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$\mathcal{SV}$ −Łukasiewicz Neutrosophic Fuzzy Matrix: A Robust Approach to Identifying an Ultimate Laptop Model

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Abstract: Neutrosophic $\mathscr{F}$uzzy Sets deal with complex, incomplete and imprecise data. For the robust approach in such data, we have constructed an $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy matrix. To approach this study with real-life problem, we took a decision-making problem on choosing an ultimate Laptop Model on behalf of the needs of an individual. We have established the operations and types of $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy matrix. These are benefited in the procedure of achieving the best outcome.

Keywords: $\mathcal{SV}$ − Neutrosophic $\mathscr{F}$uzzy Set, $\mathcal{SV}$ − Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Set, Neutrosophic $\mathscr{F}$uzzy Matrix, $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Matrix, $\mathcal{SV}$ − Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix.

1. Introduction

$\mathscr{F}$uzzy is a powerful mathematical technique that forms the basis of set theory and logic. It can handle imprecise, inconsistent and ambiguous data by providing membership grades in the interval [0,1] on the basis of truth or belongingness. In addition to this grade, Atanassov included the non-membership grade in the set theory, for considering the falsity or non-belonginess. It was named as Intuitionistic Fuzzy set. These set theories lack in handling indetermined data. [2] Smarandache handled this lack by introducing Neutrosophic set, which consists of truth, falsity and indeterminacy grades in non-standard intervals. Then, a view of Neutrosophic $\mathscr{F}$uzzy set was first proposed by S. Das et al [7]. This set theory blends with the characteristics of Neutrosophic set and $\mathscr{F}$uzzy set. The elements in the set are in the form of four tuple that defines the membership value with varying

Accuracy, Inaccuracy, and Indecisiveness degrees. This provides more accuracy in handling real-life decision-making problem. Also, it doesn't suit for complex problem due to the representation non-standard intervals for the Neutrosophic components. This issue is overcome by the proposal of $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy set [9]. As it works with the components of Neutrosophic sets in [0,1]. This helps to work in the complex field of pattern recognition, artificial intelligence and so on. Furthermore, using computational instances, the authors developed a range of set theoretic operations and properties. [10,11] Later, they extended this concept in matrix theory for the utilization of Neutrosophic $\mathscr{F}$uzzy Matrix. This contributes to the creation of Single-valued Neutrosophic $\mathscr{F}$uzzy Matrix or $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Matrix, which allows the entries of the matrix to possess the four tuples namely membership value and its degree of Accuracy, Indecisiveness and Inaccuracy [13]. As an extension of set theory to matrix theory, one can work on real life data in various ways with the use of their operations. Some authors incorporated this concept in decision making problem by conducting research in the perspective of algebraic structures and investigated the algebraic operations. Using which some authors constructed the score matrix for the application in decision-making problem by use of matrix operations [14]. This $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Matrix gives better and more accurate result in decision-making problem rather than other theories. To bring advancement in this concept, we combined this matrix theory with Łukasiewicz Fuzzy Set theory. It has both logical interpretations and a suitable basis for implementation. This was first proposed by a polish Mathematician, Jan Łukasiewicz, who was a logician and Philosopher. He gave rise to the worthwhile logic called Łukasiewicz logic. This logic provides a powerful framework with diverse application across various disciplines. As a part of it, many professionals incorporate the above logic to fuzzy set theory and presented the idea of Łukasiewicz $\mathscr{F}$uzzy Set in various algebra [15-18].

In this study, we have extended the Łukasiewicz set theory to Matrix theory and fused the concept of Łukasiewicz theory to $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Matrix for bringing the discussion on operations of the matrices to earn the betterment of result in decision making problem. We have addressed a specific problem on choosing a best laptop model under the circumstance of user's requirements and budgetary constraints.

2. Preliminaries

2.1. $\mathscr{F}$uzzy Set ($\mathfrak{FS}$)

A $\mathfrak{FS}$ $\dot{\mathbb{A}}$ over $\acute{\mathbb{U}}$ is referred as $\dot{\mathbb{A}} = \{(\ddot{\mathbf{a}}, \boldsymbol{\mu}_{\dot{\mathbb{A}}}(\ddot{\mathbf{a}}))/\ddot{\mathbf{a}} \in \acute{\mathbb{U}}\}$, and $\boldsymbol{\mu}_{\dot{\mathbb{A}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}]$ be membership function of $\dot{\mathbb{A}}$.

2.2. $\mathcal{SV}$ −Neutrosophic Set ($\mathfrak{SNS}$)

A $\mathfrak{SNS}$ $\dot{\mathbb{B}}$ over $\acute{\mathbb{U}}$ is referred as

$$\dot{\mathbb{B}} = \left\{\left(\ddot{\mathbf{a}}, \acute{\mathcal{T}}_{\dot{\mathbb{B}}}(\ddot{\mathbf{a}}), \acute{\mathcal{I}}_{\dot{\mathbb{B}}}(\ddot{\mathbf{a}}), \acute{\mathcal{F}}_{\dot{\mathbb{B}}}(\ddot{\mathbf{a}})\right) \middle| \ddot{\mathbf{a}} \in \acute{\mathbb{U}}\right\},$$

where $\acute{\mathcal{T}}_{\dot{\mathbb{B}}}, \acute{\mathcal{I}}_{\dot{\mathbb{B}}}$ and $\acute{\mathcal{F}}_{\dot{\mathbb{B}}}$ are functions of Accuracy, Indecisiveness, and Inaccuracy degrees respectively. For each point $\ddot{\mathbf{a}}$ in $\acute{\mathbb{U}}$, $\acute{\mathcal{T}}_{\dot{\mathbb{B}}}, \acute{\mathcal{I}}_{\dot{\mathbb{B}}}, \acute{\mathcal{F}}_{\dot{\mathbb{B}}} \in [\mathbf{0}, \mathbf{1}]$.

2.3. $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Set ($\mathfrak{NFS}$)

A $\mathfrak{NFS}$ $\dot{\mathbb{C}}$ on $\acute{\mathbb{U}}$ is referred as

$$\dot{\mathbb{C}} = \left\{\left(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}), \acute{\mathcal{T}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{I}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{F}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})\right) \middle| \ddot{\mathbf{a}} \in \acute{\mathbb{U}}\right\}$$

where its membership value $\acute{\boldsymbol{\mu}}_{\dot{\mathbb{C}}}$ is described by its Accuracy $\acute{\mathcal{T}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})$, Indecisiveness $\acute{\mathcal{I}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})$ and Inaccuracy $\acute{\mathcal{F}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})$functions. Moreover $\acute{\mathcal{T}}_{\dot{\mathbb{C}}}, \acute{\mathcal{I}}_{\dot{\mathbb{C}}}$ and $\acute{\mathcal{F}}_{\dot{\mathbb{C}}}$ are either standard or non-standard form of $]\mathbf{0}^-, \mathbf{1}^+[$. That is, $\acute{\mathcal{T}}_{\dot{\mathbb{C}}}: \acute{\mathbb{U}} \to]\mathbf{0}^-, \mathbf{1}^+[, \acute{\mathcal{I}}_{\dot{\mathbb{C}}}: \acute{\mathbb{U}} \to]\mathbf{0}^-, \mathbf{1}^+[$ and $\acute{\mathcal{F}}_{\dot{\mathbb{C}}}: \acute{\mathbb{U}} \to]\mathbf{0}^-, \mathbf{1}^+[$. Also $\mathbf{0}^- \leq \boldsymbol{Sup}(\acute{\mathcal{T}}_{\dot{\mathbb{C}}}) + \boldsymbol{Sup}(\acute{\mathcal{I}}_{\dot{\mathbb{C}}}) + \boldsymbol{Sup}(\acute{\mathcal{F}}_{\dot{\mathbb{C}}}) \leq \mathbf{3}^+$.

2.4. $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Set ($\mathfrak{SNFS}$)

The $\mathfrak{SNFS}$ $\dot{\mathbb{D}}$ on $\acute{\mathbb{U}}$ is defined by

$$\dot{\mathbb{D}} = \left\{\left(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}_{\dot{\mathbb{D}}}(\ddot{\mathbf{a}}), \acute{\mathcal{T}}_{\dot{\mathbb{D}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{I}}_{\dot{\mathbb{D}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{F}}_{\dot{\mathbb{D}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})\right) \middle| \ddot{\mathbf{a}} \in \acute{\mathbb{U}}\right\},$$

where $\acute{\mathcal{T}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}})$, $\acute{\mathcal{I}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{F}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}) \in [\mathbf{0}, \mathbf{1}]$ and $\mathbf{0} \leq \acute{\mathcal{T}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}) + \acute{\mathcal{I}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}) + \acute{\mathcal{F}}_{\dot{\mathbb{C}}}(\ddot{\mathbf{a}}, \acute{\boldsymbol{\mu}}) \leq \mathbf{3}$.

2.5. $\mathcal{SV}$ −Neutrosophic Matrix ($\mathfrak{SNM}$)

A $\mathfrak{SNM}$ is defined as

$$\dot{\mathbb{M}} = \left[\ddot{\mathbf{a}}_{ij}\right]_{\hat{\mathcal{p}} \times \hat{\mathcal{q}}} = \left[\langle \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{T}}}, \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{I}}}, \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{F}}} \rangle\right]_{\hat{\mathcal{p}} \times \hat{\mathcal{q}}}$$

where $\ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{T}}}, \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{I}}}, \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{F}}}$ are referred to as Accuracy, Indecisiveness and Inaccuracy degrees of the $\boldsymbol{ij^{th}}$ element in $\dot{\mathbb{M}}$. These degrees satisfy $\mathbf{0} \leq \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{T}}} + \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{I}}} + \ddot{\mathbf{a}}_{ij}^{\acute{\mathcal{F}}} \leq \mathbf{3} \ \forall\, \boldsymbol{i}, \boldsymbol{j}$.

2.6. Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{NFM}$)

The $\mathfrak{NFM}$ is defined as

$$\dot{\mathbb{H}} = \left[\ddot{\mathbf{a}}_{ij}\right]_{\hat{\mathcal{p}} \times \hat{\mathcal{q}}} = \left[\langle \ddot{\mathbf{a}}_{ij}, \acute{\boldsymbol{\mu}}_{\dot{\mathbb{H}}}(\ddot{\mathbf{a}}_{ij}), \acute{\mathcal{T}}_{\dot{\mathbb{H}}}(\ddot{\mathbf{a}}_{ij}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{I}}_{\dot{\mathbb{H}}}(\ddot{\mathbf{a}}_{ij}, \acute{\boldsymbol{\mu}}), \acute{\mathcal{F}}_{\dot{\mathbb{H}}}(\ddot{\mathbf{a}}_{ij}, \acute{\boldsymbol{\mu}}) \rangle\right]_{\hat{\mathcal{p}} \times \hat{\mathcal{q}}},$$

where its membership value has been represented by an Accuracy, Indecisiveness and Inaccuracy degrees which are subsets of either standard or non-standard form of $]\mathbf{0}^{-}, \mathbf{1}^{+}[$ and $\mathbf{0}^{-} \leq \boldsymbol{Sup}(\acute{\mathcal{T}}_{\dot{\mathbb{H}}}) + \boldsymbol{Sup}(\acute{\mathcal{I}}_{\dot{\mathbb{H}}}) + \boldsymbol{Sup}(\acute{\mathcal{F}}_{\dot{\mathbb{H}}}) \leq \mathbf{3}^{+}$.

2.7. $\mathcal{SV}$ −Neutrosophic 𝒻uzzy Matrix (𝕾𝕹𝕱𝕸)

The 𝕾𝕹𝕱𝕸 is defined as

$$\dot{\mathbb{G}} = [\ddot{\mathrm{a}}_{ij}]_{\hat{p}\times\hat{q}} = [\acute{\mu}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}), \acute{\mathcal{T}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu})]_{\hat{p}\times\hat{q}},$$

where its membership value has been represented by an Accuracy, Indecisiveness and Inaccuracy membership and $\acute{\mathcal{T}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu})$, $\acute{\mathcal{I}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}) \in [\mathbf{0}, \mathbf{1}]$ which satisfy the condition that $\mathbf{0} \leq \acute{\mathcal{T}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}) + \acute{\mathcal{I}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}) + \acute{\mathcal{F}}_{\dot{\mathbb{G}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}) \leq \mathbf{3}$.

3. $\mathcal{SV}$ −Łukasiewicz Neutrosophic 𝒻uzzy Matrix

3.1. Definitions

3.1.1. $\ddot{\boldsymbol{\theta}}-$ Łukasiewicz Neutrosophic 𝒻uzzy Set (𝕷𝕹𝕱𝕾)

Let $\dot{\mathbb{A}}$ be a 𝕱𝕾 in $\acute{\mathbb{U}}$ and let $\ddot{\boldsymbol{\theta}} = [\mathbf{0}, \mathbf{1}]$. The 𝕷𝕹𝕱𝕾 $Ł_{\dot{\mathbb{A}}}^{\ddot{\boldsymbol{\theta}}}$ is defined as

$$Ł_{\dot{\mathbb{A}}}^{\ddot{\boldsymbol{\theta}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}], \ddot{\mathrm{a}} \to \boldsymbol{max}\{\mathbf{0}, \dot{\mathbb{A}}(\ddot{\mathrm{a}}) + \ddot{\boldsymbol{\theta}} - \mathbf{1}\}.$$

3.1.2. $\mathcal{SV} - \ddot{\boldsymbol{\theta}} -$ Łukasiewicz Neutrosophic 𝒻uzzy Set (𝕾𝕷𝕹𝕱𝕾)

Let $\dot{\mathbb{D}} = (\acute{\mu}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}), \acute{\mathcal{T}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}, \acute{\mu}), \acute{\mathcal{I}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}, \acute{\mu}), \acute{\mathcal{F}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}, \acute{\mu}))$ be a 𝕾𝕹𝕱𝕾 in $\acute{\mathbb{U}}$ and let $\ddot{\boldsymbol{\theta}} = [\mathbf{0}, \mathbf{1}]$. A 𝕾𝕷𝕹𝕱𝕾 $Ł_{\dot{\mathbb{D}}}^{\ddot{\boldsymbol{\theta}}}$ is defined as

$$Ł_{\dot{\mathbb{D}}}^{\ddot{\boldsymbol{\theta}}} = \left\{(\acute{\mu}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}), \acute{\mathcal{T}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu}), \acute{\mathcal{I}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu}), \acute{\mathcal{F}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu})) \middle| \ddot{\mathrm{a}} \in \acute{\mathbb{U}}\right\},$$

where $\acute{\mu}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}], \ddot{\mathrm{a}} \to \boldsymbol{max}\{\mathbf{0}, \acute{\mu}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}) + \ddot{\boldsymbol{\theta}} - \mathbf{1}\}$, $\acute{\mathcal{T}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}], \ddot{\mathrm{a}} \to \boldsymbol{max}\{\mathbf{0}, \acute{\mathcal{T}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}) + \ddot{\boldsymbol{\theta}} - \mathbf{1}\}$,

$\acute{\mathcal{I}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}], \ddot{\mathrm{a}} \to \boldsymbol{max}\{\mathbf{0}, \acute{\mathcal{I}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}) + \ddot{\boldsymbol{\theta}} - \mathbf{1}\}$ and $\acute{\mathcal{F}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}: \acute{\mathbb{U}} \to [\mathbf{0}, \mathbf{1}], \ddot{\mathrm{a}} \to \boldsymbol{max}\{\mathbf{0}, \acute{\mathcal{F}}_{\dot{\mathbb{D}}}(\ddot{\mathrm{a}}) + \ddot{\boldsymbol{\theta}} - \mathbf{1}\}$ are membership value and its Accuracy, Indecisiveness and Inaccuracy memberships, which satisfies the condition that $\mathbf{0} \leq \acute{\mathcal{T}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu}) + \acute{\mathcal{I}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu}) + \acute{\mathcal{F}}_{Ł_{\dot{\mathbb{D}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}, \acute{\mu}) \leq \mathbf{3}$.

3.1.3. $\mathcal{SV} - \ddot{\boldsymbol{\theta}} -$ Łukasiewicz Neutrosophic 𝒻uzzy Matrix (𝕾𝕷𝕹𝕱𝕸)

The 𝕾𝕷𝕹𝕱𝕸 is defined as

$$Ł_{\dot{\mathbb{G}}}^{\ddot{\boldsymbol{\theta}}} = [\ddot{\mathrm{a}}_{ij}]_{\hat{p}\times\hat{q}} = \left[(\acute{\mu}_{Ł_{\dot{\mathbb{G}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}_{ij}), \acute{\mathcal{T}}_{Ł_{\dot{\mathbb{G}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{Ł_{\dot{\mathbb{G}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{Ł_{\dot{\mathbb{G}}}^{\ddot{\theta}}}(\ddot{\mathrm{a}}_{ij}, \acute{\mu})\right]_{\hat{p}\times\hat{q}},$$

where its membership value has been expressed by an Accuracy, Indecisiveness and Inaccuracy degrees as $\acute{\mu}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}: \acute{\mathbb{U}} \to [0,1], \ddot{a}_{ij} \to max\{0, \acute{\mu}_{\dot{\mathbb{G}}}(\ddot{a}_{ij}) + \ddot{\theta} - 1\}$, $\acute{\mathcal{T}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}: \acute{\mathbb{U}} \to [0,1], \ddot{a}_{ij} \to max\{0, \acute{\mathcal{T}}_{\dot{\mathbb{G}}}(\ddot{a}_{ij}) + \ddot{\theta} - 1\}$, $\acute{\mathcal{I}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}: \acute{\mathbb{U}} \to [0,1], \ddot{a}_{ij} \to max\{0, \acute{\mathcal{I}}_{\dot{\mathbb{G}}}(\ddot{a}_{ij}) + \ddot{\theta} - 1\}$ and $\acute{\mathcal{F}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}: \acute{\mathbb{U}} \to [0,1], \ddot{a}_{ij} \to max\{0, \acute{\mathcal{F}}_{\dot{\mathbb{G}}}(\ddot{a}_{ij}) + \ddot{\theta} - 1\}$ respectively, which satisfy the condition that $0 \le \acute{\mathcal{T}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}(\ddot{a}_{ij}, \acute{\mu}) + \acute{\mathcal{I}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}(\ddot{a}_{ij}, \acute{\mu}) + \acute{\mathcal{F}}_{Ł^{\ddot{\theta}}_{\mathbb{G}}}(\ddot{a}_{ij}, \acute{\mu}) \le 3$.

3.2. Operations on $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix

3.2.1. Matrix Addition

Let $\breve{\mathbb{P}} = [\mathbb{p}_{ij}]_{\dot{m}\times\dot{n}}$ and $\breve{\mathbb{Q}} = [\mathbb{q}_{ij}]_{\dot{m}\times\dot{n}}$ be two $\mathfrak{SLNFM}$s. The matrix addition is in the form

$$\breve{\mathbb{P}} + \breve{\mathbb{Q}} = [\langle \acute{\mu}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle]_{\dot{m}\times\dot{n}}$$

where $\acute{\mu}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}) = max\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right)$, $\acute{\mathcal{T}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = max\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$

$\acute{\mathcal{I}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = max\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$ and $\acute{\mathcal{F}}_{\breve{\mathbb{P}}+\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = min\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$.

3.2.2. Matrix Subtraction

Let $\breve{\mathbb{P}} = [\mathbb{p}_{ij}]_{\dot{m}\times\dot{n}}$ and $\breve{\mathbb{Q}} = [\mathbb{q}_{ij}]_{\dot{m}\times\dot{n}}$ be two $\mathfrak{SLNFM}$s. The matrix subtraction is in the form

$$\breve{\mathbb{P}} - \breve{\mathbb{Q}} = [\langle \acute{\mu}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle]_{\dot{m}\times\dot{n}}$$

where $\acute{\mu}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}) = \begin{cases} \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), & if\ \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}) \ge \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}) \\ 0, & otherwise \end{cases}$,

$\acute{\mathcal{T}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = \begin{cases} \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), & if\ \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \ge \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) \\ 0, & otherwise \end{cases}$,

$\acute{\mathcal{I}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = \begin{cases} \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), & if\ \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \ge \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) \\ 0, & otherwise \end{cases}$ and

$\acute{\mathcal{F}}_{\breve{\mathbb{P}}-\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = \begin{cases} \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), & if\ \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \le \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) \\ 0, & otherwise \end{cases}$.

3.2.3. Component wise Matrix Multiplication

Let $\breve{\mathbb{P}} = [\mathbb{p}_{ij}]_{\dot{m}\times\dot{n}}$ and $\breve{\mathbb{Q}} = [\mathbb{q}_{ij}]_{\dot{m}\times\dot{n}}$ be two $\mathfrak{SLNFM}$s. The component wise matrix multiplication will be

$$\breve{\mathbb{P}} \circ \breve{\mathbb{Q}} = [\langle \acute{\mu}_{\breve{\mathbb{P}}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle]_{\dot{m}\times\dot{n}},$$

where $\acute{\mu}_{\mathbb{P}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}) = min\left(\acute{\mu}_{\mathbb{P}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right)$, $\acute{\mathcal{T}}_{\mathbb{P}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = min\left(\acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$

$\acute{\mathcal{I}}_{\mathbb{P}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = min\left(\acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$ and $\acute{\mathcal{F}}_{\mathbb{P}\circ\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = max\left(\acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right)$.

3.2.4. Scalar Product

The scalar product of $\mathfrak{SLNFM}$ $\breve{\mathbb{P}} = \left[\langle \acute{\mu}_{\mathbb{P}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{n}}$ be defined to be

$$\acute{k}\breve{\mathbb{P}} = \left[\langle min\left(\acute{k}, \acute{\mu}_{\mathbb{P}}(\ddot{a}_{ij})\right), min\left(\acute{k}, \acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu})\right), min\left(\acute{k}, \acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu})\right), max\left(1-\acute{k}, \acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu})\right)\rangle\right]_{\ddot{m}\times\ddot{n}}.$$

3.2.5. Matrix Product

The matrix product of two $\mathfrak{SLNFM}$s $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\ddot{m}\times\ddot{n}}$ and $\breve{\mathbb{Q}} = [\ddot{a}_{ij}]_{\ddot{n}\times\ddot{r}}$ are defined to be

$$\breve{\mathbb{P}} * \breve{\mathbb{Q}} = \left[\langle \acute{\mu}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{r}},$$

where $\acute{\mu}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}) = max_k\left\{min\{\acute{\mu}_{\mathbb{P}}(\ddot{a}_{ik}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{kj})\}\right\}$

$\acute{\mathcal{T}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = max_k\left\{min\{\acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ik}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{kj}, \acute{\mu})\}\right\}$,

$\acute{\mathcal{I}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = min_k\left\{max\{\acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ik}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{kj}, \acute{\mu})\}\right\}$ and

$\acute{\mathcal{F}}_{\mathbb{P}*\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}) = min_k\left\{max\{\acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ik}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{kj}, \acute{\mu})\}\right\}$, $k = 1, 2, \dots, n$, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, r$.

3.2.6. Score Matrix ($\mathfrak{SM}$)

Let $\ddot{\kappa}_1$ and $\ddot{\kappa}_2$ be two $\mathfrak{SLNFM}$s. Its $\mathfrak{SM}$ is defined as

$$\mathcal{S}_c(\ddot{\kappa}_1, \ddot{\kappa}_2) = [\breve{\mathcal{U}} - \breve{\mathcal{V}}]$$

where $\breve{\mathcal{U}} = [u_{ij}]_{\ddot{m}\times\ddot{n}}$, $u_{ij} = \acute{\mu}_{\ddot{\kappa}_1}(\ddot{a}_{ij}) + \acute{\mathcal{T}}_{\ddot{\kappa}_1}(\ddot{a}_{ij}, \acute{\mu}) + \acute{\mathcal{I}}_{\ddot{\kappa}_1}(\ddot{a}_{ij}, \acute{\mu}) - \acute{\mathcal{F}}_{\ddot{\kappa}_1}(\ddot{a}_{ij}, \acute{\mu})$ and

$\breve{\mathcal{V}} = [v_{ij}]_{\ddot{m}\times\ddot{n}}$, $v_{ij} = \acute{\mu}_{\ddot{\kappa}_2}(\ddot{a}_{ij}) + \acute{\mathcal{T}}_{\ddot{\kappa}_2}(\ddot{a}_{ij}, \acute{\mu}) + \acute{\mathcal{I}}_{\ddot{\kappa}_2}(\ddot{a}_{ij}, \acute{\mu}) - \acute{\mathcal{F}}_{\ddot{\kappa}_2}(\ddot{a}_{ij}, \acute{\mu})$.

3.2.7. Transpose of the Matrix

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\ddot{m}\times\ddot{n}}$ be $\mathfrak{SLNFM}$ and its transpose is referred to be

$$\breve{\mathbb{P}}^{T} = \left[\langle \acute{\mu}_{\mathbb{P}}(\ddot{a}_{ji}), \acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ji}, \acute{\mu}), \acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ji}, \acute{\mu}), \acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ji}, \acute{\mu})\rangle\right]_{\acute{m}\times\ddot{n}}.$$

3.2.8. Complement of the Matrix

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\ddot{n}}$ be $\mathfrak{SLNFM}$ and its complement is referred to be

$$\overline{\breve{\mathbb{P}}} = \left[\langle \acute{\mathcal{F}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), 1 - \acute{\mathcal{I}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\mathbb{P}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mu}_{\mathbb{P}}(\ddot{a}_{ij})\rangle\right]_{\acute{m}\times\ddot{n}}.$$

3.2.9. Trace of the Matrix

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\acute{m}}$ be $\mathfrak{SLNFM}$ and its trace is defined as

$$tr(\breve{\mathbb{P}}) = \sum_{i=1}^{n} \ddot{a}_{ii}.$$

3.3. Types of $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix

3.3.1. $\mathcal{SV}$ −Zero Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{SZLNFM}$)

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\ddot{n}}$ be a $\mathfrak{SLNFM}$. The matrix is known to be $\mathfrak{SZLNFM}$, if all the entries of $\breve{\mathbb{P}}$ are $(0, 0, 0, 1)$ and is denoted by 0.

3.3.2. $\mathcal{SV}$ −Universal Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{SULNFM}$)

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\ddot{n}}$ be a $\mathfrak{SLNFM}$. The matrix is known to be $\mathfrak{SULNFM}$, if all the entries of $\breve{\mathbb{P}}$ are $(1, 1, 1, 0)$ and is denoted by I.

3.3.3. $\mathcal{SV}$ −Symmetric Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{SSLNFM}$)

Let $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\acute{m}}$ be a $\mathfrak{SLNFM}$. The matrix $\breve{\mathbb{P}}$ is known to be $\mathfrak{SSLNFM}$ if $\breve{\mathbb{P}}_{ij} = \breve{\mathbb{P}}_{ji}$, $i, j = 1, 2, \ldots, n$.

3.3.4. $\mathcal{SV}$ −Upper triangular Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{SUTLNFM}$)

Suppose $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\acute{m}}$ be a $\mathfrak{SLNFM}$. Then the matrix $\breve{\mathbb{P}}$ is known to be $\mathfrak{SUTLNFM}$ if $\breve{\mathbb{P}}_{ij} = (0, 0, 0, 1)$, $\forall\, i > j$ and $i, j = 1, 2, \ldots, n$.

3.3.5. $\mathcal{SV}$ −Lower Triangular Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix ($\mathfrak{SLTLNFM}$)

Suppose $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\acute{m}\times\acute{m}}$ be a $\mathfrak{SLNFM}$. Then the matrix $\breve{\mathbb{P}}$ is known to be $\mathfrak{SLTLNFM}$ if $\breve{\mathbb{P}}_{ij} = (0, 0, 0, 1)$, $\forall\, i < j$ and $i, j = 1, 2, \ldots, n$.

3.3.6. Theorem

Let $\breve{\mathbb{P}}$ be $\mathfrak{SLNFS}$. Show that $\breve{\mathbb{P}}$ is a vector space under $\mathfrak{SLNFM}$ addition and scalar multiplication.

Proof

Let us examine all the axioms of Vector space.

$\boldsymbol{A_1}$: Let $\breve{\mathbb{P}} = \left[\langle \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{m}}$ and

$\breve{\mathbb{Q}} = \left[\langle \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{m}}$ be two $\mathfrak{SLNFM}$. In accordance with matrix addition, it is obvious that $\breve{\mathbb{P}} + \breve{\mathbb{Q}}$ also belongs to the $\mathfrak{SLNFM}$ of order $\ddot{m} \times \ddot{m}$.

$\boldsymbol{A_2}$: Let $\breve{\mathbb{P}} = \left[\langle \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{m}}$

$\breve{\mathbb{Q}} = \left[\langle \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{m}}$ and

$\breve{\mathbb{R}} = \left[\langle \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]_{\ddot{m}\times\ddot{m}}$ be three $\mathfrak{SLNFM}$.

$$\breve{\mathbb{P}} + (\breve{\mathbb{Q}} + \breve{\mathbb{R}}) = \left[\langle \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right] + \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle\right]$$

$$= \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle\right]$$

$$(\breve{\mathbb{P}} + \breve{\mathbb{Q}}) + \breve{\mathbb{R}} = \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle\right] + \left[\langle \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\rangle\right]$$

$$= \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle\right]$$

Hence $\breve{\mathbb{P}} + (\breve{\mathbb{Q}} + \breve{\mathbb{R}}) = (\breve{\mathbb{P}} + \breve{\mathbb{Q}}) + \breve{\mathbb{R}}$.

In a similar manner, we can show, $\breve{\mathbb{P}} \circ (\breve{\mathbb{Q}} \circ \breve{\mathbb{R}}) = (\breve{\mathbb{P}} \circ \breve{\mathbb{Q}}) \circ \breve{\mathbb{R}}$.

$$\boldsymbol{A_3}: \breve{\mathbb{P}} + \breve{\mathbb{Q}} = \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle\right]$$

$$= \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle \right] = \breve{\mathbb{Q}} + \breve{\mathbb{P}}.$$

$\boldsymbol{A_4}$: If $\breve{\mathbb{P}} + \mathbf{0} = \breve{\mathbb{P}}$, for any $\mathfrak{SLNFM}$ $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\dot{m}\times\dot{m}}$, then $\mathfrak{SZLNFM}$, 0 is the additive identity.

If $\breve{\mathbb{P}} \circ \mathbf{I} = \breve{\mathbb{P}}$, for any $\mathfrak{SLNFM}$ $\breve{\mathbb{P}} = [\ddot{a}_{ij}]_{\dot{m}\times\dot{m}}$, then $\mathfrak{SULNFM}$, I is the identity for multiplication.

$\boldsymbol{M_1}$: Let us say $\breve{\mathbb{P}}$ be a $\mathfrak{SLNFM}$ and $\acute{\boldsymbol{k}}$ be any scalar. In accordance with the scalar multiplication for $\mathfrak{SLNFM}$, it is obvious that $\acute{\boldsymbol{k}}\breve{\mathbb{P}}$ is also a $\mathfrak{SLNFM}$.

$\boldsymbol{M_2}$: Now to prove distributive law. Suppose that $\breve{\mathbb{P}} \subseteq \breve{\mathbb{Q}}, \breve{\mathbb{R}}$. Then $\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}) \le \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})$; $\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \le \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})$; $\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \le \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})$; $\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \ge \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})$. Thus,

$$\breve{\mathbb{P}} \circ (\breve{\mathbb{Q}} + \breve{\mathbb{R}}) = \left[\left\langle \acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}) \right\rangle\right] \circ \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle \right]$$

$$= \left[\left\langle \begin{matrix} \boldsymbol{min}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \boldsymbol{max}\left(\acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right)\right), \\ \boldsymbol{min}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \boldsymbol{max}\left(\acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right), \\ \boldsymbol{min}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \boldsymbol{max}\left(\acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right), \\ \boldsymbol{max}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \boldsymbol{min}\left(\acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right) \end{matrix} \right\rangle \right] = \breve{\mathbb{P}}$$

$$(\breve{\mathbb{P}} \circ \breve{\mathbb{Q}}) + (\breve{\mathbb{P}} \circ \breve{\mathbb{R}}) = \left[\left\langle \begin{matrix} \boldsymbol{min}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right), \boldsymbol{min}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{min}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{max}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle \right]$$

$$+ \left[\left\langle \begin{matrix} \boldsymbol{min}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right), \boldsymbol{min}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \\ \boldsymbol{min}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{max}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right) \end{matrix} \right\rangle \right]$$

$$= \left[\left\langle \begin{matrix} \boldsymbol{max}\left(\boldsymbol{min}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij})\right), \boldsymbol{min}\left(\acute{\mu}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}), \acute{\mu}_{\breve{\mathbb{R}}}(\ddot{a}_{ij})\right)\right), \\ \boldsymbol{max}\left(\boldsymbol{min}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{T}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{T}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right), \\ \boldsymbol{max}\left(\boldsymbol{min}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{min}\left(\acute{\mathcal{I}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{I}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right), \\ \boldsymbol{min}\left(\boldsymbol{max}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{Q}}}(\ddot{a}_{ij}, \acute{\mu})\right), \boldsymbol{max}\left(\acute{\mathcal{F}}_{\breve{\mathbb{P}}}(\ddot{a}_{ij}, \acute{\mu}), \acute{\mathcal{F}}_{\breve{\mathbb{R}}}(\ddot{a}_{ij}, \acute{\mu})\right)\right) \end{matrix} \right\rangle \right] = \breve{\mathbb{P}}.$$

Similarly, we can prove if $\breve{\mathbb{P}} \supseteq \breve{\mathbb{Q}}, \breve{\mathbb{R}}$. Thus, the distributive law holds.

$\boldsymbol{M_3}$: For any two scalars $\acute{\boldsymbol{k}}_1$ and $\acute{\boldsymbol{k}}_2$, we have

$$\left(\acute{\boldsymbol{k}}_1 + \acute{\boldsymbol{k}}_2\right)\breve{\mathbb{P}} = \left(\acute{\boldsymbol{k}}_1 + \acute{\boldsymbol{k}}_2\right)\mathbf{I} \circ \breve{\mathbb{P}} = \left(\acute{\boldsymbol{k}}_1\mathbf{I} + \acute{\boldsymbol{k}}_2\mathbf{I}\right) \circ \breve{\mathbb{P}} = \acute{\boldsymbol{k}}_1\mathbf{I} \circ \breve{\mathbb{P}} + \acute{\boldsymbol{k}}_2\mathbf{I} \circ \breve{\mathbb{P}} = \acute{\boldsymbol{k}}_1 \circ \breve{\mathbb{P}} + \acute{\boldsymbol{k}}_2 \circ \breve{\mathbb{P}}.$$

Therefore, $\mathfrak{SLNFM}$ forms a vector space.

4. Case study on $\mathcal{SV}$ –Łukasiewicz Neutrosophic Fuzzy Matrix

The $\mathfrak{SLNFM}$ is widely applied in areas requiring decision-making under uncertainty, as it effectively handles imprecise, inconsistent, and incomplete information. We applied this idea on a decision-making problem to demonstrate how to apply it in real-life situations. In the current market, there are numerous varieties of models in electronic devices. The process of buying an electronic device deal with lot of issues due to the lack of knowledge about the device's features or confusion in selecting the appropriate model based on their specific needs. They are typically purchased infrequently. So, the decision-making process is crucial to align with both the user's requirements and their budgetary constraints. As a note of it, we have addressed the problem specifically on the selection of best Laptop model from the available data using the theory of $\mathcal{SV}$ –Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix. For the case analysis, we have collected the data from five persons $\ddot{\mathcal{P}} = \{\ddot{\mathcal{P}}_1, \ddot{\mathcal{P}}_2, \ddot{\mathcal{P}}_3, \ddot{\mathcal{P}}_4, \ddot{\mathcal{P}}_5\}$ and noted their preferences in the features such as $\ddot{\mathcal{F}} =$ {**Processor**, **Price**, **RAM**, **Display**, **Rating**} of the Laptops. Also, we have used the data of five different laptop models $\ddot{\mathcal{L}} = \{\ddot{\mathcal{L}}_1, \ddot{\mathcal{L}}_2, \ddot{\mathcal{L}}_3, \ddot{\mathcal{L}}_4, \ddot{\mathcal{L}}_5\}$ and their features. Now we have to deduce which laptop model is suitable for the persons, we have chosen. Table 6.1 is the data of persons and their preferences in laptop features and Table 6.2 is the data about the laptop models and its features.

Person	Feature Preferences				
	Processor	Price (in ₹)	DDR4 RAM	Display (in inches)	Rating
Arun $(\ddot{\mathcal{P}}_1)$	Intel Core i5 Processor	60000	8 GB	15.5	4.4
Fredrick $(\ddot{\mathcal{P}}_2)$	Intel Core i3 Processor	55000	4 GB	14	4.1
John $(\ddot{\mathcal{P}}_3)$	Apple M1 Processor	75000	16 GB	13.5	4.6
Max $(\ddot{\mathcal{P}}_4)$	Intel Core i5 Processor	65000	16 GB	14.5	4.5
Thomas $(\ddot{\mathcal{P}}_5)$	Intel Core i3 Processor	50000	8 GB	15	3.8

Table 1. Persons and their preferences in the feature of the laptops

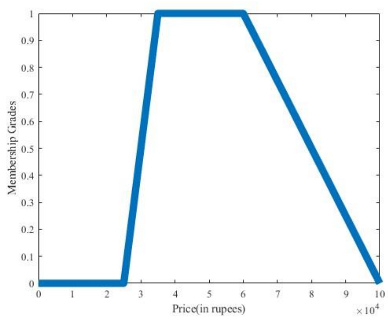
Laptop Model	Features				
	Processor	Price (in ₹)	DDR4 RAM	Display (in inches)	Rating
Lenovo V15 G2 Core i3 11th Gen $(\ddot{\mathcal{L}}_1)$	Intel Core i3 Processor	37500	8 GB	15.6	4.4

ASUS TUF Gaming F15 Core i5 10th Gen $(\ddot{\mathcal{L}}_2)$	Intel Core i5 Processor	49990	8 GB	15.6	4.4
ASUS Vivo Book 15 (2022) Core i3 10th Gen $(\ddot{\mathcal{L}}_3)$	Intel Core i3 Processor	33990	8 GB	15.6	4.3
Lenovo Athlon Dual Core $(\ddot{\mathcal{L}}_4)$	AMD Athlon Dual Core Processor	18999	4 GB	14	3.8
APPLE 2020 Mac book Air M1 $(\ddot{\mathcal{L}}_5)$	Apple M1 Processor	86990	8 GB	13.3	4.7

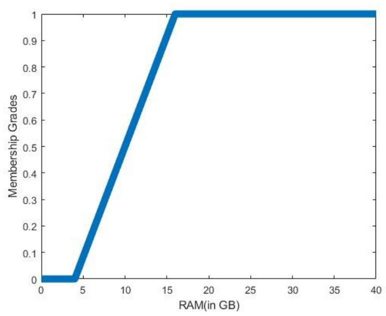
Table 2. Laptop Models and their features

We have defined the membership value for the above data sets as given below.

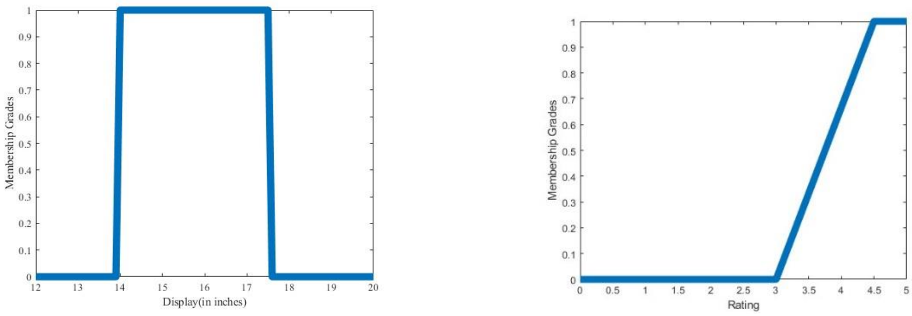
- Processor= $\left\{\frac{0.9}{Intel\ Core\ i5\ Processor}+\frac{0.7}{Intel\ Core\ i3\ Processor}+\frac{0.4}{AMD\ Athlon\ Dual\ Core\ Processor}+\frac{0.7}{Apple\ M1\ Processor}+\frac{0.8}{AMD\ Ryzen\ 5\ Hexa\ Core\ Processor}\right\}$
- Price= $\begin{cases} 0 & if\ p \leq 25000\ and\ p \geq 100000 \\ \frac{p-25000}{10000} & if\ 25000 \leq p \leq 35000 \\ 1 & if\ 35000 \leq p \leq 60000 \\ \frac{100000-p}{40000} & if\ 60000 \leq p \leq 100000 \end{cases}$
- RAM = $\begin{cases} 0 & if\ p \leq 4 \\ \frac{p-4}{12} & if\ 4 \leq p \leq 16 \\ 1 & if\ p \geq 16 \end{cases}$
- Display (in inches)= $\begin{cases} 0\ if\ p \leq 14\ and\ p \geq 17.5 \\ 1\ if\ 14 < p < 17.5 \end{cases}$
- Rating= $\begin{cases} 0\ if\ p \leq 3 \\ \frac{p-3}{1.5}\ if\ 3 \leq p \leq 4.5 \\ 1\ if\ 4.5 \leq p \leq 5 \end{cases}$



(a)



(b)



(c) **(d)**

Figure 1. Graphical Representations of membership grades. (a) Membership Grade for Price, (b) Membership Grade for RAM, (c) Membership Grade for Display, (d) Membership Grade for Rating

4.1. Algorithm

1. Construct the $\mathfrak{SNFM}$ $\ddot{\mathbb{M}}$ using Table 1 and the membership values defined.

2. Construct the $\mathfrak{SNFM}$ $\ddot{\mathbb{N}}$ using Table 2 and the membership values defined.

3. Convert the $\mathfrak{SNFM}$s $\ddot{\mathbb{M}}$ and $\ddot{\mathbb{N}}$ to $\mathfrak{SLNFM}$s $\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{M}}}$ and $\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}$.

4. Determine the matrix complement of $\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}$.

5. Determine the matrix $\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{M}}} * \text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}$.

6. Determine the matrix $\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{M}}} * {\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}}^{c}$

7. Find $\breve{\mathcal{U}}$ and $\breve{\mathcal{V}}$ of $\mathfrak{SM}$.

8. Measure the $\mathfrak{SM}$ $\mathcal{S}_c\left(\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{M}}} * \text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}, \text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{M}}} * {\text{Ł}^{\ddot{\theta}}_{\ddot{\mathbb{N}}}}^{c}\right)$.

Step-1

Construct a $\mathfrak{SNFM}$ $\ddot{\mathbb{M}}$ using Table 1 and defined Membership grades to represent the Person's preferences in the feature of the laptops.

$$\ddot{\mathbb{M}} = \begin{bmatrix} (0.9,0.6,0.2,0.1) & (1,0.7,0.1,0.2) & (0.3,0.6,0.1,0.1) & (1,0.8,0.1,0.1) & (0.93,0.6,0.2,0.1) \\ (0.7,0.7,0.1,0.2) & (1,0.5,0.2,0.1) & (0,0.3,0.8,0.2) & (0,0.6,0.3,0) & (0.73,0.5,0.4,0.2) \\ (0.7,0.9,0,0) & (0.63,0.4,0.2,0.3) & (1,0.9,0.1,0) & (0,0.8,0.1,0) & (1,0.9,0.1,0.1) \\ (0.9,0.5,0.1,0.2) & (0.88,0.6,0.1,0.1) & (1,0.8,0.2,0.2) & (1,0.7,0.1,0.1) & (1,0.8,0.1,0.2) \\ (0.7,0.6,0.2,0.1) & (1,0.3,0.5,0.1) & (0.3,0.7,0.4,0.3) & (1,0.4,0.3,0.1) & (0.53,0.5,0.5,0.2) \end{bmatrix}$$

Step-2

Construct a $\mathfrak{SNFM}$ $\ddot{\mathbb{N}}$ using Table 2 and defined Membership grades to represent the Laptop Model's features.

$$\ddot{\mathbb{N}} = \begin{bmatrix} (0.7,1,0,0) & (1,1,0,0) & (0.3,1,0,0) & (1,1,0,0) & (0.93,1,0,0) \\ (0.9,1,0,0) & (1,1,0,0) & (0.3,1,0,0) & (1,1,0,0) & (0.93,1,0,0) \\ (0.7,1,0,0) & (0.9,1,0,0) & (0.3,1,0,0) & (1,1,0,0) & (0.87,1,0,0) \\ (0.4,1,0,0) & (0,1,0,0) & (0,1,0,0) & (0,1,0,0) & (0.53,1,0,0) \\ (0.7,1,0,0) & (0.33,1,0,0) & (0.3,1,0,0) & (0,1,0,0) & (1,1,0,0) \end{bmatrix}$$

Step-3

Convert the $\mathfrak{SNFM}$s $\ddot{\mathbb{M}}$ and $\ddot{\mathbb{N}}$ to $\mathfrak{SLNFM}$s $\text{Ł}_{\ddot{\mathbb{M}}}^{\ddot{\theta}}$ and $\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}}$. Now this $\text{Ł}_{\ddot{\mathbb{M}}}^{\ddot{\theta}}$ and $\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}}$ represents the Person's preferences in the feature of the laptops and Laptop Model's features.

$$\text{Ł}_{\ddot{\mathbb{M}}}^{\ddot{\theta}} = \begin{bmatrix} (0.45,0.15,0,0) & (0.55,0.7,0,0) & (0,0.15,0,0) & (0.55,0.35,0,0) & (0.48,0.15,0,0) \\ (0.25,0.25,0,0) & (0.55,0.5,0,0) & (0,0,0.35,0) & (0,0.15,0,0) & (0.28,0.05,0,0) \\ (0.25,0.45,0,0) & (0.18,0,0,0) & (0.55,0.45,0,0) & (0,0.35,0,0) & (0.55,0.45,0,0) \\ (0.45,0.05,0,0) & (0.43,0.6,0,0) & (0.55,0.35,0,0) & (0.55,0.25,0,0) & (0.55,0.35,0,0) \\ (0.25,0.15,0,0) & (0.55,0,0.5,0) & (0,0.25,0,0) & (0.55,0,0,0) & (0.08,0.05,0.05,0) \end{bmatrix}$$

$$\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}} = \begin{bmatrix} (0.25,0.55,0,0) & (0.55,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) & (0.48,0.55,0,0) \\ (0.45,0.55,0,0) & (0.55,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) & (0.48,0.55,0,0) \\ (0.25,0.55,0,0) & (0.45,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) & (0.42,0.55,0,0) \\ (0,0.55,0,0) & (0,0.55,0,0) & (0,0.55,0,0) & (0,0.55,0,0) & (0.08,0.55,0,0) \\ (0.25,0.55,0,0) & (0,0.55,0,0) & (0,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) \end{bmatrix}$$

Step-4

Determine the defect of the laptop models by computing the complement of the matrix $\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}}$. It is represented as ${\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}}}^{c}$.

$${\text{Ł}_{\ddot{\mathbb{N}}}^{\ddot{\theta}}}^{c} = \begin{bmatrix} (0,1,0.55,0.25) & (0,1,0.55,0.55) & (0,1,0.55,0) & (0,1,0.55,0.55) & (0,1,0.55,0.48) \\ (0,1,0.55,0.45) & (0,1,0.55,0.55) & (0,1,0.55,0) & (0,1,0.55,0.55) & (0,1,0.55,0.48) \\ (0,1,0.55,0.25) & (0,1,0.55,0.45) & (0,1,0.55,0) & (0,1,0.55,0.55) & (0,1,0.55,0.42) \\ (0,1,0.55,0) & (0,1,0.55,0) & (0,1,0.55,0) & (0,1,0.55,0) & (0,1,0.55,0.08) \\ (0,1,0.55,0.25) & (0,1,0.55,0) & (0,1,0.55,0) & (0,1,0.55,0) & (0,1,0.55,0.55) \end{bmatrix}$$

Step-5

Determine the relation between feature preferences of persons and laptop features, by taking matrix multiplication between them $\left(i.e., Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}}\right)$.

$$Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}} = \begin{bmatrix} (0.45,0.55,0,0) & (0.55,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) & (0.48,0.55,0,0) \\ (0.45,0.5,0,0) & (0.55,0.5,0,0) & (0,0.5,0,0) & (0.55,0.5,0,0) & (0.48,0.5,0,0) \\ (0.25,0.45,0,0) & (0.45,0.45,0,0) & (0,0.45,0,0) & (0.55,0.45,0,0) & (0.55,0.45,0,0) \\ (0.43,0.55,0,0) & (0.45,0.55,0,0) & (0,0.55,0,0) & (0.55,0.55,0,0) & (0.55,0.55,0,0) \\ (0.45,0.25,0,0) & (0.55,0.25,0,0) & (0,0.25,0,0) & (0.55,0.25,0,0) & (0.48,0.25,0,0) \end{bmatrix}$$

Step-6

Determine the relation between feature preferences of persons and defects of laptop, by taking matrix multiplication between them $\left(i.e., Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * {Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}}}^{c}\right)$.

$$Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * {Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}}}^{c} = \begin{bmatrix} (0,0.7,0.55,0) & (0,0.7,0.55,0) & (0,0.7,0.55,0) & (0,0.7,0.55,0) & (0,0.7,0.55,0.08) \\ (0,0.5,0.55,0) & (0,0.5,0.55,0) & (0,0.5,0.55,0) & (0,0.5,0.55,0) & (0,0.5,0.55,0.08) \\ (0,0.45,0.55,0) & (0,0.45,0.55,0) & (0,0.45,0.55,0) & (0,0.45,0.55,0) & (0,0.45,0.55,0.08) \\ (0,0.6,0.55,0) & (0,0.6,0.55,0) & (0,0.6,0.55,0) & (0,0.6,0.55,0) & (0,0.6,0.55,0.08) \\ (0,0.25,0.55,0) & (0,0.25,0.55,0) & (0,0.25,0.55,0) & (0,0.25,0.55,0) & (0,0.25,0.55,0.08) \end{bmatrix}$$

Step-7

Find $\breve{\mathcal{U}}$ and $\breve{\mathcal{V}}$ of $\mathfrak{SM}$.

$$\breve{\mathcal{U}} = \begin{bmatrix} 1 & 1.1 & 0.55 & 1.1 & 1.03 \\ 0.95 & 1.05 & 0.5 & 1.05 & 0.98 \\ 0.7 & 0.9 & 0.45 & 1 & 1 \\ 0.98 & 1 & 0.55 & 1.1 & 1.1 \\ 0.7 & 0.8 & 0.25 & 0.8 & 0.73 \end{bmatrix}$$

$$\breve{\mathcal{V}} = \begin{bmatrix} 1.25 & 1.25 & 1.25 & 1.25 & 1.17 \\ 1.05 & 1.05 & 1.05 & 1.05 & 0.97 \\ 1 & 1 & 1 & 1 & 0.92 \\ 1.15 & 1.15 & 1.15 & 1.15 & 1.07 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.72 \end{bmatrix}$$

Step-8

Measure the $\mathfrak{SM}$, $\mathcal{S}_c\left(Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}}, Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}^{c}}\right)$ to determine the result.

$$\mathcal{S}_c\left(Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}}, Ł_{\ddot{\mathbb{M}}}^{\ddot{\theta}} * Ł_{\ddot{\mathbb{N}}}^{\ddot{\theta}^{c}}\right) = \begin{bmatrix} -0.25 & -0.15 & -0.7 & -0.15 & -0.14 \\ -0.1 & 0 & -0.55 & 0 & 0.01 \\ -0.3 & -0.1 & -0.55 & 0 & 0.08 \\ -0.17 & -0.15 & -0.6 & -0.05 & 0.03 \\ -0.1 & 0 & -0.55 & 0 & 0.01 \end{bmatrix}$$

From the above $\mathfrak{SM}$, the score of APPLE 2020 MacBook Air M1 is higher for all the five persons. So, we conclude that, APPLE 2020 MacBook Air M1 is the best laptop model for everyone from the given data. Utilizing $\mathfrak{SLNFM}$ enables us to identify the most accurate laptop model during the early stages of decision-making. This approach not only streamlines the selection process, but also minimizes the risk of investing in a laptop that does not align with the user's preferences, ultimately leading to cost savings. This study can also be applied for large and complex data too.

5. Conclusions

In order to address multicriteria situations with neutrophilic inputs, we have built the idea of $\mathcal{SV}-$ Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrices by combining Łukasiewicz logic to $\mathcal{SV}$ −Neutrosophic $\mathscr{F}$uzzy Matrices and discussed the operations and types associated with them. As a note of it, we address a decision-making problem on selecting the best laptop model based on the individual's feature preferences and provided a solution which is more applicable for every individual in the data. This study shows how one can carry out the benefits of $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrices in hard computations. In future, $\mathcal{SV}$ −Łukasiewicz Neutrosophic $\mathscr{F}$uzzy Matrix can be investigated in higher dimensional structures and one can incorporate this idea in Machine Learning for constructing AI Models to enhance the process of reinforcement or clustering algorithm.

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