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Complex Neutrosophic Approach for Uncertainty Management in Employment Competence for Higher Vocational College Students

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Abstract: In an era of rapid technological revolutions as well as ever-changing job market dynamics, the analysis of employment competence for higher vocational college students becomes a complicated challenge. Legacy methods become unable to manage the intrinsic uncertainty related to workforce readiness, skill relevancies, and career decision-making. In this work, we propose a multi-criteria complex neutrosophic approach integrating for optimization and uncertainty management of vocational student employment. Our approach provides a novel application of the Interval Complex Neutrosophic Set (ICNS) to handle indefinite, hesitant, and inconsistent information in vocational student employment. It also presents a novel ICNS de-neutrosophication method to provide representative and expressive scoring of ICNS decisions. Then, algorithmic analytical weighting is applied to infer the significance of ICNS-valued criteria. Following, the ICNS-VIKOR method is integrated to compute evaluation results and rank vocational employment strategies according to either compromise Measure, Regret Measure, or Q-VIKOR Index scores for each alternative. We introduce a real-world case study to optimize the evaluation of different vocational employment configurations based on ten competing criteria. The quantitative and qualitative results indicate that our methodology offers a systematic and reliable decision-support tool, that effectively optimizes trade-offs as well as uncertain judgments from policymakers, career advisors, and industry recruiters. The proof-of-concept analysis highlighted the best-ranked alternative while offering intuitions about the sensitivity of the decision process.

Keywords: Complex Neutrosophic, Interval Neutrosophic, Uncertainty, Employment Competence, Higher Vocational College Students.

1. Introduction

In today's dynamic labor market, the employment landscape for higher vocational college students is undergoing significant transformation [1]. The rise of automation, digitalization, and evolving industry demands have intensified the need for job-ready skills among graduates. Apart from common academic pathways, vocational education focuses on practical skill development and industry-specific training [2], [3]. Nevertheless, evaluating employment competence in vocational students turned out to be a complex challenge due to varied uncertainties in skill assessment, job opportunities, as well as career decision-making [4], [5].

Employment competence encompasses a broad spectrum of factors, including technical skills, soft skills, adaptability, and decision-making abilities [6]. However, these attributes are often difficult to quantify and predict using conventional evaluation techniques. Traditional competency assessment models rely on fixed criteria, rigid evaluation metrics, and deterministic decision-making, which fail to account for the uncertainty and subjectivity inherent in real-world employment scenarios [3].

Employment competence encounters many uncertainty challenges. First, modern industry usually meets rapid changes that make the acquired skills obsolete shaping emergent demand for new competencies [7]. Second, the employment landscape varies because of technological progress, economic downturns, as well as fluctuating workforce demands. Third, the vocational students vary in their unique learning capabilities, interests, and career aspirations, which made it difficult to apply standardized evaluation metrics. With these challenges in mind, there is a pressing need for a robust mathematical framework that can handle uncertainty in employment competence assessment [8].

Neutrosophic sets came into sight to provide a promising solution to handle uncertainty, thereby offering a solution for optimizing vocational student employment under realistic higher education [9], [10]. The neutrosophic approach was developed as an extension of classical logic and fuzzy set theory, explicitly designed to handle indeterminacy, incompleteness, and inconsistency in decision-making. Neutrosophic sets, apart from classical models, presented three independent components: Truth, Indeterminacy, as well as Falsity memberships [11]. Inspired by the successful application of Neutrosophic logic in employment market analytics, improving employability practices as well as personnel corresponding, which make it a suitable choice for addressing uncertainty management in vocational student employment.

In this work, we aim to investigate and develop a Multi-Criteria Complex Neutrosophic Approach to optimize and manage uncertainties in vocational student employment. Our approach aims to include multiple contradictory criteria with different ambiguity factors into an organized decision framework. Then, we aim to explore new applications of our approach to real-world vocational training scenarios, demonstrating its implications for refining employability predictions and workforce matching. By performing comparative analysis against competing methods from the literature, we demonstrate the advantage of our approach, demonstrating its practical feasibility and benefits in real-time labor market analytics, improving employability predictions and workforce matching [12].

The remaining part of this research is decomposed into five main sections. The fundamental and core concepts are reviewed in section 2. Section 3 describes the proposed method. Section 4 presents and discusses the experimental results. Section 5 concludes the findings.

2. Essentials and Fundamentals

Herin, we provide a summary of essential concepts and definitions of Neutrosophic sets and related methods.

Definition 1: Given U the universe of discourse, a neutrosophic set constituted of three membership functions, which is expressed as follows [9].

$$\overline{S} = \{(u, T_{\overline{S}}(u), I_{\overline{S}}(u), F_{\overline{S}}(u)) : u \in U\}, \quad (1)$$

such that $T_{\overline{S}}, I_{\overline{S}}, F_{\overline{S}}$ belongs to the interval [0,1], and they are summed under condition $0 \leq supT_{\overline{S}}(u) + supI_{\overline{S}}(u) + supF_{\overline{S}}(u) \leq 3$ [13].

Definition 2: **T**he Complex Fuzzy Set (CFS) extends the classical fuzzy set to incorporate complex-valued membership to better deal with oscillatory behavior. Given U is a universal set [14], the definition of CFS can be expressed as follows:

$$\overline{S} = \{(u, \eta_{\overline{S}}(u)): u \in U\}. \quad (2)$$

such that

$$\eta_{\overline{S}}(u) = r_{\overline{S}(u)} e^{i\theta_{\overline{S}(u)}} \quad (3)$$

Definition 3: Complex Intuitionistic Fuzzy Set (CIFS) was introduced as an extension to CFS by introducing complex-valued a membership function, $\eta_{\overline{S}}(u)$, as well as a non-membership function $\zeta_{\overline{S}}(u)$ to empower the modeling of decision ambiguity.

$$\overline{S} = \{(u, \eta_{\overline{S}}(u), \zeta_{\overline{S}}(u)): u \in U\}. \quad (4)$$

Such that:

$\eta_{\overline{S}}(u) = p_{\overline{S}}(u) \cdot e^{j \cdot \mu_{\overline{S}}(u)}$ and $\zeta_{\overline{S}}(u) = r_{\overline{S}}(u) \cdot e^{j \cdot \mu_{\overline{S}}(u)}$

Definition 4: A Single-Valued Complex Neutrosophic Set (SVCNS) [15] was proposed to re-design and replace the real value membership in the original NS with complex-valued truth, indeterminacy, and falsity membership functions. The SVCNS can be expressed as follows:

$$\overline{S} = \left\{\left(u, T_{\overline{S}}(u), I_{\overline{S}}(u), F_{\overline{S}}(u)\right): u \in U\right\}. \quad (5)$$

such that

$$T_{\overline{S}}(u) = p_{\overline{S}}(u) \cdot e^{j\mu_{\overline{S}}(u)}, I_{\overline{S}}(u) = q_{\overline{S}}(u) \cdot e^{j\nu_{\overline{S}}(u)} and F_{\overline{S}}(u) = r_{\overline{S}}(u) \cdot e^{j\omega_{\overline{S}}(u)},$$

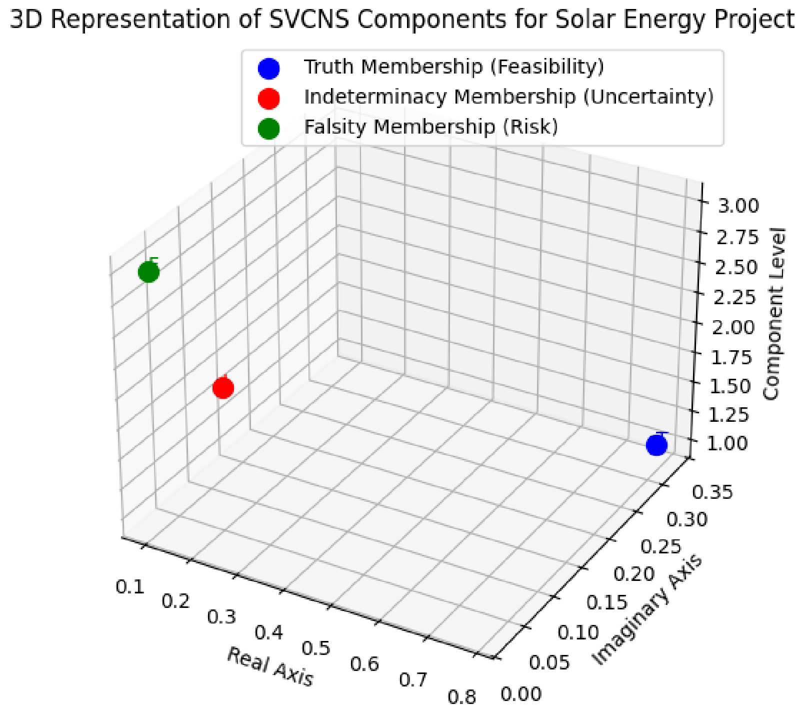

Figure 1. illustration of example SVCNS representation for the Student X $\bar{S}_X = (X, 0.85e^{j0.4}, 0.2e^{j0.3}, 0.1e^{j0.2})$

where $v_{\bar{S}}(u)$, $\mu_{\bar{S}}(u)$, $\mu_{\bar{S}}(u)$ are real values, while $p_{\bar{S}}(u), q_{\bar{S}}(u), r_{\bar{S}}(u) \in [0,1]$. In Figure 1, we provided a visualization of the example of SVCNS in the 3D coordinate system.

Definition 5: Given an SVCNS $\bar{S} = \left\{\left(u, T_{\bar{S}}(u), I_{\bar{S}}(u), F_{\bar{S}}(u)\right) : u \in U\right\}$, the complement of this set can be defined as:

$$\begin{aligned}
&\bar{S}^c = \{(u, T_{\bar{S}^c}(u), I_{\bar{S}^c}(u), F_{\bar{S}^c}(u)) : u \in U\},\\
&\textit{such that}\\
&T_{\bar{S}^c}(u) = p_{\bar{S}^c}(u) \cdot e^{j \cdot \mu_{\bar{S}^c}(u)} \rightarrow p_{\bar{S}^c}(u) = r_{\bar{S}}(u), \mu_{\bar{S}^c}(u) = \mu_{\bar{S}}(u), 2\pi - \mu_{\bar{S}}(u) \textit{ or}\\
&\qquad\qquad \mu_{\bar{S}}(u) + \pi\\
&I_{\bar{S}^c}(u) = q_{\bar{S}^c}(u) \cdot e^{j \cdot v_{\bar{S}^c}(u)} \rightarrow q_{\bar{S}^c}(u) = 1 - q_{\bar{S}}(u)\ v_{\bar{S}^c}(u) = v_{\bar{S}}(u), 2\pi - v_{\bar{S}^c}(u) \textit{or}\ v_{\bar{S}^c}(u) + \pi\\
&F_{\bar{S}^c}(u) = r_{\bar{S}^c}(u) \cdot e^{j \cdot \mu_{\bar{S}^c}(u)} \rightarrow r_{\bar{S}^c}(u) = p_{\bar{S}}(u), \mu_{\bar{S}^c}(u) = \mu_{\bar{S}}(u), 2\pi - \mu_{\bar{S}}(u) \textit{ or } \mu_{\bar{S}}(u) + \pi
\end{aligned} \quad (6)$$

Definition 6: Given two SVCNSs $\bar{P} = \left\{\left(u, T_{\bar{P}}(u), I_{\bar{P}}(u), F_{\bar{P}}(u)\right) : u \in U\right\}$, and $\bar{Z} = \left\{\left(u, T_{\bar{Z}}(u), I_{\bar{Z}}(u), F_{\bar{Z}}(u)\right) : u \in U\right\}$, the union of these two sets can be defined as:

$$\begin{aligned}
&\bar{P} \cup \bar{Z} = \left\{\left(u, T_{\bar{P}\cup\bar{Z}}(u), I_{\bar{P}\cup\bar{Z}}(u), F_{\bar{P}\cup\bar{Z}}(U)\right) : u \in U\right\},\\
&\textit{Such that}
\end{aligned} \quad (7)$$

$$T_{\overline{P}\cup\overline{Z}}(u) = \left[max\left(T_{\overline{P}}(u), T_{\overline{Z}}(u)\right)\right] \cdot e^{j\cdot\mu_{T_{\overline{P}\cup\overline{Z}}}(u)},$$
$$I_{\overline{P}\cup\overline{Z}}(u) = \left[min\left(I_{\overline{P}}(u), I_{\overline{Z}}(u)\right)\right] \cdot e^{j\cdot\nu_{I_{\overline{P}\cup\overline{Z}}}(u)},$$
$$F_{\overline{P}\cup\overline{Z}}(u) = \left[min\left(F_{\overline{P}}(u), F_{\overline{Z}}(u)\right)\right] \cdot e^{j\cdot\omega_{F_{\overline{P}\cup\overline{Z}}}(u)}$$

Definition 7: Given two SVCNSs $\overline{P} = \left\{\left(u, T_{\overline{P}}(u), I_{\overline{P}}(u), F_{\overline{P}}(u)\right): u \in U\right\}$, and $\overline{Z} = \left\{\left(u, T_{\overline{Z}}(u), I_{\overline{Z}}(u), F_{\overline{Z}}(u)\right): u \in U\right\}$, the intersection between them can be defined as:

$$\overline{P} \cap \overline{Z} = \left\{\left(u, T_{\overline{P}\cap\overline{Z}}(u), I_{\overline{P}\cap\overline{Z}}(u), F_{\overline{P}\cap\overline{Z}}(U)\right): u \in U\right\},$$

such that

$$T_{\overline{P}\cap\overline{Z}}(u) = \left[min\left(T_{\overline{P}}(u), T_{\overline{Z}}(u)\right)\right] \cdot \mathrm{e}^{j\cdot\mu_{T_{\overline{P}\cap\overline{Z}}}(u)},$$
$$I_{\overline{P}\cap\overline{Z}}(u) = \left[max\left(I_{\overline{P}}(u), I_{\overline{Z}}(u)\right)\right] \cdot \mathrm{e}^{j\cdot\nu_{I_{\overline{P}\cap\overline{Z}}}(u)}, \quad (8)$$
$$F_{\overline{P}\cap\overline{Z}}(u) = \left[max\left(F_{\overline{P}}(u), F_{\overline{Z}}(u)\right)\right] \cdot \mathrm{e}^{j\cdot\omega_{F_{\overline{P}\cap\overline{Z}}}(u)}$$

Definition 8: Interval Complex Neutrosophic Set (ICNS) was proposed to extend the SVCNS by re-designating the truth, indeterminacy, and falsity membership functions using complex numbers with interval-valued magnitudes.

$$\overline{S} = \left\{\left\langle \begin{array}{l} T_{\overline{S}}(u) = \left[t_{\overline{S_L}}(u), t_{\overline{S_U}}(u)\right] \cdot e^{j\alpha\left[\mu_{\overline{S_L}}(u), \mu_{\overline{S_U}}(u)\right]}, \\ I_{\overline{S}}(u) = \left[i_{\overline{S_L}}(u), i_{\overline{S_U}}(u)\right] \cdot e^{j\beta\left[\nu_{\overline{S_L}}(u), \nu_{\overline{S_U}}(u)\right]}, \\ F_{\overline{S}}(u) = \left[f_{\overline{S_L}}(u), f_{\overline{S_U}}(u)\right] \cdot e^{j\gamma\left[\omega_{\overline{S_L}}(u), \omega_{\overline{S_U}}(u)\right]} \end{array} \right\rangle : u \in U\right\} \quad (9)$$

Such that α, β, γ symbolizes the scaling terms and belongs to the interval $(0,2\pi]$. In Figure 2, we provided a visualization of the example of ICNS in the 3D coordinate system.

Definition 9: Given two ICNSs $\overline{P} = \left\{\left\langle T_P(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)}\right\rangle: u \in U\right\}$, and $\overline{Z} = \left\{\left\langle T_{\overline{Z}}(u) = t_{\overline{Z}}(u) \cdot e^{j\alpha\mu_{\overline{Z}}(u)}, I_{\overline{Z}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{S}}(u)}, F_{\overline{Z}}(u) = f_{\overline{Z}}(u) \cdot e^{j\gamma\omega_{\overline{Z}}(u)}\right\rangle: u \in U\right\}$, then the union between them is defined as follows

$$\overline{S} = \left\{\left\langle \begin{array}{l} T_{\overline{P}\cup\overline{Z}}(u) = [inf\,t_{\overline{P}\cup\overline{Z}}(u), sup\,t_{\overline{P}\cup\overline{Z}}(u)] \cdot e^{j\pi\mu_{\overline{P}\cup\overline{Z}}(u)}, \\ I_{\overline{P}\cup\overline{Z}}(u) = [inf\,i_{\overline{P}\cup\overline{Z}}(u), sup\,i_{\overline{P}\cup\overline{Z}}(u)] \cdot e^{j\pi\nu_{\overline{P}\cup\overline{Z}}(u)}, \\ F_{\overline{P}\cup\overline{Z}}(u) = [inf\,f_{\overline{P}\cup\overline{Z}}(u), sup\,f_{\overline{P}\cup\overline{Z}}(u)] \cdot e^{j\pi\omega_{\overline{P}\cup\overline{Z}}(u)} \end{array} \right\rangle : u \in U\right\} \quad (10)$$

Such that:

$$inf\,t_{\overline{P}\cup\overline{Z}}(u) = mau\left(inf\,t_{\overline{P}}(u), inf\,t_{\overline{Z}}(u)\right), sup\,t_{\overline{P}\cup\overline{Z}}(u) = mau\left(sup\,t_{\overline{P}}(u), sup\,t_{\overline{Z}}(u)\right);$$
$$inf\,i_{\overline{P}\cup\overline{Z}}(u) = min\left(inf\,i_{\overline{P}}(u), inf\,i_{\overline{Z}}(u)\right), sup\,i_{\overline{P}\cup\overline{Z}}(u) = min\left(sup\,i_{\overline{P}}(u), sup\,i_{\overline{Z}}(u)\right);$$
$$inf\,f_{\overline{P}\cup\overline{Z}}(u) = min\left(inf\,f_{\overline{P}}(u), inf\,f_{\overline{Z}}(u)\right), sup\,f_{\overline{P}\cup\overline{Z}}(u) = min\left(sup\,f_{\overline{P}}(u), sup\,f_{\overline{Z}}(u)\right);$$

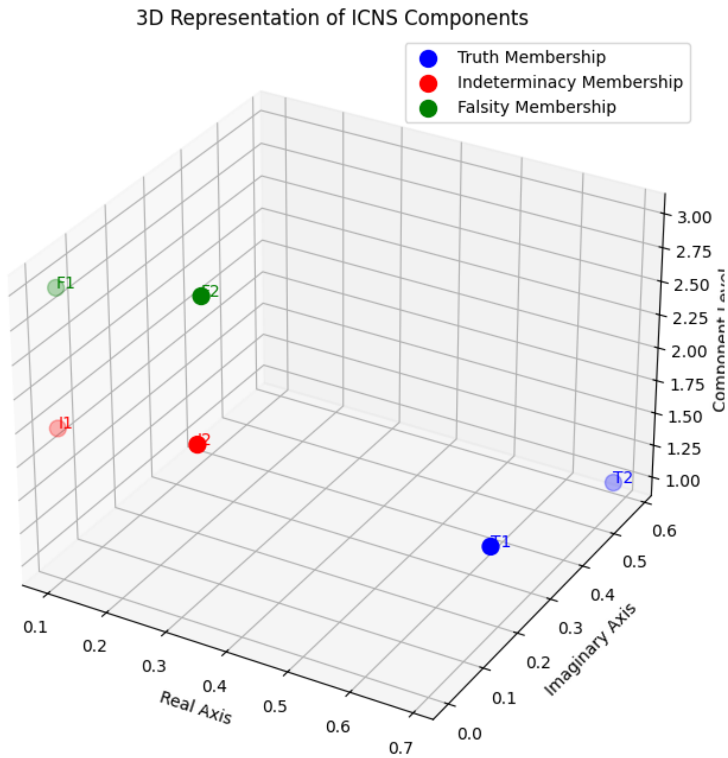

Figure 2. illustration of example IVCNS representation for student X project $\bar{S}_X = \left(X, \begin{matrix} [0.7,0.9]e^{j\alpha[0.5,0.7]}, [0.1,0.3]e^{j\beta[0.0,0.2]}, \\ [0.1,0.3]e^{j\gamma[0.1,0.3]} \end{matrix} \right)$

Definition 10: Given two ICNSs $\overline{P} = \{\langle T_P(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)} \rangle : u \in U\}$, and $\overline{Z} = \{\langle T_{\overline{Z}}(u) = t_{\overline{Z}}(u) \cdot e^{j\alpha\mu_{\overline{Z}}(u)}, I_{\overline{Z}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{S}}(u)}, F_{\overline{Z}}(u) = f_{\overline{Z}}(u) \cdot e^{j\gamma\omega_{\overline{Z}}(u)} \rangle : u \in U\}$, then the intersection between them is defined as follows

$$\overline{S} = \left\{ \left\langle \begin{matrix} T_{\overline{P}\cap\overline{Z}}(u) = [inf\,t_{\overline{P}\cap\overline{Z}}(u), sup\,t_{\overline{P}\cap\overline{Z}}(u)] \cdot e^{j\pi\mu_{\overline{P}\cap\overline{Z}}(u)}, \\ I_{\overline{P}\cap\overline{Z}}(u) = [inf\,i_{\overline{P}\cap\overline{Z}}(u), sup\,i_{\overline{P}\cap\overline{Z}}(u)] \cdot e^{j\pi\nu_{\overline{P}\cap\overline{Z}}(u)}, \\ F_{\overline{P}\cap\overline{Z}}(u) = [inf\,f_{\overline{P}\cap\overline{Z}}(u), sup\,f_{\overline{P}\cap\overline{Z}}(u)] \cdot e^{j\pi\omega_{\overline{P}\cap\overline{Z}}(u)} \end{matrix} \right\rangle : u \in U \right\} \quad (11)$$

where

$$inf\,t_{\overline{P}\cap\overline{Z}}(u) = min\left(inf\,t_{\overline{P}}(u), inf\,t_{\overline{Z}}(u)\right), sup\,t_{\overline{P}\cap\overline{Z}}(u) = min\left(sup\,t_{\overline{P}}(u), sup\,t_{\overline{Z}}(u)\right),$$

$$inf\,i_{\overline{P}\cap\overline{Z}}(u) = max\left(inf\,i_{\overline{P}}(u), inf\,i_{\overline{Z}}(u)\right), sup\,i_{\overline{P}\cap\overline{Z}}(u) = max\left(sup\,i_{\overline{P}}(u), sup\,i_{\overline{Z}}(u)\right),$$

$$inf\,f_{\overline{P}\cap\overline{Z}}(u) = max\left(inf\,f_{\overline{P}}(u), inf\,f_{\overline{Z}}(u)\right), sup\,f_{\overline{P}\cap\overline{Z}}(u) = max\left(sup\,f_{\overline{P}}(u), sup\,f_{\overline{Z}}(u)\right),$$

Definition 11: Given an ICNS $\overline{S} = \{\langle T_{\overline{S}}(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{S}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)} \rangle : u \in U\}$, then the complement of $\overline{P}$ is denoted as $\overline{P}^c$, and is defined as:

$$\overline{P}^c = \left\{ \left\langle T_{\overline{P}^c}(u) = t_{\overline{P}^c}(u) \cdot e^{j\pi\mu_{\overline{P}^c}(u)}, I_{\overline{P}^c}(u) = i_{\overline{P}^c}(u) \cdot e^{j\pi\nu_{\overline{P}^c}(u)}, F_{\overline{P}^c}(u) = f_{\overline{P}^c}(u) \cdot e^{j\pi\omega_{\overline{P}^c}(u)} \right\rangle : u \in U \right\}, \quad (12)$$

Such that

$t_{\overline{P}^c}(u) = f_{\overline{P}}(u)$, $\mu_{\overline{P}^c}(u) = 2\pi - \mu_{\overline{P}}(u)$ *or* $\mu_{\overline{P}^c}(u) = 2\pi - \mu_{\overline{P}}(u)$,

$i_{\overline{P}^c}(u) = \left(1 - sup\, i_{\overline{P}}(u), 1 - inf\, i_{\overline{P}}(u)\right)$, $\nu_{\overline{P}^c}(u) = 2\pi - \nu_{\overline{P}}(u)$ *or* $\nu_{\overline{P}}(u) + \pi$

$f_{\overline{P}^c}(u) = i_{\overline{P}^c}(u)$, $\omega_{\overline{P}^c}(u) = 2\pi - \omega_{\overline{P}}(u)$ *or* $\omega_{\overline{P}}(u) + \pi$

Definition 12: Given two ICNSs $\overline{P} = \left\{\left\langle T_P(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)}\right\rangle : u \in U\right\}$, and $\overline{Z} = \left\{\left\langle T_{\overline{Z}}(u) = t_{\overline{Z}}(u) \cdot e^{j\alpha\mu_{\overline{Z}}(u)}, I_{\overline{Z}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{S}}(u)}, F_{\overline{Z}}(u) = f_{\overline{Z}}(u) \cdot e^{j\gamma\omega_{\overline{Z}}(u)}\right\rangle : u \in U\right\}$, then the product between them is defined as follows

$$\begin{aligned}\overline{S} = \Big\{\Big\langle T_{\overline{P}*\overline{Z}}(u) &= \left[t_{\overline{P}}^{L}(u)t_{\overline{Z}}^{L}(u), t_{\overline{P}}^{U}(u)t_{\overline{Z}}^{U}(u)\right] \cdot e^{j\pi\left[\mu_{\overline{P}*\overline{Z}}^{L}(u), \mu_{\overline{P}*\overline{Z}}^{R}(u)\right]}, I_{\overline{P}*\overline{Z}}(u) \\ &= \left[i_{\overline{P}}^{L}(u) + i_{\overline{Z}}^{L}(u) - i_{\overline{P}}^{L}(u)i_{\overline{Z}}^{L}(u), i_{\overline{P}}^{R}(u) + i_{\overline{Z}}^{R}(u) - i_{\overline{P}}^{R}(u)i_{\overline{Z}}^{R}(u)\right] \\ &\cdot e^{j\pi\left[\nu_{\overline{P}*\overline{Z}}^{L}(u), \nu_{\overline{P}*\overline{Z}}^{R}(u)\right]}, F_{\overline{P}\times\overline{Z}}(u) \\ &= \left[f_{\overline{P}}^{L}(u) + f_{\overline{Z}}^{L}(u) - f_{\overline{P}}^{L}(u)f_{\overline{Z}}^{L}(u), f_{\overline{P}}^{R}(u) + f_{\overline{Z}}^{R}(u) - f_{\overline{P}}^{R}(u)f_{\overline{Z}}^{R}(u)\right] \\ &\cdot e^{j\pi\left[\omega_{\overline{P}\times\overline{Z}}^{L}(u), \omega_{\overline{P}\times\overline{Z}}^{R}(u)\right]}\Big\rangle : u \in U\Big\}\end{aligned} \tag{13}$$

Where the phase terms are defined as follows

$$\begin{aligned}\mu_{\overline{P}\times\overline{Z}}^{L}(u) &= \mu_{\overline{P}}^{L}(u)\mu_{\overline{Z}}^{L}(u), \mu_{\overline{P}\times\overline{Z}}^{U}(u) = \mu_{\overline{P}}^{U}(u)\mu_{\overline{Z}}^{U}(u) \\ \nu_{\overline{P}\times\overline{Z}}^{L}(u) &= \nu_{\overline{P}}^{L}(u)\nu_{\overline{Z}}^{L}(u), \nu_{\overline{P}\times\overline{Z}}^{U}(u) = \nu_{\overline{P}}^{U}(u)\nu_{\overline{Z}}^{U}(u) \\ \omega_{\overline{P}\times\overline{Z}}^{L}(u) &= \omega_{\overline{P}}^{L}(u)\omega_{\overline{Z}}^{L}(u), \omega_{\overline{P}\times\overline{Z}}^{U}(u) = \omega_{\overline{P}}^{U}(u)\omega_{\overline{Z}}^{U}(u).\end{aligned}$$

Definition 13: Given two ICNSs $\overline{P} = \left\{\left\langle T_P(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)}\right\rangle : u \in U\right\}$, and $\overline{Z} = \left\{\left\langle T_{\overline{Z}}(u) = t_{\overline{Z}}(u) \cdot e^{j\alpha\mu_{\overline{Z}}(u)}, I_{\overline{Z}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{S}}(u)}, F_{\overline{Z}}(u) = f_{\overline{Z}}(u) \cdot e^{j\gamma\omega_{\overline{Z}}(u)}\right\rangle : u \in U\right\}$, then the addition of them is defined as follows

$$\begin{aligned}\overline{S} = \Big\{\Big\langle &\left[t_{\overline{P}}^{L}(u) + t_{\overline{Z}}^{L}(u) - t_{\overline{P}}^{L}(u)t_{\overline{Z}}^{L}(u), t_{\overline{P}}^{U}(u) + t_{\overline{Z}}^{U}(u) - t_{\overline{P}}^{U}(u)t_{\overline{Z}}^{U}(u)\right] \cdot e^{j\pi\left[\mu_{\overline{P}+\overline{Z}}^{L}(u), \mu_{\overline{P}+\overline{Z}}^{L}(u)\right]}, I_{\overline{P}+\overline{Z}}(u) \\ &= \left[i_{\overline{P}}^{L}(u)i_{\overline{Z}}^{L}(u), i_{\overline{P}}^{U}(u)i_{\overline{Z}}^{U}(u)\right] \cdot e^{j\pi\left[\nu_{\overline{P}+\overline{Z}}^{L}(u), \nu_{\overline{P}+\overline{Z}}^{R}(u)\right]}, F_{\overline{P}+\overline{Z}}(u) \\ &= \left[f_{\overline{P}}^{L}(u)f_{\overline{Z}}^{L}(u), f_{\overline{P}}^{R}(u)f_{\overline{Z}}^{R}(u)\right] \cdot e^{j\pi\left[\omega_{\overline{P}+\overline{Z}}^{L}(u), \omega_{\overline{P}+\overline{Z}}^{R}(u)\right]}\Big\rangle : u \in U\Big\}\end{aligned} \tag{14}$$

Where the phase terms are defined as follows

$$\begin{aligned}\mu_{\overline{P}+\overline{Z}}^{L}(u) &= \mu_{\overline{P}}^{L}(u) + \mu_{\overline{Z}}^{L}(u), \mu_{\overline{P}+\overline{Z}}^{U}(u) = \mu_{\overline{P}}^{U}(u) + \mu_{\overline{Z}}^{U}(u), \nu_{\overline{P}+\overline{Z}}^{L}(u) = \nu_{\overline{P}}^{L}(u) + \nu_{\overline{Z}}^{L}(u), \nu_{\overline{P}+\overline{Z}}^{U}(u) \\ &= \nu_{\overline{P}}^{U}(u) + \nu_{\overline{Z}}^{U}(u), \omega_{\overline{P}+\overline{Z}}^{L}(u) = \omega_{\overline{P}}^{L}(u) + \omega_{\overline{Z}}^{L}(u), \omega_{\overline{P}+\overline{Z}}^{U}(u) = \omega_{\overline{P}}^{U}(u) + \omega_{\overline{Z}}^{U}(u)\end{aligned}$$

Definition 14: Given ICNS $\overline{P} = \left\{\left\langle T_P(u) = t_{\overline{P}}(u) \cdot e^{j\alpha\mu_{\overline{P}}(u)}, I_{\overline{P}}(u) = i_{\overline{P}}(u) \cdot e^{j\beta\nu_{\overline{P}}(u)}, F_{\overline{P}}(u) = f_{\overline{P}}(u) \cdot e^{j\gamma\omega_{\overline{P}}(u)}\right\rangle : u \in U\right\}$, the scalar multiplication is defined as follows:

$$\begin{aligned}T_{\overline{C}}(u) = [1 - (1 - t_{\overline{P}}^{L}(u))^k, 1 - (1 - t_{\overline{P}}^{R}(u))^k] \cdot e^{j\pi\left[\mu_{\overline{C}}^{L}(u), \mu_{\overline{C}}^{R}(u)\right]}, I_{\overline{C}}(u) = [(i_{\overline{P}}^{L}(u))^k, (i_{\overline{P}}^{R}(u))^k] \cdot \\ e^{j\pi[\nu_{\overline{C}}^{L}(u), \nu_{\overline{C}}^{R}(u)]}, I_{\overline{C}}(u) = [(i_{\overline{P}}^{L}(u))^k, (i_{\overline{P}}^{R}(u))^k] \cdot e^{j\pi[\nu_{\overline{C}}^{L}(u), \nu_{\overline{C}}^{R}(u)]}\end{aligned} \tag{15}$$

Where the phase terms are defined as follows

$$\begin{aligned}\mu_{\overline{C}}^{L}(u) &= \mu_{\overline{P}}^{L}(u) \cdot k; \mu_{\overline{C}}^{R}(u) = \mu_{\overline{P}}^{R}(u) \cdot k, \nu_{\overline{C}}^{L}(u) = \nu_{\overline{P}}^{L}(u) \cdot k; \nu_{\overline{C}}^{R}(u) = \nu_{\overline{P}}^{R}(u) \cdot k, \omega_{\overline{C}}^{L}(u) = \omega_{\overline{P}}^{L}(u) \cdot k; \omega_{\overline{C}}^{R}(u) \\ &= \omega_{\overline{P}}^{R}(u) \cdot k\end{aligned}$$

3. Proposed Method

In this section, we introduce the proposed methodology for optimizing and managing uncertainty in vocational student employment using a Multi-Criteria Complex Neutrosophic Approach. The method integrates ICNS with the MCDM technique, which enables a robust decision framework. Our approach consists of several key phases, including ICNS transformation of linguistic evaluations into ICNS numbers, weight determination, as well as alternative evaluation and ranking. In the following, we propose to explain the methodological steps involved in the proposed approach.

In step 1, we calculate the weights of different components in ICNS based on the relative dominance and uncertainty of each component. w_T, w_I, and w_F. Relative dominance implies that components with a higher average magnitude should assigned higher weight. Uncertainty implies that the component with lower uncertainty (narrower interval) should have a higher weight. The weights are computed as follows:

$$\begin{aligned} w_T &= \frac{t_S(u)\cdot(1-U_T(u))}{t_S(u)\cdot(1-U_T(u))+i_S(u)\cdot(1-U_I(u))+f_S(u)\cdot(1-U_F(u))}, \\ w_I &= \frac{i_S(u)\cdot(1-U_I(u))}{t_{\bar{S}}(u)\cdot(1-U_T(u))+i_{\bar{S}}(u)\cdot(1-U_I(u))+f_{\bar{S}}(u)\cdot(1-U_F(u))}, \\ w_F &= \frac{f_{\bar{S}}(u)\cdot(1-U_F(u))}{t_S(u)\cdot(1-U_T(u))+i_S(u)\cdot(1-U_I(u))+f_S(u)\cdot(1-U_F(u))}, \end{aligned} \tag{16}$$

where $U_T(u)$, $U_I(u)$, and $U_F(u)$ denote the normalized uncertainties:

$$\begin{aligned} U_T(u) &= \frac{t_{\bar{S}_U}(u)-t_{\bar{S}_L}(u)}{t_{\bar{S}_U}(u)+i_{\bar{S}_U}(u)+f_{\bar{S}_U}(u)}, U_I(u) = \frac{i_{S_U}(u)-i_{S_L}(u)}{t_{\bar{S}_U}(u)+i_{\bar{S}_U}(u)+f_{\bar{S}_U}(u)} \\ U_F(u) &= \frac{f_{S_U}(u)-f_{S_L}(u)}{t_{S_U}(u)+i_{S_U}(u)+f_{S_U}(u)} \end{aligned} \tag{17}$$

In step 2, we apply these weights to compute the weighted average magnitude and weighted average phase.

$$\bar{t}(u) = w_T\cdot\frac{t_{S_L}(u)+t_{S_U}(u)}{2},\ \bar{\iota}(u) = w_I\cdot\frac{i_{S_L}(u)+i_{S_U}(u)}{2},\ \bar{f}(u) = w_F\cdot\frac{f_{S_L}(u)+f_{S_U}(u)}{2} \tag{18}$$

$$\mu(u) = w_T\cdot\frac{\mu_{\bar{S}_L}(u)+\mu_{\bar{S}_U}(u)}{2},\ \nu(u) = w_I\cdot\frac{\nu_{\bar{S}_L}(u)+\nu_{\bar{S}_U}(u)}{2},\ \omega(u) = w_F\cdot\frac{\omega_{\bar{S}_L}(u)+\omega_{\bar{S}_U}(u)}{2} \tag{19}$$

In step 3, we inject phase information into the scoring mechanism:

$$\begin{gathered} S_T(u) = \bar{t}(u)\cdot\cos\big(\mu(u)\big),\ S_i(u) = \bar{\iota}(u)\cdot\cos\big(\nu(u)\big), \\ S_f(u) = \bar{f}(u)\cdot\cos\big(\omega(u)\big) \end{gathered} \tag{20}$$

In step 4, we combine the phase-sensitive scores and uncertainty measures into a final de-neutrosophication score:

$$\begin{aligned} \mathfrak{D}(P) = \Bigg(& S_T(u)\frac{\left(t_{S_U}(u)+t_{S_L}(u)\right)}{2} + S_i(u)\left(1-\frac{\left(i_{S_U}(u)+i_{S_L}(u)\right)}{2}\right)\left(i_{S_U}(u)\right) \\ & - S_i(u)\left(\frac{\left(f_{S_U}(u)+f_{S_L}(u)\right)}{2}\right)\left(1-f_{S_U}(u)\right)\Bigg) \end{aligned} \tag{21}$$

Following the de-neutrosophication of ICNS values, we introduce pseudocode for the weight calculation algorithm based on AHP [16]. As shown in Algorithm 1, the pseudocode is written in structured mathematical step terms.

Algorithm 1: pseudocode for AHP weighting

Input: Group of $\boldsymbol{n}$ criteria $\boldsymbol{C} = \{\boldsymbol{C_1}, \boldsymbol{C_2}, \dots, \boldsymbol{C_n}\}$, pairwise comparisons from DMs.
Output: A weight vector $\boldsymbol{w}= \{\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_n}\}$, where $\boldsymbol{w_i}$ Represents the weight of $\boldsymbol{i-th}$ criterion.

Aggregate pairwise comparison matrices
For each pair of criteria (C_i, C_j):
  If $i = j$
    set $\mathbb{A}_{ij} = 1$
  else
    $a_{ij} \leftarrow$ DM value on a scale of 1 to 9
  $a_{ij} = 1$: C_i is equally important to C_j.
  $a_{ij} = 9$: C_i is extremely more important than C_j
  Set $\mathbb{A}_{ji} = \frac{1}{a_{ij}}$ (reciprocal value).

$$\mathbb{A} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{bmatrix}$$

Get column sums: $S_j = \sum_{i=1}^{n} \mathbb{A}_{ij}$, for $j = 1,2, \dots, n$

Normalize each element of $\mathbb{A}$ $N_{ij} = \frac{\mathbb{A}_{ij}}{S_j}$, for $i, j = 1,2, \dots, n$ $N = \begin{bmatrix} N_{11} & N_{12} & \cdots & N_{1n} \\ N_{21} & N_{22} & \cdots & N_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ N_{n1} & N_{n2} & \cdots & N_{nn} \end{bmatrix}$

Calculate the Weights $w_i = \frac{1}{n} \sum_{j=1}^{n} N_{ij}$, for $i = 1,2, \dots, n$

Return $\boldsymbol{W} = \{\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_n}\}$

To make the multi-criteria decision, we apply VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) [17] to identify a compromise solution that balances conflicting criteria. It is particularly useful in our case as we aim to evaluate alternatives based on multiple, often competing, objectives. Below is a step-by-step description of the VIKOR method:

In step 1, we obtain the decision matrix $U_{m\times n}$, based on equation (26), along with the weight vector $W_{1\times n} = \{w_1, w_2, \dots, w_n\}$ Computed by algorithm 1. Also, we take into account the type of criterion as annotated by decision-makers. The VIKOR method uses strategy coefficient v : A scalar value between 0 and 1 (default = 0.5).

In step 2, we normalize the matrix $M_{m\times n}$ as follows. For each criterion j, determine the best (f_j^*) and worst (f_j^-) values:

$$f_j^* = \begin{cases} \max(M_{ij}) & \text{if } c_j =' \max \\ \min(M_{ij}) & \text{if } c_j =' \min \end{cases} \quad f_j^- = \begin{cases} \min(M_{ij}) & \text{if } c_j =' \max \\ \max(M_{ij}) & \text{if } c_j =' \min \end{cases} \tag{22}$$

Then, we calculate the normalized decision matrix S :

$$S_{ij} = w_j \cdot \frac{\left|f_j^* - M_{ij}\right|}{\left|f_j^* - f_j^-\right| + \epsilon} \tag{23}$$

Where ϵ is a small constant (e.g., 10^{-16}) to avoid division by zero.

In step 3, we calculate the usefulness measure S_i, as well as the regret measure R_i for each i-th alternative:

$$S_i = \sum_{j=1}^{n} S_{ij} \; R_i = \max\left(S_{ij}\right) \tag{24}$$

In step 4, we computed the VIKOR Index $(Q_i)$to decide on the best and worst values for S_i and R_i:

$$\begin{aligned} S^* &= \min(S_i), \quad S^- = \max(S_i) \\ R^* &= \min(R_i), \quad R^- = \max(R_i) \end{aligned} \tag{25}$$

In step 5, we compute the VIKOR index Q_i for i-th alternative:

$$Q_i = v \cdot \frac{S_i - S^*}{S^- - S^*} + (1 - v) \cdot \frac{R_i - R^*}{R^- - R^*} \tag{26}$$

Finally, we rank the alternatives according to the values of Q_i in ascending order. The alternative with the smallest Q_i is the best compromise solution. We use the following conditions to ensure the ranking is acceptable:

$$Q_{(2)} - Q_{(1)} \geq \frac{1}{m - 1} \tag{27}$$

where $Q_{(1)}$ and $Q_{(2)}$ are the first and second smallest Q_i Values, respectively.

4. Case Study & Results

This study introduces a case study of a Chinese university campus that plans to implement vocational student employment to enable local industry to evaluate and rank five graduating students for a technical specialist role in a manufacturing and technology firm. The employment process usually includes both qualitative and quantitative valuations, in which uncertainty in employment competence occurs because of subjective and contradictory criteria. The case study contains many conflicting criteria that govern the decision-making, some of them need to be maximized and other needs to be minimized or avoided. In Table 1, we present ten evaluation criteria used to assess vocational college students' employment competence. To facilitate a systematic and structured evaluation, Table 1 defines six linguistic terms that describe candidate performance in each criterion and maps them to ICNS, which represent uncertainty, hesitation, and imprecision in assessment are properly captured.

Table 1. Summary of vocational employment criteria and related linguistic rates

Criteria	ID	Type	Linguistic rating	
Leadership & Initiative	C1	Benefit	**Linguistic Term**	**ICNS terms**

Communication & Teamwork	C2	Benefit	Very Poor (VP)	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$
Adaptability & Problem-Solving	C3	Benefit	Poor (P)	$\left\{\left\langle [0.1,0.3]e^{j\alpha[0.2,0.4]}, [0.6,0.8]e^{j\beta[0.3,0.5]}, [0.7,0.9]e^{j\gamma[0.4,0.6]} \right\rangle\right\}$
Training Costs Incurred	C4	Cost	Fair (F)	$\left\{\left\langle [0.3,0.5]e^{j\alpha[0.3,0.5]}, [0.5,0.7]e^{j\beta[0.2,0.4]}, [0.5,0.7]e^{j\gamma[0.3,0.5]} \right\rangle\right\}$
Internship Performance Score	C5	Benefit	Good (G)	$\left\{\left\langle [0.5,0.7]e^{j\alpha[0.4,0.6]}, [0.3,0.5]e^{j\beta[0.1,0.3]}, [0.3,0.5]e^{j\gamma[0.2,0.4]} \right\rangle\right\}$
Response Time	C6	Cost	Very Good (VG)	$\left\{\left\langle [0.7,0.9]e^{j\alpha[0.5,0.7]}, [0.1,0.3]e^{j\beta[0.0,0.2]}, [0.1,0.3]e^{j\gamma[0.1,0.3]} \right\rangle\right\}$
Error Rate in Practical Tasks	C7	Cost	Excellent (E)	$\left\{\left\langle [0.9,1.0]e^{j\alpha[0.6,0.8]}, [0.0,0.2]e^{j\beta[0.0,0.1]}, [0.0,0.2]e^{j\gamma[0.0,0.2]} \right\rangle\right\}$
Employment Competence Indeterminacy Score	C8	Cost		
Technical Skill Level	C9	Benefit		
Industry Certifications	C10	Benefit		

In our case study, the decision-maker must evaluate different higher vocational students. In this case study, four policymakers are involved in evaluating the above alternatives based on the 10 criteria. Each policy maker (e.g., PM1: HR professionals, PM2: vocational training instructors, PM3: industry experts, PM4: academic counselor) provides their subjective judgment for criteria of each alternative using a linguistic rating set, which is later transformed into ICSN for decision-making. in Table 2, we present the aggregated ICNS decision matrix that consolidated assessments from the policymakers.

Table 2. Summary of aggregated ICNS decision matrix from different DMs.

Criteria	**C1**	**C2**
A1	$\left\{\left\langle [0.7,0.9]e^{j\alpha[0.5,0.7]}, [0.1,0.3]e^{j\beta[0.0,0.2]}, [0.1,0.3]e^{j\gamma[0.1,0.3]} \right\rangle\right\}$	$\left\{\left\langle [0.1,0.3]e^{j\alpha[0.2,0.4]}, [0.6,0.8]e^{j\beta[0.3,0.5]}, [0.7,0.9]e^{j\gamma[0.4,0.6]} \right\rangle\right\}$
A2	$\left\{\left\langle [0.3,0.5]e^{j\alpha[0.3,0.5]}, [0.5,0.7]e^{j\beta[0.2,0.4]}, [0.5,0.7]e^{j\gamma[0.3,0.5]} \right\rangle\right\}$	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$
A3	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$

A4	$\left\{\left\langle\begin{matrix}[0.1,0.3]e^{j\alpha[0.2,0.4]},[0.6,0.8]e^{j\beta[0.3,0.5]},\\ [0.7,0.9]e^{j\gamma[0.4,0.6]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$
A5	$\left\{\left\langle\begin{matrix}[0.7,0.9]e^{j\alpha[0.5,0.7]},[0.1,0.3]e^{j\beta[0.0,0.2]},\\ [0.1,0.3]e^{j\gamma[0.1,0.3]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$

Table 2. Summary of aggregated ICNS decision matrix from different DMs (cont.…).

Criteria	C3	C4
A1	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A2	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$
A3	$\left\{\left\langle\begin{matrix}[0.5,0.7]e^{j\alpha[0.4,0.6]},[0.3,0.5]e^{j\beta[0.1,0.3]},\\ [0.3,0.5]e^{j\gamma[0.2,0.4]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A4	$\left\{\left\langle\begin{matrix}[0.5,0.7]e^{j\alpha[0.4,0.6]},[0.3,0.5]e^{j\beta[0.1,0.3]},\\ [0.3,0.5]e^{j\gamma[0.2,0.4]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$
A5	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$

Table 2. Summary of aggregated ICNS decision matrix from different DMs (cont.…).

Criteria	C5	C6
A1	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A2	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.5,0.7]e^{j\alpha[0.4,0.6]},[0.3,0.5]e^{j\beta[0.1,0.3]},\\ [0.3,0.5]e^{j\gamma[0.2,0.4]}\end{matrix}\right\rangle\right\}$
A3	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A4	$\left\{\left\langle\begin{matrix}[0.5,0.7]e^{j\alpha[0.4,0.6]},[0.3,0.5]e^{j\beta[0.1,0.3]},\\ [0.3,0.5]e^{j\gamma[0.2,0.4]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A5	$\left\{\left\langle\begin{matrix}[0.7,0.9]e^{j\alpha[0.5,0.7]},[0.1,0.3]e^{j\beta[0.0,0.2]},\\ [0.1,0.3]e^{j\gamma[0.1,0.3]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.7,0.9]e^{j\alpha[0.5,0.7]},[0.1,0.3]e^{j\beta[0.0,0.2]},\\ [0.1,0.3]e^{j\gamma[0.1,0.3]}\end{matrix}\right\rangle\right\}$

Table 2. Summary of aggregated ICNS decision matrix from different DMs (cont.…).

Criteria	C7	C8
A1	$\left\{\left\langle\begin{matrix}[0.0,0.2]e^{j\alpha[0.1,0.3]},[0.7,0.9]e^{j\beta[0.4,0.6]},\\ [0.8,1.0]e^{j\gamma[0.5,0.7]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$
A2	$\left\{\left\langle\begin{matrix}[0.7,0.9]e^{j\alpha[0.5,0.7]},[0.1,0.3]e^{j\beta[0.0,0.2]},\\ [0.1,0.3]e^{j\gamma[0.1,0.3]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.7,0.9]e^{j\alpha[0.5,0.7]},[0.1,0.3]e^{j\beta[0.0,0.2]},\\ [0.1,0.3]e^{j\gamma[0.1,0.3]}\end{matrix}\right\rangle\right\}$
A3	$\left\{\left\langle\begin{matrix}[0.1,0.3]e^{j\alpha[0.2,0.4]},[0.6,0.8]e^{j\beta[0.3,0.5]},\\ [0.7,0.9]e^{j\gamma[0.4,0.6]}\end{matrix}\right\rangle\right\}$	$\left\{\left\langle\begin{matrix}[0.3,0.5]e^{j\alpha[0.3,0.5]},[0.5,0.7]e^{j\beta[0.2,0.4]},\\ [0.5,0.7]e^{j\gamma[0.3,0.5]}\end{matrix}\right\rangle\right\}$

A4	$\left\{\left\langle [0.1,0.3]e^{j\alpha[0.2,0.4]}, [0.6,0.8]e^{j\beta[0.3,0.5]}, [0.7,0.9]e^{j\gamma[0.4,0.6]} \right\rangle\right\}$	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$
A5	$\left\{\left\langle [0.3,0.5]e^{j\alpha[0.3,0.5]}, [0.5,0.7]e^{j\beta[0.2,0.4]}, [0.5,0.7]e^{j\gamma[0.3,0.5]} \right\rangle\right\}$	$\left\{\left\langle [0.3,0.5]e^{j\alpha[0.3,0.5]}, [0.5,0.7]e^{j\beta[0.2,0.4]}, [0.5,0.7]e^{j\gamma[0.3,0.5]} \right\rangle\right\}$

Table 2. Summary of aggregated ICNS decision matrix from different DMs (cont.…).

Criteria	C9	C10
A1	$\left\{\left\langle [0.7,0.9]e^{j\alpha[0.5,0.7]}, [0.1,0.3]e^{j\beta[0.0,0.2]}, [0.1,0.3]e^{j\gamma[0.1,0.3]} \right\rangle\right\}$	$\left\{\left\langle [0.1,0.3]e^{j\alpha[0.2,0.4]}, [0.6,0.8]e^{j\beta[0.3,0.5]}, [0.7,0.9]e^{j\gamma[0.4,0.6]} \right\rangle\right\}$
A2	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$	$\left\{\left\langle [0.3,0.5]e^{j\alpha[0.3,0.5]}, [0.5,0.7]e^{j\beta[0.2,0.4]}, [0.5,0.7]e^{j\gamma[0.3,0.5]} \right\rangle\right\}$
A3	$\left\{\left\langle [0.0,0.2]e^{j\alpha[0.1,0.3]}, [0.7,0.9]e^{j\beta[0.4,0.6]}, [0.8,1.0]e^{j\gamma[0.5,0.7]} \right\rangle\right\}$	$\left\{\left\langle [0.5,0.7]e^{j\alpha[0.4,0.6]}, [0.3,0.5]e^{j\beta[0.1,0.3]}, [0.3,0.5]e^{j\gamma[0.2,0.4]} \right\rangle\right\}$
A4	$\left\{\left\langle [0.5,0.7]e^{j\alpha[0.4,0.6]}, [0.3,0.5]e^{j\beta[0.1,0.3]}, [0.3,0.5]e^{j\gamma[0.2,0.4]} \right\rangle\right\}$	$\left\{\left\langle [0.5,0.7]e^{j\alpha[0.4,0.6]}, [0.3,0.5]e^{j\beta[0.1,0.3]}, [0.3,0.5]e^{j\gamma[0.2,0.4]} \right\rangle\right\}$
A5	$\left\{\left\langle [0.7,0.9]e^{j\alpha[0.5,0.7]}, [0.1,0.3]e^{j\beta[0.0,0.2]}, [0.1,0.3]e^{j\gamma[0.1,0.3]} \right\rangle\right\}$	$\left\{\left\langle [0.5,0.7]e^{j\alpha[0.4,0.6]}, [0.3,0.5]e^{j\beta[0.1,0.3]}, [0.3,0.5]e^{j\gamma[0.2,0.4]} \right\rangle\right\}$

To facilitate decision-making under uncertainty, we apply the de-neutrosophication process to encode ICNS values into crisp numerical values. This can ensure that the decision matrix is more interpretable and compatible with MCDM methods. The de-neutrosophicated values are presented in Table 3 for each alternative across all criteria.

Table 3. Summary of de-neutrosophication matrix derived from different criteria from ICNS decision matrix.

Alternative	C 1	C 2	C 3	C 4	C 5	C 6	C 7	C 8	C 9	C 10
A1	0.396	0.044	0.063	0.062	0.063	0.062	0.063	0.062	0.396	0.044
A2	0.062	0.063	0.063	0.063	0.063	0.162	0.396	0.396	0.063	0.062
A3	0.063	0.063	0.162	0.062	0.062	0.062	0.044	0.062	0.063	0.162
A4	0.044	0.063	0.162	0.063	0.162	0.062	0.044	0.063	0.162	0.162
A5	0.396	0.062	0.062	0.062	0.396	0.396	0.062	0.062	0.396	0.162

The AHP-based weight computation derived from Algorithm 1 determines the relative importance of each criterion. These weights, obtained using the geometric mean method, are illustrated in Figure 3.

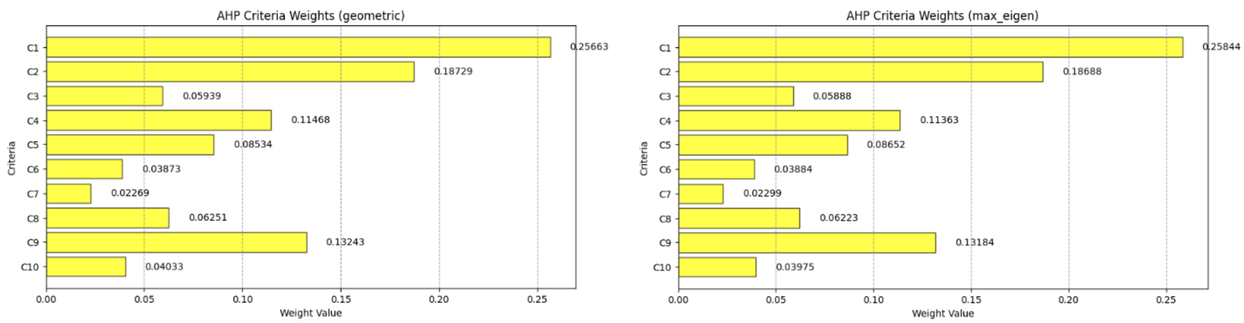

Figure 3. Qualitative results of AHP weights for different criteria from de-neutrosophied ICNS decision matrix.

In the following, we provide a visualization of ICNS-VIKOR analytical results by displaying S, R, and Q for each alternative in Figure 4. As shown, the plotted S values reflect the overall closeness of each alternative to the ideal solution, meanwhile, the R values symbolize the maximum regret of alternative w.r.t others. The Q values explain the final VIKOR ranking index. The implications of this analysis visually assess the relative performance of alternatives and support informed optimization of vocational student employment.

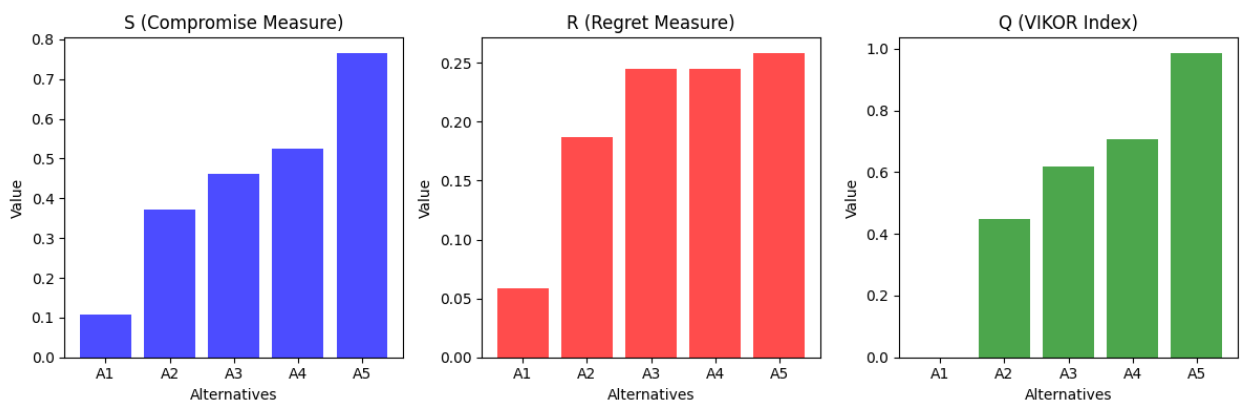

Figure 4. Qualitative results of comparing Compromise Measure, Regret Measure, and Q Index) for alternative vocational employment.

To validate the robustness of the decision-making process, we compare the AHP weights for different alternative rankings. The alternative ranking trends -under different values of the strategy coefficient - are visualized in Figure 5, which presents the prioritization of vocational student employment. This comparison ensures that our proposed ranking approach aligns with practical decision-making expectations.

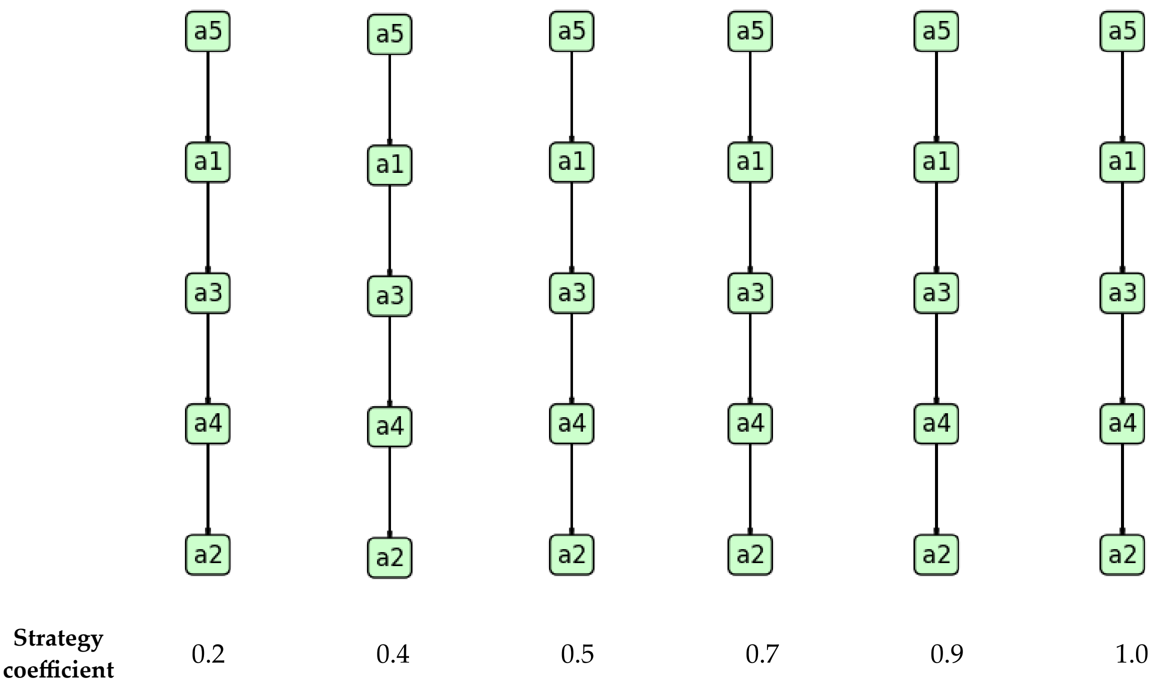

Figure 5. Qualitative ranking of alternatives from the proposed approach.

A comparative analysis of the proposed method against other MCDM techniques (such as ICNS-COPELAND [18], ICNS-CRADIS[19], and ICNS-LMAW [20]) is conducted to verify its effectiveness. The results, summarized in Table 4, demonstrate how the proposed ICNS-VIKOR framework performs compared to other approaches. The analysis highlights the accuracy, reliability, and adaptably of the model in uncertainty-driven decision environments.

Table 4: Comparative analysis of the proposed ICNS-VIKOR with previous methods for vocational student employment ranking.

	ICNS-VIKOR	**ICNS-COPELAND**	**ICNS-CRADIS**	**ICNS-LMAW**
Rank 1	A5	A3	A5	A5
Rank 2	A1	A5	A1	A1
Rank 3	A3	A4	A4	A4
Rank 4	A4	A1	A3	A3
Rank 5	A2	A2	A2	A2

To assess the stability of the decision-making process, a sensitivity analysis is performed. This analysis evaluates how variations in criteria weights affect the final ranking of alternatives. Table 5 presents the results of different weight perturbation scenarios, allowing us to determine the robustness of the ranking outcomes. The finding ensured that small changes in input parameters do not meaningfully alter the final decision, making the approach highly reliable for real-world applications.

Table 5: Sensitivity analysis showing the effect of varying criteria weights on alternative rankings.

	S-ranked	Q-ranked
+5%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2
+25%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2
+45%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2
-5%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2
-25%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2
-45%	a5 – a3 – a1 – a4 – a2	a5 – a1 – a3 – a4 – a2

Table 5: Sensitivity analysis showing the effect of varying criteria weights on alternative rankings (cont.…).

	R-ranked
+5%	a5 – a1 – a3 – a2 – a4
+25%	a5 – a1 – a3 – a2 – a4
+45%	a5 – a1 – a3 – a2 – a4
-5%	a5 – a1 – a3 – a2 – a4
-25%	a5 – a1 – a3 – a2 – a4
-45%	a5 – a1 – a3 – a2 – a4

5. Conclusion

This study proposed a Multi-Criteria Complex Neutrosophic Approach integrated for optimizing vocational student employment selection under uncertainty. With the integration of ICNS, we effectively managed uncertainty in optimizing the decision-making. A new de-neutrosophication process is presented to drive a crisp representation of uncertain data, while analytical weights are derived to ensure a structured prioritization of criteria. The results demonstrated the contribution of our framework, confirming its efficiency, sustainability, and reliability. Comparative and sensitivity analyses validated the robustness and stability of the proposed approach against traditional MCDM techniques.

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