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Uncertainty-Based Measurement of Job Satisfaction Evaluation of University Teachers: An Improved Bipolar Neutrosophic Multi-Criteria Approach for Enhanced Decision-Making

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Abstract
Assessment of Job satisfaction in higher education persists as a complicated process involving many uncertain and regularly incompatible aspects. Old-style methods for quantifying job satisfaction are no longer able to account for growing uncertainty, paradoxes, and bipolarity in human opinions, a mix of satisfaction and dissatisfaction feedback. To fill this gap, we propose on bipolar neutrosophic decision-making (BNDM) approach to quantify job satisfaction within the vague, and bipolar nature of academic life. Our approach proposes a weighted bipolar neutrosophic cross-entropy technique to quantify optimization-based weighting of criteria. Then, we drive a weighted decision matrix and propose a bi-polar edition of evaluation based on distance from the average solution method to rank alternatives in a logical and appropriate way. Besides, we introduce a case study on job satisfaction among university teachers in the Middle East based on a questionnaire survey conducted across different universities. Proof-of-concept results are derived from the application of our approach to collected data to provide an in-depth analysis of aspects that might affect job satisfaction under uncertain and contradictory perceptions.

Keywords: Neutrosophic Logic, Bi-polar Neutrosophic Sets (BNS), Job Satisfaction, Higher Education, Uncertainty Quantification, Multi-Criteria Decision Making (MCDM)

1. Introduction

Job satisfaction is a critical factor in higher education institutions, influencing faculty performance, institutional reputation, and overall workplace harmony [1]. Numerous factors—including workload, institutional policies, leadership styles, and career growth—shape job satisfaction, often leading to conflicting emotions and perceptions among educators. In real-world scenarios, job satisfaction is rarely binary; it exists on a spectrum between satisfaction and dissatisfaction [2]. An academic professional may feel satisfied with career growth opportunities but dissatisfied with administrative support at the same time. Such bipolar tendencies demand a more flexible and uncertainty-resilient evaluation framework [3]. Traditional models for measuring job satisfaction rely on deterministic and probabilistic approaches, which fail to account for the inherent uncertainty, vagueness, and contradictions in human perception [4].

To handle the ambiguity and contradiction in decision-making, this study adopts Neutrosophic Logic (NL)—a mathematical framework introduced by Florentin Smarandache to extend fuzzy and intuitionistic fuzzy logic [5], [6]. Unlike classical logic, which operates on absolute truth (0 or 1), and fuzzy logic, which represents truth as a degree between 0 and 1, NL introduces three independent degrees including Truth (T), Falsity (F), and Indeterminacy (I) [7]. With the integration of $t, f, and\ I$ in an independent and dynamic manner, NL can provide a more comprehensive and exact approach to modelling human acuity and decision-making hard ambiguity. This makes it particularly useful for evaluating job satisfaction, where individuals may simultaneously experience positive and negative emotions, alongside uncertainty. The NL theory to deliberate the evolution of dissimilar types sets namely single-valued neutrosophic sets (SVNS) [8], interval-valued neutrosophic sets (IVNS) [9], triangular neutrosophic sets (TNS) [10], bi-polar neutrosophic sets (BNSs) [11], and others.

In job satisfaction assessment, existing methods such as Likert scales, fuzzy logic, and traditional multi-criteria decision-making (MCDM) techniques struggle with the dual nature of satisfaction and dissatisfaction [12]. These methods typically assume a unipolar structure, treating satisfaction as a linear scale from negative to positive, without accounting for simultaneous conflicting perceptions.

To address this limitation, we propose an Improved Bi-Polar Neutrosophic Decision-Making (BNCDM) for uncertainty-based job satisfaction assessment in higher education. The key contributions of this study are as follows. First, we propose to integrate BNS principles to handle imprecise, contradictory, and uncertain feedback in faculty surveys, aiming to enhance the reliability of job satisfaction assessment. Second, as an improvement to [13], we proposed and Bi-Polar Neutrosophic weighted-entropy based Inter-criteria Correlation method to appropriately decide the criterion weights while considering bipolar uncertainty in BNCDM. Third, we introduce an extended edition of the Evaluation Based on the Distance from Average Solution (EDAS) method that integrates the power of bipolar neutrosophic theory to obtain better decisions about the ranking of alternatives in uncertain environments. Finally, we present a case study for an academic job satisfaction-based survey from five universities to investigate real-world factors to provide empirical validation of the proposed approach. Figure 1 displays the outline of the paper.

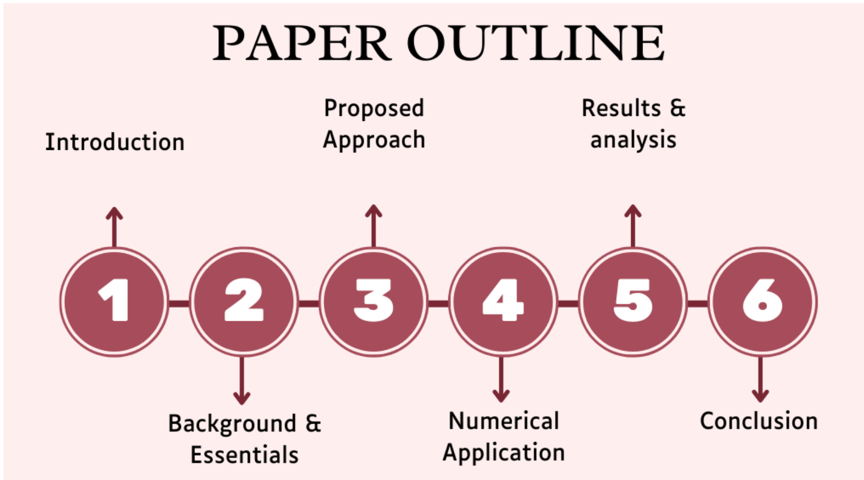

Figure 1. The visual organization of our study

2. Background & Essentials

Definition 1: A Neutrosophic Set (NS) is a mathematical framework extending the classical, fuzzy, and intuitionistic fuzzy sets with the inclusion of three independent membership functions:

$$A = \{\langle \mathfrak{y}, (\mathfrak{t}_A(\mathfrak{y}), \mathfrak{i}_A(\mathfrak{y}), \mathfrak{f}_A(\mathfrak{y}))\rangle : \mathfrak{y} \in \mathfrak{Y}\}. \quad (1)$$

These functions are defined as follows:

$$\mathfrak{t}_A: \mathfrak{Y} \to]^-0, 1^+[, \mathfrak{i}_A: \mathfrak{Y} \to]^-0, 1^+[, \mathfrak{f}_A: \mathfrak{Y} \to]^-0, 1^+[\text{ such that } 0^- \le \mathfrak{t}_A(\mathfrak{y}) + \mathfrak{i}_A(\mathfrak{y}) + \mathfrak{f}_A(\mathfrak{y}) \le 3^+ \quad (2)$$

Definition 2: A Single-Valued NSs (SVNSs) is a special case of the NS that constrains the truth, indeterminacy, and falsity membership values to the unit interval [0,1].

$$\text{SVNS} = \{\langle \mathfrak{y}, (\mathfrak{t}_A(\mathfrak{y}), \mathfrak{i}_A(\mathfrak{y}), \mathfrak{f}_A(\mathfrak{y}))\rangle : \mathfrak{y} \in \mathfrak{Y}\}, \quad (3)$$

where

$$\mathfrak{t}_A: \mathfrak{Y} \to [0,1], \mathfrak{i}_A: \mathfrak{Y} \to [0,1], \mathfrak{f}_A: \mathfrak{y} \to [0,1] \textit{ such that } 0 \le \mathfrak{t}_A(\mathfrak{y}) + \mathfrak{i}_A(\mathfrak{y}) + \mathfrak{f}_A(\mathfrak{y}) \le 3 \quad (4)$$

Definition 3: A BNS is an extension of traditional NS by introducing both positive and negative membership functions disjointedly.

$$A = \{\langle \mathfrak{y}, \mathfrak{t}^+(\mathfrak{y}), \mathfrak{i}^+(\mathfrak{y}), \mathfrak{f}^+(\mathfrak{y}), \mathfrak{t}^-(\mathfrak{y}), \mathfrak{i}^-(\mathfrak{y}), \mathfrak{f}^-(\mathfrak{y})\rangle : \mathfrak{y} \in \mathfrak{Y}\}. \quad (5)$$

where

$$\mathfrak{t}^+, \mathfrak{i}^+, \mathfrak{f}^+: \mathfrak{Y} \to [0,1], \mathfrak{t}^-, \mathfrak{i}^-, \mathfrak{f}^-: X \to [-1,0]. \quad (6)$$

Definition 4: Given two BNSs $A_1 = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{i}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{i}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y})\rangle$, and $A_2 = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{i}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{i}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\rangle$ in the discourse $\mathfrak{Y}$. Then the following operations are defined as follows:

A. Equality

$$A_1 = A_2 \; iff$$
$$\mathfrak{t}_1^+(\mathfrak{y}) = \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}) = \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}) = \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}) = \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}) = \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y}) = \mathfrak{f}_2^-(\mathfrak{y}). \tag{7}$$

B. Union

$$A_1 \cup A_2 = \left\{\left\langle \begin{matrix} \mathfrak{y}, max\left(\mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{t}_2^+(\mathfrak{y})\right), \dfrac{\mathfrak{l}_1^+(\mathfrak{y}) + \mathfrak{l}_2^+(\mathfrak{y})}{2}, min\left(\mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y})\right), \\ min\left(\mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y})\right), \dfrac{\mathfrak{l}_1^-(\mathfrak{y}) + \mathfrak{l}_2^-(\mathfrak{y})}{2}, max\left(\mathfrak{f}_1^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\right) \end{matrix} \right\rangle\right\} \tag{8}$$

C. Intersection

$$A_1 \cap A_2 = \left\{\left\langle \begin{matrix} \mathfrak{y}, min\left(\mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{t}_2^+(\mathfrak{y})\right), \dfrac{\mathfrak{l}_1^+(\mathfrak{y}) + \mathfrak{l}_2^+(\mathfrak{y})}{2}, max\left(\mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y})\right), \\ max(\mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y})), \dfrac{\mathfrak{l}_1^-(\mathfrak{y}) + \mathfrak{l}_2^-(\mathfrak{y})}{2}, min(\mathfrak{f}_1^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})) \end{matrix} \right\rangle\right\} \tag{9}$$

D. Subset

$$\begin{gathered} A_1 \subseteq A_2 \; iff \\ \mathfrak{t}_1^+(\mathfrak{y}) \leq \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}) \leq \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}) \geq \mathfrak{f}_2^+(\mathfrak{y}) \\ \&\& \\ \mathfrak{t}_1^-(\mathfrak{y}) \geq \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}) \geq \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y}) \leq \mathfrak{f}_2^-(\mathfrak{y}). \end{gathered} \tag{10}$$

E. Complement

$$A_1^c = \{\langle \mathfrak{y}, 1 - \mathfrak{t}_1^+(\mathfrak{y}), 1 - \mathfrak{l}_1^+(\mathfrak{y}), 1 - \mathfrak{f}_1^+(\mathfrak{y}), 1 - \mathfrak{t}_1^-(\mathfrak{y}), 1 - \mathfrak{l}_1^-(\mathfrak{y}), 1 - \mathfrak{f}_1^-(\mathfrak{y}) \rangle\} \tag{11}$$

Definition 5: Given two BNSs $A_1 = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y}) \rangle$, and $A_2 = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y}) \rangle$ in discourse $\mathfrak{Y}$, then the following operators of BNSs are depicted below:

$$\lambda A_1 = \langle 1 - (1 - \mathfrak{t}_1^+)^\lambda, (\mathfrak{l}_1^+)^\lambda, (\mathfrak{f}_1^+)^\lambda, -(-\mathfrak{t}_1^-)^\lambda, -(-\mathfrak{l}_1^-)^\lambda, -\left(1 - \left(1 - (\mathfrak{f}_1^-)\right)\right)^\lambda \rangle \tag{12}$$

$$A_1^\lambda = \langle (\mathfrak{t}_1^+)^\lambda, 1 - (1 - \mathfrak{l}_1^+)^\lambda, 1 - (1 - \mathfrak{f}_1^+)^\lambda, -(1 - (1 - (-\mathfrak{t}_1^-)))^\lambda, -(-\mathfrak{l}_1^-)^\lambda, -(-\mathfrak{f}_1^-)^\lambda \rangle \tag{13}$$

$$A_1 + A_2 = \langle \mathfrak{t}_1^+ + \mathfrak{t}_2^+ - \mathfrak{t}_1^+\mathfrak{t}_2^+, \mathfrak{l}_1^+\mathfrak{l}_2^+, \mathfrak{f}_1^+\mathfrak{f}_2^+, -\mathfrak{t}_1^-\mathfrak{t}_2^-, -(-\mathfrak{l}_1^- - \mathfrak{l}_2^- - \mathfrak{l}_1^-\mathfrak{l}_2^-), -(-\mathfrak{f}_1^- - \mathfrak{f}_2^- - \mathfrak{f}_1^-\mathfrak{f}_2^-) \rangle \tag{14}$$

$$A_1 A_2 = \langle \mathfrak{t}_1^+\mathfrak{t}_2^+, \mathfrak{l}_1^+ + \mathfrak{l}_2^+ - \mathfrak{l}_1^+\mathfrak{l}_2^+, \mathfrak{f}_1^+ + \mathfrak{f}_2^+ - \mathfrak{f}_1^+\mathfrak{f}_2^+, -(-\mathfrak{t}_1^- - \mathfrak{t}_2^- - \mathfrak{t}_1^-\mathfrak{t}_2^-), -\mathfrak{l}_1^-\mathfrak{l}_2^-, -\mathfrak{f}_1^-\mathfrak{f}_2^- \rangle \tag{15}$$

Definition 6: Given a BNS $A_1 = \langle \mathfrak{y}, \mathfrak{t}_1^{+}(\mathfrak{y}), \mathfrak{l}_1^{+}(\mathfrak{y}), \mathfrak{f}_1^{+}(\mathfrak{y}), \mathfrak{t}_1^{-}(\mathfrak{y}), \mathfrak{l}_1^{-}(\mathfrak{y}), \mathfrak{f}_1^{-}(\mathfrak{y})\rangle$, then, the score function $s(\tilde{b}1)$, accuracy function $a(\tilde{b}1)$ and certainty function $c(\tilde{b}1)$ are defined as follows:

$$s(\tilde{b}_1) = \frac{(\mathfrak{t}_1^{+} + 1 - \mathfrak{l}_1^{+} + 1 - \mathfrak{f}_1^{+} + 1 + \mathfrak{t}_1^{-} - \mathfrak{l}_1^{-} - \mathfrak{f}_1^{-})}{6} \tag{16}$$

$$a(\tilde{b}_1) = \mathfrak{t}_1^{+} - \mathfrak{f}_1^{+} + \mathfrak{t}_1^{-} - \mathfrak{l}_1^{-} \tag{17}$$

$$c(\tilde{b}_1) = \mathfrak{t}_1^{+} - \mathfrak{f}_1^{+} \tag{18}$$

Definition 7: Given a set of BNSs $\widetilde{\mathcal{B}} = \{B_1, B_2, B_3, \dots, B_n\}$, and the corresponding weights $W = \{w_1, w_2, \dots, w_n\}$ such that $0 \le w_j \le 1\&\&\sum_{k=1}^{n} w_k = 1$, then, the weighted average of this set is formulated as follows:

$$\mathrm{WA}_{\mathcal{W}}^{\mathrm{BNS}}(\widetilde{\mathcal{B}}) = \sum_{k=1}^{n} w_k t_k$$
$$= \left\langle \begin{array}{c} 1 - \prod_{k=1}^{n} \left(1 - \mathfrak{t}_{\theta_k}^{+}\right)^{w_k}, \prod_{k=1}^{n} \mathfrak{l}_{\theta_k}^{+w_k}, \prod_{k=1}^{n} \mathfrak{f}_{\theta_k}^{+w_k}, \\ -\prod_{k=1}^{n} \left(-\mathfrak{t}_{\theta_k}^{-}\right)^{w_k}, \\ -\left(1 - \prod_{k=1}^{n} \left(1 - \left(-\mathfrak{l}_{\theta_{t_k}^{-}}\right)\right)^{w_k}\right), -\left(1 - \prod_{k=1}^{n} \left(1 - \left(-\mathfrak{f}_{\theta_k}^{-}\right)\right)^{w_k}\right) \end{array} \right\rangle. \tag{19}$$

Definition 8: Given a set of BNSs $\widetilde{\mathcal{B}} = \{B_1, B_2, B_3, \dots, B_n\}$, and the corresponding weights $W = \{w_1, w_2, \dots, w_n\}$ such that $0 \le w_j \le 1\&\&\sum_{k=1}^{n} w_k = 1$, then, the weighted geometric operator of this set is formulated as follows:

$$\mathrm{WG}_{\mathcal{W}}^{\mathrm{BNS}}(\widetilde{\mathcal{B}}) = \prod_{k=1}^{n} \theta_k^{w_k}$$
$$= \left\langle \begin{array}{c} \prod_{k=1}^{n} \mathfrak{t}_{\theta_k}^{+w_k}, 1 - \prod_{k=1}^{n} \left(1 - \mathfrak{l}_{\theta_k}^{+}\right)^{w_k}, 1 - \prod_{k=1}^{n} \left(1 - \mathfrak{f}_{\theta_k}^{+}\right)^{w_k}, \\ -\left(1 - \prod_{k=1}^{n} \left(1 - \left(-\mathfrak{t}_{\theta_k}^{-}\right)\right)^{w_k}\right), -\prod_{k=1}^{n} \left(-\mathfrak{l}_{\theta_k}^{-}\right)^{w_k}, -\prod_{k=1}^{n} \left(-\mathfrak{f}_{\theta_k}^{-}\right)^{w_k} \end{array} \right\rangle \tag{20}$$

Definition 9: Given two BNSs $A_1 = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y})\rangle$, and $A_2 = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\rangle$, the dice similarity [14] between these two sets can be computed as follows:

$$Dice(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{2\left(\mathfrak{t}_A(\mathfrak{y}_i)\mathfrak{t}_B(\mathfrak{y}_i) + \mathfrak{l}_A(\mathfrak{y}_i)\mathfrak{l}_B(\mathfrak{y}_i) + \mathfrak{f}_A(\mathfrak{y}_i)\mathfrak{f}_B(\mathfrak{y}_i)\right)}{\begin{bmatrix}\left((\mathfrak{t}_A)^2(\mathfrak{y}_i) + (\mathfrak{l}_A)^2(\mathfrak{y}_i) + (\mathfrak{f}_A)^2(\mathfrak{y}_i)\right) + \\ \left((\mathfrak{t}_B)^2(\mathfrak{y}_i) + (\mathfrak{l}_B)^2(\mathfrak{y}_i) + (\mathfrak{f}_B)^2(\mathfrak{y}_i)\right)\end{bmatrix}}\right) \tag{21}$$

where

$$Dice(A_1, A_2) = Dice(A_1, A_2)\,;$$

$$Dice(A_1, A_2) = 1 \text{ for } A_1 = A_2$$

$$\mathfrak{t}_{A_1}(\mathfrak{y}_i) = \mathfrak{t}_{A_2}(\mathfrak{y}_i), \mathfrak{l}_{A_1}(\mathfrak{y}_i) = \mathfrak{l}_{A_2}(\mathfrak{y}_i), \mathfrak{f}_{A_1}(\mathfrak{y}_i) = \mathfrak{f}_{A_2}(\mathfrak{y}_i)(i = 1,2\ldots,n)\ \forall\ \mathfrak{y}_i(i = 1,2,\ldots,n) \in \mathfrak{Y}.$$

Definition 10: Given two BNSs $A = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y})\rangle$, and $B = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\rangle$, the Hybrid vector similarity (HVS) measures [15] between these two sets can be computed as follows:

$$\begin{aligned} HVS(A,B) &= \lambda\left(\frac{2\left(\mathfrak{t}_A(\mathfrak{y}_i)\mathfrak{t}_B(\mathfrak{y}_i) + \mathfrak{l}_A(\mathfrak{y}_i)\mathfrak{l}_B(\mathfrak{y}_i) + \mathfrak{f}_A(\mathfrak{y}_i)\mathfrak{f}_B(\mathfrak{y}_i)\right)}{[((\mathfrak{t}_A)^2(\mathfrak{y}_i) + (\mathfrak{l}_A)^2(\mathfrak{y}_i) + (\mathfrak{f}_A)^2(\mathfrak{y}_i)) + ((\mathfrak{t}_B)^2(\mathfrak{y}_i) + (\mathfrak{l}_B)^2(\mathfrak{y}_i) + (\mathfrak{f}_B)^2(\mathfrak{y}_i))]}\right) \\ &+ (1-\lambda)\left(\frac{\mathfrak{t}_A(\mathfrak{y}_i)\mathfrak{t}_B(\mathfrak{y}_i) + \mathfrak{l}_A(\mathfrak{y}_i)\mathfrak{l}_B(\mathfrak{y}_i) + \mathfrak{f}_A(\mathfrak{y}_i)\mathfrak{f}_B(\mathfrak{y}_i)}{\left[\sqrt{((\mathfrak{t}_A)^2(\mathfrak{y}_i) + (\mathfrak{l}_A)^2(\mathfrak{y}_i) + (\mathfrak{f}_A)^2(\mathfrak{y}_i))}.\sqrt{((\mathfrak{t}_B)^2(\mathfrak{y}_i) + (\mathfrak{l}_B)^2(\mathfrak{y}_i) + (\mathfrak{f}_B)^2(\mathfrak{y}_i))}\right]}\right) \end{aligned} \tag{22}$$

where

$$0 \le HVS(A,B) \le 1;$$

$$HVS(A,B) = HVS(B,A);$$

$$HVS(A,B) = 1\ \mathit{for}\ B = A\ \mathfrak{y}_i = y_i(i = 1,2\ldots,n)\forall \mathfrak{y}_i \in \mathfrak{Y}.$$

Definition 12: Given two BNSs $A_1 = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y})\rangle$, and $A_2 = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\rangle$, the weighted dice similarity [14] between these two sets can be computed as follows:

$$Dice_w(A,B) = \frac{1}{n}\sum_{i=1}^{n} w_i \times \left(\frac{2\left(\mathfrak{t}_A(\mathfrak{y}_i)\mathfrak{t}_B(\mathfrak{y}_i) + \mathfrak{l}_A(\mathfrak{y}_i)\mathfrak{l}_B(\mathfrak{y}_i) + \mathfrak{f}_A(\mathfrak{y}_i)\mathfrak{f}_B(\mathfrak{y}_i)\right)}{\begin{bmatrix}\left((\mathfrak{t}_A)^2(\mathfrak{y}_i) + (\mathfrak{l}_A)^2(\mathfrak{y}_i) + (\mathfrak{f}_A)^2(\mathfrak{y}_i)\right) + \\ \left((\mathfrak{t}_B)^2(\mathfrak{y}_i) + (\mathfrak{l}_B)^2(\mathfrak{y}_i) + (\mathfrak{f}_B)^2(\mathfrak{y}_i)\right)\end{bmatrix}}\right) \tag{23}$$

Definition 13: Given two BNSs $A = \langle \mathfrak{y}, \mathfrak{t}_1^+(\mathfrak{y}), \mathfrak{l}_1^+(\mathfrak{y}), \mathfrak{f}_1^+(\mathfrak{y}), \mathfrak{t}_1^-(\mathfrak{y}), \mathfrak{l}_1^-(\mathfrak{y}), \mathfrak{f}_1^-(\mathfrak{y})\rangle$, and $B = \langle \mathfrak{y}, \mathfrak{t}_2^+(\mathfrak{y}), \mathfrak{l}_2^+(\mathfrak{y}), \mathfrak{f}_2^+(\mathfrak{y}), \mathfrak{t}_2^-(\mathfrak{y}), \mathfrak{l}_2^-(\mathfrak{y}), \mathfrak{f}_2^-(\mathfrak{y})\rangle$, the weighted Hybrid vector similarity (HVS_w) measures [15] between these two sets can be computed as follows:

$$HVS_w(A,B) = \lambda \sum_{i=1}^{n} w_i \times \left(\frac{2\big(\mathrm{t}_A(\mathfrak{y}_i)\mathrm{t}_B(\mathfrak{y}_i) + \mathrm{l}_A(\mathfrak{y}_i)\mathrm{l}_B(\mathfrak{y}_i) + \mathrm{f}_A(\mathfrak{y}_i)\mathrm{f}_B(\mathfrak{y}_i)\big)}{\begin{bmatrix}\big((\mathrm{t}_A)^2(\mathfrak{y}_i) + (\mathrm{l}_A)^2(\mathfrak{y}_i) + (\mathrm{f}_A)^2(\mathfrak{y}_i)\big) + \\ \big((\mathrm{t}_B)^2(\mathfrak{y}_i) + (\mathrm{l}_B)^2(\mathfrak{y}_i) + (\mathrm{f}_B)^2(\mathfrak{y}_i)\big)\end{bmatrix}} \right) + (1 - \lambda) \sum_{i=1}^{n} w_i \times \left(\frac{\mathrm{t}_A(\mathfrak{y}_i)\mathrm{t}_B(\mathfrak{y}_i) + \mathrm{l}_A(\mathfrak{y}_i)\mathrm{l}_B(\mathfrak{y}_i) + \mathrm{f}_A(\mathfrak{y}_i)\mathrm{f}_B(\mathfrak{y}_i)}{\begin{bmatrix}\sqrt{\big((\mathrm{t}_A)^2(\mathfrak{y}_i) + (\mathrm{l}_A)^2(\mathfrak{y}_i) + (\mathrm{f}_A)^2(\mathfrak{y}_i)\big)} \\ \times \\ \sqrt{\big((\mathrm{t}_B)^2(\mathfrak{y}_i) + (\mathrm{l}_B)^2(\mathfrak{y}_i) + (\mathrm{f}_B)^2(\mathfrak{y}_i)\big)}\end{bmatrix}} \right) \tag{24}$$

3. Proposed Approach

In Step 1, we propose to normalize each BNS component for each pair of alternatives and criteria, in the decision matrix.

$$N_{i,j} = \langle t_{i,j}^{+}, l_{i,j}^{+}, f_{i,j}^{+}, t_{i,j}^{-}, l_{i,j}^{-}, f_{i,j}^{-} \rangle,$$

such that

$$t_{i,j}^{+} = \frac{t_{i,j}^{+}}{\sum_{i=1}^{m} t_{i,j}^{+}},\ l_{i,j}^{+} = \frac{l_{i,j}^{+}}{\sum_{i=1}^{m} l_{i,j}^{+}},\ f_{i,j}^{+} = \frac{f_{i,j}^{+}}{\sum_{i=1}^{m} f_{i,j}^{+}}$$
$$t_{i,j}^{-} = -\left|\frac{t_{i,j}^{-}}{\sum_{i=1}^{m} t_{i,j}^{-}}\right|,\ l_{i,j}^{-} = -\left|\frac{l_{i,j}^{-}}{\sum_{i=1}^{m} l_{i,j}^{-}}\right|,\ f_{i,j}^{-} = -\left|\frac{f_{i,j}^{-}}{\sum_{i=1}^{m} f_{i,j}^{-}}\right| \tag{25}$$

This operation helps ensure that all BNS components lie within the same scale, which makes them suitable for further analysis.

In step 2, for each criterion c_j, we propose the weighted BNS Cross-Entropy instead of standard entropy:

$$E_j = \left\langle E_j^{t^+}, E_j^{l^+}, E_j^{f^+}, E_j^{t^-}, E_j^{l^-}, E_j^{f^-} \right\rangle,$$

such that

$$E_j^{k} = -\frac{1}{\ln(m)} \sum_{i=1}^{m} ww_{i,j}^{k} \cdot X_{i,j}^{k} \cdot \ln\big(X_{i,j}^{k}\big), \tag{26}$$

with $k \in \{t^+, l^+, f^+, t^-, l^-, f^-\}$ represents each BNN component. $X_{i,j}^k$ is the normalized value for alternative y_i under criterion c_j. $W_{i,j}^k$ represents the adaptive weight, defined as:

$$ww_{i,j}^k = \left(\frac{1}{2} + \frac{\left|t_{i,j}^+ - t_{i,j}^-\right| + \left|f_{i,j}^+ - f_{i,j}^-\right|}{2}\right) \quad (27)$$

By applying this weight, we can amplify the entropy donations from exceedingly contradictory positive-negative interdependency (e.g., job satisfaction vs. dissatisfaction), while enhancing sensitivity for extreme paradoxes (such as high truth but high falsity).

In step 3, we compute objective weights determining the degree of reputation for each BNS component for each pair of alternatives and criteria, in the decision matrix.

$$d_j = \left\langle d_j^{t^+}, d_j^{l^+}, d_j^{f^+}, d_j^{t^-}, d_j^{l^-}, d_j^{f^-} \right\rangle,$$

such that:

$$\begin{aligned} d_j^{t^+} &= 1 - E_j^{t^+}, \quad d_j^{l^+} = 1 - E_j^{l^+}, \quad d_j^{f^+} = 1 - E_j^{f^+} \\ d_j^{t^-} &= 1 - E_j^{t^-}, \quad d_j^{l^-} = 1 - E_j^{l^-}, \quad d_j^{f^-} = 1 - E_j^{f^-}, \end{aligned} \quad (28)$$

By summing all components, we can reach the following:

$$w_j = \frac{\sum_k d_j^k}{\sum_{j=1}^n \sum_k d_j^k}, \quad (29)$$

where k represents BNS components.

In step 4, we propose to perform adjustment for BNS of cost criteria by transforming its constituting values using:

$$A'_{i,j} = \left\langle \frac{\min\left(t_{i,j}^+\right)}{t_{i,j}^+}, \frac{\min\left(l_{i,j}^+\right)}{l_{i,j}^+}, \frac{\min\left(f_{i,j}^+\right)}{f_{i,j}^+}, \frac{\max\left(t_{i,j}^-\right)}{t_{i,j}^-}, \frac{\max\left(l_{i,j}^-\right)}{l_{i,j}^-}, \frac{\max\left(f_{i,j}^-\right)}{f_{i,j}^-} \right\rangle \quad (30)$$

In step 5, we re-normalize the adjusted dataset by computing the normalized values:

$$X_{i,j} = \langle t'^{+}_{i,j}, l'^{+}_{i,j}, f'^{+}_{i,j}, t'^{-}_{i,j}, l'^{-}_{i,j}, f'^{-}_{i,j} \rangle$$

where

$$t'^{+}_{i,j} = \frac{t'^{+}_{i,j}}{\sum_{i=1}^{m} t'^{+}_{i,j}}, \text{ (similarly for } l'^{+}, f'^{+}, t'^{-}, l'^{-}, f'^{-}) \quad (31)$$

In step 6, we pairwise comparison matrix for each BNS component k in the decision matrix.

$$A_{i,j} = \left\langle \begin{matrix} \max(X_{i,,,}, t^{+}), \max(X_{i,,,l^{+}}), \max(X_{i,,,f^{+}}), \\ \max(X_{i,;,t^{-}}), \max(X_{i,;,l^{-}}), \max(X_{i,,,,f^{-}}) \end{matrix} \right\rangle \quad (32)$$

where diagonal elements are set to the maximum values in each column.

In step 7, we perform optimization of the weight distribution using a penalty matrix defined as follows:

$$P_{i,j} = \left\langle P^{t^{+}}_{i,j}, P^{l^{+}}_{i,j}, P^{f^{+}}_{i,j}, P^{t^{-}}_{i,j}, P^{l^{-}}_{i,j}, P^{f^{-}}_{i,j} \right\rangle$$

where

$$P^{k}_{i,j} = \frac{-A^{k}_{i,j} + \max(A^{k}_{:,j})}{\max(A^{k}_{:,j})} \quad (33)$$

In step 9, the weighted penalty function is to be minimized:

$$f(\mathrm{w}) = \sum_{k} \sum_{i=1}^{n} \sum_{j=1}^{n} P^{k}_{i,j} w_j,$$

such that (34)

$$\min_{\mathrm{w}} f(\mathrm{w}), \text{ subject to } \sum_{j=1}^{n} w_j = 1,\ w_j > 0$$

Then, the final weights of each criterion are derived as:

$$w^{*}_{j} = \frac{w_j}{\sum_{j=1}^{n} w_j}, \quad (35)$$

In step 10, using the calculated weights, we could calculate a weighted decision matrix as follows:

$$A_{ij}^{w_j}=\left\{t_{ij}^{w_j+},l_{ij}^{w_j+},f_{ij}^{w_j+},t_{ij}^{w_j-},l_{ij}^{w_j-},f_{ij}^{w_j-}\right\}=\left\{\begin{matrix}1-\left(1-t_{ij}^{+}\right)^{w_j},\left(l_{ij}^{+}\right)^{w_j},\left(f_{ij}^{+}\right)^{w_j},\\-\left(-t_{ij}^{-}\right)^{w_j},-\left(-l_{ij}^{-}\right)^{w_j},-\left(\left(1-\left(1-f_{ij}^{-}\right)\right)^{w_j}\right)\end{matrix}\right\} \quad (36)$$

In step 11, we compute the average BNS across all alternatives with respect to each criterion c_j:

$$AV_j=\left\langle AV_j^{t^+},AV_j^{l^+},AV_j^{f^+},AV_j^{t^-},AV_j^{l^-},AV_j^{f^-}\right\rangle$$

where:

$$AV_j^{t^+}=\frac{1}{m}\sum_{i=1}^{m}t_{ij}^{(w_j)+},\ AV_j^{l^+}=\frac{1}{m}\sum_{i=1}^{m}l_{ij}^{(w_j)+},\ AV_j^{f^+}=\frac{1}{m}\sum_{i=1}^{m}f_{ij}^{(w_j)+}$$
$$AV_j^{t^-}=\frac{1}{m}\sum_{i=1}^{m}t_{ij}^{(w_j)-},\ AV_j^{l^-}=\frac{1}{m}\sum_{i=1}^{m}l_{ij}^{(w_j)-},\ AV_j^{f^-}=\frac{1}{m}\sum_{i=1}^{m}f_{ij}^{(w_j)-} \quad (37)$$

The computed average solution acts as a reference point for evaluating the degree to which alternative deviates [16].

In step 12, we calculate the Positive Distance from Average (PDA) for each alternative y_i and criterion c_j:

$$PDA_{ij}=\left\langle PDA_{ij}^{t^+},PDA_{ij}^{l^+},PDA_{ij}^{f^+},PDA_{ij}^{t^-},PDA_{ij}^{l^-},PDA_{ij}^{f^-}\right\rangle$$

where:

$$PDA_{ij}^{t^+}=max\left(0,t_{ij}^{(w_j)+}-AV_j^{t^+}\right),\ PDA_{ij}^{l^+}=max\left(0,AV_j^{l^+}-l_{ij}^{(w_j)+}\right),\ PDA_{ij}^{f^+}=max\left(0,AV_j^{f^+}-f_{ij}^{(w_j)+}\right) \quad (38)$$

$$PDA_{ij}^{t^-}=min\left(0,t_{ij}^{(w_j)-}-AV_j^{t^-}\right),\ PDA_{ij}^{l^-}=min\left(0,AV_j^{l^-}-l_{ij}^{(w_j)^-}\right),\ PDA_{ij}^{f^-}=min\left(0,AV_j^{f^-}-f_{ij}^{(w_j)^-}\right)$$

In step 13, we calculate the Negative Distance from Average (NDA) for each alternative y_i and criterion c_j:

$$NDA_{ij} = \left\langle NDA_{ij}^{t^+}, NDA_{ij}^{l^+}, NDA_{ij}^{f^+}, NDA_{ij}^{t^-}, NDA_{ij}^{l^-}, NDA_{ij}^{f^-} \right\rangle$$

where:

$$NDA_{ij}^{t^+} = max\left(0, AV_j^{t^+} - t_{ij}^{(w_j)+}\right), NDA_{ij}^{l^+} = max\left(0, l_{ij}^{(w_j)+} - AV_j^{l^+}\right), NDA_{ij}^{f^+} = max\left(0, f_{ij}^{(w_j)+} - AV_j^{f^+}\right) \quad (39)$$

$$NDA_{ij}^{t^-} = min\left(0, AV_j^{t^-} - t_{ij}^{(w_j)-}\right), NDA_{ij}^{l^-} = min\left(0, l_{ij}^{(w_j)-} - AV_j^{l^-}\right), NDA_{ij}^{f^-} = min\left(0, f_{ij}^{(w_j)-} - AV_j^{f^-}\right)$$

In step 13, we calculate the aggregated PDA and NDA for each alternative

$$SPDA_i = \sum_{j=1}^{n} w_j \cdot s\left(PDA_{ij}^{t^+}, PDA_{ij}^{l^+}, PDA_{ij}^{f^+}, PDA_{ij}^{t^-}, PDA_{ij}^{l^-}, PDA_{ij}^{f^-}\right)$$
$$SNDA_i = \sum_{j=1}^{n} w_j \cdot s\left(PDA_{ij}^{t^+}, PDA_{ij}^{l^+}, PDA_{ij}^{f^+}, PDA_{ij}^{t^-}, PDA_{ij}^{l^-}, PDA_{ij}^{f^-}\right) \quad (40)$$

Finally, we compute the final ranking score of each alternative:

$$S_i = \frac{SPDA_i}{max(SPDA)} - \frac{SNDA_i}{max(SNDA)} \quad (41)$$

Accordingly, we find out that higher values of S_i indicates better alternative ↓.

4. Numerical Application

This research utilizes structured questionnaires to measure job satisfaction from different perspectives including but not limited to work environment, salary, career growth, academic autonomy, and institutional policies. The survey captures both qualitative and quantitative answers to amount ambiguity in faculty members' satisfaction degrees. The dataset includes answers from university faculty members within various age groups, genders, academic positions, and years of experience. Table 1 show tabular distribution of participants based on key demographic variables.

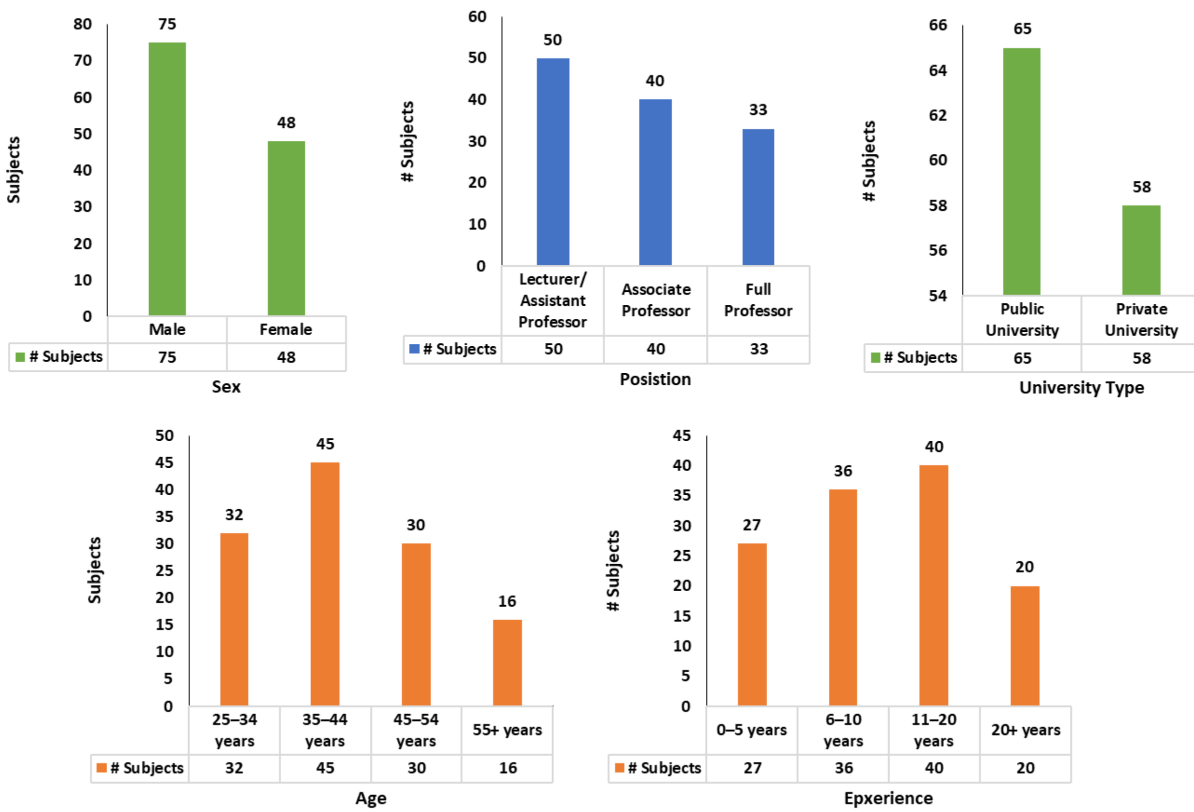

Figure 2: demographic distribution of respondents

The data collection in our case study includes broad range of criteria. Some needs to be maximized to indicate better satisfaction, and other that should be minimized to achieve better job satisfaction. These criteria include the following criteria Salary & Compensation, Career Growth Opportunities, Work Environment, Research Support [17], Teaching Autonomy, Work-Life Balance, Job Insecurity, and Bureaucratic Constraints. As part of our case study, we selected five distinctive universities from different regions in the Middle East to analyze job satisfaction. Each university vary from another one in different dimensions, thereby serving as an alternative for teachers in our MCDM along with the abovementioned criteria. We anonymously refer to these alternatives using their general university type Public Research University (U1), Private University (U2), Public Teaching University (U4), International Private Institution (U5), Newly Established University (U3). Table 1 show the summarize the aggregated BNS decision matrix from different experts.

Table 1. Aggregated BNS decision matrix for Job Satisfaction Assessment

U1	U2	U3	U4	U5

C 1	(0.887, 0.063, 0.014, -0.097, -0.09, -0.915)	(0.614, 0.221, 0.244, -0.326, -0.275, -0.593)	(0.834, 0.088, 0.081, -0.208, -0.231, -0.843)	(0.949, 0.193, 0.06, -0.053, -0.067, -0.929)	(0.127, 0.147, 0.876, -0.806, -0.173, -0.179)
C 2	(0.632, 0.326, 0.318, -0.241, -0.3, -0.665)	(0.894, 0.196, 0.021, -0.06, -0.088, -0.979)	(0.444, 0.257, 0.541, -0.54, -0.333, -0.292)	(0.733, 0.279, 0.184, -0.126, -0.21, -0.806)	(0.171, 0.105, 0.9, -0.829, -0.162, -0.076)
C 3	(0.173, 0.169, 0.938, -0.821, -0.173, -0.168)	(0.361, 0.348, 0.53, -0.467, -0.241, -0.362)	(0.912, 0.115, 0.092, -0.16, -0.133, -0.93)	(0.241, 0.158, 0.808, -0.837, -0.272, -0.16)	(0.941, 0.06, 0.131, -0.105, -0.09, -0.889)
C 4	(1.019, 0.04, 0.131, -0.191, -0.17, -0.909)	(0.806, 0.262, 0.113, -0.17, -0.112, -0.937)	(0.584, 0.395, 0.243, -0.266, -0.376, -0.582)	(0.386, 0.308, 0.578, -0.501, -0.274, -0.386)	(0.068, 0.159, 0.928, -0.828, -0.152, -0.219)
C 5	(0.975, 0.168, 0.111, -0.148, -0.153, -0.854)	(0.907, 0.188, 0.137, -0.173, -0.23, -0.928)	(0.526, 0.348, 0.276, -0.263, -0.401, -0.467)	(0.388, 0.312, 0.595, -0.496, -0.298, -0.42)	(0.177, 0.233, 0.961, -0.863, -0.081, -0.155)
C 6	(0.17, 0.203, 0.755, -0.841, -0.205, -0.11)	(0.693, 0.202, 0.17, -0.225, -0.292, -0.776)	(0.366, 0.317, 0.471, -0.558, -0.266, -0.399)	(0.15, 0.065, 0.881, -0.765, -0.213, -0.116)	(0.738, 0.28, 0.174, -0.186, -0.153, -0.835)
C 7	(0.888, 0.113, 0.101, -0.049, -0.169, -0.988)	(0.892, 0.23, 0.224, -0.18, -0.169, -0.823)	(0.11, 0.207, 0.783, -0.854, -0.11, -0.129)	(0.375, 0.292, 0.577, -0.576, -0.281, -0.304)	(0.205, 0.252, 0.863, -0.931, -0.124, -0.161)
C 8	(0.935, 0.089, 0.032, -0.098, -0.068, -0.892)	(0.915, 0.079, 0.153, -0.178, -0.134, -0.802)	(0.543, 0.387, 0.257, -0.279, -0.388, -0.503)	(0.293, 0.37, 0.518, -0.479, -0.2, -0.357)	(0.198, 0.124, 0.755, -0.936, -0.157, -0.079)

5. Results & analysis

This section presents the numerical and analytical findings of our approach for evaluating job satisfaction among university faculty members across different set of universities, bearing in mind different criteria.

First, we drive a normalized decision matrix in Table 2, where the scaled BNSs is presented for each alternative across different criteria.

Table 2: Normalized BNS Decision Matrix for Job Satisfaction Assessment

	U1	U2	U3	U4	U5
C 1	(0.26, 0.161, 0.17, -0.149, -0.173, -0.265)	(0.18, 0.218, 0.132, -0.164, -0.217, -0.171)	(0.245, 0.285, 0.152, -0.105, -0.183, -0.244)	(0.278, 0.191, 0.073, -0.177, -0.29, -0.269)	(0.037, 0.145, 0.473, -0.405, -0.137, -0.052)

C 2	(0.206, 0.26, 0.162, -0.117, -0.207, -0.2)	(0.291, 0.156, 0.011, -0.154, -0.323, -0.295)	(0.144, 0.205, 0.275, -0.264, -0.221, -0.088)	(0.238, 0.223, 0.094, -0.061, -0.142, -0.243)	(0.121, 0.156, 0.458, -0.404, -0.107, -0.174)
C 3	(0.066, 0.168, 0.347, -0.34, -0.18, -0.067)	(0.137, 0.346, 0.197, -0.193, -0.251, -0.144)	(0.347, 0.114, 0.108, -0.068, -0.139, -0.37)	(0.092, 0.157, 0.299, -0.347, -0.283, -0.066)	(0.358, 0.215, 0.048, -0.052, -0.147, -0.353)
C 4	(0.319, 0.16, 0.066, -0.097, -0.158, -0.3)	(0.269, 0.196, 0.057, -0.089, -0.103, -0.309)	(0.195, 0.295, 0.122, -0.136, -0.346, -0.192)	(0.129, 0.23, 0.29, -0.255, -0.252, -0.127)	(0.089, 0.119, 0.466, -0.422, -0.14, -0.072)
C 5	(0.328, 0.135, 0.053, -0.076, -0.112, -0.302)	(0.305, 0.151, 0.066, -0.089, -0.17, -0.328)	(0.177, 0.279, 0.133, -0.135, -0.294, -0.165)	(0.131, 0.25, 0.286, -0.255, -0.218, -0.149)	(0.06, 0.187, 0.462, -0.444, -0.206, -0.055)
C 6	(0.081, 0.16, 0.307, -0.327, -0.182, -0.091)	(0.327, 0.159, 0.073, -0.087, -0.259, -0.332)	(0.173, 0.249, 0.191, -0.217, -0.236, -0.171)	(0.072, 0.208, 0.358, -0.297, -0.189, -0.05)	(0.348, 0.224, 0.071, -0.072, -0.136, -0.357)
C 7	(0.078, 0.348, 0.534, -0.298, -0.185, -0.054)	(0.077, 0.17, 0.241, -0.406, -0.185, -0.062)	(0.324, 0.19, 0.069, -0.087, -0.265, -0.397)	(0.184, 0.135, 0.093, -0.129, -0.111, -0.168)	(0.337, 0.156, 0.062, -0.08, -0.253, -0.318)
C 8	(0.086, 0.126, 0.181, -0.166, -0.072, -0.104)	(0.087, 0.223, 0.392, -0.379, -0.331, -0.116)	(0.148, 0.126, 0.233, -0.242, -0.114, -0.185)	(0.274, 0.131, 0.116, -0.141, -0.201, -0.261)	(0.405, 0.394, 0.079, -0.072, -0.282, -0.334)

Then, we drive pairwise comparison matrix, as depicted in Table 3, which explain the relative performance of each alternative in the normalized matrix.

Table 3: Pairwise Comparison Matrix for Job Satisfaction Assessment

	C 1	**C 2**	**C 3**	**C 4**
C 1	(0.278, 0.285, 0.473, -0.105, -0.137, -0.052)	(0.238, 0.205, 0.458, -0.264, -0.107, -0.174)	(0.092, 0.114, 0.048, -0.068, -0.147, -0.353)	(0.129, 0.295, 0.466, -0.136, -0.14, -0.072)
C 2	(0.18, 0.161, 0.473, -0.177, -0.137, -0.244)	(0.291, 0.26, 0.458, -0.061, -0.107, -0.088)	(0.137, 0.168, 0.048, -0.347, -0.147, -0.37)	(0.269, 0.16, 0.466, -0.255, -0.14, -0.192)
C 3	(0.037, 0.218, 0.17, -0.405, -0.183, -0.269)	(0.121, 0.156, 0.162, -0.404, -0.221, -0.243)	(0.358, 0.346, 0.347, -0.052, -0.139, -0.066)	(0.089, 0.196, 0.066, -0.422, -0.346, -0.127)
C 4	(0.26, 0.285, 0.473, -0.164, -0.217, -0.052)	(0.206, 0.205, 0.458, -0.154, -0.323, -0.174)	(0.066, 0.114, 0.048, -0.193, -0.251, -0.353)	(0.319, 0.295, 0.466, -0.089, -0.103, -0.072)
C 5	(0.26, 0.285, 0.473, -0.149, -0.173, -0.052)	(0.206, 0.205, 0.458, -0.117, -0.207, -0.174)	(0.066, 0.114, 0.048, -0.34, -0.18, -0.353)	(0.319, 0.295, 0.466, -0.097, -0.158, -0.072)
C 6	(0.037, 0.285, 0.073, -0.405, -0.137, -0.269)	(0.121, 0.205, 0.094, -0.404, -0.107, -0.243)	(0.358, 0.114, 0.299, -0.052, -0.147, -0.066)	(0.089, 0.295, 0.29, -0.422, -0.14, -0.127)
C 7	(0.037, 0.161, 0.17, -0.405, -0.29, -0.265)	(0.121, 0.26, 0.162, -0.404, -0.142, -0.2)	(0.358, 0.168, 0.347, -0.052, -0.283, -0.067)	(0.089, 0.16, 0.066, -0.422, -0.252, -0.3)
C 8	(0.037, 0.145, 0.132, -0.405, -0.173, -0.265)	(0.121, 0.156, 0.011, -0.404, -0.207, -0.2)	(0.358, 0.215, 0.197, -0.052, -0.18, -0.067)	(0.089, 0.119, 0.057, -0.422, -0.158, -0.3)

	C 5	**C 6**	**C 7**	**C 8**

C 1	(0.131, 0.279, 0.462, -0.135, -0.206, -0.055)	(0.072, 0.249, 0.071, -0.217, -0.136, -0.357)	(0.184, 0.19, 0.062, -0.087, -0.253, -0.318)	(0.274, 0.126, 0.079, -0.242, -0.282, -0.334)
C 2	(0.305, 0.135, 0.462, -0.255, -0.206, -0.165)	(0.327, 0.16, 0.071, -0.297, -0.136, -0.171)	(0.077, 0.348, 0.062, -0.129, -0.253, -0.397)	(0.087, 0.126, 0.079, -0.141, -0.282, -0.185)
C 3	(0.06, 0.151, 0.053, -0.444, -0.294, -0.149)	(0.348, 0.159, 0.307, -0.072, -0.236, -0.05)	(0.337, 0.17, 0.534, -0.08, -0.265, -0.168)	(0.405, 0.223, 0.181, -0.072, -0.114, -0.261)
C 4	(0.328, 0.279, 0.462, -0.089, -0.17, -0.055)	(0.081, 0.249, 0.071, -0.087, -0.259, -0.357)	(0.078, 0.19, 0.062, -0.406, -0.185, -0.318)	(0.086, 0.126, 0.079, -0.379, -0.331, -0.334)
C 5	(0.328, 0.279, 0.462, -0.076, -0.112, -0.055)	(0.081, 0.249, 0.071, -0.327, -0.182, -0.357)	(0.078, 0.19, 0.062, -0.298, -0.185, -0.318)	(0.086, 0.126, 0.079, -0.166, -0.072, -0.334)
C 6	(0.06, 0.279, 0.286, -0.444, -0.206, -0.149)	(0.348, 0.249, 0.358, -0.072, -0.136, -0.05)	(0.337, 0.19, 0.093, -0.08, -0.253, -0.168)	(0.405, 0.126, 0.116, -0.072, -0.282, -0.261)
C 7	(0.06, 0.135, 0.053, -0.444, -0.218, -0.302)	(0.348, 0.16, 0.307, -0.072, -0.189, -0.091)	(0.337, 0.348, 0.534, -0.08, -0.111, -0.054)	(0.405, 0.126, 0.181, -0.072, -0.201, -0.104)
C 8	(0.06, 0.187, 0.066, -0.444, -0.112, -0.302)	(0.348, 0.224, 0.073, -0.072, -0.182, -0.091)	(0.337, 0.156, 0.241, -0.08, -0.185, -0.054)	(0.405, 0.394, 0.392, -0.072, -0.072, -0.104)

Following, we drive the penalty matrix in Table 4, in which we quantify the deviation of each alternative from the most favorable performance across all criteria, highlighting the degree of underperformance against to the best-achieving university.

Table 4: Penalty Matrix for Job Satisfaction Assessment

	C 1	**C 2**	**C 3**	**C 4**
C 1	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.182, 0.212, 0.0, -3.328, 0.0, -0.977)	(0.743, 0.671, 0.862, -0.308, -0.058, -4.348)	(0.596, 0.0, 0.0, -0.528, -0.359, 0.0)
C 2	(0.353, 0.435, 0.0, -0.686, 0.0, -3.692)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.617, 0.514, 0.862, -5.673, -0.058, -4.606)	(0.157, 0.458, 0.0, -1.865, -0.359, -1.667)
C 3	(0.867, 0.235, 0.641, -2.857, -0.336, -4.173)	(0.584, 0.4, 0.646, -5.623, -1.065, -1.761)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.721, 0.336, 0.858, -3.742, -2.359, -0.764)
C 4	(0.065, 0.0, 0.0, -0.562, -0.584, 0.0)	(0.292, 0.212, 0.0, -1.525, -2.019, -0.977)	(0.816, 0.671, 0.862, -2.712, -0.806, -4.348)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)
C 5	(0.065, 0.0, 0.0, -0.419, -0.263, 0.0)	(0.292, 0.212, 0.0, -0.918, -0.935, -0.977)	(0.816, 0.671, 0.862, -5.538, -0.295, -4.348)	(0.0, 0.0, 0.0, -0.09, -0.534, 0.0)
C 6	(0.867, 0.0, 0.846, -2.857, 0.0, -4.173)	(0.584, 0.212, 0.795, -5.623, 0.0, -1.761)	(0.0, 0.671, 0.138, 0.0, -0.058, 0.0)	(0.721, 0.0, 0.378, -3.742, -0.359, -0.764)
C 7	(0.867, 0.435, 0.641, -2.857, -1.117, -4.096)	(0.584, 0.0, 0.646, -5.623, -0.327, -1.273)	(0.0, 0.514, 0.0, 0.0, -1.036, -0.015)	(0.721, 0.458, 0.858, -3.742, -1.447, -3.167)
C 8	(0.867, 0.491, 0.721, -2.857, -0.263, -4.096)	(0.584, 0.4, 0.976, -5.623, -0.935, -1.273)	(0.0, 0.379, 0.432, 0.0, -0.295, -0.015)	(0.721, 0.597, 0.878, -3.742, -0.534, -3.167)

	C 5	**C 6**	**C 7**	**C 8**
C 1	(0.601, 0.0, 0.0, -0.776, -0.839, 0.0)	(0.793, 0.0, 0.802, -2.014, 0.0, -6.14)	(0.454, 0.454, 0.884, -0.087, -1.279, -4.889)	(0.323, 0.68, 0.798, -2.361, -2.917, -2.212)

C 2	(0.07, 0.516, 0.0, -2.355, -0.839, -2.0)	(0.06, 0.357, 0.802, -3.125, 0.0, -2.42)	(0.772, 0.0, 0.884, -0.612, -1.279, -6.352)	(0.785, 0.68, 0.798, -0.958, -2.917, -0.779)
C 3	(0.817, 0.459, 0.885, -4.842, -1.625, -1.709)	(0.0, 0.361, 0.142, 0.0, -0.735, 0.0)	(0.0, 0.511, 0.0, 0.0, -1.387, -2.111)	(0.0, 0.434, 0.538, 0.0, -0.583, -1.51)
C 4	(0.0, 0.0, 0.0, -0.171, -0.518, 0.0)	(0.767, 0.0, 0.802, -0.208, -0.904, -6.14)	(0.769, 0.454, 0.884, -4.075, -0.667, -4.889)	(0.788, 0.68, 0.798, -4.264, -3.597, -2.212)
C 5	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.767, 0.0, 0.802, -3.542, -0.338, -6.14)	(0.769, 0.454, 0.884, -2.725, -0.667, -4.889)	(0.788, 0.68, 0.798, -1.306, 0.0, -2.212)
C 6	(0.817, 0.0, 0.381, -4.842, -0.839, -1.709)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.0, 0.454, 0.826, 0.0, -1.279, -2.111)	(0.0, 0.68, 0.704, 0.0, -2.917, -1.51)
C 7	(0.817, 0.516, 0.885, -4.842, -0.946, -4.491)	(0.0, 0.357, 0.142, 0.0, -0.39, -0.82)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.0, 0.68, 0.538, 0.0, -1.792, 0.0)
C 8	(0.817, 0.33, 0.857, -4.842, 0.0, -4.491)	(0.0, 0.1, 0.796, 0.0, -0.338, -0.82)	(0.0, 0.552, 0.549, 0.0, -0.667, 0.0)	(0.0, 0.0, 0.0, 0.0, 0.0, 0.0)

Based on the above sequence of calculations, we drive the weights for job satisfaction criteria as shown in Table 5.

Table 5: Weights of Criteria for Job Satisfaction Assessment

Criteria	**C 1**	**C 2**	**C 3**	**C 4**	**C 5**	**C 6**	**C 7**	**C 8**
Weights	0.11411	0.11812	0.13514	0.12713	0.13013	0.12212	0.13213	0.12112

Based on the above sequence of calculations, we drive PDA for job satisfaction as shown in Table 6.

Table 6: PDA for Job Satisfaction Assessment

	U1	**U2**	**U3**	**U4**	**U5**
C 1	(0.058, 0.0, 0.001, 0.0, 0.0, -0.045)	(0.0, 0.011, 0.0, 0.0, -0.011, 0.0)	(0.023, 0.037, 0.0, 0.0, 0.0, -0.036)	(0.126, 0.0, 0.0, 0.0, -0.04, -0.047)	(0.0, 0.0, 0.11, -0.086, 0.0, 0.0)
C 2	(0.0, 0.029, 0.024, 0.0, -0.009, -0.008)	(0.111, 0.0, 0.0, 0.0, -0.057, -0.052)	(0.0, 0.005, 0.081, -0.048, -0.016, 0.0)	(0.022, 0.013, 0.0, 0.0, 0.0, -0.03)	(0.0, 0.0, 0.139, -0.096, 0.0, 0.0)
C 3	(0.0, 0.0, 0.093, -0.096, 0.0, 0.0)	(0.0, 0.069, 0.02, -0.024, -0.029, 0.0)	(0.136, 0.0, 0.0, 0.0, 0.0, -0.107)	(0.0, 0.0, 0.074, -0.098, -0.043, 0.0)	(0.174, 0.015, 0.0, 0.0, 0.0, -0.101)
C 4	(0.185, 0.0, 0.0, 0.0, 0.0, -0.063)	(0.043, 0.002, 0.0, 0.0, 0.0, -0.067)	(0.0, 0.048, 0.0, 0.0, -0.068, -0.009)	(0.0, 0.02, 0.075, -0.046, -0.033, 0.0)	(0.0, 0.0, 0.133, -0.106, 0.0, 0.0)

C 5	(0.216, 0.0, 0.0, 0.0, 0.0, -0.069)	(0.101, 0.0, 0.0, 0.0, 0.0, -0.079)	(0.0, 0.041, 0.0, 0.0, -0.048, 0.0)	(0.0, 0.028, 0.075, -0.051, -0.014, 0.0)	(0.0, 0.0, 0.135, -0.119, -0.008, 0.0)
C 6	(0.0, 0.0, 0.07, -0.074, 0.0, 0.0)	(0.058, 0.0, 0.0, 0.0, -0.028, -0.082)	(0.0, 0.024, 0.016, -0.026, -0.019, -0.006)	(0.0, 0.005, 0.089, -0.063, 0.0, 0.0)	(0.075, 0.013, 0.0, 0.0, 0.0, -0.09)
C 7	(0.126, 0.0, 0.0, 0.0, -0.003, -0.119)	(0.13, 0.01, 0.0, 0.0, -0.003, -0.101)	(0.0, 0.0, 0.08, -0.073, 0.0, 0.0)	(0.0, 0.036, 0.042, -0.024, -0.058, 0.0)	(0.0, 0.02, 0.093, -0.085, 0.0, 0.0)
C 8	(0.143, 0.036, 0.0, -0.003, -0.093, -0.062)	(0.119, 0.0, 0.0, 0.0, 0.0, -0.05)	(0.0, 0.035, 0.0, 0.0, -0.042, 0.0)	(0.0, 0.031, 0.041, -0.021, 0.0, 0.0)	(0.0, 0.0, 0.085, -0.098, 0.0, 0.0)

Based on the above sequence of calculations, we drive NDA for job satisfaction as shown in Table 7.

Table 7: NDA for Job Satisfaction Assessment

	U1	**U2**	**U3**	**U4**	**U5**
C 1	(0.0, 0.018, 0.0, -0.019, -0.011, 0.0)	(0.059, 0.0, 0.024, -0.01, 0.0, -0.003)	(0.0, 0.0, 0.01, -0.054, -0.006, 0.0)	(0.0, 0.002, 0.079, -0.002, 0.0, 0.0)	(0.147, 0.028, 0.0, 0.0, -0.033, -0.123)
C 2	(0.011, 0.0, 0.0, -0.037, 0.0, 0.0)	(0.0, 0.022, 0.215, -0.009, 0.0, 0.0)	(0.055, 0.0, 0.0, 0.0, 0.0, -0.08)	(0.0, 0.0, 0.03, -0.099, -0.028, 0.0)	(0.069, 0.023, 0.0, 0.0, -0.055, -0.008)
C 3	(0.119, 0.012, 0.0, 0.0, -0.007, -0.097)	(0.085, 0.0, 0.0, 0.0, 0.0, -0.011)	(0.0, 0.051, 0.051, -0.095, -0.035, 0.0)	(0.107, 0.019, 0.0, 0.0, 0.0, -0.099)	(0.0, 0.0, 0.138, -0.123, -0.029, 0.0)
C 4	(0.0, 0.019, 0.086, -0.06, -0.016, 0.0)	(0.0, 0.0, 0.1, -0.069, -0.058, 0.0)	(0.039, 0.0, 0.023, -0.025, 0.0, 0.0)	(0.085, 0.0, 0.0, 0.0, 0.0, -0.039)	(0.106, 0.049, 0.0, 0.0, -0.028, -0.101)
C 5	(0.0, 0.038, 0.109, -0.082, -0.057, 0.0)	(0.0, 0.026, 0.088, -0.066, -0.013, 0.0)	(0.072, 0.0, 0.014, -0.022, 0.0, -0.005)	(0.103, 0.0, 0.0, 0.0, 0.0, -0.017)	(0.14, 0.004, 0.0, 0.0, 0.0, -0.126)
C 6	(0.053, 0.022, 0.0, 0.0, -0.008, -0.061)	(0.0, 0.022, 0.085, -0.072, 0.0, 0.0)	(0.022, 0.0, 0.0, 0.0, 0.0, 0.0)	(0.056, 0.0, 0.0, 0.0, -0.004, -0.119)	(0.0, 0.0, 0.088, -0.091, -0.037, 0.0)
C 7	(0.0, 0.064, 0.149, -0.074, 0.0, 0.0)	(0.0, 0.0, 0.067, -0.107, 0.0, 0.0)	(0.094, 0.002, 0.0, 0.0, -0.034, -0.111)	(0.065, 0.0, 0.0, 0.0, 0.0, -0.02)	(0.095, 0.0, 0.0, 0.0, -0.029, -0.088)
C 8	(0.0, 0.0, 0.007, 0.0, 0.0, 0.0)	(0.0, 0.024, 0.085, -0.083, -0.066, 0.0)	(0.049, 0.0, 0.034, -0.037, 0.0, -0.004)	(0.098, 0.0, 0.0, 0.0, -0.017, -0.041)	(0.113, 0.079, 0.0, 0.0, -0.051, -0.067)

In Figure 3, we plot SPDA and SNDA scores for each university to offer a comparative evaluation of faculty job satisfaction across different institutions by explaining the level of deviation from the average satisfaction level, indicate fewer dissatisfaction factors.

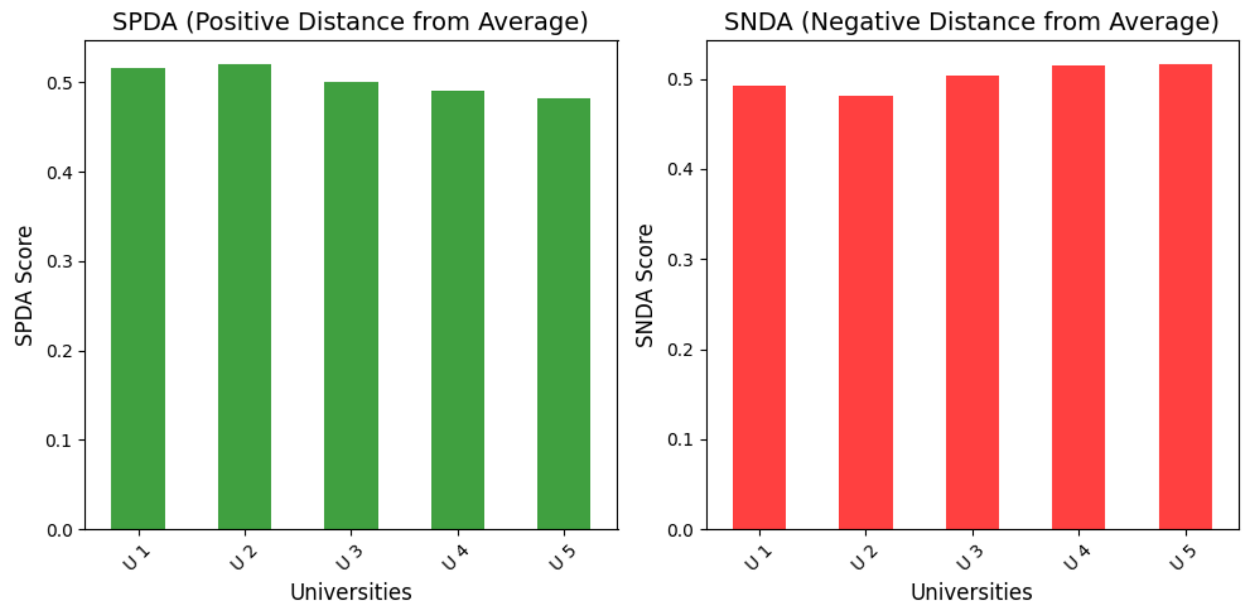

Figure 3. visualization of SPDA and SNDA scores

In Table 6, we display the performance score and the ranking of different alternatives.

Table 8. Ranking of different alternatives

Alternatives	Performance Score	Ranking
U1	**0.038700**	**2**
U2	**0.067286**	**1**
U3	**-0.014061**	**3**
U4	**-0.053327**	**4**
U5	**-.073508**	**5**

To ensure the robustness and reliability of our proposed, we conducted a sensitivity analysis by systematically varying the weights assigned to different criteria such that Scenario 1: +10%, Scenario 2: +5%, Scenario 3: 0%, Scenario 4: -5%, Scenario 5: -10%,. Then, we plot the new ranking results are displayed in Figure 4, showing the stability of the proposed model.

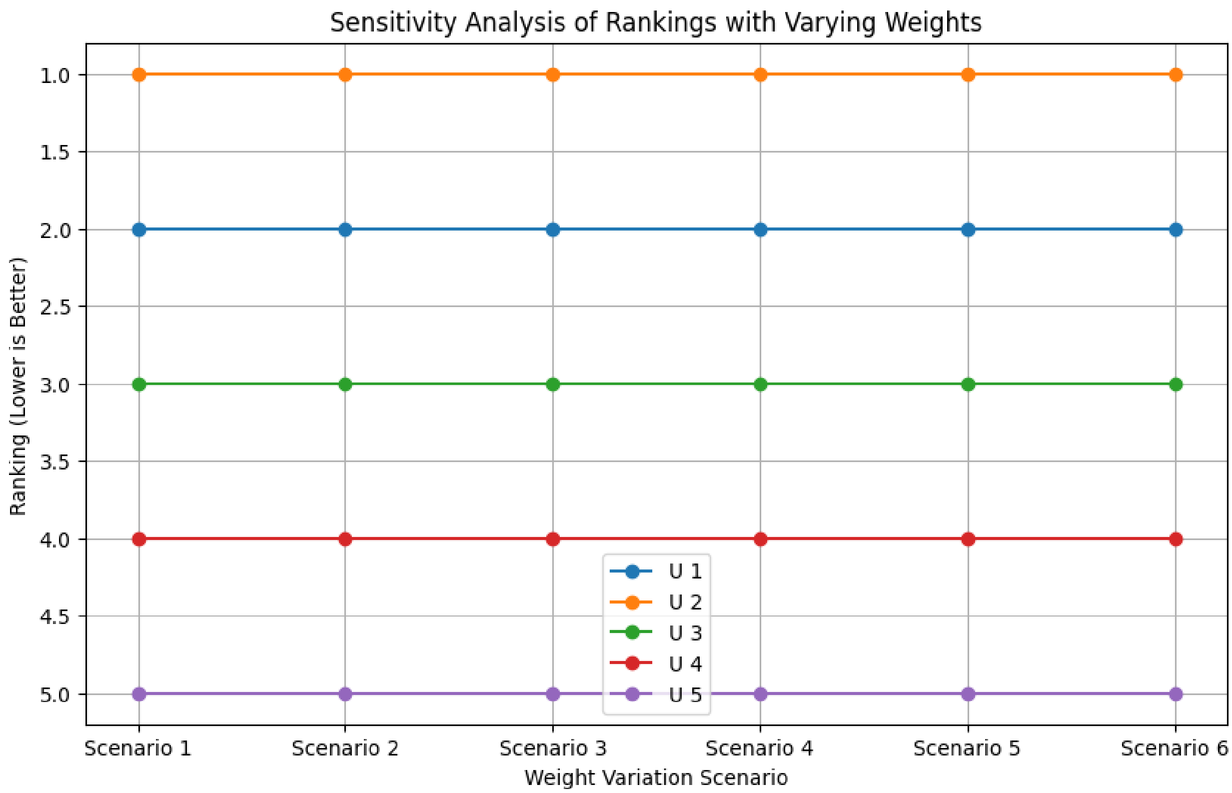

Figure 4. Sensitivity analysis of the proposed approach

6. Conclusion

This study presents an improved BNDN approach for uncertainty-based quantification of job satisfaction in higher education, addressing the limitations of traditional methods in capturing contradictory and imprecise faculty perceptions. With integration of BNSs, a bipolar entropy-based weighting method is derived to make systematically determine the importance of different criteria. Then, an BNS-based EDAS is presented to provides a more robust and comprehensive model for evaluating simultaneous satisfaction and dissatisfaction factors. We empirically validate our approach using a case study of university faculty in the Middle East, which demonstrate the effectiveness in handling uncertainty and supporting data-driven decision-making. The implications of our results offer practical insights for university administrators and policymakers to enhance faculty well-being and institutional performance.

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