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Neutrosophic Quasi-XLindley distribution with applications of COVID-19 data

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Abstract: The Quasi-XLindley distribution (QXLD) is widely used in the field of survival and reliability engineering to simulate lifespan data in different fields of human, electronic designs and other fields. However, when dealing with uncertain data, a more generalized version of this distribution is needed. To address this, a neutrosophic Quasi-XLindley distribution (NQXLD) is developed in this paper. The NQXLD is particularly useful for representing skewed uncertain data. In this study, we present some statistical characteristics of the NQXL distribution, including the neutrosophic mean time failure, neutrosophic hazard rate, neutrosophic moments, and neutrosophic survival function. We also evaluate the parameters using the maximum likelihood (ML) estimation technique in a neutrosophic context based on a simulation study. Finally, applications of three different real data sets are considered to investigate the applicability of the suggested NQXL distribution. The results show the flexibility of the NQXL distribution in fitting various types of COVID-19 data as compared to the QXLD.

Keywords: Neutrosophic; COVID-19; Maximum likelihood estimation; Probability distribution; Quasi-XLindley distribution, Uncertainty

1. Introduction

The use of lifetime probability models has revolutionized analytical tools and is now widespread across various fields., such as renewable energies, health sciences, biology and engineering. Over the past century, numerous vital distributions have been derived, as the Quasi-XLindley distribution suggested by [20] being one of the most widely used regarding reliability. Its usefulness has transformed the applied sciences, inspiring researchers to discover new and innovative ways to leverage this powerful analytical toolset. The probability density function (PDF) of the QXLD is given by:

$$f\left(x;\alpha,\theta\right)=\left\{\frac{\theta}{1+\alpha}\left(\alpha+\frac{\theta\left(1+x\right)}{1+\theta}\right)e^{-\theta x}\right\},\ x>0,\alpha,\theta>0. \tag{1}$$

The cumulative distribution function (CDF) of the QXLD is defined as:

$$F\left(x;\alpha,\theta\right)=1-\left(1+\frac{\theta x}{(1+\theta)(1+\alpha)}\right)e^{-\theta x}. \tag{2}$$

Statistical distributions are an essential tool for modeling data with exact and determinate observations, but they become unsuitable when there is uncertainty in the observations or distribution parameters. In such cases, fuzzy-logic-based distributions can be applied to datasets.

The fuzzy Rayleigh distribution suggested by Pak et al. [23] is a perfect example of this. It has been extensively studied and successfully employed for reliability analysis and biomass pyrolysis by Dhaundiyal and Singh [14]. Van Hecke has explored the estimation of a fuzzy Rayleigh distribution in [31], and Pak et al. [24] have estimated its parameter for fuzzy lifetime data. Shafiq et al. [27] provided comprehensive work on distribution reliability concerns using the fuzzy method. Chaturvedi et al. [11] have used a fuzzy method to analyze hybrid censored data. Therefore, it is safe to conclude that fuzzy-logic-based distributions offer a powerful alternative to classical probability models when uncertainty exists in the observations or distribution parameters.

The rise of neutrosophic distributions has been a significant development in recent years. These distributions are designed to simulate phenomena with ambiguous observations. Several researchers have proposed the best neutrosophic probability distributions to analyze these data sets. Alhabib et al. [5] have proposed neutrosophic Poisson, neutrosophic exponential, and neutrosophic uniform distributions. Alhasan and Smarandache [18] have introduced a wide range of neutrosophic distributions, including the neutrosophic Weibull, Rayleigh, three-parameter Weibull, five-parameter Weibull, beta Weibull, and inverse Weibull distributions. Patro and Smarandache [24] proposed neutrosophic normal and binomial distributions. Aslam [10] suggested the neutrosophic Raleigh distribution, which has been used to model wind speed data. The neutrosophic Beta distribution was proposed by Sherwani et al. [28], while the neutrosophic Kumaraswamy distribution was proposed by Ahsan-ul-Haq [1]. Aslam [7] studied the Weibull distribution under indeterminacy and used it to construct the sampling strategy for testing average wind speed. More recently, Ahsan-ul-Haq and Zafar [2] proposed a neutrosophic discrete Ramos-Louzada distribution. Duan et al. [16] suggested neutrosophic exponential distribution and investigated it in modeling and applications for complex data. Khan et al. [19] considered a development of the neutrosophic gamma distribution. Eassa et al. [17] proposed neutrosophic generalized Pareto distribution. Algamal et al. [4] introduced the neutrosophic Beta-Lindley distribution with an application to bladder cancer data. Sherwani et al. [29] offered the neutrosophic discrete geometric distribution. Rao [26] suggested the neutrosophic log-logistic distribution. Ahsan-ul-Haq et al. [3] suggested neutrosophic Topp-Leone distribution considered for interval-valued data analysis. The increasing number of neutrosophic distributions available makes it easier to simulate complex phenomena with ambiguous observations, which is crucial for solving real-world problems.

The neutrosophic logic provides information regarding determinacy, indeterminacy, and falseness as Neutrosophy [21]. Neutronsophic logic is, therefore, more effective than interval-based analysis and fuzzy logic. Later on, several writers demonstrated their work on neutrosophic logic for various issues as Pratihar et al. [25]. The introduction of neutrosophic logic is investigated by Smarandache [30] has led to the development of neutrosophic statistics, which provide a means of determining indeterminacy and determinacy (as elaborated in Aslam [8]). In the absence of data relating to the measure of indeterminacy, classical statistics can be employed as an alternative to neutrosophic statistics.

The rest of the paper is organized as follows: The suggested NQXLD is presented in Section 1 with its cdf. Some statistical properties are given in Section 3. Section 4 shows the estimation of the distribution parameters. In Section 5, three real data sets are investigated to illustrate the distribution applicability. The paper is concluded in Section 6.

2. Neutrosophic Quasi-XLindley Distribution

The authors [30] and [9] introduced neutrosophic statistics and propose the following formula for the neutrosophic variable: $X_N = X_L + I_N X_U$, $I_N \in [I_L, I_U]$ where X_L and $I_N X_U$ represent definite and indeterminate portions, respectively, and $I_N \in [I_L, I_U]$ is an arbitrary interval. Assume that the NQXLD, with neutrosophic scale parameters $\alpha_N \in [\alpha_L, \alpha_U]$ and $\theta_N \in [\theta_L, \theta_U]$, follows the neutrosophic random variable $I_N \in [I_L, I_U]$. Remember that the lower and upper values are, respectively, represented by the symbols X_L, α_L, θ_L and X_U, α_U, θ_U. The PDF of the NQXLD is provided by:

$$f(x_N;\alpha_N,\theta_N)=\left\{\frac{\theta_L}{1+\alpha_L}\left(\alpha_L+\frac{\theta_L(1+x_L)}{1+\theta_L}\right)e^{-\theta_L x_L}\right\}+\left\{\frac{\theta_U}{1+\alpha_U}\left(\alpha_U+\frac{\theta_U(1+x_U)}{1+\theta_U}\right)e^{-\theta_U x_U}\right\}I_N. \quad (3)$$

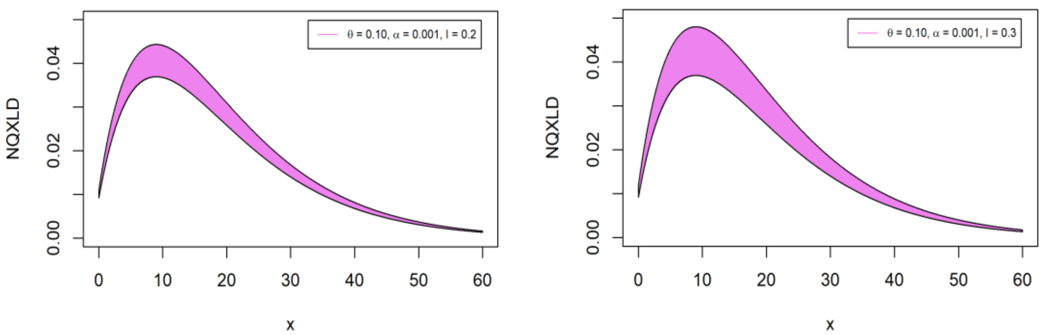
If $x_L = x_U = x_N$, then the pdf can be written as

$$f(x_N;\alpha_N,\theta_N)=\left\{\frac{\theta_N}{1+\alpha_N}\left(\alpha_N+\frac{\theta_N(1+x_N)}{1+\theta_N}\right)e^{-\theta_N x_N}\right\}(1+I_N), \quad I_N \in [I_L, I_U] \quad (4)$$

where $x_N \geq 0, \alpha_N, \theta_N > 0, I_N \in [I_L, I_U]$. Note that when $I_L = 0$, the neutrosophic quantities reduce to classical statistics. The corresponding CDF of the NQXLD is given by:

$$F(x_N;\alpha_N,\theta_N)=\left\{1-\left(1+\frac{\theta_N x_N}{(1+\theta_N)(1+\alpha_N)}\right)e^{-\theta_N x_N}\right\}(1+I_N), \; I_N \in [I_L, I_U]. \quad (5)$$

Figure 1 shows the pdf plots for the NQXL distribution for some values of the distribution parameters. It can be noted that the pdf has several shapes which make the distribution more flexible in fitting the real data. Figure 2 presents some CDF plots of the NQXLD for some parameters.



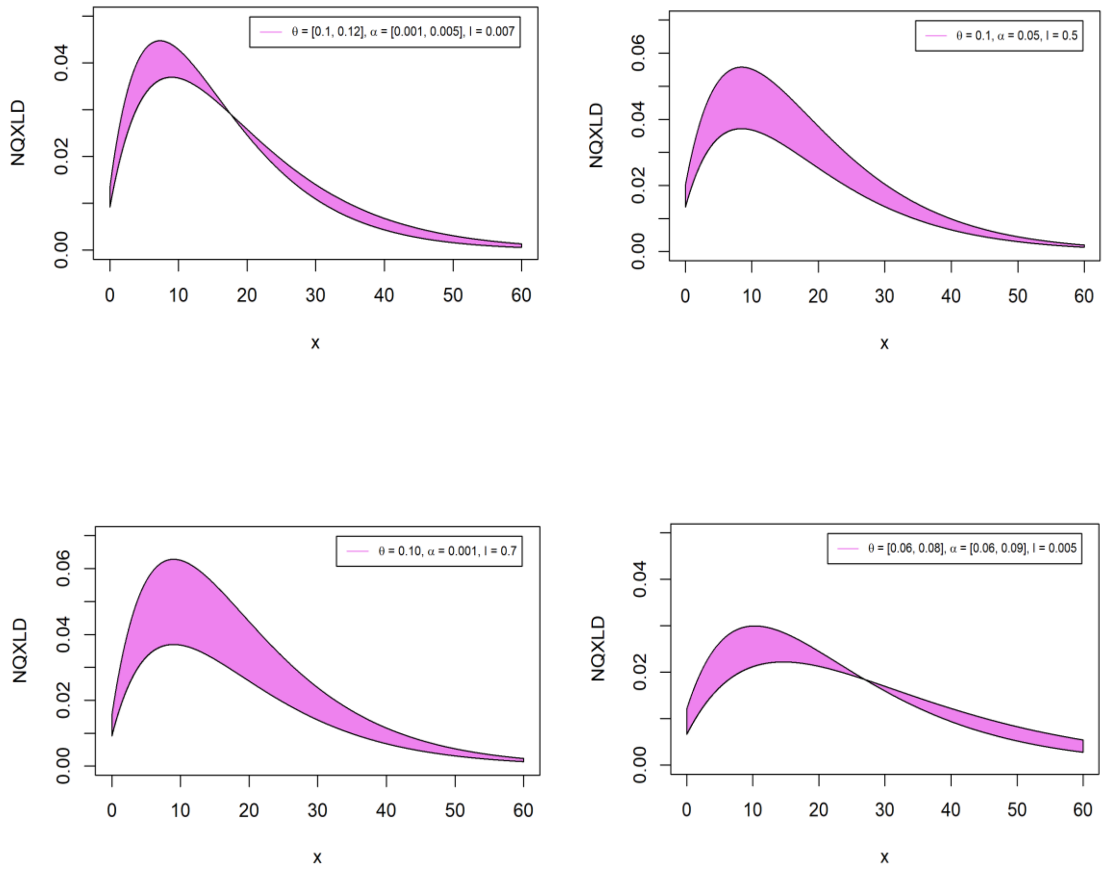


Figure 1: The pdf curves for NQXL distribution for various values of the parameters

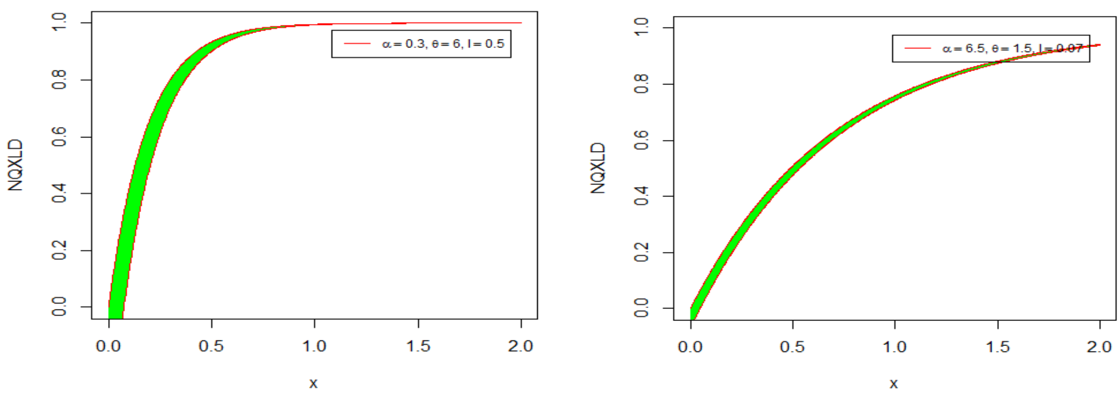


Figure 2: The cdf curves for NQXL distribution for various values of the parameters

3. Statistical Properties

The NQXLD statistical features covered in this section are now visible and available.

3.1 **Moments and associated measures**

Calculating statistical measures like variance, skewness, kurtosis, and central tendency can be simplified by utilizing the moments of a specific density function. To find the rth moment of random variable X_N around the origin, Equation (3) can be applied.

$$E\left(X_N^r\right)=\mu_{rN}^{'}=\int_0 x_N^r f\left(x_N\right)dx$$
$$=\int x_N^r\left\{\frac{\theta_N}{1+\alpha_N}\left(\alpha_N+\frac{\theta_N\left(1+x_N\right)}{1+\theta_N}\right)e^{-\theta_N x_N}\right\}\left(1+I_N\right)dx_N$$
$$=\left\{\frac{\theta_L}{1+\alpha_L}\int x_L^r\left(\alpha_L+\frac{\theta_L\left(1+x_L\right)}{1+\theta_L}\right)e^{-\theta_L x_L}\right\}+\left\{\frac{\theta_U}{1+\alpha_U}\int x_U^r\left(\alpha_U+\frac{\theta_U\left(1+x_U\right)}{1+\theta_U}\right)e^{-\theta_U x_U}\right\}I_N dx_U.$$

The following is the final expression of ordinary moments.

$$E\left(X_N^r\right)=\mu_{rN}^{'}=\left\{\frac{1}{\theta_N^r\left(1+\alpha_N\right)}\left(\left(\alpha_N+\frac{\theta_N}{1+\theta_N}\right)\Gamma(r+1)+\frac{1}{1+\theta_N}\Gamma(r+2)\right)\right\}\left(1+I_N\right). \tag{6}$$

Equation (4) can be used to get the first four moments of the random variable XN about the origin by inserting $r = 1,2,3,4$ as:

$$E(X_N)=\mu_{1N}'=\frac{\alpha_N(1+\theta_N)+(\theta_N+2)}{\theta_N(1+\alpha_N)(1+\theta_N)}(1+I_N)$$

$$E(X_N^2)=\mu_{2N}'=\frac{2\big(\alpha_N(1+\theta_N)+(\theta_N+3)\big)}{\theta_N^2(1+\alpha_N)(1+\theta_N)}(1+I_N)$$

$$E(X_N^3)=\mu_{3N}'=\frac{6\big(\alpha_N(1+\theta_N)+(\theta_N+4)\big)}{\theta_N^3(1+\alpha_N)(1+\theta_N)}(1+I_N)$$

$$E(X^4)=\mu_{4N}'=\frac{24\left(\alpha_N(1+\theta_N)+(\theta_N+5)\right)}{\theta_N^4(1+\alpha_N)(1+\theta_N)}(1+I_N)$$

Variance (Var) and index of dispersion (ID) of the NQXLD distribution are given below as:

$$\begin{aligned}\operatorname{Var}(X_N) &= \sigma_N^2 = \mu_{2N}^{'} - \left(\mu_{1N}^{'}\right)^2 \\ &= \frac{\alpha_N^2(\theta_N+1)^2 + 2\alpha_N(\theta_N+1)(\theta_N+2) + \theta_N(\theta_N+4) + 2}{\theta_N(\alpha_N+1)^2(\theta_N+1)^2}, \sigma_N^2 \in \left[\sigma_L^2, \sigma_U^2\right],\end{aligned} \tag{7}$$

$$ID(X) = \frac{\operatorname{Var}(X_N)}{\mu_{1N}^{'}} = \frac{\alpha_N^2(\theta_N+1)^2 + 2\alpha_N(\theta_N+1)(\theta_N+2) + \theta_N(\theta_N+4) + 2}{\theta_N(\alpha_N+1)(\theta_N+1)(2+\alpha_N+\theta_N+\alpha_N\theta_N)}. \tag{8}$$

The following relations can be utilized to determine the coefficient of kurtosis, represented by β_{2N},

and the coefficient of skewness, represented by $\sqrt{\beta_{1N}}$, are given by

$$\beta_{2N} = \frac{\mu_{4N}^{'} - 4\mu_{3N}^{'}\mu_{1N}^{'} + 6\mu_{2N}^{'}\left(\mu_{1N}^{'}\right)^2 - 3\left(\mu_{1N}^{'}\right)^4}{\sigma_N^4}(1+I_N),$$

$$\sqrt{\beta_{1N}} = \frac{\mu_{3N}^{'} - 3\mu_{2N}^{'}\mu_{1N}^{'} + 2\left(\mu_{1N}^{'}\right)^3}{\sigma_N^3}(1+I_N).$$

The following relations can be utilized to determine the coefficient of kurtosis, represented by β_{2N} and the coefficient of skewness, represented by β_{1N}. The neutrosophic median, neutrosophic skewness, neutrosophic kurtosis, standard deviation, and neutrosophic mean are displayed in Table (1). The results shown in Table 1 indicate that, for a given set of neutrosophic alpha parameters, different statistic values decrease when neutrosophic theta parametric values increase.

Table 1: The statistical measures of neutrosophic statistic NQXLD

Statistic	$\alpha_N = [1.5,\ 2]$	$\alpha_N = [1.5,\ 2]$	$\alpha_N = [1.5,\ 2]$	$\alpha_N = [1.5,\ 2]$	$\alpha_N = [1.5,\ 2]$
	$\theta_N = [0.5, 0.75]$	$\theta_N = [1, 1.25]$	$\theta_N = [1.75, 2]$	$\theta_N = [2, 3]$	$\theta_N = [1, 1.5]$
Mean	[2.5333,1.5873]	[1.2000,0.9185]	[0.6546,0.5556	[0.5667,0.3611]	[1.1667,0.7429]
S.D.	[2.4185,1.5461]	[1.1662,0.9031]	[0.6439,0.5500]	[0.5588,0.35897]	[1.1426,0.7350]
Median	[1.8210 1.1225]	[0.8502 0.6447]	[0.4593,0.3879]	[0.3969,0.25136]	[0.8215, 0.5189]
Skewness	[1.8162,1.8848]	[1.8764,1.9211]	[1.9234,1.9502]	[1.9332,1.9693]	[1.9056,1.9479]
Kurtosis	[7.8009,8.2116]	[8.1592,8.4438]	[8.4584,8.6389]	[8.5234,8.7724]	[8.3427,8.6228]

3.2 Features of reliability

The NQXL distribution's reliability attributes, such as the mean residual life function, stress strength reliability, neutrosophic survival and hazard functions, and neutrosophic reversed hazard function, are derived in this section.

3.2.1 Neutrosophic survival function

$$S(x_N) = 1 - F(x_N) = \left(1 + \frac{x_N\theta_N}{(\alpha_N+1)(1+\theta_N)}\right) e^{-\theta_N x}(1+I_N) + I_N, x_N > 0. \tag{9}$$

3.2.2 Hazard rate function

$$h(x_N) = \left\{\frac{\theta_N(\alpha_N(1+\theta_N)+\theta_N(1+x_N))}{(\alpha_N+1)(1+\theta_N)+x_N\theta_N}\right\}(1+I_N), x_N > 0. \tag{10}$$

3.2.3 Neutrosophic mean residual life function

$$m(t) = \left\{\frac{1}{\theta_N\left(1+\frac{t_N\theta_N}{(\alpha_N+1)(1+\theta_N)}\right)}\left[1 + \frac{1+t_N\theta_N}{(\alpha_N+1)(1+\theta_N)}\right]\right\}(1+I_N). \tag{11}$$

3.2.4 Measures of actuarial

Assessing market risk is crucial to actuarial science, mainly when dealing with asset portfolios. It is essential to evaluate the risks of any market transaction, whether it involves buying or selling. This evaluation calculates two significant actuarial measures for the NQXL distribution- Value at Risk (VaR) and Tail Value at Risk (TVaR). To determine the VaR of the NQXL model, we utilize a precise mathematical tool called the formula $x_p = F^{-1}(x_N)$, where x_N represents the direct solution of the subsequent non-linear equation.

$$1 - \frac{p}{(1+I_N)} = \left(1 + \frac{x_N\theta_N}{(\alpha_N+1)(1+\theta_N)}\right) e^{-\theta_N x},\ 0 < p < 1. \tag{12}$$

By following this formula, we can accurately calculate the conditional tail expectation, which represents the entire value at risk. It's essential to compute this value to make informed decisions that mitigate risk and ensure the safety of your investments.

$$\begin{aligned} TVaR_{Np} &= \frac{1}{1-p}\int_{VaR_{Np}}^{\infty} x f\left(x_N;\alpha_N,\theta_N\right)dx \\ &= \frac{1}{1-p}\int_{VaR_{Np}}^{\infty} x\left\{\frac{\theta_L}{1+\alpha_L}\left(\alpha_L + \frac{\theta_L(1+x_L)}{1+\theta_L}\right)e^{-\theta_L x_L}\right\} + \left\{\frac{\theta_U}{1+\alpha_U}\left(\alpha_U + \frac{\theta_U(1+x_U)}{1+\theta_U}\right)e^{-\theta_U x_U} I_N\right\}dx \\ &= \frac{e^{-\theta_N VaR}\left(2+\alpha_N+\theta_N VaR\left(2+\alpha_N\right)+\theta_N^2 VaR(\alpha_N+VaR+1)\right)}{(1-p)(\alpha_N+1)(\theta_N+1)} \end{aligned} \tag{13}$$

4. Estimation of Parameters

The neutrosophic parameters of the created NQXLD are derived by the utilization of the maximum likelihood estimation (MLE) technique. Let $X_{N1}, X_{N2},\ldots,X_{Nn}$ be a random sample of NQXLD with neutrosophic characteristics. The log- likelihood function can be represented as:

$$L(\theta_0,\alpha_0)=n\ln(\theta_0)+\sum_{i=1}^{n}\ln\left(\alpha_0+\frac{\theta_0 x_{0i}(1+\alpha_0 x_{0i})}{1+\theta_0 x_{0i}}\right)-\alpha_0\sum_{i=1}^{n}x_{0i}-n\ln(1+\alpha_0)\,. \tag{14}$$

The derivatives of Equation (14) relative to θ_N,α_N, are respectively defined as:

$$\frac{\partial L(\theta_N,\alpha_N)}{\partial\theta_N}=\frac{n}{\theta_N}+\sum_{i=1}^{n}\frac{1+x_{Ni}}{(1+\theta_N x_{Ni})\left(\alpha_N+\dfrac{\theta_N x_{Ni}(1+\alpha_N x_{Ni})}{1+\theta_N x_{Ni}}\right)}-\sum_{i=1}^{n}x_{Ni}=0\,, \tag{15}$$

$$\frac{\partial L(\theta_N,\alpha_N)}{\partial\theta_N}=\frac{n}{\theta_N}+\sum_{i=1}^{n}\frac{1+x_{Ni}}{(1+\theta_N x_{Ni})\left(\alpha_N+\dfrac{\theta_N x_{Ni}(1+\alpha_N x_{Ni})}{1+\theta_N x_{Ni}}\right)}-\sum_{i=1}^{n}x_{Ni}=0\,. \tag{16}$$

The MLE for (θ_N,α_N), $\theta_N\in[\theta_L,\theta_U]$ and $\alpha_N\in[\alpha_L,\alpha_U]$ can be found by solving Equations (15) and (16).

This section presents a comprehensive simulation study for the maximum likelihood (ML) estimation approach. The study takes into account both small and large sample sizes. The mean square error (MSE) and average bias are calculated to compare each estimator's performance. All the results in this section wer computed using the R programming language Core Team [13].

We consider the NQXL model and simulate data with $N = 10{,}000$. We also consider different sample sizes, including $n=50$ 100, 250, and 350 for certain parameter values. Table 2 provides the MSEs and simulated averages for all the simulations.

Table 2. Simulation results of the ML estimators for the NQXLD.

Sample	Parameter			MSE		NAB	
n	α	β	I_N	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
50	0.1	0.5	0	0.79970	0.00500	0.34785	0.01298
100				0.39806	0.00292	0.21414	0.01021
250				0.08422	0.00124	0.08187	0.00541
350				0.04436	0.00091	0.05619	0.00404
50	0.050	0.50	0.1	0.01387	0.23702	0.04991	0.47295
100				0.00356	0.01444	0.04941	0.11447
250				0.00250	0.01359	0.04082	0.11417

350				0.00249	0.01340	0.02292	0.11409
50	0.50	0.50	0.1	0.25532	0.03994	0.49297	0.19562
100				0.24396	0.03891	0.48718	0.19529
250				0.23966	0.03891	0.45142	0.19214
350				0.22328	0.03884	0.39219	0.18946
50	1.50	1.50	0.50	0.24992	0.30193	0.49989	0.54635
100				0.01000	0.16996	0.09995	0.40942
250				0.01000	0.16711	0.09999	0.40764
350				0.00999	0.16601	0.09888	0.40664

The average bias diminishes as sample size increases, according to the simulation results. It indicates that as sample size grows, practice and theory agree better. A larger sample size also results in a decrease in the estimators' MSEs. It is clear that the obtained estimators exhibit consistency, and the parameters' maximum likelihood estimator operates effectively, producing precise and accurate findings.

6 Applications

This section showcases the potential of the NQXLD distribution for three real-life datasets as compared to the QXLD. Some summary measures of this data set are given in Table 3. To compare the distribution, we utilize the suggested NQXLD with the classical QXL distribution. Model parameters are estimated through maximum likelihood estimation. The optimal model is chosen based on the smallest values of criteria such as log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC), $AIC = 2k - 2\ln(\hat{L})$ and $BIC = k\ln(n) - 2\ln(\hat{L})$, where k is the number of parameters, $\hat{L}$ is the maximized log-likelihood value, and n is the sample size. The results are listed in Table 4. These data can be described as follows.

Data 1: Between April 15, 2020, and June 30, 2020, COVID-19 data was collected in the United Kingdom and it is presented in Amaal and Ehab [6]. The data observations are: 0.0587, 0.0863, 0.1165, 0.1247, 0.1277, 0.1303, 0.1652, 0.2079, 0.2395, 0.2751, 0.2845 0.2992, 0.3188, 0.3317, 0.3446, 0.3553, 0.3622, 0.3926, 0.3926, 0.4110, 0.4633, 0.4690 0.4954, 0.5139, 0.5696, 0.5837, 0.6197, 0.6365, 0.7096, 0.7193, 0.7444, 0.8590, 1.0438 1.0602, 1.1305, 1.1468, 1.1533, 1.2260, 1.2707, 1.3423, 1.4149, 1.5709, 1.6017, 1.6083 1.6324, 1.6998, 1.8164, 1.8392, 1.8721, 1.9844, 2.1360, 2.3987, 2.4153, 2.5225, 2.7087 2.7946, 3.3609, 3.3715, 3.7840, 3.9042, 4.1969, 4.3451, 4.4627, 4.6477, 5.3664, 5.4500 5.7522, 6.4241, 7.0657, 7.4456, 8.2307, 9.6315, 10.1870, 11.1429, 11.2019, 11.4584.

Data II: Number of deaths per day due to COVID-19 in Nepal see Dhungana and Kumar [15]. The data are as follows: 2, 2, 2, 2, 2, 2, 3, 2, 3, 3, 4, 2, 5, 5, 3, 2, 4, 4, 8, 4, 4, 3, 2, 3, 7, 6, 6, 11, 9, 3, 8, 7, 11, 8, 12, 12, 14, 7, 11, 12, 6, 14, 9, 9, 11, 6, 6, 5, 5, 14, 9, 15, 11, 8, 4, 7, 11, 10, 16, 2, 7, 17, 6, 8, 10, 4, 10, 7, 11, 11, 8, 7, 19, 9, 15, 12, 10, 14, 22, 9, 18, 12, 19, 21, 12, 12, 18, 8, 26, 21, 17, 13, 5, 15, 14, 11, 17, 16, 17, 23, 24, 20, 30, 18, 18, 17, 21, 18, 22, 26, 15, 13, 13, 6, 9, 17, 12, 17, 22, 7, 16, 16, 24, 28, 23, 23,19, 25, 29, 21, 9, 13, 16, 10, 17, 20, 23, 14, 12, 11, 15, 9, 18, 14, 13, 6, 16, 12, 11, 7, 3, 5, 5.

Data III: The third dataset displays the number of COVID-19 deaths in China from January 23, 2020, to March 28, 2020. You can find more details about this dataset at https://www.worldometers.info/coronavirus/country/china. The numbers are as follows: 16, 15, 24, 26, 26, 38, 43, 46, 45, 57, 64, 65, 73, 73, 86, 89, 97, 108, 97, 146, 121, 143, 142, 105, 98, 136, 114, 118, 109, 97, 150, 71, 52, 29, 44, 47, 35, 42, 31, 38, 31, 30, 28, 27, 22, 17, 22, 11, 7, 13, 10, 14, 13, 11, 8, 3, 7, 6, 9, 7, 6, 6, 5, 3, 5, 5, 5, and 8.

Table 3: Some descriptive measures for the data sets

Statistic	N	Mean	Median	Q_1	Q_2	Q_3	S.D.	Min	Max
Data I	70	2.9228	1.2483	0.3926	1.2260	3.7840	4.6033	0.0587	21.1906
Data II	153	11.6144	11	6	11	16	6.7591	2	30
Data III	68	48.4559	31	11	31	73	43.8496	3	150

Table 4. The MLEs and goodness-of-fit based on the data sets

Data	Model	$\hat{\alpha}$	$\hat{\theta}$	I_N	-LogLik	AIC	BIC
I	QXLD	1124.02179	0.41056	0	287.4088	291.4088	296.0702
	NQXLD	1236.42400	0.45162	0.1	272.9216	276.9216	281.5831
		1348.82600	0.49268	0.2	259.6959	263.6959	268.3574
		1461.22800	0.53374	0.3	247.5294	251.5294	256.1909
II	QXLD	0.00001	0.16031	0	1011.2060	1015.2060	1021.2660
	NQXLD	1e-06	0.01603	0.1	982.0407	986.0407	992.1016
		2e-06	0.03206	0.2	955.4152	959.4152	965.4761
		3e-06	0.04809	0.3	930.9222	934.9222	940.9830
III	QXLD	2.91390	0.02579	0	663.5121	667.5121	671.9511
	NQXLD	0.29139	0.00258	0.1	650.5499	654.5499	658.9889
		0.58278	0.00516	0.2	638.7163	642.7163	647.1554
		0.87417	0.00774	0.3	627.8305	631.8305	636.2696

The results given in Table 4 are significant and demonstrate the importance of the indeterminacy parameter in improving the fitting of the distribution. Based on the maximum likelihood estimates and goodness-of-fit metrics for the base QXLD and NQXLD, it is evident that the NQXL distribution is a superior fit in comparison to the classical QXLD for all data sets considered in this study. Therefore, it is essential to consider the indeterminacy parameter when modeling distributions to ensure a better fit for the data considered in this study.

7. Conclusions

The neutrosophic Quasi-XLindley distribution is more flexible than the Quasi-XLindley distribution under classical statistics, as revealed by a study that examined its fundamental characteristics. Using three COVID-19 data sets, the research found that decision-makers can benefit from this distribution. This distribution could be used to construct other statistical distributions that will be useful in future research. The study suggests exploring additional properties of the neutrosophic Quasi-XLindley distribution and expanding on the multivariate distributions in subsequent analyses. Furthermore, G-families of distributions under neutrosophic statistics can be a promising area for future research based on the suggested NQXLD.

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