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Statistical Characterization of the Neutrosophic Moyal Distribution

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Abstract: The Moyal distribution is widely utilized in various fields due to its effectiveness in modeling heavy-tailed processes, such as ionization losses and certain risk assessments. In this work, we present a neutrosophic version of the Moyal distribution that extends the classical Moyal distribution while incorporating uncertainty and imprecision associated with its parameters. To better explore the neutrosophic structure, we provide the detailed derivation of some essential mathematical characteristics concerning all the properties that this distribution possesses. The proposed neutrosophic Moyal (NM) model is the one that allows it to work very well in situations where data is vague or undetermined. The proposed neutrosophic model can be effectively adopted when the Moyal distribution parameters are known and lie in the range or interval. We also develop the quantile function of the proposed model under an uncertain environment. A simulation study has been carried out to assess the adequacy of neutrosophic quantile function. Numerical results demonstrate that quantile function is quite efficient in terms of generating neutrosophic random data from the proposed model.

Keywords: Moyal model, neutrosophic probability, neutrosophic distribution, estimation, simulation

1. Introduction

Survival analysis and reliability are based on probability distributions, which describe the time-to-event data and assist researchers and engineers to calculate lifetime and survival probabilities of a system [1]. Statistical distributions have a long history in reliability and survival analysis [2]. The simplest and one of the most popular distributions used in reliability analysis is the exponential distribution [3]. It is primarily used for modelling electronic components and mechanical systems with a constant failure rate. Another basic model is the Weibull distribution which is commonly used in survival analysis, allowing for failure rates to be increasing, decreasing or constant by which its shape parameter [4]. This is particularly helpful when we model lifetimes of products or the progression of diseases. The gamma distribution is an extension of the exponential model, increasing failure time variability with the inclusion of a shape parameter that can allow for aging effects to be modeled in the system [5].

The log-normal distribution is a common model when the times to failure are influenced by multiple independent elements, which multiply their contributions to the total risk of failure [6]. It is commonly used in medical and biological survival studies where the data is right-skewed. Another right-skewed model that is especially useful in reliability testing during which failure rates accelerate exponentially over time is the inverse Gaussian distribution [7]. The normal distribution, while not used directly in survival analysis, provides an approximation of failure time distributions when lifetimes are symmetrically distributed [8]. This is inspired by the Gumbel distribution, part of the extreme value family; it best represents the breakdown of the weakest link in a system, where a component falling down initiates the collapse. Also, the Burr distribution is another candidate for heavy-tailed survival-data, meaning it can be used for high-variance lifetimes [9]. For survival data with non-monotonic hazard functions, that is when there is an initial increased risk of failure followed by a decreasing risk over time, we find the log-logistic distribution to be a generalizable option for modeling [10]. The gamma and inverse gamma distributions are common conjugate prior distributions for Bayesian survival analysis, allowing for the updating of prior beliefs about the rate of failure with new data. More sophisticated applications might use mixture distributions, in which multiple probability distributions are mixed to model highly heterogeneous populations with diverse failure modes. In reliability and survival analysis, choosing appropriate probability distributions is critical as it influences risk assessment, maintenance planning, and decision-making processes in engineering, health, and industrial applications.

The Moyal distribution, as described by Moyal [11], is a continuous probability distribution with particular applications in reliability analysis, stochastic processes, and survival studies. It is specifically applicable for modeling energy in ultrahigh-energy collisions and stochastic deviations, which is unlike the normal and heavy-tailed right distributions. The studies of Landau [12] and Möller [13] investigated the Moyal distribution as an approximation to the Landau distribution, particularly in cases where the computational performance is a crucial point. These studies showed that the Moyal distribution is a reasonable approximation for the tail of energy loss distributions so that it can be a valuable option to consider in particle physics simulations, finance, and engineering while discussing properties with other heavy tailed distributions. Zhang and Li [14] had introduced a generalized Moyal distribution that able to measure energy loss in high-energy particle collisions and yield superior fit to the standard Moyal distribution. Wang and Liu [15] used the Moyal distribution to fit heavy-tailed financial data, improving risk assessment and capturing extreme market events. The asymptotic behavior of the Moyal and Gumbel distributions in the context of extreme value theory was also analyzed by Smith and Johnson in [16], where applications in environmental science were discussed. Rodriguez et al. [17] utilized Moyal distribution to model energy loss in cosmic-ray detectors and compared it to the Landau distribution.

Neutrosophic set, proposed by Smarandache, is an extension of the classical set and fuzzy set, covering three components: truth (T), indeterminacy (I), and falsity (F), each allowed to be independent in the range of [0,1] [18]. It is an efficient abstraction of uncertainty, vagueness, and incomplete information and is applicable to a broad range of fields, including decision-making, artificial intelligence, and optimization problems [19]. Because the uncertainty is inevitable, neutrosophic sets have wide applications including medical diagnosis, image processing, risk assessment and machine learning and so on. Their flexibility allows for more precise reasoning about complex systems than classical paradigms, making them an effective means to handle ambiguous and contradictory data across a wide range of scientific and engineering domains [20].

Neutrosophic probabilistic distributions provide a natural representation encompassing the notion of indeterminacy, which makes them a fitting choice for modeling uncertainties, fluctuating decision-making or incomplete information in practical scenarios [21]. Unlike traditional probability distributions that assign a precise probability to an event, neutrosophic distributions represent probabilities as estimates represented by triplets (T, I, F), where T stands for the degree of truth, I for the degree of indeterminacy, and F for the degree of falsity [22-24]. This perspective offers greater freedom in thinking about complex systems, more than a mere uncertain process [25]. Some classical probability distributions can be generalized to obtain such distributions such as neutrosophic normal, neutrosophic exponential and neutrosophic Weibull distributions [26]. These distributions are a common choice for various applications. Neutrosophic Weibull and neutrosophic exponential distributions are applied in engineering reliability to accommodate uncertain failure times and lifetimes of systems, allowing for more robust risk assessments [27]. In medical and survival analysis, neutrosophic distributions can be used to analyze the uncertain recovery time of patients and uncertain disease progression, and which will be beneficial in personalized treatment planning. In finance and economics, neutrosophic probability models incorporate the uncertainties of market fluctuations, investment risks, and economic conditions, providing a more realistic approach to decision making [28]. As applied to artificial intelligence and machine learning, these distributions assist probabilistic models by introducing the concept of undetermined values for instances that have missing data, driving towards better decision-making across large spaces and incomplete datasets [29]. Neutrosophic p-control and other statistical process control charts can achieve diversity in production management by reflecting uncertainty in quality control and industrial applications [30]. Neutrosophic distributions have applications within the environmental sciences, including climate modeling, where uncertainty in temperature trends, pollution levels, etc., can be better represented. When it comes to dealing with data that has not only uncertainty but also vagueness, neutrosophic probability distributions play a significant role in addressing a wide range of issues.

In this article, we introduce a new generalization of the Moyal distribution with the follow-up objective of broadening its field of applicability in areas like reliability description, survival analysis, and risk evaluation. The concept for introducing this generalization derives from the idea of neutrosophy, extending applications for dealing with indeterminate, inconsistent or vague data. The uncertainty inherent in study parameters is a key feature that cannot be overlooked in concrete analyzes and needs to be housed within a mathematical framework describing a system. To the best of our knowledge, there are no research articles published in the context of the Moyal model. This difference is a primary motivation for our work.

2. Classical Structure

In this section we will discuss the classical structure of the Moyal model. The Moyal distribution is a continuous probability distribution based on the work of José Enrique Moyal. It has the most utility in particle physics (high-energy particle physics, statistical mechanics) as a good model of energy loss and momentum distributions. It is a generalization of the normal distribution but has fatter tails.
The PDF and CDF of the Moyal distribution with given parameters can be defined as:

$$f(t;\alpha,\beta)=\frac{1}{\sqrt{2\pi}\beta}\exp\left(-\frac{1}{2}\left(\frac{t-\alpha}{\beta}+e^{-(t-\alpha)/\beta}\right)\right) \tag{1}$$

$$F(t;\alpha,\beta)=F(T)=\int_{-\infty}^{t}f(t)dt \tag{2}$$

Using the appropriate transformation, Eq(2) can be rewritten as:

$$F(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{T}\exp\left(-\frac{1}{2}(t+e^{-t})\right)dt. \tag{3}$$

Applying substitution Eq (3) further can be written as:

$$F(t)=\frac{1}{\sqrt{\pi}}\int_{e^{-T/2}}^{\infty}\frac{e^{-y}}{\sqrt{y}}dy$$

Recognizing this expression as incomplete gamma function we can further write as:

$$F(t) = \frac{\Gamma\left(\frac{1}{2}, \frac{e^{-t}}{2}\right)}{\Gamma\left(\frac{1}{2}\right)},$$

which is upper incomplete gamma so we can write Eq (3) as:

$$F(t) = 1 - \gamma\left(\frac{1}{2}, \frac{e^{-t}}{2}\right)$$

where $t = \frac{x-\alpha}{\beta}$ and $\gamma(a, x)$ is upper incomplete gamma distribution.

There is not a specific closed form for the CDF. Now the R program can be used to evaluate the value of CDF. The parameter α in the Moyal distribution is the location parameter, which translates the distribution left or right on the horizontal axis and determines where its mass is centered, although the mean is not equal to this value (due to the skewness). The parameter β regulates the distribution's scale the higher the β the more spread out the distribution, and lower values concentrate it closer to α. Combined, these parameters define the behavior of the distribution, which is why it suits well the modeling of asymmetric and heavy-tailed data in a wide range of scientific applications.

Figure 1 PDF and CDF of the Moyal distribution for certain distributional parameters

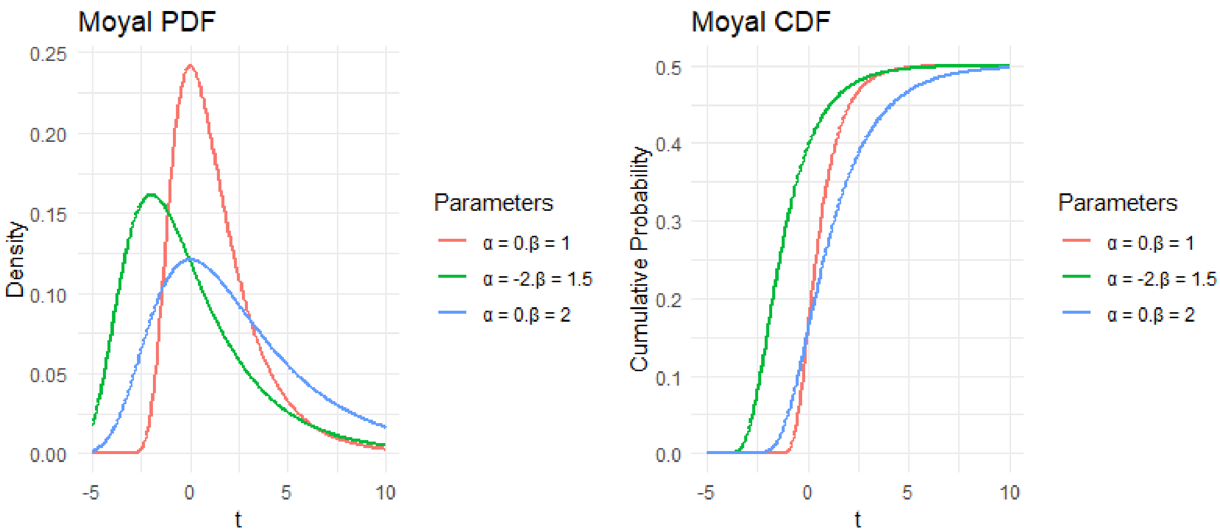

Figure 1 shows the basic plots of PDF and CDF of the Moyal distribution. The Moyal distribution is a significant probability distribution in various fields like physics, statistics, and engineering. Due to its ability to model varieties of asymmetric behavior and heavy tails, it can be useful in representing energy loss distributions in high energy physics, turbulent fluid flows, and some types of economic models. Because the probability density function (PDF) of the Moyal distribution has exponential and Gaussian-like properties, it is able to capture a number of aspects of skewed and long-tailed data that the normal distribution would otherwise be unfit to describe. While the CDF has no simple closed-form solution, it lets us understand the chances of the value being in a certain part of the space. Its flexibility encourages several applications in quantum mechanics, meteorology, and other fields, where the modeling of extreme events or non-Gaussian behaviors is required.

Similarly mean and variance of the Moyal distribution in classical context are given by:

$$E[T] = \alpha + \beta(\gamma + \ln 2) \qquad (4)$$

where ($\gamma \approx 0.5772$) is the Euler-Mascheroni constant.

$\mathrm{Var}(T) = \frac{\pi^2}{2}\beta^2$ (5)

The mean and variance of the distribution define key insights into the statistical properties of the Moyal distribution. Eq (3) indicates how the mean is displaced from the location parameter α. According to Eq(4), the variance scales quadratically with β. The appearance of $\frac{\pi^2}{2}$ in this variance is also indicative of the heavy-tailed character of the distribution, making it suitable for modeling skewed as well as extreme-value data found in a wide variety of different applications in physics and reliability analysis. More about shape of the Moyal distribution we need to study about skewness coefficient of the model which is given by:

$\text{Skewness} = \frac{2\sqrt{2}\zeta(3)}{\pi^3} \approx 0.608$ (6)

where $\zeta(3)$ is the is Apéry's constant.

Kurtosis coefficient is another important measure about tail structure of the distribution which is given by:

$\text{Kurtosis} = \frac{12}{5} \approx 2.4$ (7)

Now if assume the different values of Moyal's parameters we can calculate its basic characteristics which are given in Table 1.

Table 1 Basic characteristics of the Moyal distribution for different parameters setting

α	β	Mean	Variance	Skewness	Kurtosis
0	1	1.693	4.934	0.608	2.4
1	2	2.693	19.736	0.608	2.4
-2	1.5	1.433	11.153	0.608	2.4
3	0.5	2.265	0.493	0.608	2.4
5	2	4.693	19.736	0.608	2.4

Table 1 provides some basic properties of the Moyal model. The main statistical properties of the Moyal distribution for five pairs of α (location) and β parameters. For each pair, it computes a table with mean, variance, skewness, and kurtosis The mean is a function of α and β, while the variance is proportional to β. Moyal distribution parameters do not alter skewness and kurtosis values and maintain shape characteristics: a skewed shape with large tails useful in extreme value modeling. Now in the next section, we would overview the parallel properties of the Moyal distribution in neutrosophic structure.

3. Neutrosophic Moyal Model

In this section we will discuss the neutrosophic framework of the Moyal model. Let x be a random variable based on the neutrosophic Moyal standard probability density function (pdf) provided by:

$f(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(x + e^{-x})\right), \quad -\infty < x < \infty.$ (8)

Using a location parameter $\alpha_n = [\alpha_l, \alpha_u]$ and a scale factor $\beta_n = [\beta_l, \beta_u]$, the random variable $x = \frac{z-\alpha_n}{\beta_n}$ has a neutrosophic Moyal (NM) distribution, denoted as $\mathcal{M}_O(\alpha_n, \beta_n)$

$f(z) = \frac{1}{\sqrt{2\pi\beta_n}}\exp\left(-\frac{1}{2}\left(\frac{z-\alpha_n}{\beta_n} + e^{-\left(\frac{z-\alpha_n}{\beta_n}\right)}\right)\right), -\infty < z, \alpha_n < \infty, \beta_n > 0.$ (9)

The lower incomplete gamma function $\Upsilon(\lambda, z) = \int_0^z t^{\lambda-1}e^{-t}dt$ determines the cumulative distribution function (cdf) that corresponds to (2). It is provided by

$$\mathcal{G}(z) = 1 - \frac{\Upsilon\left(\frac{1}{2},\frac{1}{2}e^{-\left(\frac{z-\alpha_n}{\beta_n}\right)}\right)}{\Gamma\left(\frac{1}{2}\right)}, \quad (10)$$

$$= 1 - \mathcal{P}\left(\frac{1}{2},\frac{1}{2}e^{-x}\right) \quad (11)$$

where $\mathcal{P}$ is the incomplete gamma function.

By assuming the different values of neutrosophic parameters of the proposed model , PDF curves can be seen in Figure2.

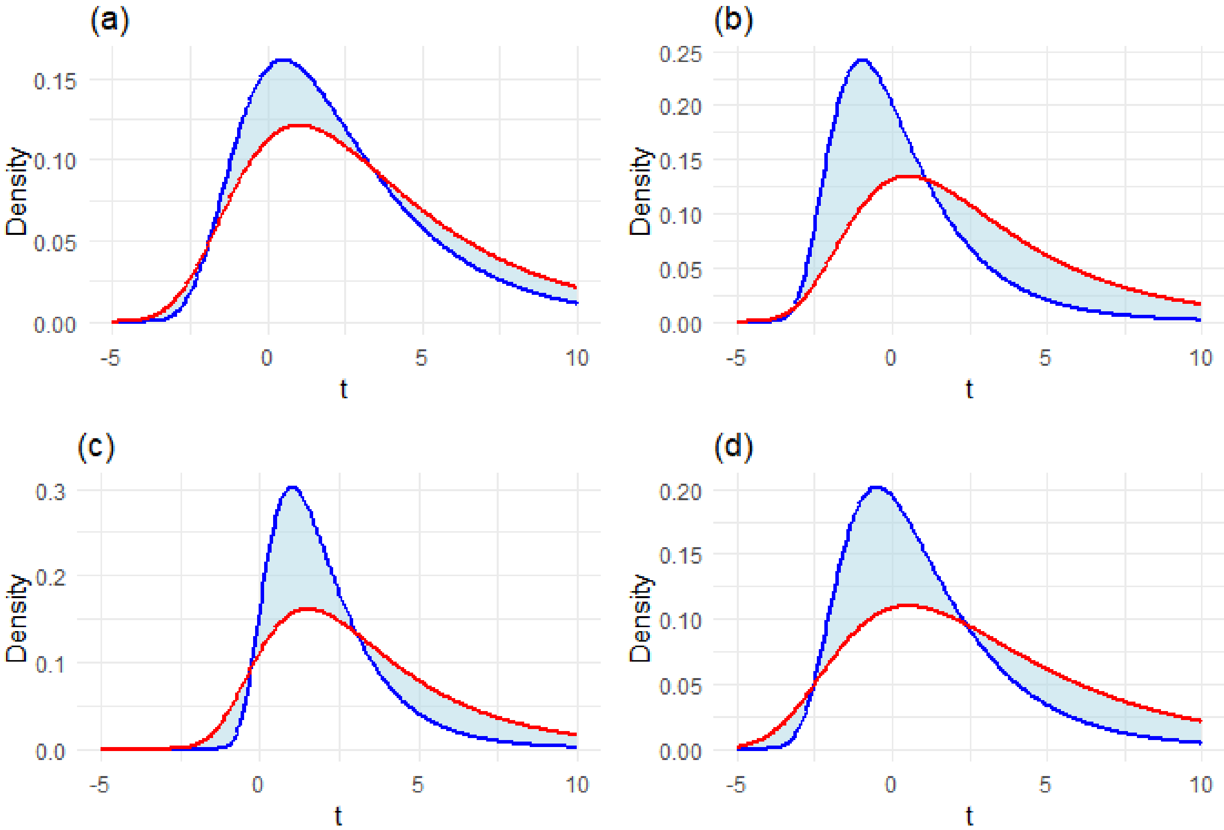

Figure 2 PDF curves of the model with (a) $\alpha_n = [0.5,1]$, $\beta_n = [1.5,2]$ (b) $\alpha_n = [-1,0.5]$, $\beta_n = [1,1.8]$(c) $\alpha_n = [1,1.5]$, $\beta_n = [0.8,1.5]$and (d) $\alpha_n = [-0.5,0.5]$, $\beta_n = [1.2,2.2]$

Figure 2 illustrates the PDF curves of the proposed model using the given neutrosophic values for the distributional parameters. Due to imprecision in the assumed parameter values, the curves appear as layers instead of single lines. The indeterminate region is highlighted with a shaded area between the curves. Similarly for same parameters setting CDF curves of the proposed model is shown in Figure 3

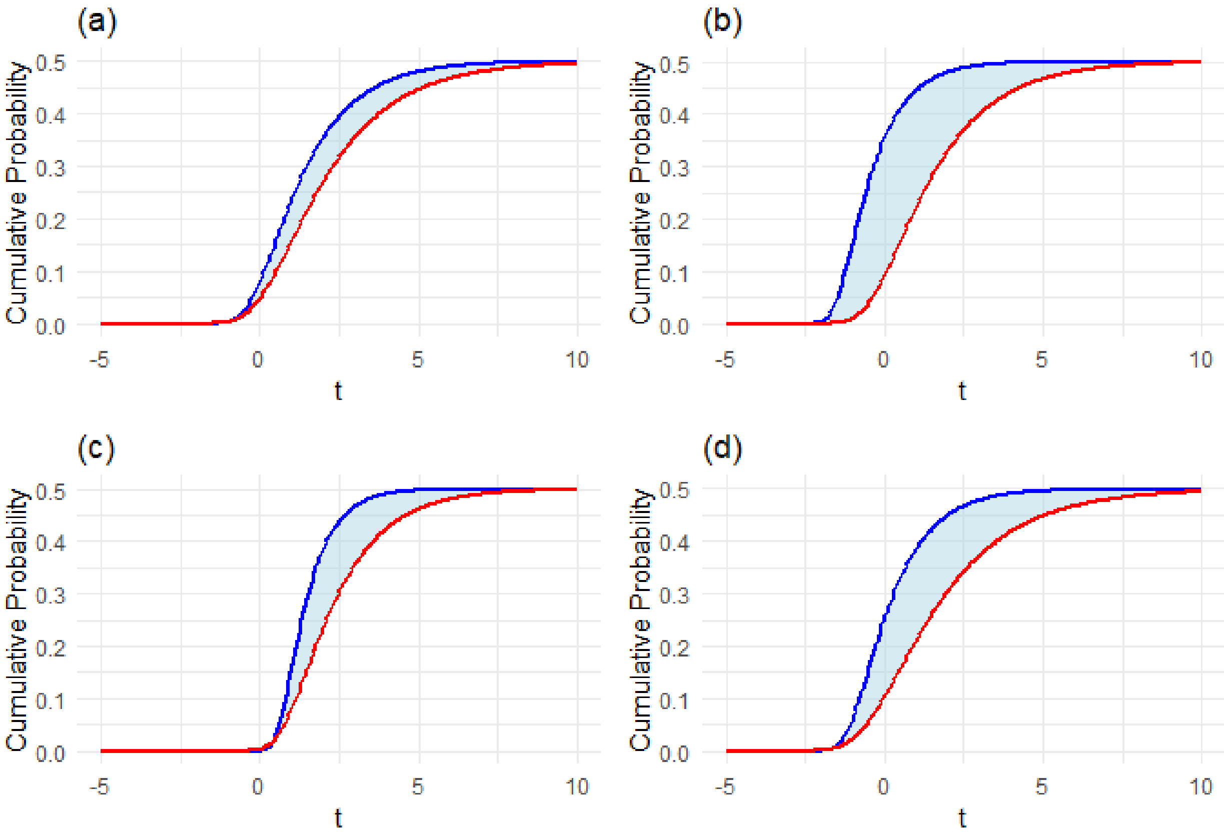

Figure 3 CDF curves of the model with (a) $\alpha_n = [0.5,1]$, $\beta_n = [1.5,2]$ (b) $\alpha_n = [-1,0.5]$, $\beta_n = [1,1.8]$(c) $\alpha_n = [1,1.5]$, $\beta_n = [0.8,1.5]$and (d) $\alpha_n = [-0.5,0.5]$, $\beta_n = [1.2,2.2]$

The CDF curves of the neutrosophic Moyal distribution are illustrated in Figure 3, demonstrating how the uncertainty in the distributional parameters affects the CDF curves. The CDF is not a single curve but a shaded region between two boundary curves. The presented neutrosophic region is covered by the wide shaded area, which means that we are not sure about the actual value, due to its neutrosophic uncertainty. The bounds are established by the lower curve and the upper curve, so the actual CDF lies between the two curves.

Definition 1. The mean of NMD is $\gamma + ln2$.

Proof: By definition the NMD is

$$E[x] = \int_{-\infty}^{\infty} xf(x)dx \tag{12}$$

$$E[x] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x + e^{-x})\right). \tag{13}$$

Define

$$z = e^{-x} \Rightarrow dz = -e^{-x}dx = -zdx$$

Thus,

$$dx = -\frac{dz}{z}$$

Rewrite x in term of z:

$$x = -lnz.$$

$$E[z] = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} -lnz\, e^{-\frac{1}{2}(-lnz+z)} \left(-\frac{dz}{z}\right)$$

Simplify

$E[z] = \frac{1}{\sqrt{2\pi}} \int_0^\infty lnz \, e^{-\frac{1}{2}(-lnz+z)} \left(\frac{dz}{z}\right) = \sqrt{2\pi}(\gamma + ln2),$

where γ is the Euler-Mascheroni constant (0.5772).

Thus,

$E[z] = (\gamma + ln2)$. Hence proved.

Definition 2. The variance of NM distributino is $\frac{\pi^2}{2}.{\beta_n}^2$

Proof: By definition the NMD is $\frac{\pi^2}{2}$

$$V(x) = E[x^2] - (E[x])^2 \quad (14)$$

First we compute $E[x^2]$:

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f(x) dx \quad (15)$$

Using integral technique

$E[x^2] = \frac{\pi^2}{2} {\beta_n}^2 + (\gamma + ln2)^2$.

Thus, the variance is;

$V(x) = \frac{\pi^2}{2} {\beta_n}^2 + (\gamma + ln2)^2 - (\gamma + ln2)^2$

$V(x) = \frac{\pi^2}{2} {\beta_n}^2 \; hence \; proved.$

Definition 3: Show that the k^{th} raw moment of the NMD is $(n-1)!\,(2^n - 1)\xi_n$

Proof: Observing that the k^{th} moment of the NMD is:

$$m_1 = -ln2 - \phi\left(\frac{1}{2}\right) = ln2 + \gamma \quad (16)$$

$$m_k = (-1)^k \phi^{(k-1)}\left(\frac{1}{2}\right) = (k-1)!\,(2^k - 1)\xi_k, for \; k \geq 2 \quad (17)$$

Using the cumulants we find the lower order moments and the coefficient of skewness and kurtosis to be

$\mu_1'' = E[x] = m_1 = ln2 + \gamma$

$\mu_2'' = V[x] = m_2 = \phi^{(1)}\left(\frac{1}{2}\right) = \frac{\pi^2}{2}$

$\mu_3'' = m_3 = -\phi^{(2)}\left(\frac{1}{2}\right) = 14\xi_3$

$\mu_4'' = m_4 + 3m_2^2 = \phi^{(3)}\left(\frac{1}{2}\right) + 3\phi^{(1)}\left(\frac{1}{2}\right)^2 = \frac{7\pi^4}{4}$

Basic on these moments skewness and kurtosis coefficients can be derived. However, we note that skewness and kurtosis coefficinets are independent of shape and location parameters so remain same as given above. Similarly other properties of the NM distribution under neutrosophic framework can be developed.

4 Quantile Function

In this section, we discuss the random generation and estimation procedure of the proposed model. The inverse cdf or the inverse transform method is a widely used technique to generate random samples from a specified probability distribution. It uses that fact that for any continuous random variable X with some F(x) there exists a way to sample from it, by drawing a uniform random variable and then applying the inverse cdf. This approach will work because the cdf "transforms" the probability range [0, 1] to the values of the random variable. First, uniform random values are generated and then, the inverse cdf is applied to transform these random values into the target distribution.

In this section, we would study about the quantile function of the proposed model and how it can be use to draw the random samples. If you use a quantile function, we can generate random samples from a proposed model. Based on inverse transform sampling, it gives the possibilities of converting random variables of uniform distribution into random variables of given distribution (in our case, with inverse transform sampling, the Moyal distribution). This is crucial for Monte Carlo simulations, bootstrapping methods, and other methods that use random sampling to estimate statistical properties. The quantile function enables sampling from the distribution in an efficient manner which is crucial for simulations that do not have conveniently computable analytical solutions. Monte Carlo simulation is also commonly applied in risk analysis, financial modeling, and reliability engineering to model and simulate outcomes based on various probabilistic assumptions. The quantile function of the proposed model can be written as:

$$Q(p;\alpha_n,\beta_n) = \alpha_n - \beta_n \cdot \ln(2\,p^{-1}(2,1-p)) \tag{18}$$

where p^{-1} is the inverse of incomplete upper gamma function.

Monte Carlo simulation can be used to generate random samples from the quantile function of the neutrosophic Moyal distribution; this can be done by first generating uniform random numbers in the range [0,1]. Now, these uniform values represent corresponding probabilities. We can then take each of these probabilities to apply the quantile function over the Moyal distribution that will correspond to each probability. This means that it correctly transforms uniform random numbers into neutrosophic Moyal distributed ones. This process is repeated as many times as should be sampled, with the values simply stored. Subsequently, these generated samples can be applied in simulations, statistical modeling, or risk analysis in order to reflect the behavior of the Moyal distribution associated with neutrosophic parameters α_n and β_n, which offers a powerful approach to simulate results according to probabilistic assumptions. A program written in R language has been used to draw these random samples. The output of these random samples is provided in Table 2.

Table 2 Random samples generated from the proposed model

Random Samples					
[-0.831,0.043]	[-0.849, 0.813]	[0.04, 1.596]	[-1.688, 0.108]	[0.47, 1.203]	[-0.423, 0.907]
[-0.177, 0.781]	[-0.592, 0.362]	[-0.173, 0.011]	[-1.273, 0.162]	[-1.435, 0.084]	[0.158, 0.967]
[-2.678, 0.526]	[-1.883, 0.415]	[0.405, 0.615]	[-0.157, 0.023]	[-0.319, 0.151]	[0.159, 0.424]
[-1.786, 0.042]	[-0.042, 0.306]	[-1.143, 0.443]	[-1.544, 0.032]	[0.013, 0.706]	[0.331, 0.807]
[-1.635, -0.13]	[-0.821, 0.153]	[0.262, 0.468]	[-0.509, 0.447]	[-0.143, 1.631]	[-0.705, 0.232]

Table 2 indicates the random samples generated from proposed model with neutrosophic values $\alpha_n = [0,0]$ and $\beta_n = [0.5, 1.5]$. Indeterminacy in the neutrosophic parameters causes imprecise random values as shown in Table 2. These random sample values are generated via R program. If we consider zero imprecision in the simulated parameters, results of neutrosophic Moyal distribution match with classical model.

5 Conclusions

In this study, the neutrosophic Moyal (NM) distribution as an extension of the classical Moyal distribution to account for uncertainty and imprecision in its parameters has been presented. By incorporating the neutrosophic logic, essential mathematical properties of the proposed distribution have been established. Mathematical results demonstrated their suitability for modeling data characterized by vagueness or indeterminacy. The quantile function of the NM distribution has been derived, enabling efficient random data generation under uncertainty. A simulation study confirmed the adequacy of the proposed quantile function, highlighting its effectiveness in generating neutrosophic random samples. The findings suggest that the NM distribution can serve as a valuable tool in various applications where the classical Moyal distribution may be insufficient due to inherent data imprecision. Further work on applications and parameter estimation methods of NM model in generic instances would be worth investigating in future research.

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