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Neutrosophic Inverse Gaussian Distribution in Economic Policy Design under Indeterminacy

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Abstract: Uncertainty and indeterminacy are central concepts in economic policy design. The inherent uncertainty is difficult to incorporate into traditional statistical models. To overcome the limitation of classical inverse Gaussian (IG) distribution for managing the imprecise data, we examine the structure of neutrosophic inverse Gaussian distribution (NIGD) which is an expansion of the classical IG distribution under neutrosophic framework. The IG distribution, which is commonly employed for reliability analysis and in financial modeling, features a positively skewed curve which makes it is an appropriate index for modeling asymmetric economic data, including risk assessments, investment returns, as well as financial duration. That model includes degrees of truth, lack of information and degrees of falsity, elements that enable policymakers to compute economic variables with incomplete / imprecise information. The mean, variance, skewness and kurtosis of NIGD are derived in neutrosophic environment. The quantile function of the proposed is derived which is further utilized to generate random samples from the proposed model. The utilization of the distribution in economic uncertainty modeling is described using two numerical examples.

Keywords: Neutrosophic probability, inverse Gaussian distribution, economic policy, indeterminacy, uncertainty modeling, financial risk.

1. Introduction

Probability distributions constitute a fundamental tool of economic policy making to recent years; their application comprises uncertainties analysis, economic indicators modelling and decision support in various correlation terms [1]. Recession predictions are essentially complex; there are stochastic factors driving the economy that are beyond economic theory, such as inflation, interest rates, stock market dynamics, and others [2]. For example, policymakers use probability models to assess risk, allocate resources, and predict economic trends. However, one of the well-known models applied in financial modeling is normal distribution, particularly used in the studies of portfolio optimization and risk and the returns of the asset are mostly assumed to have a normal distribution [3]. The exponential and Weibull distributions are also used commonly in analyzing economic data, financial data and macroeconomic data modelling, which enables policymakers to assess the lifespan of economic policies and business lifespans [4]. The Poisson distribution is another useful distribution in modeling rare economic phenomenon, like financial panics, and spikes in unemployment, helping governments plan their response to such crises [5]. The same goes for Bayesian inference with probability distributions, allowing policymakers to revise economic forecasts when new information comes into the world, thus improving the readiness of the policy structures [6]. In this way statistical distributions have long applications history in the field of economic data. One of the most useful among several other probability distributions is the IG distribution for modeling economic and financial data with positive skewness, meaning that the tail of the distribution is longer on the right side [7]. Unlike the normal distribution, which is symmetric by nature, the IG is appropriate for modelling asymmetric economic phenomena like financial returns, insurance claims, and the time-to-default events used in risk assessment of credit [8]. It is commonly used to study waiting times, so it is relevant for investigating business cycle duration, economic recovery duration, and market stabilization duration. In risk management for finance, IG distribution quantifies the likelihood of extreme losses, which are critical to banks as well as regulators, in terms of reserving capital. In industrial economics, it analyzes production processes, helping find efficient investment strategies and reducing risks of economic downturns [9]. The diverse nature of the IG distribution plays a central role in producing robust economic policies applications [10]. Fuzzy theory is an active and critical component of modern statistical inferences and data sets in programs of uncertainty, obscurity, and precision [11]. While classical statistical methods depend on exact values and point estimates of probabilities, fuzzy statistical methods provide degrees of membership rather than strictly yes or no outcomes. This is one of the reasons fuzzy statistics are particularly useful in fields such as economic, financial, or social sciences where data typically contain fuzzy or inadequate information [12]. Classical probability attempts to assign probabilities on the basis of frequency or axiomatic rules, treating all uncertainty as quantifiable in precise terms [13]. But in real-world economics issues like the prediction of inflation, modeling consumer behavior or assessing financial risk, the input for decisions often consists of linguistic or imprecise data (e.g. "high inflation", "moderate risk") that is not appropriately generated by traditional probability distributions. Fuzzy statistics are mathematical frameworks that enhance classical statistical techniques by utilizing fuzzy numbers, fuzzy sets, and theories of fuzzy logic based on inference, making it easier for policymakers and analysts to handle ambiguous data [14]. For example, in financial risk management, instead of defining rigid limits for investment risk (e.g., "low risk" below 5 percent volatility and "high risk" above 15 percent), fuzzy statistics allow for gradual transitions between levels of risk that reflect more realistic and flexible decision-making. Fittingly, also in economic policy design, especially when labor-employment rates or GDP growth needs to be forecasted, which are multiclient related uncertainties, fuzzy regression models present a better approach compared to classical regression, being able to capture the vague nature of the relationship between economic variables [15]. Another application is analyzing consumer sentiment, where responses like “somewhat satisfied” or “moderately optimistic” result in something more nuanced than fitting a data point into one of three or four arguments. Since fuzziness reflects uncertainty and linguistics captures human stories, fuzzy statistical methods are thus extremely useful for practical applicability as human implications of economic phenomena are inherently complex, subjective, and incomplete at best, providing more flexible yet stronger tools for economic policymaking under uncertainty.

The neutrosophic theory proposed by Smarandache generalizes the classical and the fuzzy logic by adding an additional component known as the “indeterminacy” to model [16-19]. Neutrosophic sets extend beyond fuzzy logic that is constrained by a single degree of membership (between 0 and 1) by introducing three independent parameters, truth (T), indeterminacy (I), and falsity (F) whose values are in the range 0 and 1 [20-22]. Neutrosophic theory has many applied areas applications including neutrosophic statistical methods. Neutrosophic statistics are more effective to study real world complex problems while data which provide solutions are ambiguous, feet and/or information missing exist in economic policymaking. For example, in finance forecasting, a market trend may be true to an extent, but indeterminate to an extent due to the impact of unforeseen geopolitical factors and false to an extent due to obsolete data sources. Fuzzy logic provides a method for reasoning with uncertain attributes, but is limited in its ability to accommodate contradictions and incomplete knowledge. Fuzzy methods overestimate the fuzziness of their inputs, under the assumption that only small-scale, gradual transitions exist between pieces of knowledge; however, in practice, there are situations in which some information is actually lost or contradictory, and these do not fade away into a smooth gradient. An example of such a situation is outlined in economic risk assessment, where the uncertainty of the financial health of a company corresponds not even to information being deficient in scholarly documentation, but also to experts presenting contradictory opinions, which neutrosophic methods can deal with much better than traditional fuzzy models can (in this sense).

Neutrosophic statistics, on the other hand, is a generalization of classical methods of statistics composed of three components: truth, indeterminacy, and falsity, which provides it with a greater capacity for analyzing data that is incomplete, inconsistent, and uncertain [23]. Neutrosophic statistics accept uncertainty, vagueness, and paradoxes, which is a natural phenomenon that occurs in the real world due to the fuzzy setup of economic and financial decision making. Neutrosophic statistical methods differ the classical statistics in terms of facilitating indeterminate and indecisive information data [24]. In macroeconomic modeling for example, it is more appropriate to handle the data inconsistencies arising from missing or unreliable sources when using neutrosophic probability distributions. This flexibility improves uncertain decision-making and gives policymakers more robust adaptive tools than classical statistical approaches can provide. Under neutrosophic probability distributions generalize the classical probability models adding new coordinates: truth (T), indeterminacy (I) and falsity (F), making it a suitable representation of random variables with uncertainty and inconsistency data [25-26]. Neutrosophic distributions which are a generalization of the traditional probability distributions capture the ambiguity related to parameters, thus are a great tool to use in fields like economic and financial forecasting, financial risk analysis, reliability modeling, etc., [27-28]. For instance, in stock market predictions, the price increase probability is sometimes true and sometimes false and partially known knowledge to be indeterminate because of externals factors and partially is false (e.g. old information).

In this work, we study a neutrosophic variant of the classical IG distribution that assumes precise data and well-defined probabilities, which might not be the case in the real world under economic conditions. Uncertainty, ambiguity, and missing information are the hallmarks of designing economic policy due to volatile markets, incomplete datasets, and diverse views from experts. The IG distribution is a special case of NIGD where: NIG is given by the complete truth T, indeterminacy (I) and complete falsity F. Such current-to-future projection dynamic captures conditionality of structural dynamics of policy environment under risk and uncertainty and contributes to risk assessment and policy optimization, as well as robustness of decision making in the face of uncertainty.

The paper is organized as follows. The description of classical IG distribution is presented in Section 2. Section 3 discusses the proposed model. Random data generating from the proposed model is presented in Section 4. Sec. Finally, the findings of the study are presented in Section 5.

2. Preliminary Results

In this section, we will view the basic results of the classical structure of the inverse Gaussian distribution and their related functions.

A random variable is said to follow the Gaussian model if it has the following PDF and CDF functions:

$$h_Y(\, y \mid \beta, \theta\,) = \sqrt{\frac{\theta}{2\pi y^3}} \exp\left(-\frac{\theta(y-\beta)^2}{2\beta^2 y}\right), y > 0 \qquad (1)$$

$$H_Y(y \mid \beta, \theta) = \Phi\left(\sqrt{\frac{\theta}{y}}\left(\frac{y}{\beta} - 1\right)\right) + \exp\left(\frac{2\theta}{\beta}\right)\Phi\left(-\sqrt{\frac{\theta}{y}}\left(\frac{y}{\beta} + 1\right)\right) \quad (2)$$

The shapes of the PDF and CDF of the NIGD are given in Figure 1 and Figure 2.

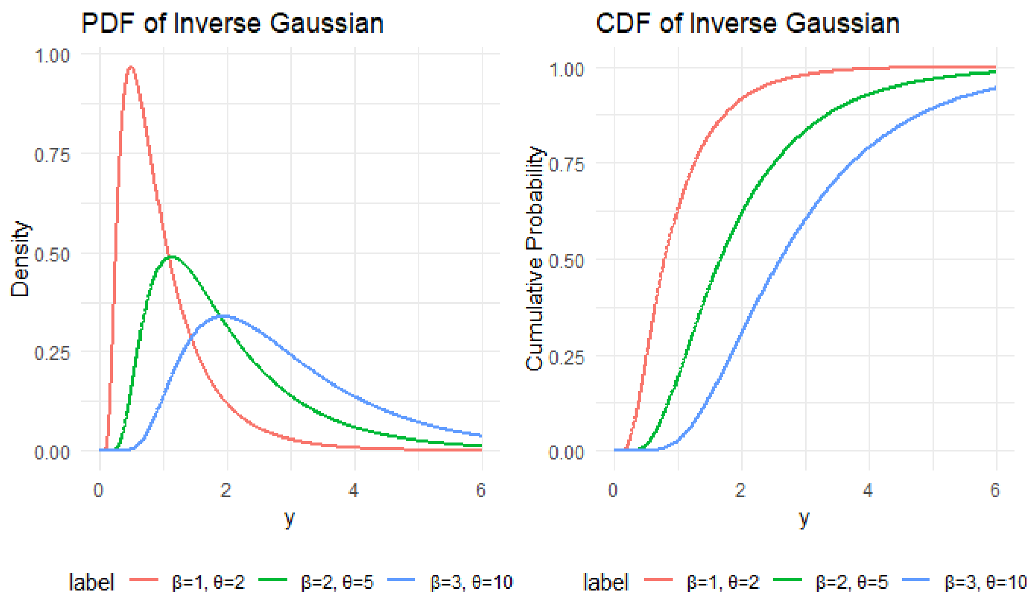


Figure 1 PDF and CDF of the NIGD with different parameters setting

Figure 1 shows the PDF and CDF function with different values of shape and scale parameters. The NIGD is a continuous probability distribution that is widely used for positively skewed data, especially as applied in reliability analysis, survival analysis, and stochastic processes. It is defined by two parameters which are the shape parameter θ and the scale parameter β. The shape parameter θ controls the skewness and spread of the distribution. For bigger values of θ the distribution is symmetric and resembles that of a normal distribution, while lower θ generates a highly skewed distribution with a long right tail. The scale parameter β defines the trajectory of central tendency, indicating how the peak moves along the domain. A larger beta shifts the distribution to the right, yielding a higher expected value, while a smaller β shifts it to the left. Because its shape is determined by both parameters, the PDF defines the likelihood of different values occurring. Knowing that, we also know from CDF that the CDF is the probability of a random variable event being less than a threshold, contributing to decision-making process scenarios.

Now other key statistical characteristics of the classical IG distribution can be established from literature as:

The mean which denotes an expected value of the random variable Y is given by:

$$E[Y] = \beta \quad (3)$$

This quantity provides essential information about the central tendency of the distribution.

To know about the spread of the distribution, variance is commonly used measure of dispersion which is given by:

$$\mathrm{Var}(Y) = \frac{\beta^3}{\theta} \quad (4)$$

The mode is also a useful measure to measure the most frequent of any distribution. It is value where we get the maximum peak of the PDF. Unlike the mean, it is not affected by the extreme values that commonly exist in the macroeconomic data. Mode of the IG distribution is given by:

$$\text{Mode} = \beta\left(\sqrt{1+\frac{9\beta}{4\theta}}-\frac{3\beta}{2\theta}\right) \quad (5)$$

Median also plays a significant role in data modelling. Median of the IG distribution can be written as:

$$\text{Median} \approx \beta\left(1-\frac{1}{3}\sqrt{\frac{\beta}{\theta}}\right) \quad (6)$$

Median is also a very important measure which is commonly recommended measure in case of skewed mode because it is also not highly affected by extreme values. It is particularly useful when data is expected to contain outliers such as real estate prices, income level making it key measure in decision-making and descriptive statistics.

Shape coefficients are also very important measures for any distribution. The skewness coefficient of the IG distribution is given by:

$$\text{Skewness} = 3\sqrt{\frac{\beta}{\theta}} \quad (7)$$

This coefficient provides information about the symmetry of the distribution. Another measure related to shape is kurtosis which can be defined as:

$$\text{Kurtosis} = \frac{15\beta}{\theta} \quad (8)$$

Kurtosis coefficient provides information about the tail thickness of the distribution.

Similarly, the moment generating function (MGF) of the IG distribution can be written as:

$$M_Y(t) = \exp\left(\frac{\theta}{\beta}\left(1-\sqrt{1-\frac{2\beta^2 t}{\theta}}\right)\right), t < \frac{\theta}{2\beta^2} \ (9)$$

MGF has a key role in mathematical statistics and is commonly derived for generating moments. These moments can then easily be obtained by differentiating the MGF, so this is particularly handy in theoretical derivations. Also, when it exists, the MGF uniquely determines the distribution, so it is possible to identify, and compare two probability distributions. Moreover, MGF also allows the distributions of sum of independent random variables which makes it a versatile tool in applied statistics and probability inference. If we assume the different values of the shape parameter such as θ =[2.5, 5, 7.5, 10] and location parameter $\beta = [1 ,2 ,3, 4]$ then the mean, variance, mode and median can be computed which is provided in Table 1.

Table 1 Statistical characteristics of the IG distribution

Location	Shape	Mean	Variance	Mode	Median
1	2.5	1	0.4	0.77	0.79
2	5	2	1.6	1.55	1.57
3	7.5	3	3.6	2.33	2.36
4	10	4	6.4	3.11	3.15

Results in Table 1 show the statistical key properties of the IG distribution. Results indicate that all the values are crips numbers because we have exact knowledge about the distributional parameters. Now in the next section, statistical model of the IG distribution with key statistical properties under neutrosophic framework are presented.

3. Neutrosophic framework for IG Model

In this section, the statistical framework for the IG distribution under the neutrosophic environment is described. Under the neutrosophic environment we assume that the parameters of the distribution are interval forms and not the crisp values. The PDF and CDF of the NIGD can be written as:

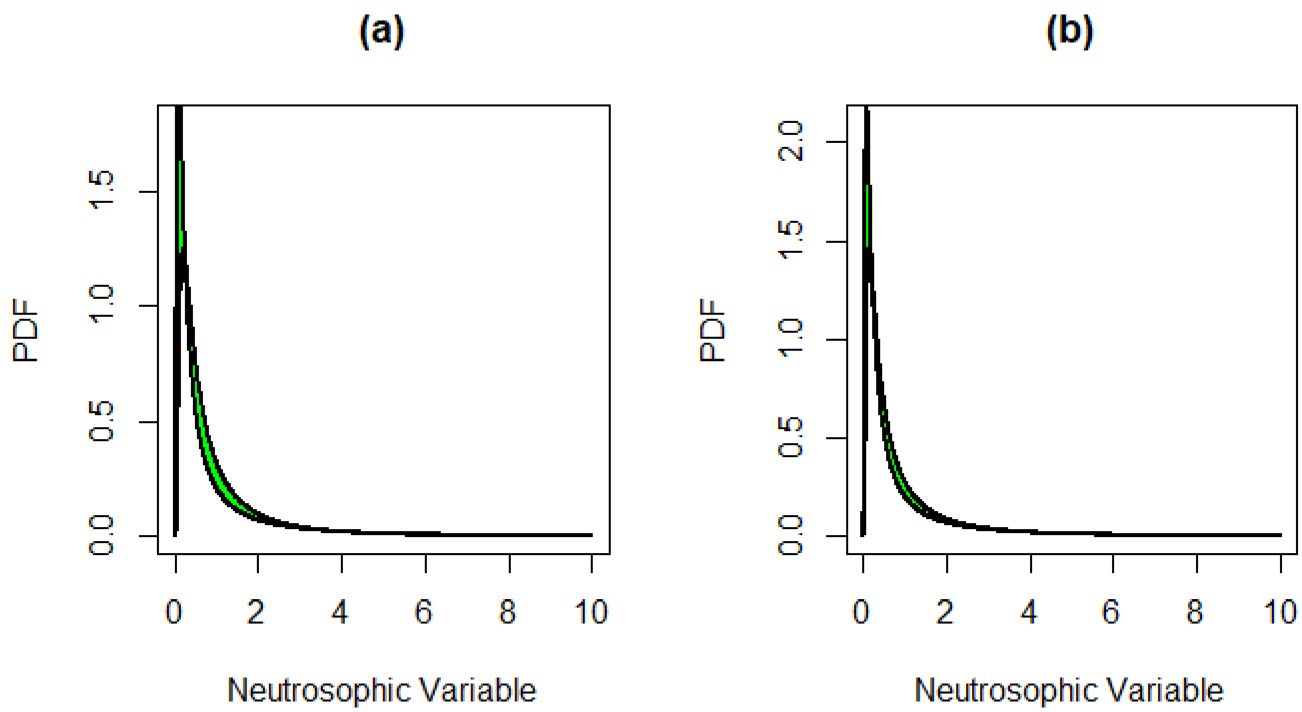
$$h_Y(y \mid \beta_N, \theta_N) = \sqrt{\frac{\theta_N}{2\pi y^3}} \exp\left(-\frac{\theta_N (y-\beta_N)^2}{2\beta_N^2 y}\right), y > 0 \tag{10}$$

$$H_Y(y \mid \beta_N, \theta_N) = \Phi\left(\sqrt{\frac{\theta_N}{y}}\left(\frac{y}{\beta_N} - 1\right)\right) + \exp\left(\frac{2\theta_N}{\beta_N}\right)\Phi\left(-\sqrt{\frac{\theta_N}{y}}\left(\frac{y}{\beta_N} + 1\right)\right) \tag{11}$$

where $\beta_N = [\beta_l, \beta_u]$ and $\theta_N = [\theta_l, \theta_u]$.
If we assume the different values of β_N and θ_N, plots of PDF and CDF are given in Figure 2 and Figure 3 respectively.

Figure 2 PDF curves of the NIGD with parameters (a) $\beta_N = [1,1]$ and $\theta_N = [1.5, 3.5]$ (b) $\beta_N = [1,1]$ and $\theta_N = [2, 3.75]$.

In Figure 2, we depict the PDF of the proposed model with varying values for the neutrosophic distributional parameters. The PDF curves are similarly right skewed in shape, consistent with the IG distribution. Then as shape parameter increases, the peak of the distribution shifts slightly to the left, and the curve becomes more spread out indicating increased variability. Also notice that the density function reaches a maximum for lower values of Y and then asymptotically decreases, resulting in a



heavy tail in the distribution. This shows that for larger location parameters, the probability of getting extreme values increases, and the distribution becomes more spread.

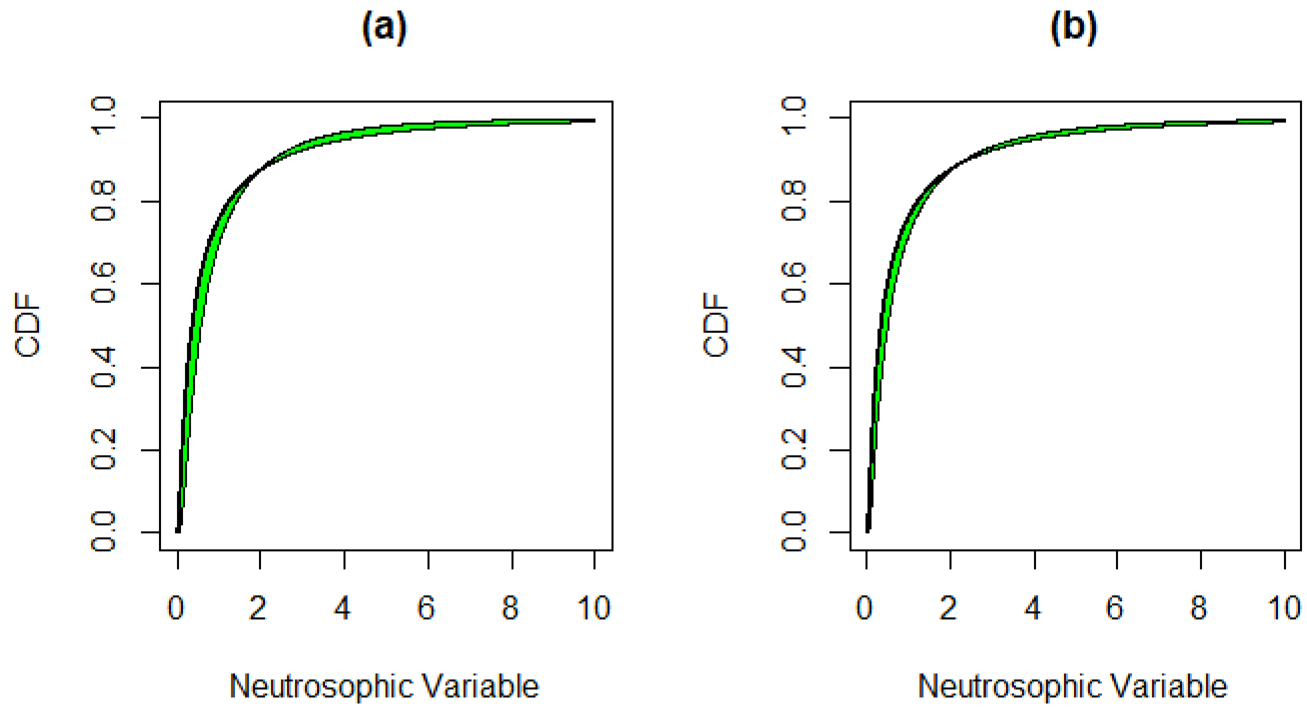


Figure 3 CDF curves of the NIGD with parameters (a) $\beta_N = [1,1]$ and $\theta_N = [1.5, 3.5]$ (b) $\beta_N = [1,1]$ and $\theta_N = [2, 3.75]$.

In Figure 3, we show the CDF of the proposed model, for the same neutrosophic parameter values. The CDF curves showing the probability that the random variable Y will be less than or equal to some given value y. As we can see, CDF is an increasing function, going to 1 when y tends to infinity. For larger shape parameter values, the CDF curve rises more slowly, indicating the greater dispersion of the distribution. Differences in CDF curves indicate that the value of shape parameters affect the distribution in overall behavior.
Mean and variance of the NIGD can be structured as follows:

$$E[Y] = \beta_N \tag{12}$$

$$\mathrm{Var}(Y) = \frac{\beta_N^3}{\theta_N} \tag{13}$$

where $\beta_N = [\beta_l, \beta_u]$ and $\theta_N = [\theta_l, \theta_u]$.
The mean and variance of the NIGD offers crucial information about the central tendency and dispersion of the distribution along with indeterminacy and uncertainty. Mean value is not fixed as it is dependent from the location parameter (β_N) which takes neutrosophic values that may be intervals or uncertain values. This versatility enables modeling of imprecise or incomplete data. The variance, defined by both the shape parameter (θ_N) and location parameter (β_N), outlines how wide the distribution is. Unlike classical IG distribution, which have well-defined mean and variance, NIGD allows for interval-based or fuzzy representations, where corresponding replications of uncertainty owned by real-world data. This makes it especially powerful in cases where data is less than perfect, open to interpretation or where more than one solution exists.
The following two examples describe the utility of the proposed model in economic policy design.

Example 1

A policy maker is interested in studying how much time it will take for an economic crises to happen in different areas. Economic conditions are still shrouded in uncertainty, and the behavior of inflation rates, interest rates as well as foreign exchange reserves varies over time, which diffuses usefulness of classical statistical models. The neutrosophic NIGD is a more flexible model that integrates truth, indeterminacy, and falsity of economic data. Let the average time until an economic crisis occurs (in years) is uncertain and estimated as given as $\theta_N = [5, 6]\, years$ and the uncertainty in economic fluctuations is modeled using the shape parameter $\beta_N = [2, 3]\, years$. Suppose the economist is interested in finding the mean and variance of the fitted model under an uncertain environment.
Using the properties of the proposed model mean and variance can be calculated as follows:

$$E(T) = \theta_N = [5, 6]\, years$$

$$var\,(T) = [41.67, 108]\, years^2$$

The time before the economic crisis is expected to be around 5 to 6 years, but because of the economic uncertainties, the predictions may vary, this variability in the predictions is captured by neutrosophic variance. High variance indicates uncertainty, so economic policy needs to be flexible in the face of fluctuations of financial stability. Because of the need to explain this imprecision in economic forecasting to policymakers, the NIGD based approach may provide important benefits to achieving their goals of risk management and responsive policymaking.

Example 2

A bank wants to know how long its customers, on average, will take to fully repay their loan. But volatile economic circumstances cast uncertainty across factors such as inflation, job security and interest rate movements and mean the period of payment is indefinite. Rather than a classical model, the bank uses the neutrosophic NIGD to account for this uncertainty. Assume that repayment time data follow the neutrosophic NIGD with $\theta_N = [7, 9]\, years$ and $\beta_N = [3, 4]\, years$.
Using the characteristics of the proposed model, mean and variance can be computed as:

$$E(T) = \theta_N = [7, 9]\, years$$

$$var\,(T) = [85.75, 243]\, years^2$$

It could take 7 to 9 years to pay back, although predicting it is imprecise because of economic uncertainties. Higher variance (in the 85.75 to 243 years^2) indicates large swings which would imply risk adjusted interest rates for borrowers to the bank. This methodology aids in improved risk assessment, lending to financial institutions to be flexible about policies by adjusting them around economic uncertainty.

Likewise, the mean, other measures used for central location are median and mode which can be defined as in context of neutrosophic environment.

$$\text{Median}(Y) \approx \beta_N \left(1 - \frac{1}{3}\sqrt{\frac{\beta_N}{\theta_N}}\right) \tag{14}$$

$$\text{Mode}(Y) = \beta_N \left(\sqrt{1 + \frac{9\beta_N}{4\theta_N}} - \frac{3\beta_N}{2\theta_N}\right) \tag{15}$$

Understanding the characteristics of data through the neutrosophic median and neutrosophic mode of the NIGD is particularly useful in cases involving uncertainty or incomplete information. The expression for median indicates that it is controlled by both the scale (θ_N) and shape (β_N) parameter. The neutrosophic median, unlike the mean which could be influenced by extreme values, is more resistant to outliers and indeterminate deviations, and so well suited for cases where data is not easy to acquire with accuracy or involve extreme values such as economic forecasting or financial market Analysis. The mode, on other hand, is the highest value of the probability density function, can be tuned by β_N and θ_N. In context of economic policy both median and mode can assess in decision making under uncertainty environment. Neutrosophic extensions enable more flexible, adaptive, and uncertainty-

resilient policies for policymakers to mitigate risk, assess impacts more comprehensively, and plan for economically robust responses in unpredictable market environments.
If the neutrosophic values of the distribution parameters are assumed, then key statistics of the NIGD can be calculated as given in Table 2.

Table 2 Statistical characteristics of the NIGD under imprecise parameters settings

β_N	θ_N	Mean	Variance	Mode	Median
[0.5, 0.5]	[2, 2.5]	[0.5, 0.5]	[0.05,0.062]	[0.452,0.463]	[0.425,0.455]
[0.5, 0.5]	[4, 75]	[0.5, 0.5]	[0.026,0.031	[0.477, 0.477]	[0.445, 0.472]
[0.5, 0.5	[6, 6.68]	[0.5, 0.5	[0.018, 0.021]	[0.482, 0.484]	[0.454, 0.479]
[0.5, 0.5	[8, 8.95]	[0.5, 0.5	[0.013,0.015]	[0.485,0.488]	[0.461, 0.483]

Results in Table 2 show that when we have imprecision in the underlying parameters, all the statistical characteristics are involved in imprecision.
Shape coefficients of the proposed model can be defined as:

$$\text{Skewness} = 3\sqrt{\frac{\beta_N}{\theta_N}} \quad 16)$$

$$\text{Kurtosis} = 15\frac{\beta_N}{\theta_N} \quad (17)$$

As we can see NIGD is like the classic form and the coefficients of skewness and coefficient of kurtosis describes it asymmetric and peakedness nature along with elements of uncertainty, indeterminacy and inconsistency. These coefficients become interval-based or fuzzy in accordance with the incomplete or imprecise conditions of data in a neutrosophic environment. The skewness coefficient measures asymmetry around central tendency. Indeed, skewness coefficient in classical form shows a specific number, whereas, in the neutrosophic setting, they can assume several possible values as there exists the uncertainty of data here, and thus, this is helpful in evaluating economic inequality. In income distribution, for example, neutrosophic skewness can provide decision-makers with the extent of potential income differences under uncertain conditions and can lead to more flexible tax policies or the means-tested adjustment of welfare programs. On the contrary, kurtosis is the coefficient of the heaviness of tails of the distribution. In standard statistics, this is an exact number, but in neutrosophic domain it considers indeterminate variations in the number, thus presenting a more valid perspective of real-world instances like financial crisis or market crash. This is incredibly useful in economic risk management, where decision-makers need to prepare for a variety of potential extreme scenarios, rather than relying on a single expected outcome. Including neutrosophic measures of skewness and kurtosis allows for improved responsiveness of economic policies and financial models to unpredictable environmental conditions, facilitating more effective risk management and sustainable growth strategies.

4 Quantile Function

In probability theory, the quantile function of a probability distribution is a function that describes the distribution in terms of an arbitrary quantile. In other words, it is useful for ascertaining the value of a random variable for a certain cumulative probability. Essentially, the quantile function provides us with thresholds or cut-off points (eg. the value below which 10% of the data sit or the value above 25% of the data). It is particularly useful in areas like risk management, economics, and decision-making, where it is important to understand the behavior of data at its extreme values at its worst-case or best-case scenario. It allows you to know the corresponding value for a desired percentile, in an inverse relationship to the CDF, which is vital to calculate measures based on percentiles that are enormously useful in making decisions in uncertain environments. The quantile function of the proposed model can be written as:

$$Q(p \mid \beta_N, \theta_N) = \beta_N\left(1 + \sqrt{\frac{9\beta_N^2}{4\theta_N} + \left(1 - \frac{4p}{\theta_N}\right)^2}\right) \tag{18}$$

This quantile function can be used to generate random samples that assumed to follow NIGD by using inverse CDF method. In this method, inverse transformation approach is used, and we generate uniformly distributed random numbers between 0 and 1 and then use the quantile function with respect to the following CDF of random samples from NIGD. We generate random numerical values between 0 and 1. These probability values are then applied to the quantile function to obtain neutrosophic NIGD samples. Doing this several times, we can produce many samples of NIGD random variable. This is particularly useful because it provides a way to generate samples from the distribution of a random variable when the distribution itself is complex and difficult to sample from. Monte Carlo simulations are beneficial when closed-form expressions of the random variable are not accessible or when the distribution is complex (e.g., in the case of the NIGD).

To generate the random samples, we have used a program written in R making use of some functions from the R library "statmod". Using these functions, we can obtain samples for the NIGD using the quantile function method. Random samples obtained from this approach are shown in Table 3.

Table 3 Random numbers generated from the proposed model using Monte Carlo experiment

Random numbers
[0.854, 1.123], [0.765, 1.045], [0.923, 1.204], [0.812, 1.112], [0.876, 1.156], [0.794, 1.098], [0.873, 1.134]
[0.902, 1.187], [0.789, 1.043], [0.865, 1.152], [0.847, 1.129], [0.888, 1.203], [0.823, 1.097], [0.904, 1.214]
[0.911, 1.176], [0.765, 1.054], [0.856, 1.126], [0.822, 1.093], [0.931, 1.223], [0.875, 1.153], [0.897, 1.190]
[0.877, 1.145], [0.853, 1.118], [0.928, 1.217], [0.802, 1.075], [0.872, 1.138], [0.794, 1.056], [0.883, 1.164]
[0.894, 1.194], [0.872, 1.136], [0.910, 1.199], [0.765, 1.051], [0.842, 1.115], [0.801, 1.089], [0.932, 1.228]
[0.919, 1.204], [0.812, 1.097], [0.866, 1.143], [0.839, 1.110], [0.926, 1.211], [0.794, 1.068], [0.903, 1.190]
[0.884, 1.172], [0.803, 1.085], [0.918, 1.201], [0.866, 1.150], [0.879, 1.146], [0.799, 1.081], [0.902, 1.185]

Results in Table 3 show interval value samples from the proposed model at parameters setting $\beta_N = [1,1]$ and $\theta_N = [1.5, 2.5]$.These random samples are generated using specific seed value 222 in the program. If we assume that indeterminacy in the dispersion parameter is zero, yielded samples would coincide with the classical approach of the IG distribution.

5 Conclusions

Based on classical inverse Gaussian distribution and neutrosophic logic, the neutrosophic inverse Gaussian distribution (NIGD) has been described in this study. The suggested model presents a powerful alternative to its classical statistical variant with a view to capture uncertainty and imprecision in economic data. This allows the neutrosophic NIGD to characterize asymmetric processes by truth, information deficit, and falsity, leading to a versatile continuum to analyze economic processes with a focus on risk measures, return on investment, and financial time scales. The key statistical properties of the proposed model have been discussed both in theoretical and practical aspects. The proposed model extended the classical IG distribution to better account for partial or imprecise information when trying to model economic variables which are intimately linked to proper policy design. The quantile function of the proposed model has been derived further utilized to random samples from the proposed model. Numerical findings indicated that the proposed model is a versatile model to capture uncertainty data that often occurs in financial risk management and economic growth data modeling.

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