



Multineutrosophic Analysis of Financial Statements as a Basis for Strategic Decision-Making in Uncertain Environments

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Abstract. The analysis of financial statements has been essential for managerial decision-making, guiding strategies in areas such as budgeting, investments, and risk management. However, traditional models have faced challenges due to the uncertainty and incomplete information inherent in real financial data. Therefore, this study focused on analyzing the key strategic guidelines to strengthen organizational resilience in volatile environments by integrating predictive and proactive financial analysis along with risk management. To achieve this, tools such as the Multineutrosophic Set and the ARAS multineutrosophic method were employed to evaluate the strategic guidelines and their impact. The results have indicated that the most relevant guidelines are the strengthening of predictive financial analysis and risk management, with an emphasis on organizational innovation. In conclusion, it has been observed that the integration of these guidelines enhances organizational capacity to adapt to unforeseen changes, facilitating resilient decision-making in dynamic environments.

Keywords: Financial analysis, dynamic environments, Multineutrosophic Set, ARAS multineutrosophic method.

1 Introduction

The analysis of financial statements plays a crucial role in managerial decision-making by providing a clear and detailed view of an organization's economic situation [1, 2]. These documents not only reflect the company's past performance but also serve as fundamental instruments for strategic planning, allowing managers to assess profitability, solvency, liquidity, and operational efficiency [3]. Therefore, in an increasingly volatile and uncertain business environment, financial statements face challenges related to uncertainty when identifying emerging trends. Moreover, they contribute to the timely adjustment of strategies, thereby ensuring long-term sustainability and growth [4].

In fact, the ability to interpret and effectively use financial statements facilitates key decision-making in areas such as budgeting, resource allocation in investments, and risk management [5, 6]. These analyses enable executives not only to assess the viability of projects and investments but also to anticipate the effects of various decisions under different economic scenarios. In this way, financial information becomes an essential component for informed decision-making that optimizes organizational performance and promotes adaptability in dynamic markets [7].

Based on the aforementioned information, this study focuses on analyzing key strategic guidelines for strengthening organizational resilience in volatile environments through the integration of predictive and proactive financial analysis, as well as financial and operational risk management. Additionally, it examines the impact of financial statement analysis on managerial decision-making by evaluating its influence on budgeting, investment management, and risk mitigation [8]. This approach aims to optimize business strategies in commercial environments characterized by uncertainty and constant change. To achieve this, tools such as the Multineutrosophic Set and the modeling of the multineutrosophic ARAS method are integrated for the selection of key strategic guidelines in dynamic and uncertain environments.

In this context, the present study seeks to expand the understanding of how the implementation of neutrosophic analysis can strengthen strategic decision-making in highly volatile financial environments. Specifically, it examines how these methods can optimize financial evaluation, reduce biases in analysis, and enhance organizational adaptive capacity in situations characterized by uncertainty and complexity, thus consolidating business resilience and sustainability.

2 Preliminaries

2.1 MultiNeutrosophic Set

Definition 1. The *Neutrosophic set* N is characterized by three membership functions [9], which are the truth-membership function T_A , indeterminacy-membership function I_A , and falsehood-membership function F_A , where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^{-}0, 1^{+}[$, and $^{-}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$ [10].

See that according to the definition, $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ and hence, $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^{-}0$ and 1^{+} belong to the set of hyperreal numbers.

Definition 2[10,11]. The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in U \}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$ and $F_A: U \rightarrow [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The SVNS emerged with the idea of applying neutrosophic sets for practical purposes. Some operations between SVNN are expressed below:

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ two SVNN, the sum between A_1 and A_2 is defined as:

$$A_1 \oplus A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \quad (1)$$

Given $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ two SVNN, the multiplication between A_1 and A_2 is defined as:

$$A_1 \otimes A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \quad (2)$$

The product of a positive scalar with an SVNN, $A = (a, b, c)$ is defined as:

$$A = (1 - (1 - a), b, c) \quad (3)$$

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3 [11]. The *Subset Refined Neutrosophic Set* (SRNS).

Let \mathcal{U} be a universe of discourse, and a set $R \subset \mathcal{U}$. Then a Subset Refined Neutrosophic R is defined as follows: $R = \{ \langle x, x(T, I, F), x \in U \rangle$, where T is refined/split into p sub-truths, $T = \langle T_1, T_2, \dots, T_p \rangle$, $T_j \subseteq [0, 1]$, $1 \leq j \leq p$; I is refined/split into r sub-indeterminacies, $I = \langle I_1, I_2, \dots, I_r \rangle$, $I_k \subseteq [0, 1]$, $1 \leq k \leq r$, and F is refined/split into s sub-falsehoods, $F = \langle F_1, F_2, \dots, F_s \rangle$, $F_s \subseteq [0, 1]$, $1 \leq l \leq s$, where $p, r, s \geq 0$ are integers, and $p + r + s = n \geq 2$, and at least one of p, r, s is ≥ 2 in order to ensure the existence of refinement (splitting).

Definition 4 ([12]). The *MultiNeutrosophic Set* (or *Subset MultiNeutrosophic Set* SMNS).

Let \mathcal{U} be a universe of discourse and M a subset of it. Then, a MultiNeutrosophic Set is: $M = \{ \langle x, x(T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s) \rangle, x \in U$,

where p, r, s are integers ≥ 0 , $p + r + s = n \geq 2$ and at least one of p, r, s is ≥ 2 , in order to ensure the existence of multiplicity of at least one neutrosophic component: truth/membership, indeterminacy, or falsehood/non-membership; all subsets $T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s \subseteq [0, 1]$;

$$0 \leq \sum_{j=1}^p \inf T_j + \sum_{k=1}^r \inf I_k + \sum_{l=1}^s \inf F_l \leq \sum_{j=1}^p \sup T_j + \sum_{k=1}^r \sup I_k + \sum_{l=1}^s \sup F_l \leq n.$$

No other restrictions apply on these neutrosophic multicomponents.

T_1, T_2, \dots, T_p are multiplicities of the truth, each one provided by a different source of information (expert).

Similarly, I_1, I_2, \dots, I_r are multiplicities of the indeterminacy, each one provided by a different source.

And F_1, F_2, \dots, F_s are multiplicities of the falsehood, each one provided by a different source.

The Degree of MultiTruth (MultiMembership), also called *MultiDegree of Truth*, of the element x with respect to the set M are T_1, T_2, \dots, T_p .

the Degree of MultiIndeterminacy (MultiNeutrality), also called *MultiDegree of Indeterminacy*, of the element x with respect to the set M are I_1, I_2, \dots, I_r .

and the Degree of MultiFalsehood (MultiNonmembership), also called *MultiDegree of Falsehood*, of element x with respect to the set M are F_1, F_2, \dots, F_s .

All these $p + r + s = n \geq 2$ are assigned by n sources (experts) that may be:

- either totally independent;
- or partially independent and partially dependent;
- or totally dependent; according or as needed to each specific application.

A generic element x with regard to the MultiNeutrosophic Set A has the form:

$$\begin{array}{ccc} x(T_1, T_2, \dots, T_p; & I_1, I_2, \dots, I_r; & F_1, F_2, \dots, F_s) \\ \text{multi-truth} & \text{multi-indeterminacy} & \text{multi-falsehood} \end{array}$$

In many particular cases $p = r = s$, and a source (expert) assigns all three degrees of truth, indeterminacy, and falsehood T_j, I_j, F_j for the same element.

Definition 5 [12]. Ranking of n -valued MultiNeutrosophic types of the same (p, r, s) -form.

$(T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s)$, where p, r, s are integers ≥ 0 , and $p + r + s = n \geq 2$, and at least one of $p, r, s \geq 2$ to be sure that it has multiplicity for at least one neutrosophic component (either truth, or indeterminacy, or falsehood).

It offers an easier n-ranking, but this is rather an approximation. Let's compute the following.

Average positivity (4).

$$\frac{\sum_{j=1}^p T_j + \sum_{k=1}^r (1 - I_k) + \sum_{e=1}^s (1 - F_e)}{p + r + s} \quad (4)$$

Average (Truth-Falsehood) (5)

$$\frac{\sum_{j=1}^p T_j + \sum_{e=1}^s (1 - F_e)}{p + s} \quad (5)$$

Average Truth (6).

$$\frac{\sum_{j=1}^p T_j}{p} \quad (6)$$

Definition 6 [12]. Ranking n-valued MultiNeutrosophic tuples of different (p, r, s) -forms.

Let's consider two n-valued multi neutrosophic tuples of the forms (p_1, r_1, s_1) and respectively (p_2, r_2, s_2) , where $p_1, r_1, s_1, p_2, r_2, s_2$ are integers ≥ 0 , and $p_1 + r_1 + s_1 = n_1 \geq 2$, and at least one of p_1, r_1, s_1 is ≥ 2 , to be sure that there is multiplicity for at least one neutrosophic component (either truth, or indeterminacy, or falsehood); similarly $p_2 + r_2 + s_2 = n_2 \geq 2$, and at least one of p_2, r_2, s_2 is ≥ 2 .

Let's take the following Single-Valued Multi Neutrosophic Tuples (SVMNT):

$SVMNT = (T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s)$ of (p_1, r_1, s_1) - form, and

$SVMNT' = (T'_1, T'_2, \dots, T'_p; I'_1, I'_2, \dots, I'_r; F'_1, F'_2, \dots, F'_s)$ of (p_1, r_1, s_1) - form.

It makes the classical averages of truth (T_a), indeterminacies (I_a) and falsehood (F_a), respectively for $SVMNT = (T_a, I_a, F_a)$ and the averages of truths (T_a), indeterminacies (I_a) and falsehood (F_a) respectively for: $SVMNT = (T'_a, I'_a, F'_a)$. And then it applies the Score (S), Accuracy (A), and Certainty (C) Functions, as for the single valued neutrosophic set:

Compute the Score Function (average of positiveness) (7).

$$S(T_a, I_a, F_a) = \frac{T_a + (1 - I_a) + (1 - F_a)}{3}$$

$$S(T'_a, I'_a, F'_a) = \frac{T'_a + (1 - I'_a) + (1 - F'_a)}{3} \quad (7)$$

- i. If $S(T_a, I_a, F_a) \geq S(T'_a, I'_a, F'_a)$ then $SVMNT \geq SVMNT'$,
- ii. If $S(T_a, I_a, F_a) \leq S(T'_a, I'_a, F'_a)$ then $SVMNT \leq SVMNT'$,
- iii. And if $S(T_a, I_a, F_a) = S(T'_a, I'_a, F'_a)$ then $SVMNT = SVMNT'$, then go to the second step.

Compute the Accuracy Function (difference between the truth and falsehood) (8).

$$A(T_a, I_a, F_a) = T_a - F_a$$

$$A(T'_a, I'_a, F'_a) = T'_a - F'_a \quad (8)$$

- i. If $A(T_a, I_a, F_a) \geq A(T'_a, I'_a, F'_a)$ then $SVMNT \geq SVMNT'$,
- ii. If $A(T_a, I_a, F_a) \leq A(T'_a, I'_a, F'_a)$ then $SVMNT \leq SVMNT'$,
- iii. And if $A(T_a, I_a, F_a) = A(T'_a, I'_a, F'_a)$ then $SVMNT = SVMNT'$, then go to the third step.

3. Compute the Certainty Function (truth) (9).

$$C(T_a, I_a, F_a) = T_a$$

$$C(T'_a, I'_a, F'_a) = T'_a \quad (9)$$

- i. If $C(T_a, I_a, F_a) \geq C(T'_a, I'_a, F'_a)$ then $SVMNT \geq SVMNT'$,
- ii. If $C(T_a, I_a, F_a) \leq C(T'_a, I'_a, F'_a)$ then $SVMNT \leq SVMNT'$,
- iii. And if $C(T_a, I_a, F_a) = C(T'_a, I'_a, F'_a)$ then $SVMNT = SVMNT'$ are multi-neutrosophically equal, i.e. $T_a = T'_a, I_a = I'_a, F_a = F'_a$, or their corresponding truth, indeterminacy, and falsehood averages are equal.

Definition 7 [12]. In cases some sources have a greater weight in evaluation than others, one uses the weighted averages, indexed as T_{wa}, I_{ua}, F_{va} and $T'_{wa}, I'_{ua}, F'_{va}$, respectively. Because the sources may be independent or partially independent, the sum of weights should not necessarily be equal to 1. As such:

- i. $w_1, w_2, \dots, w_p \in [0,1]$, while the sum $w_1 + w_2 + \dots + w_p$ may be < 1 , or $= 1$, or > 1 .
- ii. $u_1, u_2, \dots, u_p \in [0,1]$, while the sum $u_1 + u_2 + \dots + u_p$ may be < 1 , or $= 1$, or > 1 .
- iii. $v_1, v_2, \dots, v_p \in [0,1]$, while the sum $v_1 + v_2 + \dots + v_p$ may be < 1 , or $= 1$, or > 1 .

And, similarly, one applies the Score, Accuracy, and Certainty Functions on these weighted averages to rank them.

In 2013, Smarandache [13] refined and split the neutrosophic components (T, I, F) into more detailed neutrosophic subcomponents ($T_1, T_2, \dots; I_1, I_2, \dots; F_1, F_2, \dots$), allowing for greater granularity in modeling uncertainty and imprecision. This refinement is based on the need to more accurately represent the levels of truth, indeterminacy, and falsehood in complex systems.

This refined structure has been key in the development of new applications in artificial intelligence, decision-making, and computational modeling, as it enables a more detailed analysis of uncertain information. Additionally, it has been demonstrated that the MultiNeutrosophic Set is isomorphic to the Refined Neutrosophic Set [12], implying that both models can represent the same underlying mathematical structure, providing different approaches to managing uncertainty.

3. Materials and Methods

The ARAS (Additive Ratio Assessment) method is a multi-criteria decision-making technique used to select the best option from a set of alternatives [15]. In this case, the study establishes among its objectives a series of strategic guidelines aimed at enhancing decision-making in financial analysis. To this end, an extension of the traditional method is proposed through evaluation using multineutrosophic sets. Consequently, it is reformulated as the multineutrosophic ARAS method to determine the complex relative efficiency of each strategic guideline. This involves evaluating each strategic guideline through multiple sources (experts) based on the corresponding criteria (Figure 1).

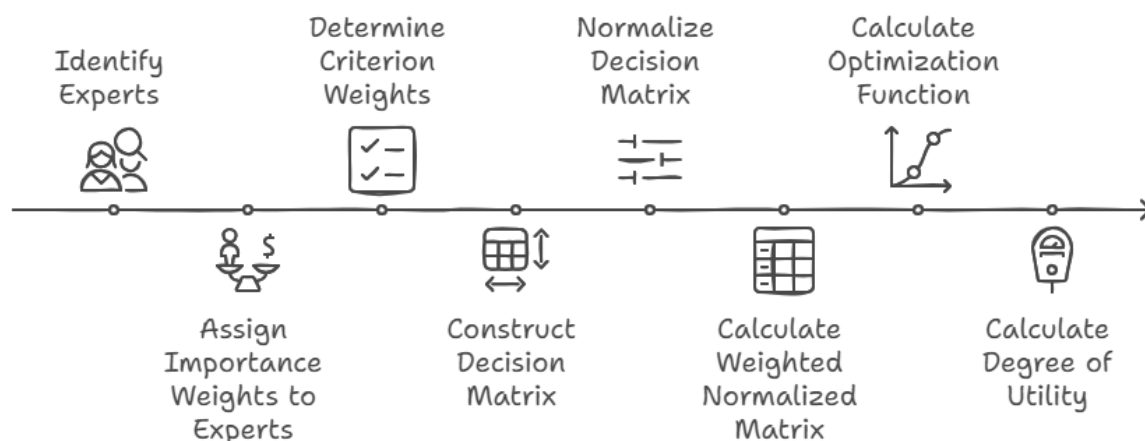


Figure 1. Multicriteria Decision Making Process

By integrating the analysis of multineutrosophic sets into the ARAS method, the following steps are defined:

Step 1: Identify multiple sources (experts) for the multi-criteria evaluation and assign an importance weight to each expert based on their knowledge and contribution to financial statement analysis (according to Definition 7 in Section 2.1). For this purpose, the neutrosophic AHP method by Saaty is applied (following procedures referenced in bibliographic sources [13,15]).

Step 2: Determine the importance weights of each criterion in decision-making for each source (expert).

Step 3: Construct the decision matrix L_{ij} (see Figure 2), where the element L_{ij} represents each strategic guideline (SG) evaluated by multiple sources (experts (Exp.), according to Definitions 5 and 6 in Section 2.1) based on an identified criterion (C).

$$\begin{bmatrix} l_{11} & l_{12} & \dots & l_{1j} & \dots & l_{1n} \\ l_{21} & l_{22} & \dots & l_{2j} & \dots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \dots & l_{ij} & \dots & l_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{m1} & l_{m2} & \dots & l_{mj} & \dots & l_{mn} \end{bmatrix}$$

Figure 2: Decision matrix L_{ij} for the multineutrosophic ARAS method. Source: Own elaboration.

Step 4: The normalized decision matrix \bar{L}_{ij} , considering the benefit and cost values, is calculated using Equation (10) and (11):

$$\bar{L}_{ij} = \frac{l_{ij}}{\sum_{i=0}^m l_{ij}} \quad (10)$$

$$L_{ij} = \frac{1}{l_{ij}^*} \quad (11)$$

Step 5: The weighted normalized decision matrix is calculated using Equation (12).

$$\hat{L}_{ij} = \bar{L}_{ij} \cdot W_j \quad (12)$$

The weight values W_j are determined by the Entropy method. Where W_j is the weight of criterion j and \bar{L}_{ij} is the normalized ranking of each criterion.

Step 6: Calculation of the optimization function S_i using Equation (13).

$$G_i = \sum \hat{L}_{ij} \quad (13)$$

Where G_i is the value of the optimization function of the alternative i . This calculation has a directly proportional relationship with the process of the values \hat{L}_{ij} and weights W_j of the investigated criteria and their relative influence on the result.

Step 7: Calculation of the degree of utility. This degree is determined by comparing the variant being analyzed with the best one G_o , according to Equation (14).

$$K_i = \frac{G_i}{G_o} \quad (14)$$

Where G_i and G_o are the values of the optimization function. These values range from 0 to 100%, therefore, the alternative with the highest K_i is the best of the alternatives analyzed.

4 Case Study

4.1 Impact of financial information on strategic decision-making.

The quality and relevance of financial information are crucial for effective managerial decision-making. An analysis of financial statements must evaluate three key aspects: relevance, reliability, and timeliness. Relevance ensures that the information influences strategic and operational decisions by providing meaningful data aligned with corporate objectives, such as key performance indicators including EBITDA (Earnings Before Interest, Taxes, Depreciation, and Amortization), ROE (Return on Equity), and net profit margin. In contrast, reliability guarantees that the data accurately reflect the company's financial situation by preventing biases and material errors, which is essential for financial stability. Meanwhile, timeliness ensures that information is available promptly, enabling proactive decision-making in response to threats or opportunities and preventing outdated decisions that could impact organizational performance.

Additionally, financial analysis is critical for budgeting and investment planning, as it allows for assessing an organization's economic performance and making informed strategic decisions. In budgeting, it facilitates the realistic projection of revenues, expenses, and capital needs while ensuring control and identifying deviations. In investment planning, financial analysis evaluates the feasibility and profitability of projects using tools such as Net Present Value (NPV) and Internal Rate of Return (IRR), while managing risks related to liquidity, solvency, and capital structure. Key profitability, liquidity, solvency, and operational efficiency indicators guide strategic and operational decisions, optimizing resource allocation and maximizing organizational profitability.

Similarly, financial analysis is essential in corporate risk management, as it helps identify potential threats and adopt proactive measures to mitigate risks. By analyzing key indicators such as liquidity, solvency, profitability, and operational efficiency, financial, operational, and strategic risks can be detected. Early detection of these risks facilitates the implementation of preventive and corrective strategies. Furthermore, financial analysis enables forecasting future risks through simulations and projections, optimizing resource allocation and enhancing organizational stability. This continuous process, supported by monitoring tools and predictive analysis, ensures the resilience and adaptability of the organization in a changing environment.

Moreover, integrating expert judgment, financial analysis, and MultiNeutrosophy optimizes strategic decision-making by combining the rigor of financial metrics with contextual experience and a comprehensive analysis of uncertainty. Thus, it enables the processing of multiple sources of information by balancing quantitative and qualitative dimensions, strengthening organizational resilience and proactive risk management in dynamic environments. Its practical application is evident in the combined evaluation of profitability and risks, the implementation of dynamic dashboards, scenario simulations, comprehensive opportunity identification, and multidisciplinary participation, all supported by continuous monitoring and iterative feedback. This approach enables organizations to make more accurate decisions, avoiding those based solely on numerical data or unsubstantiated intuition.

4.2 Strategic guidelines: Integration of MultiNeutrosophy in financial analysis.

The integration of financial analysis with multi-expert judgment provided by the MultiNeutrosophic Set and advanced technological tools enables companies to develop a more informed, adaptable, and resilient decision-making process. By adopting these strategic guidelines, organizations not only strengthen their ability to face uncertainty and constant change but also promote continuous improvement in business management, ensuring sustainable and competitive growth (see Table 1).

No	Strategic guideline	Objective	Strategies	Impact
LE1	Strengthening predictive and proactive financial analysis	Anticipate scenarios and potential risks through financial projections adjusted to the variability of possible scenarios.	Implement predictive analysis models based on time series and machine learning to project revenue, expenses, and cash flows while accounting for uncertainty. Develop advanced multineutrosophic sensitivity analyses and scenario simulations (optimistic, moderate, pessimistic, and neutrosophic), evaluated by multi-experts. Conduct quarterly projections adjusted according to market evolution and internal variations.	Enhances the ability to make informed, adaptive, and resilient decisions in the face of economic uncertainties and variable scenarios.
LE2	Encouraging a culture of innovation and financial adaptability	Promote organizational flexibility through agile and adaptive financial practices.	Establish flexible budgets that allow dynamic adjustments in response to macroeconomic changes. Encourage continuous training of the financial team in advanced tools and multineutrosophic approaches for decision-making. Promote the adoption of emerging technologies, such as financial process automation and artificial intelligence.	Increases organizational innovation and adaptability by maintaining financial stability amid unpredictable market changes.
LE3	Prioritizing key financial and non-financial performance indicators (KPIs)	Comprehensively monitor organizational performance using both quantitative and qualitative metrics.	Define key financial KPIs such as EBITDA, profit margin, asset turnover, and cash conversion cycle, considering uncertain scenarios. Complement financial KPIs with non-financial indicators such as customer satisfaction, operational efficiency, and organizational commitment, incorporating neutrosophic variations in their analysis. Automate real-time data collection and analysis, adapting it to projected scenarios.	Improves organizational visibility and flexibility by optimizing resource allocation and operational efficiency through quantitative and qualitative metrics.
LE4	Integrating expert judgment into financial evaluation	Complement quantitative analysis with the contextualized expertise of professionals to maximize evaluation accuracy.	Establish flexible multidisciplinary committees to evaluate investment projects, combining quantitative analysis with expert judgment from key areas (operations, marketing, legal). Incorporate feedback from industry experts and external consultants to integrate perspectives on uncertainty. Develop interactive dashboards that combine financial indicators with qualitative interpretations by integrating multineutrosophic evaluation from multiple sources.	Enhances decision accuracy and reduces cognitive biases, promoting more comprehensive and adaptive choices in uncertain contexts.
LE5	Promoting continuous improvement through financial audits and feedback	Ensure the accuracy, transparency, and relevance of financial information for long-term decision-making.	Implement periodic internal and external audits to validate the reliability of information, incorporating variability analysis. Establish continuous feedback mechanisms in which financial results are analyzed and adjusted according to uncertain scenarios. Develop dynamic financial reports accessible at all management levels, adjusting them in real time.	Strengthens organizational transparency by aligning decisions with evolving business environments.
LE6	Focus on financial and operational risk management	Identify, mitigate, and manage financial and operational risks through a multi-source (multineutrosophic) analysis that considers	Create a financial risk map that considers variables such as exchange rate exposure and credit risk while accounting for uncertainties. Implement early warning alerts based on key metrics adjustable to internal and external uncertainties. Design contingency plans by simulating adverse	Mitigates negative impacts from unforeseen events by improving resilience against complex financial and operational risks.

LE7	Implementing a dynamic working capital management approach	environmental uncertainties. Optimize organizational liquidity and solvency through a flexible working capital management approach.	scenarios to reformulate strategies in the face of potential crises. Establish collection and payment policies aligned with cash flow projections adjusted for uncertainty analysis. Continuously monitor inventory turnover and adjust purchasing decisions through demand simulations, integrating multineutrosophic analysis. Apply financial analysis to identify cost-reduction opportunities without compromising quality.	Increases organizational liquidity and enhances responsiveness to uncertain scenarios with a flexible approach.
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Table 1: Strategic guidelines for continuous improvement in business decision-making. Source: own elaboration.

3.3 Multineutrosophic ARAS Modeling. Selection of the strategic guideline.

Step 1: Selection of multiple sources (experts) in the multicriteria evaluation. Modeling of the neutrosophic Saaty AHP method.

To provide an effective interpretation of financial statements, it is crucial to involve experts from various disciplines who can offer a comprehensive and detailed view of an organization's financial situation (see Table 2). Below, eight expert professions are evaluated to assess each strategic guideline.

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Profession	Certified Public Accountant	Internal Auditor	Economist	Financial risk manager	Financial Analyst	Tax advisor	Financial Consultant	Corporate Lawyer

Table 2: Multi-sources for the neutrosophic evaluation (experts). Source: Own elaboration.

To assign an importance weight to each expert in relation to their knowledge and contribution to the analysis of financial statements, several factors must be considered, such as technical specialization, the ability to interpret financial data, and experience in the practical application of such knowledge. Below, a weight distribution is proposed, based on the modeling of Saaty's neutrosophic AHP method (see Tables 3 and 4). Therefore, it should be noted that some roles have a more direct impact on the interpretation and analysis of financial statements, while others provide complementary perspectives.

Source	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Exp-1	(0.54,0.51,0.52)	(0.64,0.41,0.42)	(0.94,0.11,0.12)	(0.94,0.11,0.12)	(0.54,0.51,0.52)	(0.94,0.11,0.12)	(0.74,0.31,0.32)	(0.84,0.21,0.22)
Exp-2	(0.44,0.61,0.62)	(0.54,0.51,0.52)	(0.74,0.31,0.32)	(0.74,0.31,0.32)	(0.24,0.81,0.82)	(0.74,0.31,0.32)	(0.54,0.51,0.52)	(0.74,0.31,0.32)
Exp-3	(0.14,0.91,0.92)	(0.34,0.71,0.72)	(0.54,0.51,0.52)	(0.64,0.41,0.42)	(0.34,0.71,0.72)	(0.64,0.41,0.42)	(0.44,0.61,0.62)	(0.64,0.41,0.42)
Exp-4	(0.14,0.91,0.92)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.54,0.51,0.52)	(0.24,0.81,0.82)	(0.64,0.41,0.42)	(0.44,0.61,0.62)	(0.54,0.51,0.52)
Exp-5	(0.54,0.51,0.52)	(0.84,0.21,0.22)	(0.74,0.31,0.32)	(0.84,0.21,0.22)	(0.54,0.51,0.52)	(0.84,0.21,0.22)	(0.64,0.41,0.42)	(0.84,0.21,0.22)
Exp-6	(0.14,0.91,0.92)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.44,0.61,0.62)	(0.24,0.81,0.82)	(0.54,0.51,0.52)	(0.54,0.51,0.52)	(0.54,0.51,0.52)
Exp-7	(0.34,0.71,0.72)	(0.54,0.51,0.52)	(0.64,0.41,0.42)	(0.64,0.41,0.42)	(0.44,0.61,0.62)	(0.54,0.51,0.52)	(0.54,0.51,0.52)	(0.74,0.31,0.32)
Exp-8	(0.24,0.81,0.82)	(0.34,0.71,0.72)	(0.44,0.61,0.62)	(0.54,0.51,0.52)	(0.24,0.81,0.82)	(0.54,0.51,0.52)	(0.34,0.71,0.72)	(0.54,0.51,0.52)

Table 3: Neutrosophic AHP paired matrix. Source: Own elaboration.

Source	A x Weight	Weight	Approximate Eigenvalues
Exp-1	2.93	0.32	9.151851
Exp-2	1.21	0.13	9.227916
Exp-3	0.53	0.06	8.537562
Exp-4	0.33	0.04	8.504645
Exp-5	2.87	0.30	9.535476
Exp-6	0.30	0.03	8.657509
Exp-7	0.88	0.10	9.121778

The analysis of the consistency of the method showed that its eigenvalue is 8.90, IC=0.13 and RC=0.09, which confirms that the exercise was consistent.

Exp-8	0.26	0.03	8.470680
Eigenvalue = 8.900927018			

Table 4: Analysis of the consistency of the paired matrix. Source: Own elaboration.

The results of the modeling of Saaty's AHP method highlight two experts with the greatest weight in the multineutrosophic evaluation of each strategic guideline, based on their experience in dynamic environments and financial and operational risks. Among them, the *certified public accountant* stands out, ensuring the accuracy and compliance of financial reports with a weight of 0.32. Meanwhile, the financial analyst complements this role through projections and strategic analysis, with a weight of 0.30.

Step 2: Selection and evaluation of criteria by each multi-source (expert).

The evaluation criteria focus on the ability of strategic guidelines to influence managerial decisions and achieve organizational objectives. These include:

- Relevance for decision-making (C1): Measures the impact on the quality of decisions.
- Improvement of transparency and accuracy (C2): Values the reliability and clarity of the resulting decisions.
- Flexibility and agility (C3): Determines the ability to respond to new situations
- Ability to adapt to the economic environment (C4): Evaluates flexibility in the face of changes and risks.
- Contribution to organizational stability (C5): Analyzes the effectiveness in strengthening financial stability.

Therefore, once each criterion is defined, the next step is to evaluate the weight of importance based on evaluations from multiple sources. As shown in Table 5, the highest-weighted criterion is C4, followed by C1. Since the scores in the score function are different, there is no need to apply the Accuracy and Certainty functions in the multineutrosophic analysis.

$(\{T_1, T_2, T_3\}, \{I_1, I_2\}, \{F_1, F_2, F_3, F_4\})$	(T_a, I_a, F_a)	Weight	Score (S)
C1($\{0.9, 0.5, 0.6\}, \{0.6, 0.1\}, \{0.5, 0.6, 0.7, 0.9\}$)	(0.67, 0.35, 0.68)	0.25	0.55
C2($\{0.2, 0.4, 0.1\}, \{0.2, 0.1\}, \{0.3, 0.5, 0.3, 0.4\}$)	(0.23, 0.35, 0.38)	0.09	0.50
C3($\{0.4, 0.4, 0.6\}, \{0.4, 0.1\}, \{0.8, 0.8, 0.4, 0.5\}$)	(0.47, 0.45, 0.63)	0.17	0.46
C4($\{0.8, 0.7, 0.9\}, \{0.2, 0.1\}, \{0.3, 0.6, 0.3, 0.2\}$)	(0.8, 0.45, 0.35)	0.30	0.67
C5($\{0.2, 0.9, 0.5\}, \{0.7, 0.1\}, \{0.2, 0.4, 0.4, 0.8\}$)	(0.53, 0.6, 0.45)	0.20	0.49

Table 5: Multineutrosophic evaluation of each criterion. Source: Own elaboration.

Step 3 to 7: Calculation of the optimization function S_i , from the formation of the multineutrosophic decision matrix L_{ij} .

Once the weight of the set of criteria and the evaluation sources is defined, the multineutrosophic ARAS method is applied to evaluate each strategic guideline. First, the multineutrosophic decision matrix L_{ij} must be defined, and then the resulting optimization function S_j of each strategic guideline is determined based on its impact on managerial decision-making (see Tables 6 and 7). For the modeling, it was determined that all criteria have been classified as "B" (Benefit) in the evaluation of each guideline.

$(\{T_1, T_2, T_3\}, \{I_1, I_2\}, \{F_1, F_2, F_3, F_4\})$	$l_{ij} = (T_a, I_a, F_a)$	Score (S)
Criterion: C1		
LE1($\{0.8, 0.5, 0.4\}, \{0.8, 0.9\}, \{0.5, 0.6, 0.2, 0.2\}$)	(0.6, 0.84, 0.34)	0.47
LE2($\{0.5, 0.5, 0.4\}, \{0.7, 0.9\}, \{0.1, 0.3, 0.3, 0.9\}$)	(0.48, 0.7, 0.53)	0.42
LE3($\{0.6, 0.7, 0.1\}, \{0.4, 0.9\}, \{0.5, 0.1, 0.5, 0.2\}$)	(0.51, 0.29, 0.25)	0.66
LE4($\{0.8, 0.3, 0.9\}, \{0.3, 0.9\}, \{0.2, 0.9, 0.1, 0.7\}$)	(0.66, 0.34, 0.6)	0.57
LE5($\{0.3, 0.1, 0.8\}, \{0.4, 0.9\}, \{0.4, 0.9, 0.1, 0.8\}$)	(0.36, 0.29, 0.65)	0.47
LE6($\{0.9, 0.4, 0.2\}, \{0.6, 0.9\}, \{0.1, 0.3, 0.5, 0.3\}$)	(0.56, 0.53, 0.33)	0.57
LE7($\{0.7, 0.8, 0.8\}, \{0.6, 0.9\}, \{0.5, 0.9, 0.9, 0.6\}$)	(0.76, 0.53, 0.75)	0.49
Criterion: C2		
LE1($\{0.7, 0.6, 0.3\}, \{0.2, 0.9\}, \{0.8, 0.7, 0.8, 0.5\}$)	(0.57, 0.43, 0.65)	0.50
LE2($\{0.5, 0.6, 0.3\}, \{0.3, 0.9\}, \{0.5, 0.6, 0.1, 0.9\}$)	(0.48, 0.45, 0.61)	0.47
LE3($\{0.2, 0.6, 0.7\}, \{0.8, 0.9\}, \{0.7, 0.3, 0.7, 0.6\}$)	(0.46, 0.69, 0.53)	0.41
LE4($\{0.6, 0.5, 0.6\}, \{0.1, 0.9\}, \{0.4, 0.8, 0.7, 0.7\}$)	(0.57, 0.33, 0.71)	0.51
LE5($\{0.4, 0.6, 0.9\}, \{0.7, 0.9\}, \{0.4, 0.8, 0.3, 0.5\}$)	(0.59, 0.74, 0.54)	0.44
LE6($\{0.7, 0.9, 0.2\}, \{0.2, 0.9\}, \{0.7, 0.2, 0.4, 0.7\}$)	(0.64, 0.28, 0.48)	0.63
LE7($\{0.1, 0.1, 0.8\}, \{0.1, 0.9\}, \{0.3, 0.4, 0.7, 0.9\}$)	(0.28, 0.4, 0.66)	0.41
Criterion: C3		
LE1($\{0.2, 0.7, 0.4\}, \{0.5, 0.9\}, \{0.3, 0.5, 0.5, 0.1\}$)	(0.43, 0.35, 0.35)	0.58
LE2($\{0.3, 0.1, 0.9\}, \{0.9, 0.9\}, \{0.3, 0.2, 0.2, 0.1\}$)	(0.43, 0.5, 0.2)	0.58

LE3({0.4,0.7,0.9},{0.2,0.9},{0.9,0.3,0.3,0.7})	(0.67,0.45,0.55)	0.56
LE4({0.7,0.2,0.8},{0.8,0.9},{0.5,0.8,0.1,0.5})	(0.57,0.65,0.48)	0.48
LE5({0.6,0.6,0.1},{0.9,0.9},{0.4,0.9,0.2,0.8})	(0.43,0.65,0.58)	0.40
LE6({0.6,0.5,0.2},{0.3,0.9},{0.9,0.6,0.5,0.2})	(0.43,0.55,0.55)	0.44
LE7({0.7,0.8,0.6},{0.3,0.9},{0.8,0.2,0.8,0.4})	(0.7,0.35,0.55)	0.60
Criterion: C4		
LE1({0.2,0.3,0.2},{0.8,0.9},{0.7,0.5,0.3,0.5})	(0.23,0.45,0.5)	0.43
LE2({0.9,0.9,0.4},{0.1,0.9},{0.6,0.5,0.2,0.8})	(0.73,0.1,0.53)	0.70
LE3({0.2,0.7,0.3},{0.8,0.9},{0.1,0.3,0.9,0.7})	(0.4,0.55,0.5)	0.45
LE4({0.4,0.8,0.6},{0.7,0.9},{0.2,0.9,0.1,0.2})	(0.6,0.55,0.35)	0.57
LE5({0.5,0.4,0.3},{0.4,0.9},{0.8,0.4,0.4,0.8})	(0.4,0.45,0.6)	0.45
LE6({0.5,0.9,0.8},{0.5,0.9},{0.8,0.3,0.5,0.4})	(0.73,0.55,0.5)	0.56
LE7({0.5,0.3,0.1},{0.5,0.9},{0.5,0.5,0.1,0.9})	(0.3,0.3,0.5)	0.50
Criterion: C5		
LE1({0.7,0.4,0.6},{0.5,0.9},{0.4,0.2,0.9,0.4})	(0.57,0.35,0.48)	0.58
LE2({0.7,0.4,0.8},{0.1,0.9},{0.1,0.8,0.8,0.6})	(0.63,0.35,0.58)	0.57
LE3({0.6,0.2,0.4},{0.1,0.9},{0.7,0.7,0.1,0.6})	(0.4,0.5,0.53)	0.46
LE4({0.7,0.1,0.1},{0.1,0.9},{0.1,0.3,0.4,0.7})	(0.3,0.35,0.38)	0.52
LE5({0.9,0.2,0.7},{0.7,0.9},{0.5,0.3,0.3,0.1})	(0.6,0.8,0.3)	0.50
LE6({0.8,0.5,0.7},{0.3,0.9},{0.7,0.5,0.4,0.2})	(0.67,0.45,0.45)	0.59
LE7({0.5,0.3,0.4},{0.2,0.9},{0.1,0.5,0.3,0.5})	(0.4,0.5,0.35)	0.52

Table 6: Multineutrosophic ARAS decision matrix of each strategic guideline. Source: Own elaboration.

	C1	C2	C3	C4	C5	Gi	Ki	
Weight	0.25	0.09	0.17	0.30	0.20			
LE1	0.0322	0.0134	0.0271	0.1045	0.0310	0.2082	100.00%	$G_0 = 0.2082$
LE2	0.0288	0.0126	0.0271	0.0452	0.0305	0.1440	69.19%	
LE3	0.0452	0.0109	0.0262	0.0247	0.0246	0.1317	63.24%	
LE4	0.0390	0.0136	0.0224	0.0371	0.0278	0.1400	67.25%	
LE5	0.0322	0.0118	0.0187	0.0247	0.0267	0.1141	54.81%	
LE6	0.0390	0.0168	0.0205	0.0452	0.0316	0.1531	73.55%	
LE7	0.0336	0.0109	0.0280	0.0186	0.0278	0.1189	57.11%	

Table 7: Optimization function G_i from the assignment of the criterion weight W_j . Source: Own elaboration.

Through the application of the multineutrosophic ARAS method, it is determined that the strategic guidelines with the highest weight of importance are the strengthening of predictive and proactive financial analysis and the focus on managing financial and operational risks. These guidelines should be prioritized due to their direct impact on making informed and resilient decisions, especially in volatile environments. Meanwhile, fostering a culture of financial innovation and adaptability also stands out as a significant guideline to promote organizational agility.

5 Discussion.

The results obtained through the Multineutrosophic ARAS method align with the existing literature on the priority of predictive financial analysis and risk management as key factors in organizational decision-making. In fact, in an environment characterized by economic uncertainties and rapid market changes, organizations that have implemented a proactive approach to financial analysis manage to stay ahead.

On the other hand, the research has shown that the ability to adapt to changes, combined with constant innovation, not only improves organizational agility but also increases resilience to potential crises. These results support the need for organizations to cultivate a culture of innovation in all areas, particularly in the financial sector, to remain competitive in the long term.

Finally, the integration of these elements into organizational strategy, as observed in the study's results, allows decisions to have a broader reach and be less vulnerable to external fluctuations. The interrelationship between predictive analysis, risk management, strategic innovation, and the use of tools like multineutrosophic analysis constitutes an essential factor that not only improves decision-making but also contributes to the sustainable growth of organizations. Therefore, it enables companies to not only respond to current challenges but also anticipate future trends with characteristics of uncertainty and market opportunities, ensuring their stability and long-term success.

In line with these findings, recent studies have demonstrated that the application of neutrosophic methods effectively addresses the inherent uncertainty in supplier evaluation, organizational performance, and corporate strategies. For instance, Sallam and Mohamed[16] highlight the utility of neutrosophic sets for evaluating supplier quality under uncertain environments, providing a robust tool for managing uncertainty in a structured manner.

Similarly, Alarcón et al. [17] employed sentiment analysis combined with NeutroAlgebra to evaluate organizational strategies and performance levels among basic education teachers, emphasizing how this methodology facilitates informed strategic decisions in complex situations.

Furthermore, other investigations have reinforced the practical applicability of neutrosophic approaches in critical areas such as healthcare and engineering. Abdullah [18] demonstrated that integrating neutrosophic sets with deep learning models significantly improves accuracy in breast cancer classification, enabling more precise and earlier clinical decisions. In the same vein, Cevallos-Torres et al. [19] developed neutrosophic control charts to detect cardiac arrhythmias in electrocardiograms, demonstrating the efficacy of these methods in highly sensitive contexts. Additionally, Mohamed, Smarandache, and Voskoglou [20] implemented a neutrosophic decision tree model to evaluate obstacles in electric mobility adoption, offering a clear analytical framework in the face of multiple uncertainties.

Conclusion

The analysis conducted through the multineutrosophic ARAS method has shown that strengthening predictive and proactive financial analysis, along with solid financial and operational risk management, are essential for informed and resilient organizational decision-making, especially in volatile economic environments. Furthermore, it has been confirmed that promoting a culture of innovation and financial adaptability significantly improves organizational agility, enabling companies to respond quickly to market changes. Therefore, the use of tools derived from the MultiNeutrosophic Set, integrated between financial analysis, risk management, and organizational culture, has demonstrated its capacity to enhance business resilience and sustainability. This opens the door to future research that delves into its application and effectiveness across various sectors immersed in financial uncertainty.

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