_2I_1 + c_2I_2)^2}$$

$$= \sqrt{(a_2 + b_2I_1 + c_2I_2)^2(tan^2\vartheta + 1)}$$

$$= (a_2 + b_2I_1 + c_2I_2)\,sec\,\vartheta$$

$$\Rightarrow \int \frac{a_1 + b_1I_1 + c_1I_2}{\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}}dx = \int \frac{a_1 + b_1I_1 + c_1I_2}{(a_2 + b_2I_1 + c_2I_2)\,sec\,\vartheta}(a_2 + b_2I_1 + c_2I_2)sec^2\theta\ d\theta$$

$$= \int (a_1 + b_1I_1 + c_1I_2)\,sec\,\vartheta\ d\vartheta = (a_1 + b_1I_1 + c_1I_2)\int sec\,\vartheta\ d\vartheta$$

$$= (a_1 + b_1I_1 + c_1I_2)ln|sec\,\vartheta + tan\,\vartheta| = (a_1 + b_1I_1 + c_1I_2)ln\left|\sqrt{tan^2\vartheta + 1} + tan\,\vartheta\right|$$

$$= (a_1 + b_1I_1 + c_1I_2)ln\left|\frac{x}{a_2 + b_2I_1 + c_2I_2} + \sqrt{\left(\frac{x}{a_2 + b_2I_1 + c_2I_2}\right)^2 + 1}\right|$$

$$= (a_1 + b_1I_1 + c_1I_2)ln\left|\frac{x}{a_2 + b_2I_1 + c_2I_2} + \frac{1}{a_2 + b_2I_1 + c_2I_2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right|$$

$$= (a_1 + b_1I_1 + c_1I_2)ln\left|\frac{1}{a_2 + b_2I_1 + c_2I_2}\left(x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right)\right|$$

$$= (a_1 + b_1I_1 + c_1I_2)\left[ln\left|\left(x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right)\right| - ln|a_2 + b_2I_1 + c_2I_2| + C_1\right]$$

Hence:

$$\int \frac{a_1 + b_1I_1 + c_1I_2}{\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}}dx = (a_1 + b_1I_1 + c_1I_2)ln\left|x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right| + C$$

Whereas:

$$sec\,\vartheta = \frac{x}{a_2 + b_2I_1 + c_2I_2}\ and\ \ C = -ln|a_2 + b_2I_1 + c_2I_2| + C_1$$

Example 4

Evaluate:

$$\int \frac{12-14I_1-92I_2}{\sqrt{x^2+25+13I_1+11I_2}}dx$$

Solution:

$$x^2+25+13I_1+11I_2 = x^2+\left(\sqrt{25+13I_1+11I_2}\right)^2$$

$$x^2+25+13I_1+11I_2 = x^2-(5+I_1+I_2)^2$$

Whereas: $\sqrt{25+13I_1+11I_2} = 5+\left[\sqrt{25+13+11}-\sqrt{36}\right]I_1+\left[\sqrt{36}-5\right]I_2$

$$\Rightarrow \sqrt{25+13I_1+11I_2} = 5+I_1+I_2$$

$$\int \frac{12-14I_1-92I_2}{\sqrt{x^2+25+13I_1+11I_2}}dx = \int \frac{12-14I_1-92I_2}{\sqrt{x^2+(5+I_1+I_2)^2}}dx$$

$$= (12-14I_1-92I_2)ln\left|x+\sqrt{x^2+(5+I_1+I_2)^2}\right|+C$$

Let's check the answer:

$$\frac{d}{dx}\left[(12-14I_1-92I_2)ln\left|x+\sqrt{x^2+(5+I_1+I_2)^2}\right|+C\right]$$

$$= (12-14I_1-92I_2)\frac{1+\frac{2x}{2\sqrt{x^2+(5+I_1+I_2)^2}}}{x+\sqrt{x^2+(5+I_1+I_2)^2}} = (12-14I_1-92I_2)\frac{\frac{\sqrt{x^2+(5+I_1+I_2)^2}+x}{\sqrt{x^2+(5+I_1+I_2)^2}}}{x+\sqrt{x^2+(5+I_1+I_2)^2}}$$

$$= (12-14I_1-92I_2)\frac{1}{\sqrt{x^2+(5+I_1+I_2)^2}}$$

$$= \frac{12-14I_1-92I_2}{\sqrt{x^2+25+13I_1+11I_2}} \quad (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral V:**

$$\int \sqrt{(a_2+b_2I_1+c_2I_2)^2-x^2}\,dx = \frac{x}{2}\sqrt{(a_2+b_2I_1+c_2I_2)^2-x^2} + \frac{(a_2+b_2I_1+c_2I_2)^2}{2}sin^{-1}\left(\left(\frac{1}{a_2}+\left[\frac{-b_2}{(a_2+c_2)(a_2+b_2+c_2)}\right]I_1-\left[\frac{c_2}{a_2(a_2+c_2)}\right]I_2\right)x\right) + C$$

Whereas $a_2 \neq 0,\ a_2 \neq -c_2,\ a_2 \neq -b_2-c_2$

Proof:

Let's put: $x = (a_2+b_2I_1+c_2I_2)\,sin\,\vartheta \quad \Rightarrow dx = (a_2+b_2I_1+c_2I_2)\cos\vartheta\ d\vartheta$

Then:

$$(a_2+b_2I_1+c_2I_2)^2-x^2 = (a_2+b_2I_1+c_2I_2)^2-(a_2+b_2I_1+c_2I_2)^2sin^2\vartheta$$

$$= (a_2 + b_2 I_1 + c_2 I_2)^2 (1 - sin^2\vartheta)$$

$$= (a_2 + b_2 I_1 + c_2 I_2)^2 cos^2\vartheta$$

$$\Rightarrow \int \sqrt{(a_2 + b_2 I_1 + c_2 I_2)^2 - x^2}\, dx = \int \sqrt{(a_2 + b_2 I_1 + c_2 I_2)^2 cos^2\vartheta}\, (a_2 + b_2 I_1 + c_2 I_2) \cos\vartheta \; d\vartheta$$

$$= \int (a_2 + b_2 I_1 + c_2 I_2)^2 cos^2\vartheta \, d\vartheta = \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} \int (cos\, 2\vartheta + 1) d\vartheta$$

$$= \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} \int (cos\, 2\vartheta + 1) d\vartheta = \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} \left(\frac{1}{2} \sin 2\vartheta + \vartheta\right)$$

$$= \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} (sin\,\vartheta\, cos\,\vartheta + \vartheta)$$

$$= \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} \left(\frac{x}{a_2 + b_2 I_1 + c_2 I_2} \frac{\sqrt{(a_2 + b_2 + c_2)^2 - x^2}}{(a_2 + b_2 + c_2)} + sin^{-1}\left(\left(\frac{1}{a_2} + \left[\frac{-b_2}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right] I_1 - \left[\frac{c_2}{a_2(a_2 + c_2)}\right] I_2\right) x\right)\right)$$

Whereas:

$$\vartheta = sin^{-1}\left(\frac{x}{a_2 + b_2 I_1 + c_2 I_2}\right)$$

Hence:

$$\int \sqrt{(a_2 + b_2 I_1 + c_2 I_2)^2 - x^2}\, dx$$

$$= \frac{x}{2}\sqrt{(a_2 + b_2 I_1 + c_2 I_2)^2 - x^2} + \frac{(a_2 + b_2 I_1 + c_2 I_2)^2}{2} sin^{-1}\left(\left(\frac{1}{a_2} + \left[\frac{-b_2}{(a_2 + c_2)(a_2 + b_2 + c_2)}\right] I_1 - \left[\frac{c_2}{a_2(a_2 + c_2)}\right] I_2\right) x\right) + C$$

Example 5

$$\int \sqrt{25 + 80 I_1 + 39 I_2 - x^2}\, dx$$

Solution:

$$25 + 80 I_1 + 39 I_2 - x^2 = \left(\sqrt{25 + 80 I_1 + 39 I_2}\right)^2 - x^2$$

$$25 + 80 I_1 + 39 I_2 - x^2 = (5 + 4 I_1 + 3 I_2)^2 - x^2$$

Whereas: $\sqrt{25 + 80 I_1 + 39 I_2} = \sqrt{25} + \left[\sqrt{25 + 80 + 39} - \sqrt{25 + 39}\right] I_1 + \left[\sqrt{25 + 39} - \sqrt{25}\right] I_2$

$$\Rightarrow \sqrt{1 + 72 I_1 + 8 I_2} = 5 + 4 I_1 + 3 I_2$$

$$\int \sqrt{25+80I_1+39I_2-x^2}\,dx = \int \sqrt{(5+4I_1+3I_2)^2-x^2}\,dx$$

$$= \frac{x}{2}\sqrt{(5+4I_1+3I_2)^2-x^2} + \frac{(5+4I_1+3I_2)^2}{2} sin^{-1}\left(\left(\frac{1}{5+4I_1+3I_2}\right)x\right) + C$$

$$= \frac{x}{2}\sqrt{25+80I_1+39I_2-x^2} + \left(\frac{25}{2}+40I_1+\frac{39}{2}I_2\right) sin^{-1}\left(\left(\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2\right)x\right) + C$$

Let's check the answer:

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{25+80I_1+39I_2-x^2} + \left(\frac{25}{2}+40I_1+\frac{39}{2}I_2\right) sin^{-1}\left(\left(\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2\right)x\right) + C\right]$$

$$= \frac{1}{2}\sqrt{25+80I_1+39I_2-x^2} + \frac{x}{2}\frac{-2x}{2\sqrt{25+80I_1+39I_2-x^2}}$$
$$+ \left(\frac{25}{2}+40I_1+\frac{39}{2}I_2\right)\frac{\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2}{\left(\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2\right)\sqrt{1-\left(\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2\right)^2 x^2}}$$

$$= \frac{1}{2}\left(\frac{25+80I_1+39I_2-2x^2}{\sqrt{25+80I_1+39I_2-x^2}}\right) + \frac{\frac{25}{2}+40I_1+\frac{39}{2}I_2}{\sqrt{\frac{1}{\left(\frac{1}{5}-\frac{1}{24}I_1+\frac{3}{40}I_2\right)^2}-x^2}}$$

$$= \frac{1}{2}\left(\frac{25+80I_1+39I_2-2x^2}{\sqrt{25+80I_1+39I_2-x^2}}\right) + \frac{1}{2}\frac{25+80I_1+39I_2}{\sqrt{25+80I_1+39I_2-x^2}} = \frac{25+80I_1+39I_2-x^2}{\sqrt{25+80I_1+39I_2-x^2}}$$

$$= \sqrt{25+80I_1+39I_2-x^2} \quad (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral VI:**

$$\int \sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\,dx$$

$$= \frac{x}{2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2} + \frac{(a_2+b_2I_1+c_2I_2)^2}{2} ln\left|x+\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right| + C$$

Proof:
Put: $x = (a_2+b_2I_1+c_2I_2)\,sec\,\vartheta \quad \Longrightarrow dx = (a_2+b_2I_1+c_2I_2)\,sec\,\vartheta\,tan\,\vartheta\ d\vartheta$

Then:

$$\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2} = \sqrt{(a_2+b_2I_1+c_2I_2)^2 sec^2\vartheta-(a_2+b_2I_1+c_2I_2)^2}$$

$$= \sqrt{(a_2+b_2I_1+c_2I_2)^2(sec^2\vartheta-1)}$$

$$= (a_2+b_2I_1+c_2I_2)\,tan\,\vartheta$$

$$\Rightarrow \int \sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}dx = \int (a_2+b_2I_1+c_2I_2)\tan\vartheta\,(a_2+b_2I_1+c_2I_2)\sec\vartheta\tan\vartheta\,d\vartheta$$

$$= (a_2+b_2I_1+c_2I_2)^2\int \sec\vartheta\tan^2\vartheta\,d\vartheta = (a_2+b_2I_1+c_2I_2)^2\int \sec\vartheta\,(\sec^2\theta-1)d\vartheta$$

$$= (a_2+b_2I_1+c_2I_2)^2\int (\sec^3\vartheta-\sec\vartheta)d\vartheta$$

$$= (a_2+b_2I_1+c_2I_2)^2\left[\frac{1}{2}\sec\vartheta\tan\vartheta+\frac{1}{2}ln|\sec\vartheta+\tan\vartheta|-ln|\sec\vartheta+\tan\vartheta|\right]+C_1$$

$$= (a_2+b_2I_1+c_2I_2)^2\left[\frac{1}{2}\sec\vartheta\sqrt{\sec^2\vartheta-1}-\frac{1}{2}ln\left|\sec\vartheta+\sqrt{\sec^2\theta-1}\right|\right]+C_1$$

$$= (a_2+b_2I_1+c_2I_2)^2\left[\frac{1}{2}\sec\vartheta\sqrt{\sec^2\vartheta-1}-\frac{1}{2}ln|\sec\vartheta+\tan\vartheta|\right]+C_1 \quad (1)$$

But:

$$\sec\vartheta = \frac{x}{a_2+b_2I_1+c_2I_2}$$

$$\sqrt{\sec^2\vartheta-1} = \sqrt{\left(\frac{x}{a_2+b_2I_1+c_2I_2}\right)^2-1} = \frac{1}{a_2+b_2I_1+c_2I_2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}$$

By substitution in (1), we get:

$$\int \sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}dx$$
$$= (a_2+b_2I_1+c_2I_2)^2\left[\frac{1}{2}\frac{x}{(a_2+b_2I_1+c_2I_2)^2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right.$$
$$\left.-\frac{1}{2}ln\left|\frac{x}{a_2+b_2I_1+c_2I_2}+\frac{1}{a_2+b_2I_1+c_2I_2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right|\right]+C_1$$
$$= \frac{x}{2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}$$
$$-\frac{(a_2+b_2I_1+c_2I_2)^2}{2}ln\left|\frac{1}{a_2+b_2I_1+c_2I_2}\left(x+\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right)\right|$$

$$= \frac{x}{2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}-\frac{(a_2+b_2I_1+c_2I_2)^2}{2}ln\left|x+\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right|$$
$$-ln|a_2+b_2I_1+c_2I_2|+C_1$$

Hence:

$$\int \sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}dx$$
$$= \frac{x}{2}\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}-\frac{(a_2+b_2I_1+c_2I_2)^2}{2}ln\left|x+\sqrt{x^2-(a_2+b_2I_1+c_2I_2)^2}\right|$$
$$+C$$

Whereas:

$$C = -ln|a_2+b_2I_1+c_2I_2|+C_1$$

Example 6

$$\int \sqrt{x^2-(1-4I_1-3I_2)^2}dx$$

$$=\frac{x}{2}\sqrt{x^2-(1-4I_1-3I_2)^2}-\frac{(1-4I_1-3I_2)^2}{2}ln\left|\left(x+\sqrt{x^2-(1-4I_1-3I_2)^2}\right)\right|+C$$

$$=\frac{x}{2}\sqrt{x^2-(1-4I_1-3I_2)^2}-(1+32I_1+3I_2)ln\left|\left(x+\sqrt{x^2-(1-4I_1-3I_2)^2}\right)\right|+C$$

Let's check the answer:

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{x^2-(1-4I_1-3I_2)^2}-(1+32I_1+3I_2)ln\left|\left(x+\sqrt{x^2-(1-4I_1-3I_2)^2}\right)\right|+C\right]$$

$$=\frac{1}{2}\sqrt{x^2-(1-4I_1-3I_2)^2}+\frac{x}{2}.\frac{2x}{2\sqrt{x^2-(1-4I_1-3I_2)^2}}-\frac{(1-4I_1-3I_2)^2}{2}\frac{1+\frac{2x}{2\sqrt{x^2-(1-4I_1-3I_2)^2}}}{x+\sqrt{x^2-(1-4I_1-3I_2)^2}}$$

$$=\frac{1}{2}\left(\sqrt{x^2-(1-4I_1-3I_2)^2}+\frac{x^2}{\sqrt{x^2-(1-4I_1-3I_2)^2}}-(1-4I_1-3I_2)^2.\frac{\frac{x+\sqrt{x^2-(1-4I_1-3I_2)^2}}{\sqrt{x^2-(1-4I_1-3I_2)^2}}}{x+\sqrt{x^2-(1-4I_1-3I_2)^2}}\right)$$

$$=\frac{1}{2}\left(\sqrt{x^2-(1-4I_1-3I_2)^2}+\frac{x^2}{\sqrt{x^2-(1-4I_1-3I_2)^2}}-(1-4I_1-3I_2)^2.\frac{1}{\sqrt{x^2-(1-4I_1-3I_2)^2}}\right)$$

$$=\frac{1}{2}\left(\sqrt{x^2-(1-4I_1-3I_2)^2}+\frac{x^2-(1-4I_1-3I_2)^2}{\sqrt{x^2-(1-4I_1-3I_2)^2}}\right)$$

$$=\frac{1}{2}\left(\frac{x^2-(1-4I_1-3I_2)^2+x^2-(1-4I_1-3I_2)^2}{\sqrt{x^2-(1-4I_1-3I_2)^2}}\right)=\frac{1}{2}\left(\frac{2(x^2-(1-4I_1-3I_2)^2)}{\sqrt{x^2-(1-4I_1-3I_2)^2}}\right)$$

$$=\sqrt{x^2-(1-4I_1-3I_2)^2}\ (The\ same\ integral\ function)$$

- **Standard 2- refined irrational neutrosophic integral VII:**

$$\int \sqrt{x^2+(a_2+b_2I_1+c_2I_2)^2}dx$$

$$=\frac{x}{2}\sqrt{x^2+(a_2+b_2I_1+c_2I_2)^2}+\frac{(a_2+b_2I_1+c_2I_2)^2}{2}ln\left|\left(x+\sqrt{x^2+(a_2+b_2I_1+c_2I_2)^2}\right)\right|+C$$

Proof:

Put $x=(a_2+b_2I_1+c_2I_2)\,tan\,\vartheta \;\Longrightarrow\; dx=(a_2+b_2I_1+c_2I_2)sec^2\vartheta\,d\vartheta$

Then:

$$\sqrt{x^2+(a_2+b_2I_1+c_2I_2)^2}=\sqrt{(a_2+b_2I_1+c_2I_2)^2tan^2\vartheta+(a_2+b_2I_1+c_2I_2)^2}$$

$$=\sqrt{(a_2+b_2I_1+c_2I_2)^2(tan^2\vartheta+1)}$$

$$= (a_2 + b_2I_1 + c_2I_2)\,sec\,\vartheta$$

$$\Rightarrow \int \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}dx = \int (a_2 + b_2I_1 + c_2I_2)\,sec\,\vartheta\,(a_2 + b_2I_1 + c_2I_2)sec^2\vartheta\,d\vartheta$$

$$= (a_2 + b_2I_1 + c_2I_2)^2 \int sec^3\vartheta\,d\vartheta$$

$$= (a_2 + b_2I_1 + c_2I_2)^2 \left[\frac{1}{2}sec\,\vartheta\,tan\,\vartheta + \frac{1}{2}ln|sec\,\vartheta + tan\,\vartheta|\right] + C_1$$

$$= (a_2 + b_2I_1 + c_2I_2)^2 \left[\frac{1}{2}tan\,\vartheta\sqrt{tan^2\vartheta + 1} + \frac{1}{2}ln\left|tan\,\vartheta + \sqrt{tan^2\vartheta + 1}\right|\right] + C_1$$

$$= (a_2 + b_2I_1 + c_2I_2)^2 \left[\frac{1}{2}tan\,\vartheta\sqrt{tan^2\vartheta + 1} + \frac{1}{2}ln\left|tan\,\vartheta + \sqrt{tan^2\vartheta + 1}\right|\right] + C_1 \quad (2)$$

But:

$$tan\,\vartheta = \frac{x}{(a_2 + b_2I_1 + c_2I_2)}$$

$$\sqrt{tan^2\theta + 1} = \sqrt{\left(\frac{x}{(a_2 + b_2I_1 + c_2I_2)}\right)^2 + 1} = \frac{1}{(a_2 + b_2I_1 + c_2I_2)}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}$$

By substitution in (2), we get:

$$\int \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}dx$$

$$= (a_2 + b_2I_1 + c_2I_2)^2 \left[\frac{1}{2}\frac{x}{(a_2 + b_2I_1 + c_2I_2)^2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2} + \frac{1}{2}ln\left|\frac{x}{a_2 + b_2I_1 + c_2I_2} + \frac{1}{a_2 + b_2I_1 + c_2I_2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right|\right] + C_1$$

$$= \frac{x}{2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2} + \frac{(a_2 + b_2I_1 + c_2I_2)^2}{2}ln\left|\frac{1}{a_2 + b_2I_1 + c_2I_2}\left(x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right)\right| + C_1$$

$$= \frac{x}{2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2} + \frac{(a_2 + b_2I_1 + c_2I_2)^2}{2}ln\left|x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right| - ln|a_2 + b_2I_1 + c_2I_2| + C_1$$

Hence:

$$\int \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}dx$$

$$= \frac{x}{2}\sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2} + \frac{(a_2 + b_2I_1 + c_2I_2)^2}{2}ln\left|x + \sqrt{x^2 + (a_2 + b_2I_1 + c_2I_2)^2}\right| + C$$

Whereas:

$$C = -ln|a_2 + b_2 I_1 + c_2 I_2| + C_1$$

Example 7

1) $\int \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} dx$

$$= \frac{x}{2}\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \frac{(8 + 2I_1 - 5I_2)^2}{2} ln\left|\left(x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}\right)\right| + C$$

$$= \frac{x}{2}\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \left(32 + 8I_1 - \frac{55}{2}I_2\right) ln\left|\left(x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}\right)\right| + C$$

Let's check the answer:

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \left(32 + 8I_1 - \frac{55}{2}I_2\right) ln\left|\left(x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}\right)\right| + C\right]$$

$$= \frac{1}{2}\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \frac{x}{2}.\frac{2x}{2\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}} + \frac{(8 + 2I_1 - 5I_2)^2}{2}\frac{1 + \frac{2x}{2\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}}{x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}$$

$$= \frac{1}{2}\left(\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \frac{x^2}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}} + (8 + 2I_1 - 5I_2)^2.\frac{\frac{x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}}{x + \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}\right)$$

$$= \frac{1}{2}\left(\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \frac{x^2}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}} + (8 + 2I_1 - 5I_2)^2.\frac{1}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}\right)$$

$$= \frac{1}{2}\left(\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2} + \frac{x^2 + (8 + 2I_1 - 5I_2)^2}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}\right)$$

$$= \frac{1}{2}\left(\frac{x^2 + (8 + 2I_1 - 5I_2)^2 + x^2 + (8 + 2I_1 - 5I_2)^2}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}\right) = \frac{1}{2}\left(\frac{2(x^2 + (8 + 2I_1 - 5I_2)^2)}{\sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}}\right)$$

$$= \sqrt{x^2 + (8 + 2I_1 - 5I_2)^2}\ (The\ same\ integral\ function)$$

3. Conclusions

The importance of this paper lies in the fact that it provides rules for calculating the integral of 2- refined irrational neutrosophic functions directly, without the need to use long methods to solve these integrals, in addition to use the division of refined neutrosophic numbers. We reached accurate results in the solution, which were verified. Finally, we discussed a set of examples that illustrate the ideas presented.

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