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Quadripartitioned Bipolar Neutrosophic Competition Graph with Novel Application

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Abstract. This paper introduces the concept of quadripartitioned neutrosophic bipolar competition graphs. The properties of operations like Cartesian and direct products are investigated within these graphs. Furthermore, this work presents an algorithm that applies the competition model to quadripartitioned bipolar neutrosophic competition graphs for depicting applicant competition. Finally, we compare the results with existing literature, emphasizing the novelty and contributions of the proposed method.

Keywords: Quadripartitioned Bipolar Neutrosophic; Competition Graph; Uncertainty.

1. Introduction

Cohen [1] developed the concept of competition graphs in an ecological context to depict the link between species in the food chain. Competition graphs are frequently utilised in a variety of applications, including channel assignment, complex economic modelling, phytogenetic tree reconstruction, coding, and energy systems. Much work has been done on competition graphs and their variations since the initial inspiration of ecological application of species competition. Each of these representations necessitates the explicit definition of vertices and edges within the networks, a methodology deemed inadequate for fully encapsulating all competitions within real-world challenges. The study of intuitionistic fuzzy graphs involves developing algorithms, methodologies, and techniques to analyze and manipulate these structures effectively in [30, 31]. Researchers explore properties, operations, and algorithms specific to intuitionistic fuzzy graphs to extend the reach of multi polar graph theory into the realm of uncertainty and ambiguity in [29].

The single-valued neutrosophic set, an expansion of intuitionistic fuzzy sets, presents a rapid resolution for real-world challenges, notably in decision support [3], as investigated in [4]. Few authors choose to frame the behavior of indeterminacy in the same manner that truth-membership is modelled, while others infer to frame it in the similar fashion with falsity-membership. Authors in [3] introduced the notion of a single-valued neutrosophic set and described its features. Now it is applied in many domains such as sensors [5], decision making [6], image processing [7], control theory [8], medical diagnosis [9] and graph theory [10–12]. The following Figure 1 represents the distinction of the quadripartioned bipolar neutrosophic competition graphs with previously introduced graphs.

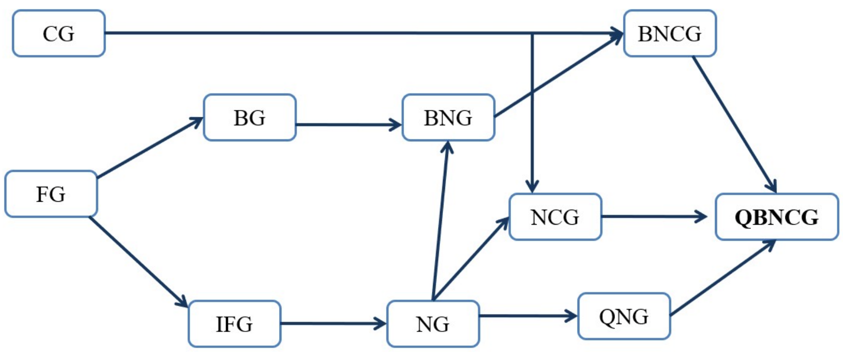


FIGURE 1. CG-Competition Graphs, FG-Fuzzy Graphs, BG-Bipolar Graphs, IFG-Intuitionistic Fuzzy Graphs, NG-Neutrosophic Graphs, BNG-Bipolar Neutrosophic Graphs, NCG-Neutrosophic Competition Graphs, BNCG-Bipolar Neutrosophic Competition Graphs, QNG-Quadripartitioned Neutrosophic Graphs, QBNCG-Quadripartitioned Neutrosophic Competition Graphs

Nowadays, advanced level of neutrosophic sets and graphs received much attention in the application viewpoint. The notion of bipolar neutrosophic competition graphs is insufficient in real-world applications involving competitions and relationships with opposing side facts. This approach pushes us to use bipolar neutrosophic quadripartitioned sets [15] to model bipolar neutrosophic quadripartitioned competition graphs. When we separate the indeterminacy section of the neutrosophic set into two sections, we get four parts: 'Contradiction' (both true and false) and 'Unknown' (neither true nor false), that is $\mathcal{T}, \mathcal{C}, \mathcal{U}$ and $\mathcal{F}$ which defines a new set called 'quadripartitioned single valued neutrosophic set', introduced by Chatterjee., et al. [16]. This study is fundamentally grounded in "Belnap's four-valued logic" [17] and Smarandache's "Four Numerical valued neutrosophic logic" [18]. It introduces quadripartitioned neutrosophic competition graphs by incorporating the concepts of Quadripartitioned Neutrosophic Set (QNS) and competition graphs. In [10], researchers established bipolar quadripartitioned single-valued neutrosophic graphs. Recently, there has been significant attention given to the quadripartitioned neutrosophic domain, with authors further developing concepts related to graph properties [10–12].

Some operations, regular and irregular neutrosophic graphs and significant real-life applications are discussed in [13]. A lot of works have been done on competition graphs in various domains. In that one can assume that the nodes and arcs of the graphs are defined perfectly, but in reality, one can observe that sometimes the vertices and edges of graphs are not defined precisely. Algorithms are constructed in this domain to model the real-life application [14]. For instance, in predator-prey model, specious and prey are assumed as fuzzy sets and the preys are demonstrated as a fuzzy graphs [1]. Certain bipolar neutrosophic competition graphs are studied in [19] with characterizations of bipolar neutrosophic out-neighborhoods, bipolar neutrosophic in-neighborhoods, bipolar neutrosophic open neighborhood graphs, bipolar neutrosophic closed neighborhood graphs, bipolar neutrosophic p-competition graphs. The quadripartitioned neutrosophic soft graphs for treating neutrosophic soft information by employing the theory of quadripartitioned neutrosophic soft sets with graphs. Operations like strong product, Cartesian product, cross product and lexicographic product, of quadripartitioned neutrosophic soft graphs are established in [11]. In [2], the matrix representation and generalised neutrosophic competition graph are examined. Additionally, with practical applications, the minimal graph and competition number corresponding to the generalised neutrosophic competition graph are defined. Based on the above motivations, to deal with all the competitions of world, quadripartitioned bipolar neutrosophic competition graphs are introduced in this paper. Predicting a suitable job by the applicant and selecting a suitable applicant is the common issue. Based on this motivation, we established the job competition model in quadripartitioned bipolar neutrosophic competition graphs to represent competition between the applicants along with algorithm.

In recent years, neutrosophic theories have been widely explored for their ability to address uncertainty and imprecision in real-world problems. Chakraborty et al. (2020) introduced the concept of cylindrical neutrosophic single-valued numbers, applying them to networking problems, multi-criterion group decision-making, and graph theory, showcasing their versatility in complex systems [20]. Similarly, Chakraborty, Mondal, and Broumi (2019) developed a de-neutrosophication technique for pentagonal neutrosophic numbers, demonstrating its utility in solving minimal spanning tree problems [21]. These advancements underline the growing relevance of neutrosophic models in various domains, motivating further exploration of their applications. Building on this foundation, we propose the Quadripartitioned Bipolar Neutrosophic Competition Graph (QBNCG), which enhances existing models by dividing the indeterminacy component into contradiction and ignorance, thereby enabling more precise modeling of competitive scenarios such as job applicant competition. More concepts related to our study in different fields have been studied in [22–26]

To the best of the authors' knowledge, no research has been done on the notions of quadripartioned neutrosophic bipolar competition graphs with the application, as a result of the above-mentioned works.

The significant contributions in this article are provided as follows:

(1) Discussion is made regarding quadripartitioned bipolar neutrosophic competition graphs. The application in quadripartitioned bipolar neutrosophic field is covered for the first time in this work of literature.
(2) The operations like a Cartesian product and direct product of quadripartitioned neutrosophic bipolar competition graphs with their properties are discussed.
(3) The proposed work is illustrated with an application of quadripartitioned neutrosophic bipolar competition graphs in selection of designation for applicant.

The main advantages for the proposed results are listed as follows:

(1) In many domains, including algebra, geometry, topology, and optimization, the stated results in the quadripartitioned neutrosophic field are utilised as a significant tool for conceiving and solving combinatorial and decision-making problems.
(2) The proposed graph is the generalization of neutrosophic competition graphs and it will generalize the existing works in the literature (see [19, 27]).
(3) In this paper, modelling of job competition problem is discussed. The positive membership function denotes the applicant's eligibility and the negative membership function represents the ineligible percentage of applicants.
(4) Accurate finding of suitable jobs by the applicants can be done by employing the quadripartitioned bipolar neutrosophic competition graphs.
(5) One can apply the proposed work in various quadripartitioned neutrosophic fields.

The following describes the article: Preliminaries Section 2 provides a few necessary definitions. In Section 3, the operations and quadripartioned neutrosophic bipolar competition graphs are introduced. In Section 4, applications are described for the developed results.

2. Preliminaries

Definition 2.1. [4] A neutrosophic set Ψ in Λ is defined by three functions: the truth-membership function (TMF) $\alpha_\Psi(\varrho)$, the indeterminacy-membership function (IMF) $\omega_\Psi(\varrho)$, and the falsity-membership function (FMF) $\delta_\Psi(\varrho)$.

For every point ϱ in Λ, $\alpha_\Psi(\varrho)$, $\omega_\Psi(\varrho)$, and $\delta_\Psi(\varrho)$ fall within the range of [0,1].

$\alpha_\Psi(\varrho), \omega_\Psi(\varrho), \delta_\Psi(\varrho) \in [0,1]$. Also

$$\Psi = \{\varrho, \alpha_\Psi(\varrho), \omega_\Psi(\varrho), \delta_\Psi(\varrho)\} \text{ and } 0 \leq \alpha_\Psi(\varrho) + \omega_\Psi(\varrho) + \delta_\Psi(\varrho) \leq 3.$$

Definition 2.2. [4] A neutrosophic graph is represent as a pair $\mathbb{G}^* = (\mathbb{Y}, \mathbb{D})$ where
(i) $\mathbb{Y} = \{y_1, y_2, .., y_n\}$ such that $\alpha_\Psi : \mathbb{Y} \to [0,1]$, $\omega_\Psi : \mathbb{Y} \to [0,1]$ and $\delta_\Psi : \mathbb{Y} \to [0,1]$ denote the degrees of TMF, IMF and FMF

$$0 \leq \alpha_\Psi(y) + \omega_\Psi(y) + \delta_\Psi(y) \leq 3,\ \forall y \in \mathbb{Y}.$$

(ii) $\mathbb{D} \subseteq \mathbb{Y} \times \mathbb{Y}$ where $\alpha_\mathbb{D} : \mathbb{D} \to [0,1]$, $\omega_\mathbb{D} : \mathbb{D} \to [0,1]$ and $\delta_\mathbb{D} : \mathbb{D} \to [0,1]$ are such that

$$\alpha_\Upsilon(pq) \leq \alpha_\Psi(p) \wedge \alpha_\Psi(q),$$
$$\omega_\Upsilon(pq) \leq \omega_\Psi(p) \wedge \omega_\Psi(q),$$
$$\delta_\Upsilon(pq) \leq \delta_\Psi(p) \vee \delta_\Psi(q),$$
$$\text{and } 0 \leq \alpha_\Upsilon(pq) + \omega_\Upsilon(pq) + \delta_\Upsilon(pq) \leq 3,\ \forall pq \in \mathbb{D}.$$

See article [16] for further information on the following definitions and outcomes.

Definition 2.3. [16] Let Λ be the universal set. A Quadripartitioned Neutrosophic Set (QNS) Ψ on Λ is represented as

$$\Psi = \{\langle p, \alpha_\Psi(p), \beta_\Psi(p), \gamma_\Psi(p), \delta_\Psi(p) : p \in \Lambda\rangle\}$$

where $\alpha_\Psi, \beta_\Psi, \gamma_\Psi, \delta_\Psi : \varrho \to [0,1]$ and $0 \leq \alpha_\Psi(p) + \beta_\Psi(p) + \gamma_\Psi(p) + \delta_\Psi(p) \leq 4$. Here, $\alpha_\Psi(p)$-Truth Membership Function, $\beta_\Psi(p)$-Contradiction Membership Function, $\gamma_\Psi(p)$-Ignorance Membership Function, $\delta_\Psi(p)$-False Membership Function.

Definition 2.4. [15] Let Λ be the universal set. A Quadripartitioned Bipolar Neutrosophic Set (QBNS) Ψ on Λ is defined as

$$\Psi = \{\langle p, \alpha_\Psi^P(p), \beta_\Psi^P(p), \gamma_\Psi^P(p), \delta_\Psi^P(p), \alpha_\Psi^N(p), \beta_\Psi^N(p), \gamma_\Psi^N(p), \delta_\Psi^N(p) : p \in \Lambda\rangle\}$$

where $\alpha_\Psi^P, \beta_\Psi^P, \gamma_\Psi^P, \delta_\Psi^P : \Lambda \to [0,1]$, $\alpha_\Psi^N, \beta_\Psi^N, \gamma_\Psi^N, \delta_\Psi^N : \Lambda \to [-1,0]$ and $0 \leq \alpha_\Psi^P(p) + \beta_\Psi^P(p) + \gamma_\Psi^P(p) + \delta_\Psi^P(p) \leq 4$ and $-4 \leq \alpha_\Psi^N(p) + \beta_\Psi^N(p) + \gamma_\Psi^N(p) + \delta_\Psi^N(p) \leq 0$.

Definition 2.5. [10] A QBSVNG of a crisp graph $\mathbb{G} = (\mathbb{Y}, \mathbb{D})$ is said to be a pair $\mathtt{G} = (\mathcal{C}, \mathcal{H})$ with $\mathcal{C} = (\mathcal{C}^P, \mathcal{C}^N)$ and $\mathcal{H} = (\mathcal{H}^P, \mathcal{H}^N)$, where
(i) the functions $(\alpha_\mathcal{C})^P, (\beta_\mathcal{C})^P, (\gamma_\mathcal{C})^P, (\delta_\mathcal{C})^P : \mathbb{Y} \to [0,1]$ and $(\alpha_\mathcal{C})^N, (\beta_\mathcal{C})^N, (\gamma_\mathcal{C})^N, (\delta_\mathcal{C})^N : \mathbb{Y} \to [-1,0]$ represent the degree of TMF, CMF, IMF and FMF of the element $p \in \mathbb{Y}$, respectively, there is no restriction on the sum $0 \leq (\alpha_\mathcal{C})^P(p) + (\beta_\mathcal{C})^P(p) + (\gamma_\mathcal{C})^P(p) + (\delta_\mathcal{C})^P(p) \leq 4$, $-4 \leq (\alpha_\mathcal{C})^N(p) + (\beta_\mathcal{C})^N(p) + (\gamma_\mathcal{C})^N(p) + (\delta_\mathcal{C})^N(p) \leq 0$ for each $p \in \mathbb{Y}$,
(ii) the functions $(\alpha_\mathcal{H})^P, (\beta_\mathcal{H})^P, (\gamma_\mathcal{H})^P, (\delta_\mathcal{H})^P : \mathbb{D} \subseteq \mathbb{Y} \times \mathbb{Y} \to [0,1]$ and

$(\alpha_{\mathcal{H}})^N, (\beta_{\mathcal{H}})^N, (\gamma_{\mathcal{H}})^N, (\delta_{\mathcal{H}})^N : \mathbb{D} \subseteq \mathbb{Y} \times \mathbb{Y} \to [-1, 0]$ are defined as:

$$(\alpha_{\mathcal{H}})^P(pq) \leq \{(\alpha_{\mathcal{C}})^P(p) \wedge (\alpha_{\mathcal{C}})^P(q)\}$$
$$(\beta_{\mathcal{H}})^P(pq) \leq \{(\beta_{\mathcal{C}})^P(p) \wedge (\beta_{\mathcal{C}})^P(q)\}$$
$$(\gamma_{\mathcal{H}})^P(pq) \leq \{(\gamma_{\mathcal{C}})^P(p) \vee (\gamma_{\mathcal{C}})^P(q)\}$$
$$(\delta_{\mathcal{H}})^P(pq) \leq \{(\delta_{\mathcal{C}})^P(p) \vee (\delta_{\mathcal{C}})^P(q)\}, \quad \forall \quad pq \in \mathbb{D},$$

$$(\alpha_{\mathcal{H}})^N(pq) \geq \{(\alpha_{\mathcal{C}})^N(p) \vee (\alpha_{\mathcal{C}})^N(q)\}$$
$$(\beta_{\mathcal{H}})^N(pq) \geq \{(\beta_{\mathcal{C}})^N(p) \vee (\beta_{\mathcal{C}})^N(q)\}$$
$$(\gamma_{\mathcal{H}})^N(pq) \geq \{(\gamma_{\mathcal{C}})^N(p) \wedge (\gamma_{\mathcal{C}})^N(q)\}$$
$$(\delta_{\mathcal{H}})^N(pq) \geq \{(\delta_{\mathcal{C}})^N(p) \wedge (\delta_{\mathcal{C}})^N(q)\}, \quad \forall \quad pq \in \mathbb{D}.$$

Here $\mathcal{C}$ is the QBN vertex set of $\mathtt{G}$ and $\mathcal{H}$ is the QBN edge set of $\mathtt{G}$.

3. Quadripartitioned Bipolar Neutrosophic Competition Graphs (QBNCG)

Definition 3.1. A QBND on a non-empty set ϱ is a pair $\mathtt{G} = (\Psi, \overrightarrow{\Upsilon})$, where Ψ is a QBNS on ϱ and Υ is a QBN relation on Λ, such that

$$\alpha^p_\Upsilon(\overrightarrow{p,q}) \leq \alpha^p_\Psi(p) \wedge \alpha^p_\Psi(q), \quad \beta^p_\Upsilon(\overrightarrow{p,q}) \leq \beta^p_\Psi(p) \wedge \beta^p_\Psi(q),$$
$$\gamma^p_\Upsilon(\overrightarrow{p,q}) \leq \gamma^p_\Psi(p) \vee \gamma^p_\Psi(q), \quad \delta^p_\Upsilon(\overrightarrow{p,q}) \leq \delta^p_\Psi(p) \vee \delta^p_\Psi(q),$$
$$\alpha^n_\Upsilon(\overrightarrow{p,q}) \geq \alpha^n_\Psi(p) \vee \alpha^n_\Psi(q), \quad \beta^n_\Upsilon(\overrightarrow{p,q}) \geq \beta^n_\Psi(p) \vee \beta^n_\Psi(q),$$
$$\gamma^n_\Upsilon(\overrightarrow{p,q}) \geq \gamma^n_\Psi(p) \wedge \gamma^n_\Psi(q), \quad \delta^n_\Upsilon(\overrightarrow{p,q}) \geq \delta^n_\Psi(p) \wedge \delta^n_\Psi(q), \forall \quad p, q \in \Lambda$$

Example 3.2. Consider QBND $\mathtt{G} = (\Psi, \overrightarrow{\Upsilon})$ on $\Lambda = (p, q, r)$ as shown in figure 2.

Definition 3.3. Let $\overrightarrow{\mathtt{G}}$ be a QBND the QBN out neighbourhood of a vertex q is a QBNS $\mathtt{Q}^+(p) = (\Lambda^+_a, \alpha^{(p)^+}_p, \beta^{(p)^+}_p, \gamma^{(p)^+}_p, \delta^{(p)^+}_p, \alpha^{(n)^+}_p, \beta^{(n)^+}_p, \gamma^{(n)^+}_p, \delta^{(n)^+}_p)$, where
$\Lambda^+_p = \{q | \Upsilon^p_1\overrightarrow{(p,q)} > 0, \Upsilon^p_2\overrightarrow{(p,q)} > 0, \Upsilon^p_3\overrightarrow{(p,q)} > 0, \Upsilon^p_4\overrightarrow{(p,q)} > 0, \Upsilon^n_1\overrightarrow{(p,q)} < 0, \Upsilon^n_2\overrightarrow{(p,q)} < 0, \Upsilon^n_3\overrightarrow{(p,q)} < 0, \Upsilon^n_4\overrightarrow{(p,q)} < 0, \}$
such that $\alpha^{(p)^+}_p : \Lambda^+_p \to [0,1]$ represent by $\alpha^{(p)^+}_p(q) = \Upsilon^p_1\overrightarrow{(p,q)}$, $\beta^{(p)^+}_p : \Lambda^+_p \to [0,1]$ represent by $\beta^{(p)^+}_p(q) = \Upsilon^p_2\overrightarrow{(p,q)}$, $\gamma^{(p)^+}_p : \varrho^+_p \to [0,1]$ represent by $\gamma^{(p)^+}_p(q) = \Upsilon^p_3\overrightarrow{(p,q)}$, $\delta^{(p)^+}_p : \Lambda^+_p \to [0,1]$ represent by $\delta^{(p)^+}_p(q) = \Upsilon^p_4\overrightarrow{(p,q)}$, $\alpha^{(n)^+}_p : \Lambda^+_p \to [-1,0]$ represent by $\alpha^{(n)^+}_p(q) = \Upsilon^n_1\overrightarrow{(p,q)}$, $\beta^{(n)^+}_p : \Lambda^+_p \to [-1,0]$ represent by $\beta^{(n)^+}_p(q) = \Upsilon^n_2\overrightarrow{(p,q)}$, $\gamma^{(n)^+}_p : \Lambda^+_p \to [-1,0]$ represent by $\gamma^{(n)^+}_p(q) = \Upsilon^n_3\overrightarrow{(p,q)}$, $\delta^{(n)^+}_p : \Lambda^+_p \to [-1,0]$ represent by $\delta^{(n)^+}_p(q) = \Upsilon^n_4\overrightarrow{(p,q)}$,

Definition 3.4. Let $\overrightarrow{\mathtt{G}}$ be a QBND the QBN in-neighbourhood of a vertex p is a QBNS $\mathtt{Q}^-(p) = (\Lambda^-_q, \alpha^{(p)^-}_p, \beta^{(p)^-}_p, \gamma^{(p)^-}_p, \delta^{(p)^-}_p, \alpha^{(n)^-}_p, \beta^{(n)^-}_p, \gamma^{(n)^-}_p, \delta^{(n)^-}_p)$, where

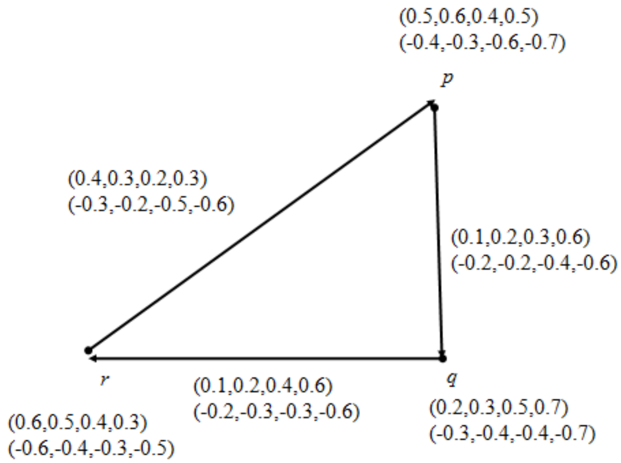


FIGURE 2. Quadripartitioned Bipolar Neutrosophic Digraph (QBND)

$\Lambda_p^- = \{p|\Upsilon_1^p\overrightarrow{(p,q)} > 0, \Upsilon_2^p\overrightarrow{(p,q)} > 0, \Upsilon_3^p\overrightarrow{(p,q)} > 0, \Upsilon_4^p\overrightarrow{(p,q)} > 0, \Upsilon_1^n\overrightarrow{(p,q)} < 0, \Upsilon_2^n\overrightarrow{(p,q)} < 0, \Upsilon_3^n\overrightarrow{(p,q)} < 0, \Upsilon_4^n\overrightarrow{(p,q)} < 0, \}$
such that $\alpha_p^{(p)^-} : \Lambda_p^- \to [0,1]$ represent by $\alpha_p^{(p)^-}(q) = \Upsilon_1^p\overrightarrow{(p,q)}$, $\beta_p^{(p)^-} : \Lambda_q^- \to [0,1]$ represent by $\beta_p^{(p)^-}(q) = \Upsilon_2^p\overrightarrow{(p,q)}$, $\gamma_p^{(p)^-} : \Lambda_p^- \to [0,1]$ represent by $\gamma_p^{(p)^-}(q) = \Upsilon_3^p\overrightarrow{(p,q)}$, $\delta_p^{(p)^-} : \Lambda_p^- \to [0,1]$ represent by $\delta_p^{(p)^-}(q) = \Upsilon_4^p\overrightarrow{(p,q)}$, $\alpha_p^{(n)^-} : \Lambda_p^- \to [-1,0]$ represent by $\alpha_p^{(n)^-}(q) = \Upsilon_1^n\overrightarrow{(p,q)}$, $\beta_p^{(n)^-} : \Lambda_p^- \to [-1,0]$ represent by $\beta_p^{(n)^-}(q) = \Upsilon_2^n\overrightarrow{(p,q)}$, $\gamma_p^{(n)^-} : \Lambda_q^- \to [-1,0]$ represent by $\gamma_p^{(n)^-}(q) = \Upsilon_3^n\overrightarrow{(p,q)}$, $\delta_p^{(n)^-} : \Lambda_p^- \to [-1,0]$ represent by $\delta_p^{(n)^-}(q) = \Upsilon_4^n\overrightarrow{(p,q)}$,

Example 3.5. Let an QBND $\mathtt{G} = (\Psi, \overrightarrow{\Upsilon})$ on $\Lambda = (p,q,r)$ as explained in figure 2. Tables 1 and 2 show the QBN in and out neighbourhoods, respectively.

QBN Out-neighbourhoods

Table 1.

ϱ	$\mathtt{Q}^+(\varrho)$
p	$\{q(0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6), r(0.4, 0.3, 0.2, 0.3, -0.3, -0.2, -0.5, -0.6)\}$
q	$\emptyset$
r	$\{q(0.1, 0.2, 0.4, 0.6, -0.2, -0.3, -0.3, -0.6)\}$

Table 2. QBN in-neighbourhoods

ϱ	$\mathtt{Q}^-(\varrho)$
p	$\emptyset$
q	$\{p(0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6), r(0.1, 0.2, 0.4, 0.6, -0.2, -0.3, -0.3, -0.6)\}$
r	$\{p(0.4, 0.3, 0.2, 0.3, -0.3, -0.2, -0.5, -0.6)\}$

Definition 3.6. The height of a QBNS $h(\mathtt{Q}) = (\Lambda, \alpha^p, \beta^p, \gamma^p, \delta^p, \alpha^n, \beta^n, \gamma^n, \delta^n,)$ and

$$h(\mathtt{Q}) = (h_1(\mathtt{Q}), h_2(\mathtt{Q}), h_3(\mathtt{Q}), h_4(\mathtt{Q}), h_5(\mathtt{Q}), h_6(\mathtt{Q}), h_7(\mathtt{Q}), h_8(\mathtt{Q}))$$

$=(\sup_{r\in\mathbb{X}} \alpha^p_{\mathtt{Q}(r)}, \sup_{r\in\mathbb{X}} \beta^p_{\mathtt{Q}(r)}, \inf_{r\in\mathbb{X}} \gamma^p_{\mathtt{Q}(r)}, \inf_{r\in\mathbb{X}} \delta^p_{\mathtt{Q}(r)}, \sup_{r\in\mathbb{X}} \alpha^p_{\mathtt{Q}(r)}, \sup_{r\in\mathbb{X}} \beta^p_{\mathtt{Q}(r)} \inf_{r\in\mathbb{X}} \gamma^p_{\mathtt{Q}(r)}, \inf_{r\in\mathbb{X}} \delta^p_{\mathtt{Q}(r)})$.

Example 3.7. The height of a QBNS

$\Psi = \{(p, (0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6)), (q, (0.4, 0.3, 0.4, 0.7, -0.1, -0.2, -0.1, -0.6)),$

$(r, (0.2, 0.2, 0.4, 0.5, -0.2, -0.3, -0.3, -0.6))\}$ in $\varrho = \{p, q, r\}$ is $h(\Psi) = \{(0.4, 0.3, 0.3, 0.5, 0.4, 0.3, 0.3, 0.5)\}$

Definition 3.8. A QBNCG of a QBNG $\overrightarrow{\mathtt{G}} = (\Psi, \Upsilon)$ is an undirected graph QBNG $\beta(\overrightarrow{\mathtt{G}}) = (\Psi, \mathtt{Q})$ which has the same vertex set as in $\overrightarrow{\mathtt{G}}$ and there is an edge between two vertices p and q if and only if $\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)$ is non-empty. The positive TMF, CMF, IMF, FMF and negative TMF, CMF, IMF, FMF value of the edge (p, q) are defined as,

$$\alpha^p_{\mathtt{Q}}(p,q) = (\alpha^p_{\Psi}(p) \wedge \alpha^p_{\Psi}(q))h_1(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)), \quad \beta^p_{\mathtt{Q}}(p,q) = (\beta^p_{\Psi}(p) \wedge \beta^p_{\Psi}(q))h_2(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)),$$

$$\gamma^p_{\mathtt{Q}}(p,q) = (\gamma^p_{\Psi}(p) \vee \gamma^p_{\Psi}(q))h_3(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)), \quad \delta^p_{\mathtt{Q}}(p,q) = (\delta^p_{\Psi}(p) \vee \beta^p_{\Psi}(q))h_4(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)),$$

$$\alpha^n_{\mathtt{Q}}(p,q) = (\alpha^n_{\Psi}(p) \vee \alpha^n_{\Psi}(q))h_5(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)), \quad \beta^n_{\mathtt{Q}}(p,q) = (\beta^n_{\Psi}(p) \vee \beta^n_{\Psi}(q))h_6(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)),$$

$$\gamma^n_{\mathtt{Q}}(p,q) = (\gamma^n_{\Psi}(p) \wedge \gamma^n_{\Psi}(q))h_7(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)), \quad \delta^n_{\mathtt{Q}}(p,q) = (\delta^n_{\Psi}(p) \wedge \beta^n_{\Psi}(q))h_8(\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)), \quad \forall p, q \in \Lambda$$

Example 3.9. Let $\mathtt{G} = (\Psi, \Upsilon)$ is a QBND on $\Lambda = (p, q, r)$, such that $\Psi = \{(p, 0.5, 0.6, 0.4, 0.5, -0.4, -0.3, -0.6, -0.7),$
$(q, 0.2, 0.3, 0.5, 0.7, -0.3, -0.4, -0.4, -0.7), (r, 0.6, 0.5, 0.4, 0.3, -0.6, -0.4, -0.3, -0.5)\}$, and $\Upsilon = \{(\overrightarrow{pq}, 0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6), (\overrightarrow{pr}, 0.4, 0.3, 0.2, 0.2, -0.3, -0.2, -0.5, -0.6),$
$(\overrightarrow{rq}, 0.1, 0.2, 0.4, 0.6, -0.2, -0.3, -0.3, -0.6)\}$ as shown in figure 3. By direct calculations Tables 3 and 4 show the out and in-neighborhoods of QBN, respectively.

Table 3. QBN Out-neighbourhoods

ϱ	$\mathtt{Q}^+(\varrho)$
p	$\{q(0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6), r(0.4, 0.3, 0.2, 0.3, -0.3, -0.2, -0.5, -0.6)\}$
q	$\emptyset$
r	$\{q(0.1, 0.2, 0.4, 0.6, -0.2, -0.3, -0.3, -0.6)\}$

Table 4. QBN in-neighbourhoods

ϱ	$\mathtt{Q}^-(\varrho)$
p	$\emptyset$
q	$\{p(0.1, 0.2, 0.3, 0.6, -0.2, -0.2, -0.4, -0.6), r(0.1, 0.2, 0.4, 0.6, -0.2, -0.3, -0.3, -0.6)\}$
r	$\{p(0.4, 0.3, 0.2, 0.3, -0.3, -0.2, -0.5, -0.6)\}$

The QBNCG of Figure 3 is shown in Figure 4

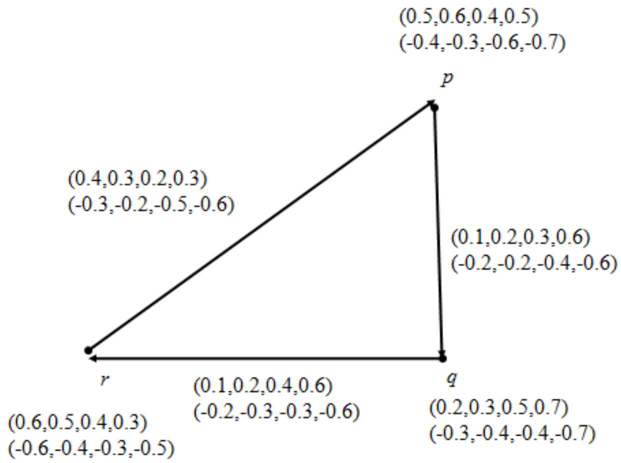


FIGURE 3. QBND

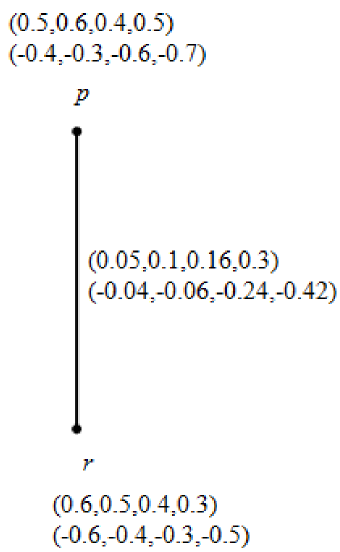


FIGURE 4. QBNCG

Definition 3.10. A QBN $\mathtt{G} = (\Psi, \Upsilon)$, where
$\Psi = (\Psi_1^p, \Psi_2^p, \Psi_3^p, \Psi_4^p, \Psi_1^n, \Psi_2^n, \Psi_3^n, \Psi_4^n)$, and $\Upsilon = (\Upsilon_1^p, \Upsilon_2^p, \Upsilon_3^p, \Upsilon_4^p, \Upsilon_1^n, \Upsilon_2^n, \Upsilon_3^n, \Upsilon_4^n)$ then, an edge (p, q),$p, q \in \Lambda$ is called independent strong if

$$\frac{1}{2}[\Psi_1^p(p) \wedge \Psi_1^p(q)] < \Upsilon_1^p(p,q), \quad \frac{1}{2}[\Psi_1^n(p) \vee \Psi_1^n(q)] > \Upsilon_1^n(p,q),$$
$$\frac{1}{2}[\Psi_2^p(p) \wedge \Psi_2^p(q)] < \Upsilon_2^p(p,q), \quad \frac{1}{2}[\Psi_2^n(p) \vee \Psi_2^n(q)] > \Upsilon_1^n(p,q),$$
$$\frac{1}{2}[\Psi_3^p(p) \vee \Psi_3^p(q)] > \Upsilon_3^p(p,q), \quad \frac{1}{2}[\Psi_3^n(p) \wedge \Psi_3^n(q)] < \Upsilon_3^n(p,q),$$
$$\frac{1}{2}[\Psi_4^p(p) \vee \Psi_4^p(q)] > \Upsilon_4^p(p,q), \quad \frac{1}{2}[\Psi_4^n(p) \wedge \Psi_4^n(q)] < \Upsilon_4^n(p,q).$$

In otherwords, one can say it is weak.

Theorem 3.11. *Let* Λ *be a QBND. If the intersection* $\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)$ *contains only one element of* $\overrightarrow{\mathtt{G}}$*, then the edge* (p,q) *of* $\mathbb{C}(\overrightarrow{\mathtt{G}})$ *is independently strong if and only if*

$$|[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\alpha^p} > 0.50 \quad |[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\alpha^n} < 0.50$$
$$|[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\beta^p} > 0.50 \quad |[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\beta^n} < 0.50$$
$$|[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\gamma^p} < 0.50 \quad |[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\gamma^n} < 0.50$$
$$|[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\delta^p} < 0.50 \quad |[\mathtt{Q}^+(p) \cap \mathtt{Q}^+(q)]|_{\delta^n} < 0.50$$

Proof.
Consider $\overrightarrow{\mathcal{X}}$ is a QBND. let $\varrho, \zeta \in \Lambda$.Suppose $\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta) = (p, p^p, q^p, r^p, s^p, p^n, q^n, r^n, s^n)$, where,$(p^p, q^p, r^p, s^p, p^n, q^n, r^n, s^n)$ are the positive TMF, CMF, IMF and FMF and negative TMF, CMF, IMF and FMF values of either the lines (ϱ, p) or the lines (ζ, p), respectively. Here,

$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\alpha^p} = p^p = h_1(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\beta^p} = q^p = h_2(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\gamma^p} = r^p = h_3(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\delta^p} = s^p = h_4(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\alpha^n} = p^n = h_5(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\beta^n} = q^n = h_6(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\gamma^n} = r^n = h_7(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$
$$|\ [\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\ |_{\delta^n} = s^n = h_8(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta))$$

Then,

$$\Upsilon_1^p(\varrho,\zeta) = p^p \times [\Psi_1^p(\varrho) \wedge \Psi_1^p(\zeta)], \quad \Upsilon_1^n(\varrho,\zeta) = p^n \times [\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta)],$$
$$\Upsilon_2^p(\varrho,\zeta) = q^p \times [\Psi_2^p(\varrho) \wedge \Psi_2^p(\zeta)], \quad \Upsilon_2^n(\varrho,\zeta) = q^n \times [\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta)],$$
$$\Upsilon_3^p(\varrho,\zeta) = r^p \times [\Psi_3^p(\varrho) \vee \Psi_3^p(\zeta)], \quad \Upsilon_3^n(\varrho,\zeta) = r^n \times [\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta)],$$
$$\Upsilon_4^p(\varrho,\zeta) = s^p \times [\Psi_4^p(\varrho) \vee \Psi_4^p(\zeta)], \quad \Upsilon_4^n(\varrho,\zeta) = s^n \times [\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta)].$$

Hence, the line (ϱ,ζ) in $C(\overrightarrow{\mathtt{G}})$ is independent strong if and only if $p^p > 0.50, q^p > 0.50, r^p < 0.50, s^p < 0.50$ $p^n < 0.50, q^n < 0.50, r^n > 0.50, s^n > 0.50$ Therefore, the line (ϱ,ζ) of $C(\overrightarrow{G})$ is independent strong if and only if

$$|\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\alpha^p} > 0.50 \quad |\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\alpha^n} < 0.50$$
$$|\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\beta^p} > 0.50 \quad |\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\beta^n} < 0.50$$
$$|\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\gamma^p} < 0.50 \quad |\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\gamma^n} < 0.50$$
$$|\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\delta^p} < 0.50 \quad |\,[\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)]\,|_{\delta^n} < 0.50$$

We visualize the theorem with an instance as verified in as shown in Figure 5 □

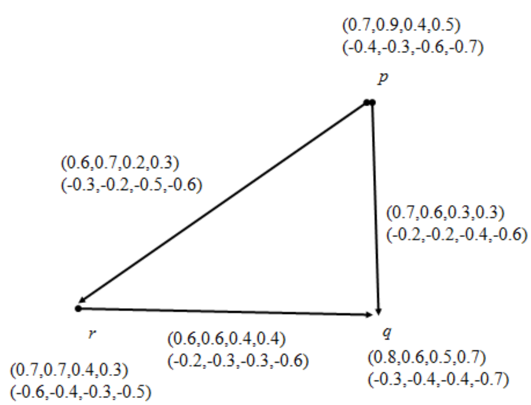
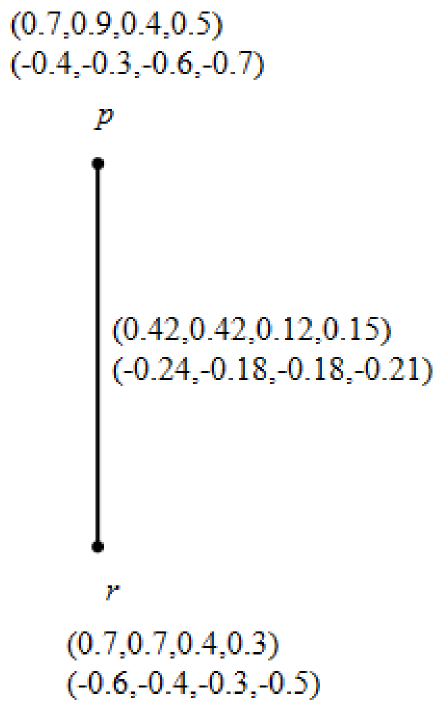


Figure 5. (5a)



(b)

FIGURE 6. (5b)
QBNCG (1) QBNCD (2)

Theorem 3.12. *If all the lines of QBND $\overrightarrow{\mathtt{G}}$ are independent strong, then*

$$\begin{array}{ll}
\frac{\Upsilon_1^p(\varrho,\zeta)}{(\Psi_1^p(\varrho)\wedge\Psi_1^p(\zeta))^2} > 0.50 & \frac{\Upsilon_1^n(\varrho,\zeta)}{(\Psi_1^n(\varrho)\vee\Psi_1^n(\zeta))^2} < 0.50 \\
\frac{\Upsilon_2^p(\varrho,\zeta)}{(\Psi_2^p(\varrho)\wedge\Psi_2^p(\zeta))^2} > 0.50 & \frac{\Upsilon_2^n(\varrho,\zeta)}{(\Psi_2^n(\varrho)\vee\Psi_2^n(\zeta))^2} < 0.50 \\
\frac{\Upsilon_3^p(\varrho,\zeta)}{(\Psi_3^p(\varrho)\vee\Psi_3^p(\zeta))^2} < 0.50 & \frac{\Upsilon_3^n(\varrho,\zeta)}{(\Psi_3^n(\varrho)\wedge\Psi_3^n(\zeta))^2} < 0.50 \\
\frac{\Upsilon_4^p(\varrho,\zeta)}{(\Psi_4^p(\varrho)\vee\Psi_4^p(\zeta))^2} < 0.50 & \frac{\Upsilon_4^n(\varrho,\zeta)}{(\Psi_4^n(\varrho)\wedge\Psi_4^n(\zeta))^2} < 0.50
\end{array}$$

for all edges (ϱ,ζ) *in* $C(\overrightarrow{\mathtt{G}})$.

Proof. Assume that QBND $\overrightarrow{\mathtt{G}}$ has independent, strong edges all around it.

$$\frac{1}{2}[\Psi_1^p(\varrho)\wedge\Psi_1^p(\zeta)] < \Upsilon_1^p(\varrho,\zeta), \quad \frac{1}{2}[\Psi_1^n(\varrho)\vee\Psi_1^n(\zeta)] > \Upsilon_1^n(\varrho,\zeta),$$
$$\frac{1}{2}[\Psi_2^p(\varrho)\wedge\Psi_2^p(\zeta)] < \Upsilon_2^p(\varrho,\zeta), \quad \frac{1}{2}[\Psi_2^n(\varrho)\vee\Psi_2^n(\zeta)] > \Upsilon_2^n(\varrho,\zeta),$$
$$\frac{1}{2}[\Psi_3^p(\varrho)\vee\Psi_3^p(\zeta)] > \Upsilon_3^p(\varrho,\zeta), \quad \frac{1}{2}[\Psi_3^n(\varrho)\wedge\Psi_3^n(\zeta)] < \Upsilon_3^n(\varrho,\zeta),$$
$$\frac{1}{2}[\Psi_4^p(\varrho)\vee\Psi_4^p(\zeta)] > \Upsilon_4^p(\varrho,\zeta), \quad \frac{1}{2}[\Psi_4^n(\varrho)\wedge\Psi_4^n(\zeta)] < \Upsilon_4^n(\varrho,\zeta).$$

for every lines (ϱ,ζ) in $\overrightarrow{\mathtt{G}}$. Let the corresponding QBNCG be $C(\overrightarrow{\mathtt{G}})$.

Then their arises two cases:

Case (1):

when $\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)=\emptyset$ for all $\varrho,\zeta\in\Lambda$. Then there exists no edge in $C(\overrightarrow{\mathtt{G}})$ between ϱ and ζ. Thus, we have nothing to claim this case.

Case (2):

when $\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)\neq\emptyset$. Let $\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta) = (a_1,m_1^p,r_1^p,p_1^p,q_1^p,m_1^n,r_1^n,p_1^n,q_1^n)$, $(a_2,m_2^p,r_2^p,p_2^p,q_2^p,m_2^n,r_2^n,p_2^n,q_2^n)$, $(a_3,m_3^p,r_3^p,p_3^p,q_3^p,m_3^n,r_3^n,p_3^n,q_3^n)$,...., $(a_l,m_l^p,r_l^p,p_l^p,q_l^p,m_l^n,r_l^n,p_l^n,q_l^n)$, where $(m_i^p,r_i^p,p_i^p,q_i^p,m_i^n,r_i^n,p_i^n,q_i^n)$ are the positive and negative TMF, CMF, IMF and FMF membership values of either $(\overrightarrow{\varrho,a_i})$ or $(\overrightarrow{\zeta,a_i})$ for $i=1,2,...,l$, respectively, Thus,

$$m_i^p = [\Upsilon_1^p(\overrightarrow{\varrho,a_i})\wedge\Upsilon_1^p(\overrightarrow{\zeta,a_i})], \quad m_i^n = [\Upsilon_1^n(\overrightarrow{\varrho,a_i})\vee\Upsilon_1^n(\overrightarrow{\zeta,a_i})]$$
$$r_i^p = [\Upsilon_2^p(\overrightarrow{\varrho,a_i})\wedge\Upsilon_2^p(\overrightarrow{\zeta,a_i})], \quad r_i^n = [\Upsilon_2^n(\overrightarrow{\varrho,a_i})\vee\Upsilon_2^n(\overrightarrow{\zeta,a_i})]$$
$$p_i^p = [\Upsilon_3^p(\overrightarrow{\varrho,a_i})\vee\Upsilon_3^p(\overrightarrow{\zeta,a_i})], \quad p_i^n = [\Upsilon_3^n(\overrightarrow{\varrho,a_i})\wedge\Upsilon_3^n(\overrightarrow{\zeta,a_i})]$$
$$q_i^p = [\Upsilon_4^p(\overrightarrow{\varrho,a_i})\vee\Upsilon_4^p(\overrightarrow{\zeta,a_i})], \quad q_i^n = [\Upsilon_4^n(\overrightarrow{\varrho,a_i})\wedge\Upsilon_4^n(\overrightarrow{\zeta,a_i})]$$

for all $i=1,2,...,l$.

Consider,

$$h_1(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \max\{m_i^p, \quad i=1,2,..,l\} = m_{\max}^p$$
$$h_2(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \max\{r_i^p, \quad i=1,2,..,l\} = r_{\max}^p$$
$$h_3(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{r_i^p, \quad i=1,2,..,l\} = r_{\min}^p$$
$$h_4(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{q_i^p, \quad i=1,2,..,l\} = q_{\min}^p$$
$$h_5(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{m_i^n, \quad i=1,2,..,l\} = m_{\max}^n$$
$$h_6(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{r_i^n, \quad i=1,2,..,l\} = r_{\max}^n$$
$$h_7(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{r_i^n, \quad i=1,2,..,l\} = r_{\min}^n$$
$$h_8(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)) = \min\{q_i^n, \quad i=1,2,..,l\} = q_{\min}^n$$

Obviously, $m^p_{\max} > \Upsilon^p_1(\overrightarrow{\varrho,\zeta})$, $r^p_{\max} > \Upsilon^p_2(\overrightarrow{\varrho,\zeta})$, $p^p_{\min} < \Upsilon^p_3(\overrightarrow{\varrho,\zeta})$, $q^p_{\min} < \Upsilon^p_4(\overrightarrow{\varrho,\zeta})$, $m^n_{\max} < \Upsilon^n_1(\overrightarrow{\varrho,\zeta})$, $r^n_{\max} < \Upsilon^n_2(\overrightarrow{\varrho,\zeta})$, $p^p_{\min} < \Upsilon^n_3(\overrightarrow{\varrho,\zeta})$, $q^p_{\min} < \Upsilon^p_4(\overrightarrow{\varrho,\zeta})$, for all edges $(\overrightarrow{\varrho,\zeta})$ shows that,

$$\begin{aligned}
&\frac{m^p_{\max}}{\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta)} > \frac{\Upsilon^p_1(\overrightarrow{\varrho,\zeta})}{\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta)} > 0.50, \quad \frac{m^n_{\max}}{\Psi^n_1(\varrho)\vee\Psi^n_1(\zeta)} < \frac{\Upsilon^n_1(\overrightarrow{\varrho,\zeta})}{\Psi^n_1(\varrho)\vee\Psi^n_1(\zeta)} < 0.50\\
&\frac{r^p_{\max}}{\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta)} > \frac{\Upsilon^p_2(\overrightarrow{\varrho,\zeta})}{\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta)} > 0.50, \quad \frac{r^n_{\max}}{\Psi^n_2(\varrho)\vee\Psi^n_2(\zeta)} < \frac{\Upsilon^n_2(\overrightarrow{\varrho,\zeta})}{\Psi^n_2(\varrho)\vee\Psi^n_2(\zeta)} < 0.50\\
&\frac{p^p_{\min}}{\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta)} < \frac{\Upsilon^p_3(\overrightarrow{\varrho,\zeta})}{\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta)} < 0.50, \frac{p^n_{\min}}{\Psi^n_3(\varrho)\wedge\Psi^n_3(\zeta)} < \frac{\Upsilon^n_3(\overrightarrow{\varrho,\zeta})}{\Psi^n_3(\varrho)\wedge\Psi^n_3(\zeta)} < 0.50\\
&\frac{q^p_{\min}}{\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta)} < \frac{\Upsilon^p_4(\overrightarrow{\varrho,\zeta})}{\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta)} < 0.50, \quad \frac{q^n_{\min}}{\Psi^n_4(\varrho)\wedge\Psi^n_4(\zeta)} < \frac{\Upsilon^n_4(\overrightarrow{\varrho,\zeta})}{\Psi^n_4(\varrho)\wedge\Psi^n_4(\zeta)} < 0.50
\end{aligned}$$

therefore,

$$\begin{aligned}
\Upsilon^p_1(\varrho,\zeta) &= (\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta))h_1(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon^p_1(\varrho,\zeta) = [\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta)]\times m^p_{\max},\\
\text{or} \quad &\frac{\Upsilon^p_1(\varrho,\zeta)}{(\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta))} = m^p_{\max}, \text{or} \quad \frac{\Upsilon^p_1(\varrho,\zeta)}{(\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta))^2} = \frac{m^p_{\max}}{(\Psi^p_1(\varrho)\wedge\Psi^p_1(\zeta))} > 0.50\\
\Upsilon^p_2(\varrho,\zeta) &= (\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta))h_2(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon^p_2(\varrho,\zeta) = [\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta)]\times r^p_{\max},\\
\text{or} \quad &\frac{\Upsilon^p_2(\varrho,\zeta)}{(\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta))} = r^p_{\max}, \text{or} \quad \frac{\Upsilon^p_2(\varrho,\zeta)}{(\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta))^2} = \frac{r^p_{\max}}{(\Psi^p_2(\varrho)\wedge\Psi^p_2(\zeta))} > 0.50\\
\Upsilon^p_3(\varrho,\zeta) &= (\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta))h_3(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon^p_3(\varrho,\zeta) = [\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta)]\times p^p_{\min},\\
\text{or} \quad &\frac{\Upsilon^p_3(\varrho,\zeta)}{(\Psi^p_3(\varrho)\wedge\Psi^p_3(\zeta))} = p^p_{\min}, \text{or} \quad \frac{\Upsilon^p_3(\varrho,\zeta)}{(\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta))^2} = \frac{p^p_{\min}}{(\Psi^p_3(\varrho)\vee\Psi^p_3(\zeta))} < 0.50\\
\Upsilon^p_4(\varrho,\zeta) &= (\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta))h_4(\mathtt{Q}^+(\varrho)\cap\mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon^p_4(\varrho,\zeta) = [\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta)]\times q^p_{\min},\\
\text{or} \quad &\frac{\Upsilon^p_4(\varrho,\zeta)}{(\Psi^p_4(\varrho)\wedge\Psi^p_4(\zeta))} = q^p_{\min}, \text{or} \quad \frac{\Upsilon^p_4(\varrho,\zeta)}{(\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta))^2} = \frac{q^p_{\min}}{(\Psi^p_4(\varrho)\vee\Psi^p_4(\zeta))} < 0.50
\end{aligned}$$

$$\Upsilon_1^n(\varrho,\zeta) = (\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta))h_1(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon_1^n(\varrho,\zeta) = [\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta)] \times m_{\max}^n,$$
$$\text{or} \quad \frac{\Upsilon_1^n(\varrho,\zeta)}{(\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta))} = m_{\max}^n, \text{or} \quad \frac{\Upsilon_1^n(\varrho,\zeta)}{(\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta))^2} = \frac{m_{\max}^n}{(\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta))} < 0.50$$
$$\Upsilon_2^n(\varrho,\zeta) = (\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta))h_2(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon_2^n(\varrho,\zeta) = [\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta)] \times r_{\max}^n,$$
$$\text{or} \quad \frac{\Upsilon_2^n(\varrho,\zeta)}{(\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta))} = r_{\max}^n, \text{or} \quad \frac{\Upsilon_2^n(\varrho,\zeta)}{(\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta))^2} = \frac{r_{\max}^n}{(\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta))} < 0.50$$
$$\Upsilon_3^n(\varrho,\zeta) = (\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta))h_3(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon_3^n(\varrho,\zeta) = [\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta)] \times p_{\min}^n,$$
$$\text{or} \quad \frac{\Upsilon_3^n(\varrho,\zeta)}{(\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta))} = p_{\min}^n, \text{or} \quad \frac{\Upsilon_3^n(\varrho,\zeta)}{(\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta))^2} = \frac{p_{\min}^n}{(\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta))} < 0.50$$
$$\Upsilon_4^n(\varrho,\zeta) = (\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta))h_4(\mathtt{Q}^+(\varrho) \cap \mathtt{Q}^+(\zeta)), \text{or} \quad \Upsilon_4^n(\varrho,\zeta) = [\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta)] \times q_{\min}^n,$$
$$\text{or} \quad \frac{\Upsilon_4^n(\varrho,\zeta)}{(\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta))} = q_{\min}^n, \text{or} \quad \frac{\Upsilon_4^n(\varrho,\zeta)}{(\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta))^2} = \frac{q_{\min}^n}{(\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta))} < 0.50$$

Hence,

$$\frac{\Upsilon_1^p(\varrho,\zeta)}{(\Psi_1^p(\varrho) \wedge \Psi_1^p(\zeta))^2} > 0.50 \quad \frac{\Upsilon_1^n(\varrho,\zeta)}{(\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta))^2} < 0.50$$
$$\frac{\Upsilon_2^p(\varrho,\zeta)}{(\Psi_2^p(\varrho) \wedge \Psi_2^p(\zeta))^2} > 0.50 \quad \frac{\Upsilon_2^n(\varrho,\zeta)}{(\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta))^2} < 0.50$$
$$\frac{\Upsilon_3^p(\varrho,\zeta)}{(\Psi_3^p(\varrho) \vee \Psi_3^p(\zeta))^2} < 0.50 \quad \frac{\Upsilon_3^n(\varrho,\zeta)}{(\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta))^2} < 0.50$$
$$\frac{\Upsilon_4^p(\varrho,\zeta)}{(\Psi_4^p(\varrho) \vee \Psi_4^p(\zeta))^2} < 0.50 \quad \frac{\Upsilon_4^n(\varrho,\zeta)}{(\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta))^2} < 0.50,$$

for every lines (ϱ,ζ) in $C(\overrightarrow{\mathtt{G}})$. □

Definition 3.13. The QBN open-neighborhood of a vertex ϱ of a QBNG $\mathtt{G} = (\Psi, \Upsilon)$ is QBNS $\mathtt{Q}(\varrho) = (\Lambda, \alpha_\varrho^p, \beta_\varrho^p, \gamma_\varrho^p, \delta_\varrho^p, \alpha_\varrho^n, \beta_\varrho^n, \gamma_\varrho^n, \delta_\varrho^n)$, where, $\Lambda_\varrho = (\zeta | \Upsilon_1^p(\varrho,\zeta) > 0, \Upsilon_2^p(\varrho,\zeta) > 0, \Upsilon_3^p(\varrho,\zeta) > 0, \Upsilon_4^p(\varrho,\zeta)) > 0, \Upsilon_1^n(\varrho,\zeta) < 0, \Upsilon_2^n(\varrho,\zeta) < 0, \Upsilon_3^n(\varrho,\zeta) < 0, \Upsilon_4^p(\varrho,\zeta)) < 0$, and $\alpha_\varrho^p : \Lambda_\varrho \to [0,1]$ represent by $\alpha_\varrho^p = \Upsilon_1^p(\varrho,\zeta)$, $\beta_\varrho^p : \Lambda_\varrho \to [0,1]$ represent by $\beta_\varrho^p = \Upsilon_2^p(\varrho,\zeta)$, $\gamma_\varrho^p : \Lambda_\varrho \to [0,1]$ represent by $\gamma_\varrho^p = \Upsilon_3^p(\varrho,\zeta)$, $\delta_\varrho^p : \Lambda_\varrho \to [0,1]$ represent by $\delta_\varrho^p = \Upsilon_4^p(\varrho,\zeta)$, $\alpha_\varrho^n : \Lambda_\varrho \to [-1,0]$ represent by $\alpha_\varrho^n = \Upsilon_1^n(\varrho,\zeta)$, $\beta_\varrho^n : \Lambda_\varrho \to [-1,0]$ represent by $\beta_\varrho^n = \Upsilon_2^n(\varrho,\zeta)$, $\gamma_\varrho^n : \Lambda_\varrho \to [-1,0]$ represent by $\gamma_\varrho^n = \Upsilon_3^n(\varrho,\zeta)$, $\delta_\varrho^n : \Lambda_\varrho \to [-1,0]$ represent by $\delta_\varrho^n = \Upsilon_4^n(\varrho,\zeta)$. For every vertex $\varrho \in \Lambda$, the QBN singleton set, $\Psi_\varrho^{|} = (\varrho, \Psi_1^{p'}, \Psi_2^{p'}, \Psi_3^{p'}, \Psi_4^{p'}, \Psi_1^{n'}, \Psi_2^{n'}, \Psi_3^{n'}, \Psi_4^{n'})$ such that $\Psi_1^{p'} : \varrho \to [0,1], \Psi_2^{p'} : \varrho \to [0,1], \Psi_3^{p'} : \varrho \to [0,1], \Psi_4^{p'} : \varrho \to [0,1], \Psi_1^{n'} : \varrho \to [-1,0], \Psi_2^{n'} : \varrho \to [-1,0], \Psi_3^{n'} : \varrho \to [-1,0], \Psi_4^{n'} : \varrho \to [-1,0]$, represent by $\Psi_1^{p'} = \Psi_1^p(\varrho), \Psi_2^{p'} = \Psi_2^p(\varrho), \Psi_3^{p'} = \Psi_3^p(\varrho), \Psi_4^{p'} = \Psi_4^p(\varrho)$ and $\Psi_1^{n'} = \Psi_1^n(\varrho), \Psi_2^{n'} = \Psi_2^n(\varrho), \Psi_3^{n'} = \Psi_3^n(\varrho), \Psi_4^{n'} = \Psi_4^n(\varrho)$, respectively. The QBN closed neighbourhood of a vertex ϱ is $\mathtt{Q}[\varrho] = \mathtt{Q}(\varrho) \cup \Psi_\varrho$

Definition 3.14. Consider $\mathtt{G} = (\Psi, \Upsilon)$ is a QBNG. QBN open neighbourhood graph of $\mathtt{G}$ is a QBNG $\mathtt{Q}(\mathtt{G}) = (\Psi, \Upsilon^{'})$ which has the same QBNS of a vertex in $\mathtt{G}$ and has a QBN edge between two vertices $\varrho, \zeta \in \Lambda$ in $(\mathtt{Q})$ if and only if $\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)$ is a non-empty QBNS in $\mathtt{G}$. The positive TMF, CMF, IMF and FMF, negative TMF, CMF, IMF and FMF value of the edge (ϱ, ζ) are given by:

$$\Upsilon_1^{p^{'}}(\varrho,\zeta) = [\Psi_1^p(\varrho) \wedge \Psi_1^p(\zeta)]h_1(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)), \quad \Upsilon_1^{n^{'}}(\varrho,\zeta) = [\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta)]h_1(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)),$$
$$\Upsilon_2^{p^{'}}(\varrho,\zeta) = [\Psi_2^p(\varrho) \wedge \Psi_2^p(\zeta)]h_2(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)), \quad \Upsilon_2^{n^{'}}(\varrho,\zeta) = [\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta)]h_2(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)),$$
$$\Upsilon_3^{p^{'}}(\varrho,\zeta) = [\Psi_3^p(\varrho) \vee \Psi_3^p(\zeta)]h_3(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)), \quad \Upsilon_3^{n^{'}}(\varrho,\zeta) = [\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta)]h_3(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)),$$
$$\Upsilon_4^{p^{'}}(\varrho,\zeta) = [\Psi_4^p(\varrho) \vee \Psi_4^p(\zeta)]h_4(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)), \quad \Upsilon_4^{n^{'}}(\varrho,\zeta) = [\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta)]h_4(\mathtt{Q}(\varrho) \cap \mathtt{Q}(\zeta)).$$

Definition 3.15. Consider $\mathtt{G} = (\Psi, \Upsilon)$ is a QBNG. QBN open neighbourhood graph of $\mathtt{G}$ is a QBNG $\mathtt{Q}[\mathtt{G}] = (\Psi, \Upsilon^{'})$ which has the same QBNS of a vertex in $\mathtt{G}$ and has a QBN edge between two vertices $\varrho, \zeta \in \Lambda$ in $\mathtt{Q}[\mathtt{G}]$ if and only if $\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]$ is a non-empty QBNS in $\mathtt{G}$. The positive TMF, CMF, IMF and FMF, negative TMF, CMF, IMF and FMF value of the edge (ϱ, ζ) are given by:

$$\Upsilon_1^{p^{'}}(\varrho,\zeta) = [\Psi_1^p(\varrho) \wedge \Psi_1^p(\zeta)]h_1(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]), \quad \Upsilon_1^{n^{'}}(\varrho,\zeta) = [\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta)]h_1(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]),$$
$$\Upsilon_2^{p^{'}}(\varrho,\zeta) = [\Psi_2^p(\varrho) \wedge \Psi_2^p(\zeta)]h_2(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]), \quad \Upsilon_2^{n^{'}}(\varrho,\zeta) = [\Psi_2^n(\varrho) \vee \Psi_2^n(\zeta)]h_2(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]),$$
$$\Upsilon_3^{p^{'}}(\varrho,\zeta) = [\Psi_3^p(\varrho) \vee \Psi_3^p(\zeta)]h_3(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]), \quad \Upsilon_3^{n^{'}}(\varrho,\zeta) = [\Psi_3^n(\varrho) \wedge \Psi_3^n(\zeta)]h_3(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]),$$
$$\Upsilon_4^{p^{'}}(\varrho,\zeta) = [\Psi_4^p(\varrho) \vee \Psi_4^p(\zeta)]h_4(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]), \quad \Upsilon_4^{n^{'}}(\varrho,\zeta) = [\Psi_4^n(\varrho) \wedge \Psi_4^n(\zeta)]h_4(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]).$$

Example 3.16. Let $\mathtt{G} = (\Psi, \Upsilon)$ is a QBND, such that, $\Lambda = \{p, q, r, s\}$, $\Psi = \{(p, 0.3, 0.4, 0.5, 0.6, -0.4, -0.6, -0.7, -0.4),$
$(q, 0.6, 0.7, 0.6, 0.5, -0.6, -0.7, -0.5, -0.4), (r, 0.4, 0.3, 0.4, 0.4, -0.6, -0.7, -0.6, -0.5),$
$(s, 0.4, 0.6, 0.7, 0.6, -0.4, -0.3, -0.4, -0.3)\}$, and
$\Upsilon = \{(pq, 0.2, 0.3, 0.5, 0.5, -0.3, -0.5, -0.6, -0.3),$
$(qr, 0.3, 0.2, 0.5, 0.4, -0.5, -0.6, -0.4, -0.3), (sr, 0.3, 0.2, 0.6, 0.5, -0.3, -0.2, -0.4, -0.4),$
$(ps, 0.2, 0.3, 0.5, 0.5, -0.3, -0.2, -0.6, -0.3)\}$, according to figure 6.
Figures 7 and 8 show the corresponding QBN open and closed neighbourhood graphs.

Theorem 3.17. *For each edge of a QBNG* $\mathtt{G}$*, there exist an edge in* $\mathtt{Q}[\mathtt{G}]$*.*

Proof. If (ϱ, ζ) is an line of a QBNG $\mathtt{G} = (\Psi, \Upsilon)$. Suppose $\mathtt{Q}[\mathtt{G}] = (\Psi, \Upsilon^{'})$ is the corresponding closed neighbourhood of a QBNG. Suppose $\varrho, \zeta \in \mathtt{Q}[\varrho]$ and $\varrho, \zeta \in \mathtt{Q}[\zeta]$. Then $\varrho, \zeta \in \mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]$. Hence,

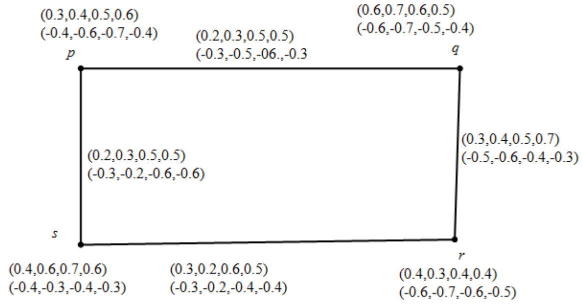


FIGURE 7. QBNG

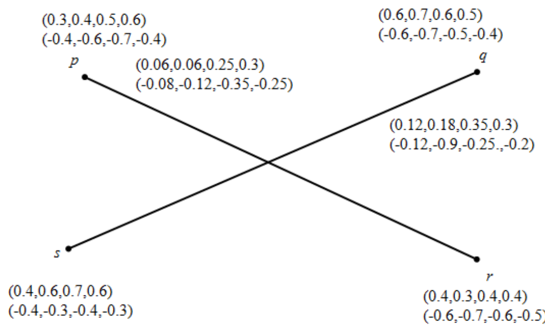


FIGURE 8. Q(G)

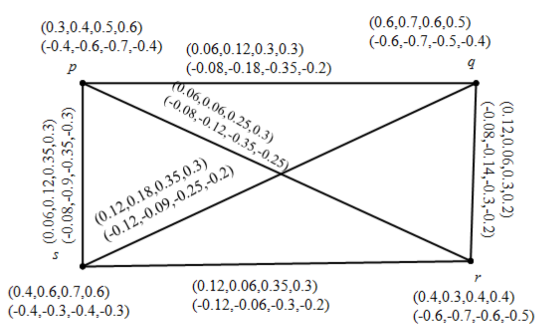


FIGURE 9. Q[G]

$$h_1(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0 \quad h_2(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0$$
$$h_3(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0 \quad h_4(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0$$
$$h_5(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0 \quad h_6(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$

Then,

$$\Upsilon_1^{p'}(\varrho,\zeta) = [\Psi_1^p(\varrho) \wedge \Psi_1^p(\zeta)]h_1(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_2^{p'}(\varrho,\zeta) = [\Psi_2^p(\varrho) \wedge \Psi_2^p(\zeta)]h_2(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_3^{p'}(\varrho,\zeta) = [\Psi_3^p(\varrho) \vee \Psi_3^p(\zeta)]h_3(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_4^{p'}(\varrho,\zeta) = [\Psi_4^p(\varrho) \vee \Psi_4^p(\zeta)]h_4(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_1^{n'}(\varrho,\zeta) = [\Psi_1^n(\varrho) \vee \Psi_1^n(\zeta)]h_1(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_2^{n'}(\varrho,\zeta) = [\Psi_2^n(\varrho) \wedge \Psi_2^n(\zeta)]h_2(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_3^{n'}(\varrho,\zeta) = [\Psi_3^n(\varrho) \vee \Psi_3^n(\zeta)]h_3(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$
$$\Upsilon_4^{n'}(\varrho,\zeta) = [\Psi_4^n(\varrho) \vee \Psi_4^n(\zeta)]h_4(\mathtt{Q}[\varrho] \cap \mathtt{Q}[\zeta]) \neq 0.$$

Thus, for each edge (ϱ,ζ) in QBNG $\mathtt{G}$, there exists an edge (ϱ,ζ) in $\mathtt{Q}[\mathtt{G}]$. □

Theorem 3.18. *Let $C(\overrightarrow{\mathtt{G}_1}) = (\Psi_1,\Upsilon_1)$ and $C(\overrightarrow{\mathtt{G}_2}) = (\Psi_1,\Upsilon_2)$ be two QBNCG of QBND $\overrightarrow{\mathtt{G}_1} = (\Psi_1,l_1)$ and $\overrightarrow{\mathtt{G}_2} = (\Psi_1,l_2)$, respectively. Then $C(\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2}) = \mathtt{G}_{C(\overrightarrow{\mathtt{G}_1})^*\square C(\overrightarrow{\mathtt{G}_2})^*} \cup \mathtt{G}^\square$, where, $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1})^*\square C(\overrightarrow{\mathtt{G}_2})^*}$ is a QBNG on crisp graph $(\varrho_1\times\varrho_2, E^*_{C(\overrightarrow{\mathtt{G}_1})}\square E^*_{C(\overrightarrow{\mathtt{G}_2})})$, $C(\overrightarrow{\mathtt{G}_1})^*$ and $C(\overrightarrow{\mathtt{G}_2})^*$ are the crisp competition graphs of $\overrightarrow{\mathtt{G}_1}$ and $\overrightarrow{\mathtt{G}_2}$, respectively. $\mathtt{G}^\square$ is a QBNG on $(Y_1\times Y_2, E^\square)$ such that:*

(1.) $E^\square = \{(\varrho_1,\varrho_2)(\zeta_1,\zeta_2) : \zeta_1 \in \mathtt{Q}^-(\varrho_1^*), \zeta_2 \in \mathtt{Q}^+(\varrho_2^*)\}$

$$E^*_{C(\overrightarrow{\mathtt{G}_1})}\square E^*_{C(\overrightarrow{\mathtt{G}_2})} = \{(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) : \varrho_1 \in Y_1, \varrho_2\zeta_2 \in E^*_{C(\overrightarrow{\mathtt{G}_2})}\} \cup \{(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) : \varrho_2 \in Y_2, \varrho_1\zeta_1 \in E^*_{C(\overrightarrow{\mathtt{G}_1})}\}$$

(2.)
$$\alpha^p_{\Psi_1\square\Psi_2} = \alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\Psi_2}(\varrho_2) \quad \beta^p_{\Psi_1\square\Psi_2} = \beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\Psi_2}(\varrho_2)$$
$$\gamma^p_{\Psi_1\square\Psi_2} = \gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\Psi_2}(\varrho_2) \quad \delta^p_{\Psi_1\square\Psi_2} = \delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\Psi_2}(\varrho_2)$$
$$\alpha^n_{\Psi_1\square\Psi_2} = \alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\Psi_2}(\varrho_2) \quad \beta^n_{\Psi_1\square\Psi_2} = \beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\Psi_2}(\varrho_2)$$
$$\gamma^n_{\Psi_1\square\Psi_2} = \gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\Psi_2}(\varrho_2) \quad \delta^n_{\Psi_1\square\Psi_2} = \delta^p_{\Psi_1}(\varrho_1) \wedge \delta^n_{\Psi_2}(\varrho_2)$$

(3) $\alpha^p_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\Psi_2}(\varrho_2) \wedge \alpha^p_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\overrightarrow{l_2}}(\varrho_2 a_2) \wedge \alpha^p_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(4) $\beta^p_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\Psi_2}(\varrho_2) \wedge \beta^p_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\overrightarrow{l_2}}(\varrho_2 a_2) \wedge \beta^p_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(5) $\gamma^p_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\Psi_2}(\varrho_2) \wedge \gamma^p_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\overrightarrow{l_2}}(\varrho_2 a_2) \vee \gamma^p_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(6) $\delta^p_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\Psi_2}(\varrho_2) \wedge \delta^p_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\overrightarrow{l_2}}(\varrho_2 a_2) \vee \delta^p_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(7) $\alpha^n_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\Psi_2}(\varrho_2) \vee \alpha^n_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\overrightarrow{l_2}}(\varrho_2 a_2) \vee \alpha^n_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(8) $\beta^n_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\Psi_2}(\varrho_2) \vee \beta^n_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\overrightarrow{l_2}}(\varrho_2 a_2) \vee \beta^n_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(9) $\gamma^n_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\Psi_2}(\varrho_2) \wedge \gamma^n_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\overrightarrow{l_2}}(\varrho_2 a_2) \wedge \gamma^n_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(10) $\delta^n_{\Upsilon}((\varrho_1,\varrho_2),(\varrho_1,\zeta_2)) = [\delta^n_{\Psi_1}(\varrho_1) \wedge \delta^n_{\Psi_2}(\varrho_2) \wedge \delta^n_{\Psi_2}(\zeta_2)] \times \vee_{a_2}\{\delta^n_{\Psi_1}(\varrho_1) \wedge \delta^n_{\overrightarrow{l_2}}(\varrho_2 a_2) \wedge \delta^n_{\overrightarrow{l_2}}(\zeta_2 a_2)\}$

$(\varrho_1,\varrho_2)(\varrho_1,\zeta_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_2 \in (Q^+(\varrho_2) \cap Q^+(\zeta_2))^*$

(11) $\alpha^p_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\Psi_1}(\zeta_1) \wedge \alpha^p_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\alpha^p_{\Psi_2}(\varrho_2) \wedge \alpha^p_{\overrightarrow{l_1}}(\varrho_1 a_1) \wedge \alpha^p_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1 \cap Q^+(\zeta_1))^*$

(12) $\beta^p_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\Psi_1}(\zeta_1) \wedge \beta^p_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\beta^p_{\Psi_2}(\varrho_2) \wedge \beta^p_{\overrightarrow{l_1}}(\varrho_1 a_1) \wedge \beta^p_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1) \cap Q^+(\zeta_1))^*$

(13) $\gamma^p_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\Psi_1}(\zeta_1) \vee \gamma^p_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\gamma^p_{\Psi_2}(\varrho_2) \vee \gamma^p_{\overrightarrow{l_1}}(\varrho_1 a_1) \vee \gamma^p_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1) \cap Q^+(\zeta_1))^*$

(14) $\delta^p_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\Psi_1}(\zeta_1) \vee \delta^p_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\delta^p_{\Psi_2}(\varrho_2) \vee \delta^p_{\overrightarrow{l_1}}(\varrho_1 a_1) \vee \delta^p_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1) \cap Q^+(\zeta_1))^*$

(15) $\alpha^n_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\Psi_1}(\zeta_1) \vee \alpha^n_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\alpha^n_{\Psi_2}(\varrho_2) \vee \alpha^n_{\overrightarrow{l_1}}(\varrho_1 a_1) \vee \alpha^n_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1) \cap Q^+(\zeta_1))^*$

(16) $\beta^n_{\Upsilon}((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\Psi_1}(\zeta_1) \vee \beta^n_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\beta^n_{\Psi_2}(\varrho_2) \vee \beta^n_{\overrightarrow{l_1}}(\varrho_1 a_1) \vee \beta^n_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{G_1}^*)} \Box E_{C(\overrightarrow{G_2}^*)}, a_1 \in (Q^+(\varrho_1) \cap Q^+(\zeta_1))^*$

(17) $\gamma^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\Psi_1}(\zeta_1) \wedge \gamma^n_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\gamma^n_{\Psi_2}(\varrho_2) \wedge \gamma^n_{\overrightarrow{l_1}}(\varrho_1 a_1) \wedge \gamma^n_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{\mathtt{G}_1}^*)} \square E_{C(\overrightarrow{\mathtt{G}_2}^*)}, a_1 \in (\mathtt{Q}^+(\varrho_1) \cap \mathtt{Q}^+(\zeta_1))^*$

(18) $\delta^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\varrho_2)) = [\delta^n_{\Psi_1}(\varrho_1) \wedge \delta^n_{\Psi_1}(\zeta_1) \wedge \delta^n_{\Psi_2}(\varrho_2)] \times \vee_{a_1}\{\delta^n_{\Psi_2}(\varrho_2) \wedge \delta^n_{\overrightarrow{l_1}}(\varrho_1 a_1) \wedge \delta^n_{\overrightarrow{l_1}}(\zeta_1 a_1)\}$

$(\varrho_1,\varrho_2)(\zeta_1,\varrho_2) \in E_{C(\overrightarrow{\mathtt{G}_1}^*)} \square E_{C(\overrightarrow{\mathtt{G}_2}^*)}, a_1 \in (\mathtt{Q}^+(\varrho_1) \cap \mathtt{Q}^+(\zeta_1))^*,$

(19) $\alpha^p_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\Psi_1}(\zeta_1) \wedge \alpha^p_{\Psi_2}(\varrho_2) \wedge \alpha^p_{\Psi_2}(\zeta_2)] \times \{\alpha^p_{\Psi_1}(\varrho_1) \wedge \alpha^p_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \wedge \alpha^p_{\Psi_2}(\zeta_2) \wedge \alpha^p_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(20) $\beta^p_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\Psi_1}(\zeta_1) \wedge \beta^p_{\Psi_2}(\varrho_2) \wedge \beta^p_{\Psi_2}(\zeta_2)] \times \{\beta^p_{\Psi_1}(\varrho_1) \wedge \beta^p_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \wedge \beta^p_{\Psi_2}(\zeta_2) \wedge \beta^p_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(21) $\gamma^p_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\Psi_1}(\zeta_1) \vee \gamma^p_{\Psi_2}(\varrho_2) \vee \gamma^p_{\Psi_2}(\zeta_2)] \times \{\gamma^p_{\Psi_1}(\varrho_1) \vee \gamma^p_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \vee \gamma^p_{\Psi_2}(\zeta_2) \vee \gamma^p_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(22) $\delta^p_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\Psi_1}(\zeta_1) \vee \delta^p_{\Psi_2}(\varrho_2) \vee \delta^p_{\Psi_2}(\zeta_2)] \times \{\delta^p_{\Psi_1}(\varrho_1) \vee \delta^p_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \vee \delta^p_{\Psi_2}(\zeta_2) \vee \delta^p_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(23) $\alpha^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\Psi_1}(\zeta_1) \vee \alpha^n_{\Psi_2}(\varrho_2) \vee \alpha^n_{\Psi_2}(\zeta_2)] \times \{\alpha^n_{\Psi_1}(\varrho_1) \vee \alpha^n_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \vee \alpha^n_{\Psi_2}(\zeta_2) \vee \alpha^n_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(24) $\beta^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\Psi_1}(\zeta_1) \vee \beta^n_{\Psi_2}(\varrho_2) \vee \beta^n_{\Psi_2}(\zeta_2)] \times \{\beta^n_{\Psi_1}(\varrho_1) \vee \beta^n_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \vee \beta^n_{\Psi_2}(\zeta_2) \vee \beta^n_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(25) $\gamma^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\Psi_1}(\zeta_1) \wedge \gamma^n_{\Psi_2}(\varrho_2) \wedge \gamma^n_{\Psi_2}(\zeta_2)] \times \{\gamma^n_{\Psi_1}(\varrho_1) \wedge \gamma^n_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \wedge \gamma^n_{\Psi_2}(\zeta_2) \wedge \gamma^n_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square$

(26) $\delta^n_\Upsilon((\varrho_1,\varrho_2),(\zeta_1,\zeta_2)) = [\delta^n_{\Psi_1}(\varrho_1) \wedge \delta^n_{\Psi_1}(\zeta_1) \wedge \delta^n_{\Psi_2}(\varrho_2) \wedge \delta^n_{\Psi_2}(\zeta_2)] \times \{\delta^n_{\Psi_1}(\varrho_1) \wedge \delta^n_{\overrightarrow{l_1}}(\zeta_1\varrho_1) \wedge \delta^n_{\Psi_2}(\zeta_2) \wedge \delta^n_{\overrightarrow{l_2}}(\varrho_2\zeta_2)\}$

$(\varrho_1,\zeta_1)(\varrho_2,\zeta_2) \in E^\square.$

Example 3.19. Consider $\overrightarrow{\mathtt{G}_1} = (\Psi_1, l_1)$ and $\overrightarrow{\mathtt{G}_2} = (\Psi_2, l_2)$ be two QBND, as shown in figure 9. The QBN out and in-neighbourhood of $\overrightarrow{\mathtt{G}_1}$ and $\overrightarrow{\mathtt{G}_2}$ are given in tables 5 and 6. The QBNC graphs $C(\overrightarrow{\mathtt{G}_1})$ and $C(\overrightarrow{\mathtt{G}_2})$ are given in Figure 10.

Table 5. QBN Out neighbourhoods of $\overrightarrow{\mathtt{G}_1}$

ϱ	$\mathtt{Q}^+(\varrho)$
p_1	$\{q_1(0.4, 0.3, 0.2, 0.1, -0.2, -0.1, -0.2, -0.3)\}$
q_1	$\emptyset$
r_1	$\{q_1(0.5, 0.3, 0.6, 0.5, -0.2, -0.1, -0.4, -0.5)\}$
s_1	$\{r_1(0.4, 0.4, 0.6, 0.5, -0.3, -0.2, -0.4, -0.5)\}$

Table 6. QBN Out neighbourhoods of $\overrightarrow{\mathtt{G}_2}$

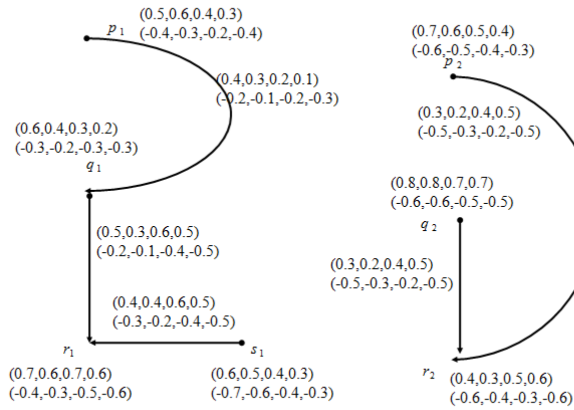


FIGURE 10. QBND

ϱ	$\mathtt{Q}^{+}(\varrho)$
p_2	$\{r_2(0.3, 0.2, 0.4, 0.5, -0.5, -0.3, -0.2, -0.2)\}$
q_2	$\{r_2(0.3, 0.2, 0.4, 0.5, -0.5, -0.3, -0.2, -0.5)$
r_2	$\{q_1(0.4, 0.3, 0.5, 0.6, -0.6, -0.4, -0.3, -0.6)\}$

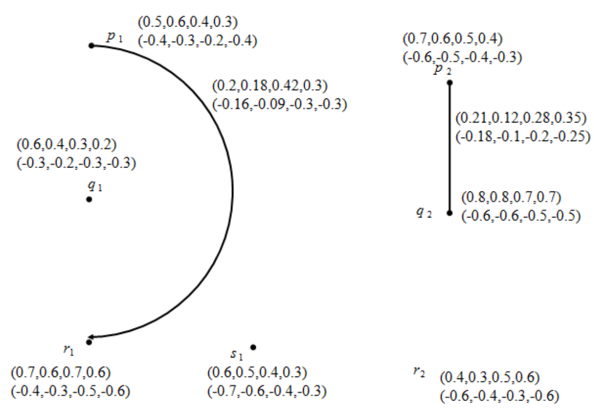


FIGURE 11. QBNCG

We now construct the QBNC graph $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}*)\square C(\overrightarrow{\mathtt{G}_2^*})} \cup \mathtt{G}^{\square} = (W, B)$, where, $W = (\alpha_W^p, \beta_W^p, \gamma_W^p, \delta_W^p, \alpha_W^n, \beta_W^n, \gamma_W^n, \delta_W^n)$ and $B = (\alpha_B^p, \beta_B^p, \gamma_B^p, \delta_B^p, \alpha_B^n, \beta_B^n, \gamma_B^n, \delta_B^n)$ from $C(\overrightarrow{\mathtt{G}_1}*)$ and

$C(\overrightarrow{\mathtt{G}_2}^*)$ using Theorem(3.13). We obtain the following set of edges by using condition (1).

$$E_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})} = \{(p_1,p_2)(p_1,q_2)(q_1,p_2)(q_1,q_2),(r_1,p_2),(r_1,q_2),$$
$$(s_1,p_2),(s_1,q_2),(p_1,p_2)(r_1,p_2),$$
$$(p_1,q_2)(p_1,r_2),(p_1,r_2)(r_1,r_2)\},$$
$$E^{\square} = \{(q_1,p_2)(p_1,r_2),(q_1,p_2)(r_1r_2),(q_1,q_2)(p_1,r_2),$$
$$(q_1,q_2)(r_1,r_2),(r_1,p_2)(s_1,r_2),(r_1,q_2)(s_1,r_2)\}$$

According to condition (3) to (28), the degree of positive TMF, CMF, IMF and FMF, negative TMF, CMF, IMF and FMF of the adjacent edges of $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})}$ and $\mathtt{G}^{\square}$ are given in Table 7.

Table 7. Adjacent edges of $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})} \cup \mathtt{G}^{\square} = (W,B)$

$(\varrho_1,\varrho_2)(\zeta_1,\zeta_2)$	$\Upsilon(\varrho_1,\varrho_2)(\zeta_1,\zeta_2)$
$(p_1,p_2)(p_1,q_2)$	$\{(0.15,0.12,0.28,0.35,-0.12,-0.06,-0.2,-0.25)\}$
$(q_1,p_2)(q_1q_2)$	$\{(0.18,0.08,0.28,0.35,-0.09,-0.04,-0.2,-0.25)\}$
$(r_1,p_2)(r_1q_2)$	$\{(0.21,0.12,0.49,0.42,-0.12,-0.06,-0.35,-0.36)\}$
$(s_1,p_2)(s_1,q_2)$	$\{(0.18,0.1,0.28,0.35,-0.18,-0.1,-0.20,-0.25)\}$
$(p_1,p_2)(r_1,p_2)$	$\{(0.2,0.18,0.42,0.3,-0.16,-0.09,-0.24,-0.20)\}$
$(p_1,q_2)(r_1,q_2)$	$\{(0.2,0.18,0.49,0.49,-0.16,-0.09,-0.35,-0.42)\}$
$(p_1,r_2)(r_1,r_2)$	$\{(0.16,0.09,0.42,0.36,-0.16,-0.09,-0.30,-0.36)\}$
$(q_1,p_2)(p_1,r_2)$	$\{(0.12,0.06,0.2,0.36,-0.09,-0.04,-0.2,-0.24)\}$
$(q_1,p_2)(r_1,r_2)$	$\{(0.12,0.06,0.42,0.42,-0.09,-0.04,-0.3,-0.36)\}$
$(q_1,q_2)(p_1,r_2)$	$\{(0.12,0.06,0.35,0.42,-0.09,-0.04,-0.25,-0.30)\}$
$(q_1,q_2)(r_1,r_2)$	$\{(0.12,0.06,0.42,0.42,-0.09,-0.3,-0.25,-0.36)\}$
$(r_1,p_2)(s_1,r_2)$	$\{(0.12,0.06,0.49,0.36,-0.12,-0.06,-0.35,-0.36)\}$
$(r_1,q_2)(s_1,r_2)$	$\{(0.12,0.06,0.49,0.42,-0.12,-0.06,-0.35,-0.36)\}$

The QBNCG obtained using this model is given in Figure 11. where, the solid line indicate the part of QBNCG obtained from $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})}$, the dotted lines represent the part $\mathtt{G}^{\square}$. The Cartesian product $\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2}$ of QBND $\overrightarrow{\mathtt{G}_1}$ and $\overrightarrow{\mathtt{G}_2}$ is shown in figure 12. The QBN out-neighborhood of $\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2}$ are calculate in Table 8. The QBNCG of $\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2}$ is shown in figure 13.

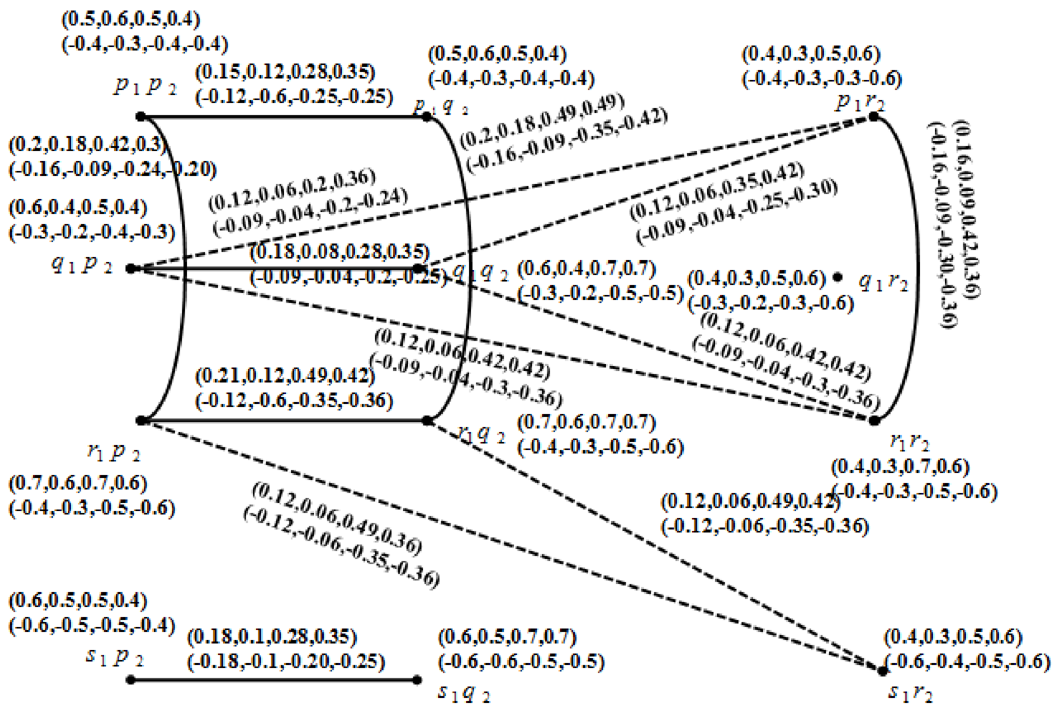


FIGURE 12

$\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})} \cup \mathtt{G}^{\square}$

Table 8. QBN out-neighborhood of $\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2}$

(ϱ, ζ)	$\mathtt{Q}^+(\varrho, \zeta)$
(p_1, p_2)	$\{(p_1, r_2)(0.3, 0.2, 0.4, 0.5, -0.4, -0.3, -0.2, -0.4), (\mathtt{Q}_1, p_2)(0.4, 0.3, 0.5, 0.4, -0.2, -0.1, -0.4, -0.3)\}$
(p_1, q_2)	$\{(p_1, r_2)(0.3, 0.2, 0.4, 0.5, -0.4, -0.3, -0.2, -0.5), (\mathtt{Q}_1, q_2)(0.4, 0.3, 0.7, 0.7, -0.2, -0.1, -0.5, -0.5)\}$
(p_1, r_2)	$\{(q_1 r_2)(0.4, 0.3, 0.5, 0.6, -0.2, -0.1, -0.3, -0.6)\}$
(q_1, p_2)	$\{(q_1, r_2)(0.3, 0.2, 0.4, 0.5, -0.3, -0.2, -0.3, -0.3)\}$
(q_1, q_2)	$\{(q_1 r_2)(0.3, 0.2, 0.4, 0.5, -0.3, -0.2, -0.3, -0.5)\}$
(r_1, p_2)	$\{(r_1 r_2)(0.3, 0.2, 0.7, 0.6, -0.4, -0.3, -0.5, -0.6)\}$
(r_1, q_2)	$\{(r_1, r_2)(0.3, 0.2, 0.7, 0.6, -0.4, -0.3, -0.5, -0.6), (q_1, q_2)(0.5, 0.3, 0.7, 0.7, -0.2, -0.1, -0.5, -0.5)\}$
(r_1, r_2)	$\{(q_1, r_2)(0.4, 0.3, 0.6, 0.6, -0.2, -0.1, -0.4, -0.6)\}$
(q_1, r_2)	$\emptyset$
(s_1, p_2)	$\{(r_1 p_2)(0.4, 0.4, 0.6, 0.5, -0.3, -0.2, -0.4, -0.5), (s_1 r_2)(0.3, 0.2, 0.4, 0.5, -0.5, -0.3, -0.5, -0.4)\}$
(s_1, q_2)	$\{(r_1, q_2)(0.4, 0.4, 0.7, 0.7, -0.3, -0.2, -0.5, -0.5), (s_1, r_2)(0.3, 0.2, 0.4, 0.5, -0.5, -0.3, -0.5, -0.4)\}$
(s_1, r_2)	$\{(r_1 r_2)(0.4, 0.3, 0.6, 0.6, -0.3, -0.2, -0.4, -0.6)\}$

It is clear from figure 11 and figure 13 that $\mathtt{G}_{C(\overrightarrow{\mathtt{G}_1}^*)\square C(\overrightarrow{\mathtt{G}_2^*})} \cup \mathtt{G}^{\square} \cong C(\overrightarrow{\mathtt{G}_1}\square\overrightarrow{\mathtt{G}_2})$.

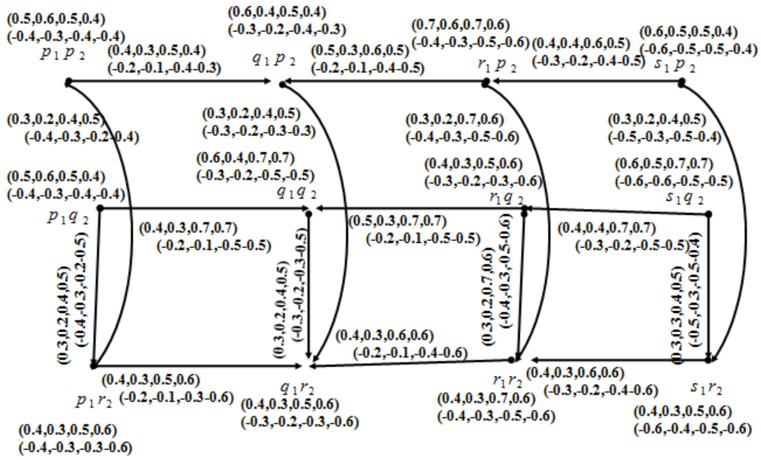


FIGURE 13
$\overrightarrow{G_1} \square \overrightarrow{G_2}$

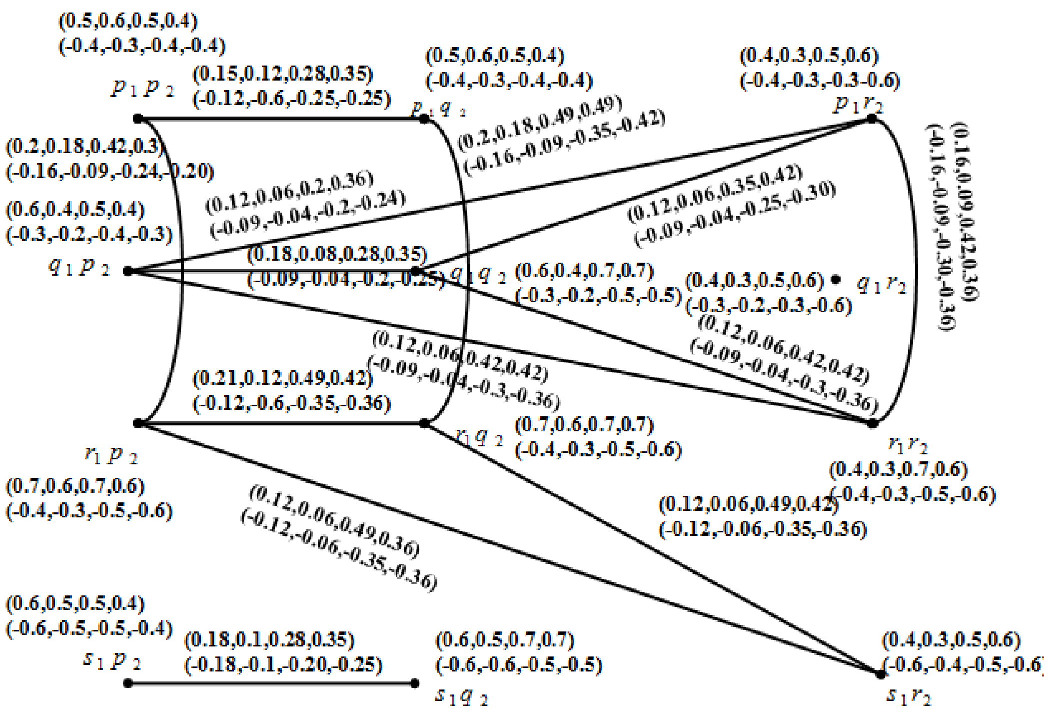


FIGURE 14
$C(\overrightarrow{G_1} \square \overrightarrow{G_2})$

4. **Application**

4.1. *Job competition on QBNCG*

Let the bipolar quadripartitioned neutrosophic competition digraph in figure 14 represent the competition amongst job seekers in a company. Let (Hussain, Aslam, Mohammed, Satham, Durga) be the set of applicants for the jobs Vice Chancellor (VC), Pro Vice Chancellor (PVC), Academic Dean (AD), Professor (PR). The positive degree of truth membership $\alpha(\varrho)$ of each applicant denote the percentage of academic performance, contradiction membership degree $\beta(\varrho)$ of each applicant represent the percentage of background knowledge, $\gamma(\varrho)$ and $\delta(\varrho)$ represent the research and teaching aptitude percentage. The negative degree of membership $\alpha(\varrho)$ represents the percentage that the applicant in not effective in order to fulfil the goals of the organization, etc. The bipolar quadripartitioned neutrosophic competition graph can be utilized in order to find the suitable jobs of the applicants.

Table 9. QBN out-neighborhood of Applicant and Jobs

$\varrho \in Y$	$\mathtt{Q}^+(\varrho,\zeta)$
Hussain	{(VC)(0.4,0.3,0.3,0.3,-0.3,-0.2,-0.4,-0.4), (PVC)(0.4,0.3,0.3,0.3,-0.3,-0.2,-0.4,-0.6), (AD)(0.3,0.3,0.3,0.3,-0.2,-0.1,-0.5,-0.5)}
Durga	{(PR)(0.2,0.2,0.2,0.2,-0.1,-0.1,-0.5,-0.5), (PVC)(0.6,0.6,0.6,0.6,-0.2,-0.2,-0.2,-0.2)}
Aslam	{(VC)(0.3,0.4,0.2,0.5,-0.5,-0.3,-0.3,-0.3), (PVC)(0.2,0.2,0.2,0.2,-0.1,-0.1,-0.1,-0.1), (AD)(0.3,0.2,0.5,0.5,-0.2,-0.2,-0.2,-0.2)}
Satham	{(AD)(0.2,0.1,0.3,0.3,-0.1,-0.1,-0.1,-0.1), (PR)(0.2,0.2,0.2,0.2,-0.1,-0.1,-0.1,-0.1)}
Mohammed	{(PR)(0.3,0.3,0.3,0.3,-0.3,-0.2,-0.2,-0.2)}

Here, $\mathtt{Q}^+(Hussain)\cap\mathtt{Q}^+(Mohammed)=\emptyset$, $\mathtt{Q}^+(Aslam)\cap\mathtt{Q}^+(Mohammed)=\emptyset$,
$\mathtt{Q}^+(Hussain)\cap\mathtt{Q}^+(Durga)=(PVC,0.4,0.3,0.6,0.6,-0.2,-0.2,-0.4,-0.6)$, $\mathtt{Q}^+(Hussain)\cap\mathtt{Q}^+(Satham)=(AD,0.3,0.2,0.5,0.5,-0.1,-0.1,-0.5,-0.5)$
$\mathtt{Q}^+(Mohammed)\cap\mathtt{Q}^+(Durga)=(PR,0.2,0.2,0.3,0.3,-0.1,-0.1,-0.5,-0.5)$,
$\mathtt{Q}^+(Mohammed)\cap\mathtt{Q}^+(Satham)=(PR,0.2,0.2,0.3,0.3,-0.1,-0.1,-0.2,-0.2)$
$\mathtt{Q}^+(Satham)\cap\mathtt{Q}^+(Durga)=(PR,0.2,0.2,0.2,0.2,-0.1,-0.1,-0.5,-0.5)$, $\mathtt{Q}^+(Aslam)\cap\mathtt{Q}^+(Durga)=(PVC,0.2,0.2,0.6,0.6,-0.1,-0.1,-0.2,-0.2)$
$\mathtt{Q}^+(Satham)\cap\mathtt{Q}^+(Aslam)=(AD,0.2,0.1,0.5,0.5,-0.1,-0.1,-0.2,-0.2)$,
$\mathtt{Q}^+(Hussain)\cap\mathtt{Q}^+(Aslam)=(PVC,0.2,0.2,0.3,0.3,-0.1,-0.1,-0.4,-0.6)$,
$(VC,0.3,0.3,0.3,0.5,-0.2,-0.1,-0.5,-0.5)$, $(AD,0.3,0.2,0.5,0.5,-0.2,-0.1,-0.5,-0.5)$.
Figure 15 depicts a bipolar quadripartitioned neutrosophic competition graph. The dotted lines reflect the applicant competing for the specific employment, and the solid

lines show how fiercely two applicants are competing. For instance, Hussain and Mohammed both are competing for the jobs, AD and strength of competition between them $(0.12, 0.18, 0.21, 0.24, -0.12, -0.09, -0.21, -0.24)$.

Here table 10, $\alpha(p, q)$ indicates the value of strength of competition of applicant p for Job q with respect to academic performance in order fulfill the goals of that jobs. In a similar fashion, the table 10 is constructed as follows:

Table 10. QBN out-neighborhood of Applicant and Jobs

Applicant, Job	Competition	α(Applicant, Job)	Sr(Applicant, Job)
Hussain,VC	Aslam	(0.12,0.18,0.21,0.24,-0.12,-0.09,-0.21,-0.24)	2.09
Aslam,VC	Hussain	(0.12,0.18,0.21,0.24,-0.12,-0.09,-0.21,-0.24)	2.09
Hussain,AD	Aslam, Satham	(0.105,0.15,0.28,0.32,-0.105,-0.065,-0.28,-0.32)	2.085
Aslam,AD	Hussain, Satham	(0.09,0.125,0.18,0.33,-0.09,-0.055,-0.28,-0.36)	2.2
Satham,AD	Hussain, Aslam	(0.075,0.095,0.25,0.41,-0.075,-0.055,-0.35,-0.44)	2.17
Hussain,PVC	Aslam, Durga	(0.16,0.18,0.345,0.36,-0.1,-0.075,-0.31,-0.36)	2.13
Aslam,PVC	Hussain, Durga	(0.1,0.17,0.345,0.36,-0.08,-0.065,-0.315,-0.36)	2.095
Durga,PVC	Hussain, Aslam	(0.14,0.17,0.48,0.48,-0.06,-0.05,-0.42,-0.48)	2.14
Mohammed,PR	Satham, Durga	(0.06,0.08,0.18,0.225,-0.05,-0.04,-0.12,-0.12)	1.885
Durga,PR	Mohammed, Satham	(0.06,0.12,0.20,0.20,-0.04,-0.04,-0.08,-0.08)	1.86
Satham,PR	Mohammed, Durga	(0.06,0.12,0.14,0.185,-0.05,-0.04,-0.08,-0.08)	1.925

Algorithm

(i) For a set of r applicants, enter the values for truth positive membership, contradiction membership, ignorance membership and false membership..

(ii) If for every two points p_i and p_j, $\alpha^+(r_ir_j) > 0.$ $\beta^+(r_ir_j) > 0.$, $\gamma^+(r_ir_j) > 0.$ $\delta^+(r_ir_j) > 0.$ and $\alpha^-(r_ir_j) < 0.$, $\beta^-(r_ir_j) < 0.$, $\gamma^-(r_ir_j) < 0.$ $\delta^-(r_ir_j) < 0.$, then $(r_j, \alpha^+(r_ir_j), \beta^+(r_ir_j), \gamma^+(r_ir_j), \delta^+(r_ir_j),$
$\alpha^-(r_ir_j), \beta^-(r_ir_j), \gamma^+(r_ir_j), \delta^-(r_ir_j) \in \mathtt{Q}^+(r_i))$

(iii) Repeat the step (ii) for all points r_i and r_j to calculate bipolar quadripartitioned neutrosophic out-neighbourhoods $\mathtt{Q}^+(r_i)$.

(iv) Calculate $\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)$ for every pair of distinct points r_i and r_j.

(v) Calculate $h[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)]$

(vi) If $\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j) \neq \emptyset$ then draw an edge r_ir_j

(vii) Repeat step (vi) for every pair of distinct points.

(viii) Provide each edge membership values. With the conditions r_ir_j

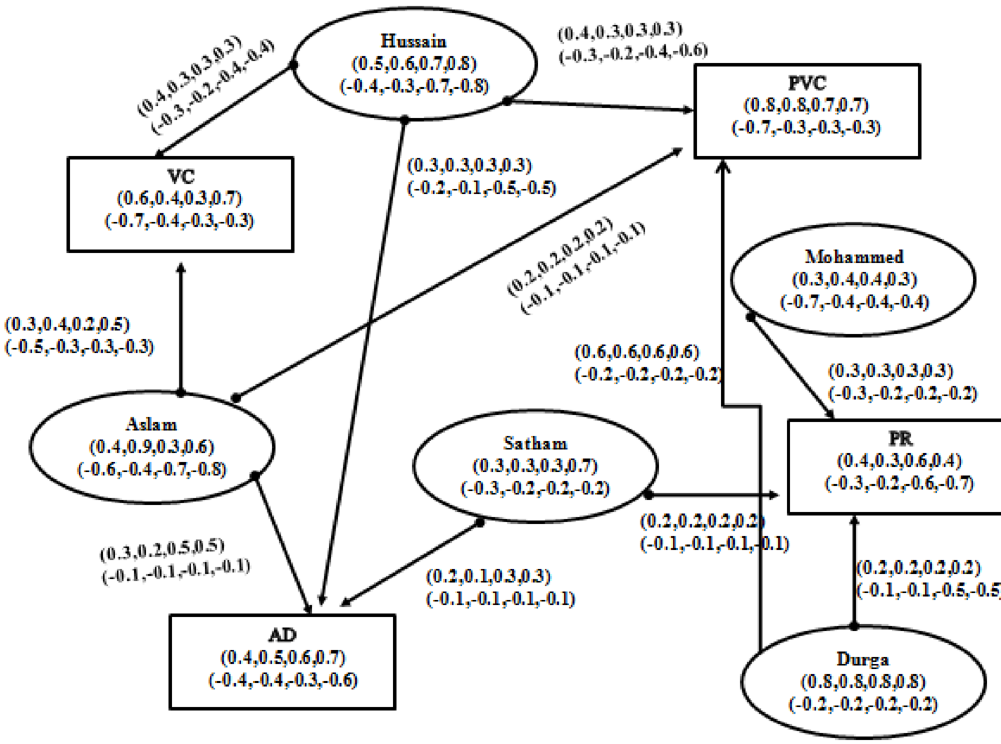


FIGURE 15. Bipolar Quadripartitioned Neutrosophic Digraph

$$\alpha^+(r_ir_j) = (r_i \wedge r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)], \alpha^-(r_ir_j) = (r_i \vee r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)]$$
$$\beta^+(r_ir_j) = (r_i \wedge r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)], \beta^-(r_ir_j) = (r_i \vee r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)]$$
$$\gamma^+(r_ir_j) = (r_i \vee r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)], \gamma^-(r_ir_j) = (r_i \wedge r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)]$$
$$\delta^+(r_ir_j) = (r_i \vee r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)], \delta^-(r_ir_j) = (r_i \wedge r_j)h_1[\mathtt{Q}^+(r_i) \cap \mathtt{Q}^+(r_j)].$$

(ix) If $r, s_1, s_2, s_3, ..., s_p$ are the applicants computing for designation t, then strength of competition $\alpha(r,t) = (\alpha^+(r,t), \beta^+(r,t), \gamma^+(r,t), \delta^+(r,t), \alpha^-(r,t), \beta^-(r,t), \gamma^-(r,t), \delta^-(r,t))$ of each applicant r for the designation t is

$$\alpha(r,t) = \frac{\alpha^+(rs_1) + ... + \alpha^+(ys_1), \beta^+(ts_1) + ... + \beta^+(ys_1), \gamma^+(rs_1) + ... + \gamma^+(ys_1), \delta^+(rs_1) + ... + \delta^+(ys_1)}{s},$$
$$\frac{\alpha^-(rs_1) + ... + \alpha^-(ys_1), \beta^-(rs_1) + ... + \beta^-(ys_1), \gamma^-(rs_1) + ... + \gamma^-(ys_1), \delta^-(rs_1) + ... + \delta^-(ys_1)}{s}$$

(x) Calculate $Sr(r,t)$, the strength of competition of each applicant r and t.

$Sr(r,t) = \alpha^+(r,t) + \beta^+(r,t) - \gamma^+(r,t) - \delta^+(r,t) + 2 + \alpha^-(r,t) + \beta^-(r,t) - (\gamma^+(r,t) + \delta^+(r,t))$.

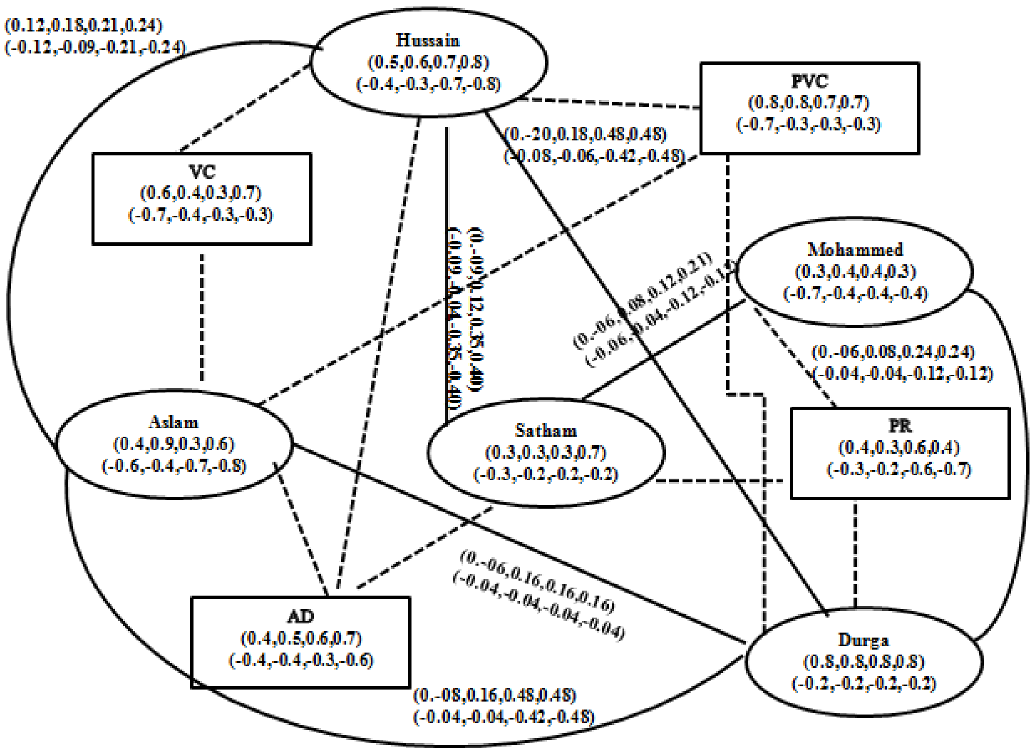


FIGURE 16. Bipolar Quadripartitioned Neutrosophic Competition Graph

5. Results and Discussion

In this study, quadripartitioned bipolar neutrosophic competition graphs are introduced as the generalisation of bipolar neutrosophic competition graphs. The sum of positive and negative truth, contradiction, ignorance and falsity membership functions are lies between -4 and 4. Finding a suitable job by the applicant and selecting a suitable applicant is the major issue. The proposed work is applied to find the suitable designation for the applicants with algorithm. We established the job competition model in quadripartitioned bipolar neutrosophic competition graphs to represent competition between the applicants along with algorithm. It generalises the existing works in the literature [19, 27]. A quadripartitioned neutrosophic pythagorean set is a powerful general format framework that generalizes the concept of quadripartitioned neutrosophic sets and neutrosophic pythagorean sets [32]. One can interpret the proposed method which yields the new concept in quadripartitioned bipolar neutrosophic pythagorean graphs. Pentapartitioned neutrosophic sets are a generalization of the single-valued and quadri-partitioned single-valued neutrosophic sets, and utilizes five symbol-valued neutrosophic logic. Pentapartitioned neutrosophic graphs and applications are established in [33]. Using the generalised score function, one can derive the application of pentapartitioned bipolar neutrosophic competition graphs. A new ranking function [34, 35] can be defined to apply the proposed concept in quadripartitioned triangular neutrosophic graphs. Also the proposed concepts are extended to quadripartitioned neutrosophic statistical models [36].

Conclusion and Future Direction

This manuscript dealt with the new concept of quadripartitioned bipolar neutrosophic competition graphs and the operations like a Cartesian product and direct product of quadripartitioned neutrosophic bipolar competition graphs with their properties have been discussed. The proposed concepts are illustrated with examples and application. The positive membership function denotes the applicant's eligibility and the negative membership function represents the ineligible percentage of applicants. The significant advantage of the proposed work is to find accuracy of suitable jobs by the applicants can be done by employing the quadripartitioned bipolar neutrosophic competition graphs. One can apply the proposed work in various quadripartitioned neutrosophic fields. The concept of considered graphs provided sufficient feasibility to suppress the related concepts on imprecise information. In future, we can extend the proposed graphs based on the properties of quadripartitioned neutrosophic rough competition graphs, interval valued quadripartitioned neutrosophic competition graphs, quadripartitioned neutrosophic trees and applications. Also, one can extend the developed concepts into isomorphic properties and regularity properties in the proposed graph structures. Furthermore, authors planned to use the ideas mentioned in the articles [37,38] and extend as Quadripartitioned trapezoidal bipolar neutrosophic competition graphs and application in decision making problems.

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