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# A Concise Formalization of Partial Falsifiability, Water Logic, and Neither Nor Logic with Neutrosophic logic

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**Abstract**. This paper explores the application of neutrosophic logic to Partial Falsifiability, Water Logic, and Neither Nor Logic through a mathematical perspective. Neutrosophic logic, as an extension of classical logic, introduces truth, indeterminacy, and falsehood as independent components, offering a framework to handle uncertainty more effectively [34].

Partial Falsifiability refers to a hypothesis that can be partially refuted under certain conditions without being completely disproven [36]. Water Logic represents a flexible reasoning system that, like water, adapts and flows around obstacles rather than adhering to strict true/false dichotomies. Neither Nor Logic challenges binary choices, allowing for an indeterminate middle state to accommodate uncertainty and ambiguity(cf. [8]).

By incorporating neutrosophic logic, this paper provides an initial mathematical examination of how these alternative logical systems can be formally expressed and analyzed.

**Keywords:** Partial Falsifiability, Neutrosophic Logic, Neutrosophic set, Water Logic, Neither Nor Logic

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## 1. Partial Falsifiability

In this section, we discuss Partial Falsifiability.

### 1.1. *Partial Falsifiability*

In classical logic, a hypothesis is considered falsifiable if there exists a potential observation that can refute it. Karl Popper introduced falsifiability as a demarcation criterion for scientific theories. In the realm of multi-valued logics—such as neutrosophic logic—propositions are characterized by degrees of truth, indeterminacy, and falsehood. Here, we extend the classical concept to accommodate *partial falsifiability*, where a hypothesis may be only partially refuted [36].

**Definition 1.1** (Classical Binary Logic)**.** Classical binary logic is a formal system where each proposition $p$ is assigned a truth value from the set $L = \{0, 1\}$, where:

- 1 represents **true** (`T`),
- 0 represents **false** (`F`).

**Definition 1.2** (Classical Falsifiability)**.** (cf. [14, 25, 26]) A classical (binary) hypothesis $H$ is said to be *falsifiable* if there exists a possible observation $O$ such that $O$ contradicts $H$. In other words, $H$ is falsifiable if there exists a condition under which $H$ can be proven false.

**Example 1.3** (Observation of a Black Swan)**.** (cf. [32]) Consider the hypothesis: "All swans are white." This hypothesis is falsifiable because the observation of a single black swan would directly contradict it.

**Example 1.4** (Boiling Point of Water)**.** (cf. [30]) Consider the hypothesis: "Water boils at 100°C at sea level." Under controlled conditions, if water is observed to boil at a temperature other than 100°C at sea level, the hypothesis would be refuted.

**Example 1.5** (Gravitational Attraction)**.** (cf. [38]) Consider the hypothesis: "Gravity attracts all objects towards the Earth." This hypothesis is falsifiable in principle, as an observation contradicting expected gravitational behavior (such as an object repelling from Earth under normal conditions) would refute it.

### 1.2. *Neutrosophic Hypothesis*

In neutrosophic logic, a hypothesis is represented as a triple that encodes partial truth, indeterminacy, and falsehood. Several related concepts in logic have been introduced, including fuzzy logic [17, 18, 45], intuitionistic fuzzy logic [3], and vague logic [20]. These frameworks can be viewed as special cases within the broader generalization provided by neutrosophic logic.

**Definition 1.6** (Neutrosophic Hypothesis)**.** [19, 33, 34] Let $X$ be a non-empty set. A *Neutrosophic Hypothesis* is a proposition represented as

$$\mathrm{NLH}(t, i, f),$$

where $t, i, f \in [0, 1]$ denote, respectively, the degree of truth, indeterminacy, and falsehood of the hypothesis, and these values satisfy

$$0 \leq t + i + f \leq 3.$$

**Example 1.7** (Drug Efficacy Hypothesis)**.** (cf. [23, 46]) Consider the hypothesis: "The new drug is effective for 80% of patients, uncertain for 10%, and ineffective for 10%." This can be represented as NLH(0.8, 0.1, 0.1). In a clinical trial, if the observed outcomes under certain conditions deviate significantly towards the negated hypothesis (indicating ineffectiveness), the hypothesis is partially falsifiable.

**Example 1.8** (Economic Forecast Hypothesis)**.** (cf. [12, 24, 39, 41]) Let NLH(0.7, 0.2, 0.1) represent an economic hypothesis predicting robust growth (70% chance of growth, 20% indeterminacy, 10% chance of recession). If, during an economic downturn, data indicate that the economy is weak (approximating a falsified state), the hypothesis is considered partially falsifiable.

**Example 1.9** (Weather Prediction Hypothesis)**.** (cf. [4, 31, 37]) Consider a weather forecast given as NLH(0.75, 0.15, 0.10), meaning there is a 75% chance of a sunny day, 15% uncertainty, and 10% chance of rain. If, under specific meteorological conditions, observations yield results closer to the negated hypothesis (e.g., a higher likelihood of rain), the original weather hypothesis is partially falsifiable.

### 1.3. *The negation of a neutrosophic hypothesis*

A natural candidate for the negation of a neutrosophic hypothesis is to swap the degrees of truth and falsehood and complement the degree of indeterminacy.

**Definition 1.10** (Negation Operator in Neutrosophic Logic)**.** [36] Define the negation operator $\mathcal{N}$ on a neutrosophic hypothesis by

$$\mathcal{N}(t, i, f) = (f,\, 1 - i,\, t).$$

This operator provides the neutrosophic complement of the hypothesis.

**Example 1.11** (Environmental Health Assessment)**.** Consider a neutrosophic hypothesis about the health of an ecosystem:

$$\text{NLH}(0.8,\, 0.1,\, 0.1).$$

This suggests that the ecosystem is considered healthy with an 80% truth value, 10% indeterminacy, and 10% falsehood. Its neutrosophic negation is given by

$$\mathcal{N}(0.8,\, 0.1,\, 0.1) = (0.1,\, 0.9,\, 0.8).$$

In practical terms, if during a period of severe pollution the environmental measurements yield values approximating (0.1, 0.9, 0.8), then the original hypothesis is effectively refuted under those conditions.

**Example 1.12** (Economic Forecast)**.** (cf. [12, 24, 39, 41]) Suppose an economist proposes the hypothesis:

$$\text{NLH}(0.7,\ 0.2,\ 0.1),$$

which indicates a 70% likelihood of robust economic growth, with 20% uncertainty and a 10% chance of recession. The negation of this hypothesis is

$$\mathcal{N}(0.7,\ 0.2,\ 0.1) = (0.1,\ 0.8,\ 0.7).$$

If subsequent economic data during a downturn show results close to (0.1, 0.8, 0.7), then the hypothesis is partially falsified, demonstrating how the negation operator helps in assessing the validity of economic predictions.

**Example 1.13** (Medical Diagnosis)**.** Consider a medical hypothesis regarding the efficacy of a new treatment:

$$\text{NLH}(0.9,\ 0.05,\ 0.05),$$

which suggests that the treatment is highly effective (90% true), with very low uncertainty (5%) and a low chance of ineffectiveness (5%). Its negation is

$$\mathcal{N}(0.9,\ 0.05,\ 0.05) = (0.05,\ 0.95,\ 0.9).$$

If clinical trial results under specific conditions indicate that the treatment is largely ineffective, with values approaching (0.05, 0.95, 0.9), then the original hypothesis is partially falsified. This exemplifies the application of the negation operator in a medical context.

### 1.4. *Partial Falsifiability in Neutrosophic Logic*

We now extend the notion of falsifiability to neutrosophic hypotheses.

**Definition 1.14** (Partial Falsifiability in Neutrosophic Logic)**.** [36] A neutrosophic hypothesis $\text{NLH}(t, i, f)$ is said to be *partially falsifiable* if there exists a set of conditions $C$ (such as specific space-time conditions or experimental setups) and a tolerance $\epsilon > 0$ such that under at least one condition $c \in C$, the observed evaluation $O(c)$ of the hypothesis satisfies

$$d\big(O(c),\, \mathcal{N}(t, i, f)\big) < \epsilon,$$

where $d : [0, 1]^3 \times [0, 1]^3 \to \mathbb{R}_{\geq 0}$ is a suitable distance (or discrepancy) function (e.g., the Euclidean distance). In other words, the hypothesis is partially falsifiable if there exists empirical evidence under some conditions that nearly or fully supports the negated hypothesis $\mathcal{N}(t, i, f)$.

There are two common interpretations of partial falsifiability in neutrosophic logic:

(1) **Logical Interpretation:** A neutrosophic hypothesis $\text{NLH}(t, i, f)$ is partially falsifiable if there exists a condition under which its negation, $\mathcal{N}(t, i, f) = (f, 1-i, t)$, is observed or supported by evidence.

(2) **Probabilistic Interpretation:** Consider a neutrosophic probabilistic hypothesis NPH$(t, i, f)$, where $t, i, f$ represent the probabilities (or chances) that the hypothesis is true, indeterminate, or false, respectively. The hypothesis is partially falsifiable if there exists a nonzero probability that, under certain conditions, the observed probabilities approximate $\mathcal{N}(t, i, f) = (f, 1 - i, t)$.

### 1.5. *Theoretical Properties of Partial Falsifiability*

In this section, we present the theoretical properties of Partial Falsifiability.

**Theorem 1.15** (Involutiveness of the Negation Operator)**.** *For any neutrosophic hypothesis NLH$(t, i, f)$, the negation operator $\mathcal{N}$ is involutive; that is,*

$$\mathcal{N}\big(\mathcal{N}(t, i, f)\big) = (t, i, f).$$

*Proof.* By definition,

$$\mathcal{N}(t, i, f) = (f,\, 1 - i,\, t).$$

Applying $\mathcal{N}$ a second time, we obtain:

$$\mathcal{N}\big(f,\, 1 - i,\, t\big) = \big(t,\, 1 - (1 - i),\, f\big) = (t, i, f).$$

Thus, the operator $\mathcal{N}$ is involutive. □

**Theorem 1.16** (Symmetry of Partial Falsifiability)**.** *If a neutrosophic hypothesis NLH$(t, i, f)$ is partially falsifiable, then its negation $\mathcal{N}(t, i, f)$ is also partially falsifiable.*

*Proof.* Assume that NLH$(t, i, f)$ is partially falsifiable, meaning there exists a condition $c \in C$ and $\epsilon > 0$ such that

$$d\big(O(c), \mathcal{N}(t, i, f)\big) < \epsilon.$$

Since $\mathcal{N}$ is involutive, applying it twice gives the original hypothesis. Thus, under conditions where the negated hypothesis is observed, applying $\mathcal{N}$ again yields that the negation of $\mathcal{N}(t, i, f)$ (i.e., the original hypothesis) is also subject to a similar level of falsifiability. Hence, the property of partial falsifiability is symmetric. □

1.6. *Some Examples of Partial Falsifiability*

In this section, we present several examples of Partial Falsifiability.

**Example 1.17** (Environmental Hypothesis)**.** (cf. [7, 22, 29]) Consider a hypothesis about the health of an ecosystem represented by

$$\mathrm{NLH}(0.8,\ 0.1,\ 0.1),$$

indicating that the ecosystem is mostly healthy (with a truth degree of 0.8, indeterminacy 0.1, and falsehood 0.1). The negation of this hypothesis is

$$\mathcal{N}(0.8,\ 0.1,\ 0.1) = (0.1,\ 0.9,\ 0.8).$$

If, under certain environmental conditions (e.g., during a period of severe pollution), observations yield a neutrosophic evaluation close to (0.1, 0.9, 0.8) within a tolerance $\epsilon$, then the original hypothesis is partially falsifiable.

**Example 1.18** (Economic Forecast Hypothesis)**.** Let NLH(0.7, 0.2, 0.1) represent an economic hypothesis predicting robust growth. Its negation is

$$\mathcal{N}(0.7,\ 0.2,\ 0.1) = (0.1,\ 0.8,\ 0.7).$$

If, during an economic downturn, empirical data indicate that the economy is weak (i.e., the evaluation approximates (0.1, 0.8, 0.7)) with a discrepancy less than $\epsilon$, then the economic forecast hypothesis is partially falsifiable.

**Example 1.19** (Medical Treatment Hypothesis)**.** (cf. [2, 27]) Consider a hypothesis regarding the efficacy of a new medical treatment:

$$\mathrm{NLH}(0.9,\ 0.05,\ 0.05),$$

which indicates that the treatment is 90% effective, with only 5% uncertainty and 5% ineffectiveness. Its neutrosophic negation is

$$\mathcal{N}(0.9,\ 0.05,\ 0.05) = (0.05,\ 0.95,\ 0.9).$$

Suppose that in a clinical trial under particular adverse conditions, the treatment's observed performance is measured to be approximately (0.1, 0.9, 0.85) (i.e., low effectiveness with high falsity). Since these observations are close to the negated hypothesis, the original hypothesis is partially falsifiable.

**Example 1.20** (Weather Forecast Hypothesis)**.** (cf. [4, 31, 37]) Let a weather forecast be represented by the hypothesis:

$$\mathrm{NLH}(0.6,\ 0.2,\ 0.2),$$

meaning there is a 60% chance of clear weather, with 20% indeterminacy and 20% chance of rain. The negation is then

$$\mathcal{N}(0.6,\, 0.2,\, 0.2) = (0.2,\, 0.8,\, 0.6).$$

If, during a storm, meteorological observations yield an evaluation close to (0.2, 0.8, 0.6) within an acceptable error margin $\epsilon$, the original weather forecast hypothesis is partially falsifiable.

**Example 1.21** (Bridge Safety Hypothesis)**.** (cf. [9,42–44]) Consider an engineering hypothesis about the safety of a bridge:

$$\mathrm{NLH}(0.85,\, 0.1,\, 0.05),$$

which asserts that the bridge is highly safe (85% safe, with 10% uncertainty and 5% risk). Its negation is

$$\mathcal{N}(0.85,\, 0.1,\, 0.05) = (0.05,\, 0.9,\, 0.85).$$

If, following an unexpected event such as a minor earthquake, structural analysis data suggest that the bridge's safety is significantly compromised—yielding measurements close to (0.05, 0.9, 0.85)—then the hypothesis is partially falsifiable under those conditions.

## 2. **Water Logic and Neither Nor Logic**

In this section, we introduce two alternative logical frameworks designed to extend classical binary logic(cf. [8])..

### 2.1. *Water Logic*

*Water Logic* emphasizes the fluidity and lateral thinking inherent in creative problem solving, while *Neither Nor Logic* offers a three-valued approach that accommodates indeterminacy by rejecting strict dichotomies(cf. [8]).

**Definition 2.1** (Water Logic)**.** [11] Let $L = [0, 1]$ be the set of truth values. In Water Logic, we define two binary operations on $L$:

(1) *Flow Operation* ($\nabla$): For any $a, b \in L$,

$$a \nabla b = \min\{1,\, a + b\}.$$

This operation models the idea that truth values can "flow" together but are capped at 1.

(2) *Bypass Operation* ($\Delta$): For any $a, b \in L$,

$$a \Delta b = \max\{0,\, a + b - 1\}.$$

This operation represents the residual "gap" when the combined truth values do not reach full certainty.

A proposition $p$ in Water Logic is assigned a truth value $\mu(p) \in L$. The logical connectives are defined as follows:

- **Disjunction (OR):** $\mu(p \vee q) = \mu(p)\nabla\mu(q)$.
- **Conjunction (AND):** $\mu(p \wedge q) = \mu(p)\Delta\mu(q)$.

**Remark 2.2** (Correspondence with Fuzzy Logic)**.** It is important to note that Water Logic is closely related to *Fuzzy Logic* when the operators are defined via the standard **min** and **max** functions [45]. In Fuzzy Logic, the disjunction (union) and conjunction (intersection) are given by:

$$a \vee b = \min\{1,\, a+b\} \quad \text{and} \quad a \wedge b = \max\{0,\, a+b-1\},$$

respectively. Hence, the Flow Operation $\nabla$ (which may also be referred to as the *Floor Operator*) serves as the union operator, while the Bypass Operation $\Delta$ acts as the conjunction operator in Fuzzy Logic.

**Example 2.3** (Creative Problem Solving)**.** Suppose we have two propositions:

- $p$: "A creative solution exists," with $\mu(p) = 0.6$.
- $q$: "Alternative methods are viable," with $\mu(q) = 0.5$.

Then:

$$\mu(p \vee q) = 0.6\nabla 0.5 = \min\{1,\, 0.6+0.5\} = 1,$$

indicating a strong overall potential for a creative solution. Conversely,

$$\mu(p \wedge q) = 0.6\Delta 0.5 = \max\{0,\, 0.6+0.5-1\} = 0.1,$$

revealing that the conjunction of these propositions yields a low certainty when considering the residual gap.

**Theorem 2.4** (Associativity of the Flow Operation)**.** *For all $a, b, c \in L$, the flow operation $\nabla$ is associative:*

$$(a\nabla b)\nabla c = a\nabla(b\nabla c).$$

*Proof.* By definition, $a\nabla b = \min\{1,\, a+b\}$. Then:

$$(a\nabla b)\nabla c = \min\{1,\, \min\{1, a+b\}+c\} = \min\{1,\, a+b+c\},$$

and similarly,

$$a\nabla(b\nabla c) = \min\{1,\, a+\min\{1, b+c\}\} = \min\{1,\, a+b+c\}.$$

Thus, $(a\nabla b)\nabla c = a\nabla(b\nabla c)$. □

### 2.2. *Neither Nor Logic*

The definition of Neither-Nor Logic is provided below. Considering real-world applications, improvements have been made by incorporating an intermediate component to enhance its applicability (cf. [8]).

**Definition 2.5** (Neither-Nor Connective)**.** Let $A$ and $B$ be propositions with truth values $a, b \in [0, 1]$. The *neither–nor* connective, denoted by

$$N(a, b),$$

is defined by

$$N(a,b) = \begin{cases} \dfrac{(1-a)+(1-b)}{2}, & \text{if } (a,b) \neq (0.5, 0.5), \\ 0, & \text{if } a = b = 0.5. \end{cases}$$

**Definition 2.6** (Properties of Neither Nor Logic)**.** Next, the definition of Properties of Neither Nor Logic is provided below.

- **Distinctness from Operands:** For any proposition $A$ with truth value $a \neq 0.5$,

$$N(a,a) = \frac{(1-a)+(1-a)}{2} = 1 - a,$$

  ensuring that $N(a, a) \neq a$. For instance, if $a = 1$ (True) then $N(1, 1) = 0$ (False), and if $a = 0$ (False) then $N(0, 0) = 1$ (True).
- **Resolution of Indeterminacy:** When $a = b = 0.5$, we define

$$N(0.5, 0.5) = 0,$$

  so that "Neither Indeterminacy nor Indeterminacy" resolves to False rather than remaining indeterminate.
- **Mixed Cases:** For distinct truth values $a$ and $b$, $N(a, b)$ yields the arithmetic mean of their complements. For example:
  - If $a = 1$ (True) and $b = 0$ (False), then

$$N(1,0) = \frac{(1-1)+(1-0)}{2} = \frac{0+1}{2} = 0.5.$$

  - If $a = 1$ and $b = 0.5$, then

$$N(1,0.5) = \frac{(1-1)+(1-0.5)}{2} = \frac{0+0.5}{2} = 0.25.$$

  - If $a = 0$ and $b = 0.5$, then

$$N(0,0.5) = \frac{(1-0)+(1-0.5)}{2} = \frac{1+0.5}{2} = 0.75.$$

**Example 2.7** (Product Review Scenario)**.** Consider a situation where two experienced product reviewers evaluate a newly released smartphone. Reviewer A thoroughly tests the device—examining its hardware performance, software responsiveness, and overall design—and is completely convinced of its excellence. Therefore, Reviewer A assigns a truth value of

$$a = 1,$$

which represents a definitively positive evaluation. In contrast, Reviewer B notices some unresolved technical issues, such as occasional lag and inconsistent battery performance, which leave them uncertain about the product's overall quality. As a result, Reviewer B assigns a truth value of

$$b = 0.5,$$

indicating an indeterminate evaluation.

To combine these assessments using the Neither–Nor connective, we calculate:

$$N(1, 0.5) = \frac{(1-1) + (1-0.5)}{2}.$$

The calculation proceeds as follows:

(1) Compute the complement of $a$: $1 - 1 = 0$.
(2) Compute the complement of $b$: $1 - 0.5 = 0.5$.
(3) Sum the two complements: $0 + 0.5 = 0.5$.
(4) Divide the sum by 2: $\frac{0.5}{2} = 0.25$.

Thus, the final combined value is 0.25. This outcome means that even though one reviewer is completely positive, the uncertainty from the other reduces the overall evaluation to a moderately positive level. Consequently, potential buyers might be encouraged to consult additional reviews before making a purchase decision.

**Example 2.8** (Project Deadline Evaluation)**.** In this scenario, two project managers assess the likelihood of meeting a critical project deadline. Manager A, relying on detailed project planning and a strong past performance record, is fully confident that the deadline will be met and therefore assigns a truth value of

$$a = 1.$$

However, Manager B is concerned about possible unforeseen delays—such as supply chain issues or unexpected technical difficulties—and thus expresses uncertainty by assigning a truth value of

$$b = 0.5.$$

The combined evaluation using the Neither–Nor connective is computed as:

$$N(1, 0.5) = \frac{(1-1) + (1-0.5)}{2}.$$

Let us break down the calculation:

(1) Complement of Manager A's evaluation: $1 - 1 = 0$.
(2) Complement of Manager B's evaluation: $1 - 0.5 = 0.5$.
(3) Sum the two results: $0 + 0.5 = 0.5$.
(4) Divide by 2: $\frac{0.5}{2} = 0.25$.

The final value of 0.25 indicates that despite one manager's strong optimism, the overall assessment is moderated by uncertainty. This suggests that additional contingency planning may be necessary.

**Example 2.9** (Risk Mitigation Evaluation)**.** In a software development project, two risk assessors evaluate the effectiveness of a mitigation strategy for a potential technical issue. Assessor A, after reviewing test results and mitigation measures, is convinced that the risk has been completely addressed, and therefore assigns a truth value of

$$a = 1.$$

Conversely, Assessor B remains uncertain about the strategy's complete effectiveness—perhaps due to incomplete test coverage or potential hidden flaws—and assigns a truth value of

$$b = 0.5.$$

The Neither–Nor connective is applied as follows:

$$N(1, 0.5) = \frac{(1-1) + (1-0.5)}{2}.$$

Step-by-step, we compute:

(1) Compute the complement of $a$: $1 - 1 = 0$.
(2) Compute the complement of $b$: $1 - 0.5 = 0.5$.
(3) Add these values: $0 + 0.5 = 0.5$.
(4) Divide by 2: $\frac{0.5}{2} = 0.25$.

The result, 0.25, shows that the positive evaluation is significantly tempered by uncertainty, indicating that further analysis and monitoring of the risk mitigation strategy is advisable.

**Example 2.10** (Political Opinion Poll)**.** Imagine a political survey conducted to assess public opinion on a new government policy. One segment of the population expresses strong support for the policy, corresponding to a truth value of

$$a = 1.$$

Another segment, however, remains uncertain about the policy's implications and effectiveness, leading to an indeterminate evaluation with a truth value of

$$b = 0.5.$$

The combined evaluation is given by:

$$N(1, 0.5) = \frac{(1-1) + (1-0.5)}{2}.$$

Let us detail the calculation:

(1) Calculate the complement for the strong support: $1 - 1 = 0$.
(2) Calculate the complement for the uncertainty: $1 - 0.5 = 0.5$.
(3) Add the two results: $0 + 0.5 = 0.5$.
(4) Divide by 2: $\frac{0.5}{2} = 0.25$.

The final score of 0.25 indicates that although there is strong backing from one group, the uncertainty of the other dilutes the overall support. This result may prompt policymakers to undertake further research or more focused communication with the uncertain voters.

**Example 2.11** (Movie Review Scenario)**.** Consider a scenario where two professional film critics review a newly released movie. Critic A is highly dissatisfied with the movie—citing issues such as a weak storyline, poor acting, and subpar production quality—and assigns a truth value of

$$a = 0,$$

which indicates a definitively negative evaluation. In contrast, Critic B, while noting several problems, remains unsure about the overall quality due to some redeeming aspects and assigns a truth value of

$$b = 0.5.$$

The Neither–Nor connective is applied as follows:

$$N(0, 0.5) = \frac{(1-0) + (1-0.5)}{2}.$$

We calculate step-by-step:

(1) Compute the complement of Critic A's evaluation: $1 - 0 = 1$.
(2) Compute the complement of Critic B's evaluation: $1 - 0.5 = 0.5$.
(3) Sum the two results: $1 + 0.5 = 1.5$.
(4) Divide by 2: $\frac{1.5}{2} = 0.75$.

A final value of 0.75 implies that the overall evaluation is largely negative, suggesting that potential viewers might be cautious and seek further opinions before watching the movie.

**Example 2.12** (Customer Service Evaluation)**.** Imagine a scenario in which two customers provide feedback on their experience with a company's customer service. The first customer encountered several issues, such as long wait times and unhelpful responses, leading them to assign a definitively negative score with a truth value of

$$a = 0.$$

The second customer had a mixed experience, finding some aspects acceptable while remaining uncertain overall; hence, they assign a truth value of

$$b = 0.5.$$

To combine these evaluations using the Neither–Nor connective, we perform the calculation:

$$N(0, 0.5) = \frac{(1 - 0) + (1 - 0.5)}{2}.$$

The detailed steps are:

(1) Determine the complement of the first customer's score: $1 - 0 = 1$.
(2) Determine the complement of the second customer's score: $1 - 0.5 = 0.5$.
(3) Sum the complements: $1 + 0.5 = 1.5$.
(4) Divide by 2: $\frac{1.5}{2} = 0.75$.

The overall value of 0.75 indicates that the customer service is generally perceived as unsatisfactory. This outcome could prompt the company to investigate and improve its service quality.

**Example 2.13** (Budget Sufficiency Evaluation)**.** Consider a scenario in which two finance officers review the sufficiency of the allocated budget for a major project. Finance Officer A, after analyzing all projected expenses, is convinced that the current budget is insufficient and assigns a truth value of

$$a = 0.$$

Finance Officer B, however, is uncertain about the adequacy of the budget due to potential fluctuations in costs and unforeseen expenditures, and thus assigns a truth value of

$$b = 0.5.$$

The Neither–Nor connective is used to combine their evaluations:

$$N(0, 0.5) = \frac{(1 - 0) + (1 - 0.5)}{2}.$$

We detail the computation:

(1) Compute the complement of $a$: $1 - 0 = 1$.
(2) Compute the complement of $b$: $1 - 0.5 = 0.5$.
(3) Add the two results: $1 + 0.5 = 1.5$.
(4) Divide by 2: $\frac{1.5}{2} = 0.75$.

The result of 0.75 reflects a predominantly negative evaluation, indicating that the current budget is likely insufficient, and suggesting that additional funds or budget adjustments may be necessary.

**Example 2.14** (Quality Assurance Evaluation)**.** In a quality assurance review for a project deliverable, two team members evaluate whether the output meets the required quality standards. One team member, after a detailed inspection and testing, finds that the deliverable falls significantly short of the standards and assigns a truth value of

$$a = 0.$$

Meanwhile, another team member is uncertain about the quality of the deliverable due to mixed test results and ambiguous performance metrics, assigning a truth value of

$$b = 0.5.$$

The combined evaluation via the Neither–Nor connective is computed as:

$$N(0, 0.5) = \frac{(1 - 0) + (1 - 0.5)}{2}.$$

The step-by-step calculation is:

(1) Determine the complement of the negative evaluation: $1 - 0 = 1$.
(2) Determine the complement of the uncertain evaluation: $1 - 0.5 = 0.5$.
(3) Sum these values: $1 + 0.5 = 1.5$.
(4) Divide the sum by 2: $\frac{1.5}{2} = 0.75$.

A final value of 0.75 indicates that the overall quality evaluation is largely negative. This suggests that the deliverable does not meet the quality requirements and further improvements are necessary.

**Example 2.15** (Weather Forecast Ambiguity)**.** Imagine a local weather service that provides forecasts for tomorrow. Two independent forecasts are reported:

- Forecast $A$: "It will definitely rain tomorrow" (truth value $a = 1$).
- Forecast $B$: "It will definitely not rain tomorrow" (truth value $b = 0$).

Given the conflicting information, applying the Neither–Nor connective we compute:

$$N(1, 0) = \frac{(1 - 1) + (1 - 0)}{2} = \frac{0 + 1}{2} = 0.5.$$

The outcome 0.5 represents an indeterminate state, meaning that the overall forecast is ambiguous—it is neither certainly rainy nor certainly dry. This reflects the real-life scenario in which conflicting meteorological data leave the public uncertain, suggesting that one should prepare for a variety of weather conditions (e.g., perhaps carrying an umbrella just in case).

**Example 2.16** (Restaurant Dish Quality Evaluation)**.** Consider a scenario where two well-known food critics review a new dish at a restaurant:

- Critic $A$ gives the dish a perfect score, effectively saying it is excellent (truth value $a = 1$).

- Critic $B$ gives the dish the lowest score, asserting that it is unsatisfactory (truth value $b = 0$).

Using Neither Nor Logic, the overall evaluation is given by:

$$N(1,0) = \frac{(1-1)+(1-0)}{2} = \frac{0+1}{2} = 0.5.$$

Here, the result 0.5 indicates that the dish is rated as indeterminate—neither definitively good nor definitively bad. This captures the real-life impact of polarized opinions, where mixed reviews lead to an ambiguous overall rating. In practice, such an evaluation might prompt potential diners to seek additional opinions or sample the dish themselves before making a final decision.

The following are several theorems related to Neither-Nor Logic.

**Theorem 2.17** (Commutativity)**.** *For all $a, b \in [0,1]$, the Neither–Nor connective is commutative:*

$$N(a,b) = N(b,a).$$

*Proof.* If $(a,b) \neq (0.5, 0.5)$, then by definition we have

$$N(a,b) = \frac{(1-a)+(1-b)}{2}.$$

Since addition is commutative,

$$\frac{(1-a)+(1-b)}{2} = \frac{(1-b)+(1-a)}{2} = N(b,a).$$

In the special case $a = b = 0.5$, we have $N(0.5, 0.5) = 0$, and the equality holds trivially. □

**Theorem 2.18** (Non-Idempotence)**.** *For any $a \in [0,1]$ with $a \neq 0.5$,*

$$N(a,a) = 1 - a \quad \textit{and hence} \quad N(a,a) \neq a.$$

*Proof.* For $a \neq 0.5$, by substituting $a$ for both arguments we have

$$N(a,a) = \frac{(1-a)+(1-a)}{2} = \frac{2(1-a)}{2} = 1 - a.$$

Since $1 - a = a$ if and only if $a = 0.5$, it follows that for $a \neq 0.5$, $N(a,a) \neq a$. □

**Theorem 2.19** (Extreme Opposites Yield Indeterminacy)**.** *For $a = 1$ (True) and $b = 0$ (False) (or vice versa), the Neither–Nor connective yields:*

$$N(1,0) = 0.5.$$

*Proof.* Substitute $a = 1$ and $b = 0$ into the definition:

$$N(1,0) = \frac{(1-1)+(1-0)}{2} = \frac{0+1}{2} = 0.5.$$

By commutativity, the case $N(0,1)$ yields the same result. □

**Theorem 2.20** (Monotonicity)**.** *For fixed $b \in [0,1]$ and for any $a_1, a_2 \in [0,1]$ (with the pair $(a_i, b) \neq (0.5, 0.5)$ for $i = 1, 2$) such that $a_1 \leq a_2$, it holds that*

$$N(a_1, b) \geq N(a_2, b).$$

*Proof.* When $(a, b) \neq (0.5, 0.5)$, the connective is given by

$$N(a,b) = \frac{(1-a)+(1-b)}{2} = \frac{2-(a+b)}{2}.$$

For a fixed $b$, the expression $2 - (a + b)$ is a decreasing linear function in $a$. Hence, if $a_1 \leq a_2$ then

$$2 - (a_1 + b) \geq 2 - (a_2 + b).$$

Dividing by 2 (a positive constant) preserves the inequality, so

$$N(a_1, b) = \frac{2-(a_1+b)}{2} \geq \frac{2-(a_2+b)}{2} = N(a_2, b).$$

□

**Theorem 2.21** (Range)**.** *For all $a, b \in [0,1]$, the output of the Neither–Nor connective satisfies*

$$N(a,b) \in [0,1],$$

*and if $a, b$ are extreme values (i.e., $a, b \in \{0,1\}$), then $N(a,b)$ takes an intermediate value.*

*Proof.* For any $a, b \in [0,1]$, observe that both $1 - a$ and $1 - b$ lie in $[0,1]$. Hence, their sum lies in $[0,2]$ and dividing by 2 yields a number in $[0,1]$. In particular, if $a = 1$ and $b = 0$ (or vice versa), then

$$N(1,0) = \frac{0+1}{2} = 0.5.$$

Thus, extreme truth values produce the intermediate value 0.5. □

### 2.3. *Distinct Neither-Nor Logic*

Now, let us consider a different framework where we have a set $\{A, B, C\}$ of three propositions that are pairwise distinct. In this setting, the neither–nor connective $N$ is defined directly on the propositions as follows.

**Definition 2.22** (Distinct Neither-Nor Connective on $\{A, B, C\}$)**.** Let $A$, $B$, and $C$ be three distinct propositions. The *neither–nor* connective $N$ is defined by:

(1) For any two distinct propositions, the connective returns the third proposition:

$$N(A, B) = C, \quad N(B, C) = A, \quad N(C, A) = B.$$

(2) For identical inputs, the operation returns a weighted combination of the other two propositions. Let $\alpha, \beta \in (0, 1)$ be fixed parameters. Then,

$$N(A, A) = \alpha \cdot B + \beta \cdot C,$$

$$N(B, B) = \alpha \cdot A + \beta \cdot C,$$

$$N(C, C) = \alpha \cdot A + \beta \cdot B.$$

Here, "$\cdot$" denotes scalar multiplication and the $+$ operation represents a fuzzy (or probabilistic) sum, indicating that the result is a combination of the contributions from the other two propositions.

**Example 2.23** (Distinct Propositions)**.** Assume that $A$, $B$, and $C$ are distinct. Then, by definition,

$$N(A, B) = C, \quad N(B, C) = A, \quad N(C, A) = B.$$

**Example 2.24** (Diagonal Case with Weighted Combination)**.** Let $\alpha = 0.3$ and $\beta = 0.7$. Then, for proposition $A$, we have

$$N(A, A) = 0.3 \cdot B + 0.7 \cdot C.$$

This expression represents a combination in which $B$ contributes 30% and $C$ contributes 70%. Similarly,

$$N(B, B) = 0.3 \cdot A + 0.7 \cdot C, \quad N(C, C) = 0.3 \cdot A + 0.7 \cdot B.$$

**Example 2.25** (Dining Options)**.** Consider a scenario where a person must choose among three distinct dining options:

- $A$: An Italian restaurant known for its traditional pasta and pizza.
- $B$: A Japanese restaurant celebrated for its sushi and ramen.
- $C$: A Mexican restaurant famous for its vibrant flavors and festive ambiance.

According to the distinct neither–nor connective, if the person rules out both the Italian and Japanese options, then

$$N(A,B) = C,$$

which means that the choice is the Mexican restaurant.

Furthermore, suppose the person repeatedly leans toward choosing the Italian restaurant (i.e., considering $N(A,A)$) but seeks a compromise rather than a repetitive choice. Then, the logic dictates a weighted combination of the remaining options:

$$N(A,A) = \alpha \cdot B + \beta \cdot C,$$

for example, with $\alpha = 0.4$ and $\beta = 0.6$. This result can be interpreted as the individual opting for a dining experience that blends aspects of Japanese and Mexican cuisines—perhaps choosing a fusion restaurant that offers dishes inspired by both culinary traditions.

**Example 2.26** (Car Purchase Decision)**.** Imagine a car buyer evaluating three distinct models:

- $A$: A fuel-efficient hybrid car that emphasizes eco-friendly performance.
- $B$: A luxurious sedan that prioritizes comfort and high-end features.
- $C$: A rugged SUV designed for off-road capability and durability.

If the buyer eliminates both the hybrid and the sedan, then by the neither–nor connective we have

$$N(A,B) = C,$$

indicating that the SUV is the chosen option.

Alternatively, if the buyer consistently shows a preference for the hybrid (i.e., considering $N(A,A)$) but wishes to temper that preference, the decision might be refined to a compromise:

$$N(A,A) = \alpha \cdot B + \beta \cdot C,$$

for instance, with $\alpha = 0.5$ and $\beta = 0.5$. This balanced weighting suggests that the buyer may consider a crossover vehicle that combines eco-friendly features with elements of luxury and ruggedness from the sedan and SUV, respectively.

The following are several theorems related to Distinct Neither-Nor Logic.

**Theorem 2.27** (Commutativity)**.** *If $A$ and $B$ are two distinct propositions (with $A, B, C$ pairwise distinct), then*

$$N(A,B) = N(B,A) = C.$$

*Proof.* By definition, for two distinct propositions the connective returns the unique third proposition. Hence, by definition we have

$$N(A,B) = C \quad \text{and} \quad N(B,A) = C.$$

Thus, $N(A, B) = N(B, A)$, which proves commutativity in the case of distinct inputs. □

**Theorem 2.28** (Non-Idempotence)**.** *Let $A$, $B$, and $C$ be three pairwise distinct propositions, and let $\alpha, \beta \in (0, 1)$ with $\alpha + \beta = 1$. Then for any proposition $X \in \{A, B, C\}$,*

$$N(X, X) = \alpha \cdot Y + \beta \cdot Z \quad \text{with } \{X, Y, Z\} = \{A, B, C\},$$

*and in particular,*

$$N(X, X) \neq X.$$

*Proof.* Assume without loss of generality that $X = A$; then by definition,

$$N(A, A) = \alpha \cdot B + \beta \cdot C.$$

Since $A$, $B$, and $C$ are pairwise distinct and $\alpha, \beta > 0$, the weighted combination $\alpha \cdot B + \beta \cdot C$ cannot equal $A$ (assuming that the combination operation preserves distinctness). Hence, $N(A, A) \neq A$. The same reasoning applies to $B$ and $C$. □

**Theorem 2.29** (Reversibility)**.** *Let $A$, $B$, and $C$ be three pairwise distinct propositions such that*

$$N(A, B) = C.$$

*Then the connective is reversible in the sense that if one of the inputs and the result are known, the other input is uniquely determined. In particular, given $A$ and $C$, one can recover $B$ as the unique proposition distinct from both $A$ and $C$.*

*Proof.* By definition, the connective $N$ on distinct propositions is defined such that for any two distinct propositions $X$ and $Y$,

$$N(X, Y)$$

returns the unique third proposition $Z$ with $\{X, Y, Z\} = \{A, B, C\}$. Thus, if $N(A, B) = C$, then the only proposition that is neither $A$ nor $C$ is $B$. This uniqueness guarantees the reversibility: knowing $A$ and $C$ implies that the other input must be $B$. The same holds for any permutation of the three propositions. □

**Theorem 2.30** (Uniqueness of Diagonal Representation)**.** *Let $A$, $B$, and $C$ be three pairwise distinct propositions and let $\alpha, \beta \in (0, 1)$ with $\alpha + \beta = 1$. Then the representation of $N(A, A)$ as a weighted combination*

$$N(A, A) = \alpha \cdot B + \beta \cdot C$$

*is unique. In particular, no other convex combination of $B$ and $C$ equals $A$.*

*Proof.* By the definition of the connective for identical inputs, $N(A, A)$ is defined to be exactly $\alpha \cdot B + \beta \cdot C$. Suppose there exists another pair $(\alpha', \beta')$ with $\alpha', \beta' \in (0, 1)$ and $\alpha' + \beta' = 1$ such that

$$N(A, A) = \alpha' \cdot B + \beta' \cdot C.$$

Then, by the uniqueness of convex combinations in a space where $B$ and $C$ are distinct and the combination is non-degenerate, we must have $\alpha' = \alpha$ and $\beta' = \beta$. Moreover, since $A$ is distinct from both $B$ and $C$, no convex combination of $B$ and $C$ can equal $A$. Therefore, the weighted representation is unique and $N(A, A) \neq A$. □

## 2.4. *Water Neutrosophic Logic*

Water Neutrosophic Logic extends traditional Water Logic by applying fluid-like aggregation operations to neutrosophic evaluations.

**Definition 2.31** (Water Neutrosophic Logic)**.** Let a neutrosophic hypothesis be represented as

$$\mathrm{NLH}(t, i, f),$$

with $t, i, f \in [0, 1]$. Define two binary operations on neutrosophic values:

(1) **Neutrosophic Flow Operation** ($\nabla_N$):

$$\mathrm{NLH}(t_1, i_1, f_1)\nabla_N \mathrm{NLH}(t_2, i_2, f_2) = \mathrm{NLH}\Big(\min\{1,\ t_1 + t_2\},\ \max\{i_1, i_2\},\ \min\{1,\ f_1 + f_2\}\Big).$$

This operation models the idea that when combining information, the truth and falsehood values ”flow” together (added and capped at 1), while the overall indeterminacy is taken as the higher (worst-case) value.

(2) **Neutrosophic Bypass Operation** ($\Delta_N$):

$$\mathrm{NLH}(t_1, i_1, f_1)\Delta_N \mathrm{NLH}(t_2, i_2, f_2) = \mathrm{NLH}\Big(\max\{0,\ t_1 + t_2 - 1\},\ \min\{i_1, i_2\},\ \max\{0,\ f_1 + f_2 - 1\}\Big).$$

This operation captures the residual (or ”bypassed”) truth and falsehood values when the aggregated measure does not reach full certainty, while the indeterminacy decreases to reflect improved precision.

**Example 2.32** (Water Neutrosophic Logic: Decision-Making Scenario)**.** Decision-making is the process of selecting the best option among alternatives based on logic, analysis, and judgment (cf. [5, 6, 10, 16, 21]). Suppose two experts evaluate a project using neutrosophic assessments:

$$\mathrm{NLH}_1 = \mathrm{NLH}(0.6,\ 0.1,\ 0.2), \quad \mathrm{NLH}_2 = \mathrm{NLH}(0.5,\ 0.2,\ 0.3).$$

Applying the Neutrosophic Flow Operation:

$$\mathrm{NLH}_1 \nabla_N \mathrm{NLH}_2 = \mathrm{NLH}\Big(\min\{1,\ 0.6{+}0.5\},\ \max\{0.1, 0.2\},\ \min\{1,\ 0.2{+}0.3\}\Big) = \mathrm{NLH}(1,\ 0.2,\ 0.5).$$

This result reflects a combined evaluation where the project is fully considered effective in truth, but with significant residual falsehood.

**Theorem 2.33** (Associativity of Neutrosophic Flow)**.** *For all neutrosophic evaluations* $NLH(t_1, i_1, f_1)$, $NLH(t_2, i_2, f_2)$, *and* $NLH(t_3, i_3, f_3)$, *the flow operation* $\nabla_N$ *is associative:*

$$\Big(NLH(t_1, i_1, f_1) \nabla_N NLH(t_2, i_2, f_2)\Big) \nabla_N NLH(t_3, i_3, f_3) = NLH(t_1, i_1, f_1) \nabla_N \Big(NLH(t_2, i_2, f_2) \nabla_N NLH(t_3, i_3, f_3)\Big).$$

*Proof.* Using the definition,

$$\mathrm{NLH}(t_1, i_1, f_1) \nabla_N \mathrm{NLH}(t_2, i_2, f_2) = \mathrm{NLH}\Big(\min\{1, t_1 + t_2\},\ \max\{i_1, i_2\},\ \min\{1, f_1 + f_2\}\Big).$$

Then, applying the operation with $\mathrm{NLH}(t_3, i_3, f_3)$ on either side yields:

$$\min\{1, t_1 + t_2 + t_3\}, \quad \max\{i_1, i_2, i_3\}, \quad \min\{1, f_1 + f_2 + f_3\}.$$

Since addition is associative and the min and max functions are associative over finite sets, the operation is associative. □

**Notation 1** (Classical embedding)**.** *Define the* classical embedding $E : [0, 1] \to \{\mathrm{NLH}(t, i, f) : t, i, f \in [0, 1]\}$ *by*

$$E(a) = \mathrm{NLH}(a, 0, 1 - a),$$

*for any* $a \in [0, 1]$*. Here, the indeterminacy is set to* $0$ *and the falsehood is the complement* $1 - a$*. Under this embedding, a classical proposition with truth value* $a$ *is represented as* $E(a)$*.*

**Theorem 2.34** (Generalization of Water Logic)**.** *Let* $a, b \in [0, 1]$ *and let*

$$X = E(a) = \mathrm{NLH}(a, 0, 1 - a), \quad Y = E(b) = \mathrm{NLH}(b, 0, 1 - b).$$

*Then:*

(1) *The neutrosophic flow operation* $\nabla_N$ *reduces to the Water Logic flow operation on the truth values:*

$$X \nabla_N Y = \mathrm{NLH}\Big(\min\{1,\ a + b\},\ 0,\ 1 - \min\{1,\ a + b\}\Big).$$

(2) *The neutrosophic bypass operation* $\Delta_N$ *reduces to the Water Logic bypass operation on the truth values:*

$$X \Delta_N Y = \mathrm{NLH}\Big(\max\{0,\ a + b - 1\},\ 0,\ 1 - \max\{0,\ a + b - 1\}\Big).$$

*In this sense, Water Neutrosophic Logic is a generalization of Water Logic.*

*Proof.* Let $a, b \in [0,1]$ and consider the classical embeddings:

$$X = \mathrm{NLH}(a, 0, 1-a), \quad Y = \mathrm{NLH}(b, 0, 1-b).$$

**(i) Neutrosophic Flow Operation:** By definition,

$$X \nabla_N Y = \mathrm{NLH}\Big(\min\{1,\, a+b\},\, \max\{0, 0\},\, \min\{1, (1-a)+(1-b)\}\Big).$$

Since $\max\{0, 0\} = 0$ and

$$(1-a) + (1-b) = 2 - (a+b),$$

we have

$$\min\{1, 2-(a+b)\} = \begin{cases} 1, & \text{if } a+b \leq 1, \\ 2-(a+b), & \text{if } a+b > 1. \end{cases}$$

Notice that

$$1 - \min\{1, a+b\} = \begin{cases} 1-(a+b), & \text{if } a+b \leq 1, \\ 0, & \text{if } a+b > 1. \end{cases}$$

It is straightforward to check that in both cases

$$\min\{1, 2-(a+b)\} = 1 - \min\{1, a+b\}.$$

Thus,

$$X \nabla_N Y = \mathrm{NLH}\Big(\min\{1,\, a+b\},\, 0,\, 1 - \min\{1,\, a+b\}\Big).$$

This is exactly the classical Water Logic flow operation applied to $a$ and $b$, embedded back into the neutrosophic framework.

**(ii) Neutrosophic Bypass Operation:** Similarly, by definition,

$$X \Delta_N Y = \mathrm{NLH}\Big(\max\{0,\, a+b-1\},\, \min\{0, 0\},\, \max\{0, (1-a)+(1-b)-1\}\Big).$$

Since $\min\{0, 0\} = 0$, we simplify the falsehood component:

$$(1-a) + (1-b) - 1 = 1 - (a+b-1).$$

Thus,

$$\max\{0, 1-(a+b-1)\} = 1 - \min\{1,\, a+b-1\}.$$

But note that $\min\{1,\, a+b-1\} = a+b-1$ when $a+b-1 \geq 0$ (i.e. $a+b \geq 1$), and is $a+b-1 = 0$ when $a+b < 1$. Hence, the expression simplifies to:

$$\max\{0,\, a+b-1\}$$

for the truth component, and correspondingly the falsehood component becomes

$$1 - \max\{0,\, a+b-1\}.$$

Thus,

$$X \Delta_N Y = \mathrm{NLH}\Big(\max\{0,\, a+b-1\},\, 0,\, 1-\max\{0,\, a+b-1\}\Big).$$

This coincides with the Water Logic bypass operation on $a$ and $b$.

Therefore, under the classical embedding $E(a) = \mathrm{NLH}(a, 0, 1-a)$, the neutrosophic operations $\nabla_N$ and $\Delta_N$ reduce to the Water Logic operations $\nabla$ and $\Delta$ on the truth values. This completes the proof that Water Neutrosophic Logic generalizes Water Logic. □

## 3. Future Directions of This Research

This section outlines the future prospects of this research. In subsequent studies, we aim to investigate the properties, behavior, and practical applications of partial falsifiability when extended to the following advanced logical frameworks:

- **Bipolar Neutrosophic Set (Bipolar Neutrosophic Logic)**: Exploring how the concept of partial falsifiability applies in contexts where positive and negative components coexist within a bipolar neutrosophic framework [1, 13, 28, 40].
- **HyperNeutrosophic Set (HyperNeutrosophic Logic)**: Examining how higher-dimensional uncertainty representations affect the falsifiability of hypotheses under complex multi-layered uncertainty [15, 35].
- **SuperHyperNeutrosophic Set (SuperHyperNeutrosophic Logic)**: Investigating partial falsifiability in an even more generalized framework that incorporates multi-level hyperstructures in uncertainty modeling [15, 35].

By analyzing these extensions, we aim to further enhance the theoretical foundation and practical applicability of partial falsifiability in decision-making processes across various domains.

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### Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

### Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

### Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

### Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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