NSS Neutrosophic Sets and Systems, Vol. 83, 2025

University of New Mexico

A CHARACTERIZATION OF NEUTROSOPHIC σ BAIRE SPACE ON FUZZY SETTING

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Abstract. In this paper we investigate several characterization of F. neu. σ-baire spaces and study conditions under which a fy. neu. top. space we becomes a F. neu. σ-B.S. and also discussed F. neu. almost resolvable space, F. neu. hyperconnected space, F. neu. submaximal space.

Keywords: F. neu.F_σ-set, F. neu. G_δ-set, F. neu. σ-first and second category set, F. neu. nowhere dense set, fuzzy dense set, F. neu. σ-nwd set, F. neu. σ-first category space, F. neu. σ- second category space, F. neu. σ-B.S, F. neu. almost resolvable space , F. neu. hyperconnected , F. neu. submaximal space.)

1. **Introduction**

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [10]. The important concept of fuzzy topological space was offered by C.L. Chang [2]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [8]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [9] introduced Fy. neutrosophic topological spaces. The idea of fy. neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [3].

In this paper, we discussed and established several characterization of neutrosophic σ-baire space on fuzzy setting. Also, we introduce to study the conditions under witch a neu. top. space becomes a neu. σ-B.S on fuzzy setting.

2. Preliminaries

Definition 2.1. [1] A fy. neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \le T_A(x) + I_A(x), F_A(x) \le 3$. With the condition $0 \le T_{A^*}(x) + I_{A^*}(x), F_{A^*}(x) \le 2$

Definition 2.2. [1] A fy. neutrosophic set A is a subset of a fy. neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \le T_B(x),\ I_A(x) \le I_B(x),\ F_A(x) \ge F_B(x)$

Definition 2.3. [1] Let X be a non-empty set, and
$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fy. neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [1] The difference between two fy. neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [1] A fy. neutrosophic set A over the universe X is said to be null or empty fy. neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.6. [1] A fy. neutrosophic set A over the universe X is said to be absolute (universe) fy. neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.7. [1] The complement of a fy. neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fy. neutrosophic set A an also be defined as $A^C = 1_N - A$.

Definition 2.8. [1] A fy. neutrosophic topology on a non-empty set X is a τ of fy. neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fy. neutrosophic top. space and any fy. neutrosophic set in τ is known as fy. neutrosophic open set in X

Definition 2.9. [1] The complement A^C of a fy. neutrosophic set A in a fy. neutrosophic top. space (X, τ) is called fy. neutrosophic closed set in X.

Definition 2.10. [1] Let (X, τ) be a fy. neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fy. neutrosophic set in X. Then the closure and Interior of A are defined by

$I.(A) = \cup\{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ inX\ and\ G \subseteq A\}$

$C.(A) = \cap\{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ inX\ and\ A \subseteq G\}$

Definition 2.11. [1] Let (X, τ) be a fy. neutrosophic topological space over X. Then (i) $C.(A^C) = (I.\ A)^C$, (ii) $I.(A^C) = (cl\ A)^C$

Definition 2.12. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS$ B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15. [3] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS$ B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is $fn(((A_N)^-)^+) = 0_N$

Definition 2.16. [3] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17. [3] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

3. A Characterization of Fuzzy Neutrosophic σ Baire Space on Fuzzy Setting

Definition 3.1. Let (X_N, T_N) be a Fy. neutrosophic topological space then (X_N, T_N) is called a F. neutrosophic σ-Baire space if $I.(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$, where λ_{N_i}'s are F. neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 3.2. *[5] Let (X_N, TN) be a Fy. neutrosophic topological space. Then the following are equivalent*

(1) (X_N, T_N) *is a F. neutrosophic σ-Baire space.*

(2) *$I.(\lambda_N) = o$ for every F. neutrosophic σ- first category set λ_N in (X_N, T_N).*

(3) *$C.(\mu_N) = 1$ for every F. neutrosophic σ-residual set μ_N in (X_N, T_N)*

Theorem 3.3. *[5] In a Fy. neutrosophic topological space (X_N, T_N) a fy. neutrosophic set λ_N is F. neutrosophic σ-nowhere dense in (X_N, T_N) if and only if $1 - \lambda_N$ F. neutrosophic dense and F. neutrosophic G_δ-set in (X_N, T_N).*

Theorem 3.4. *[3] If λ_N is a F. neu. dense and F. neu. G_δ-set in a fy. neu. top. space (X_N, T_N) then $1 - \lambda_N$ is a F. neu. first category set in (X_N, T_N).*

Proposition 3.5. *If the fuzzy neu. top. space (X_N, T_N) is a F. neu. σ-Baire space, then $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$, where the fuzzy sets (λ_{N_i})'s ($i = 1$ to ∞) are F. neu. dense set and F. neu. G_δ-set in (X_N, T_N).*

Proof. Let (λ_{N_i})'s ($i = 1$ *to* ∞) be a F. neu. dense set. and F. neu. G_δ-set in (X_N, T_N) By theorem 3.3 $(1 - \lambda_{N_i})$'s are fuzzy neu. σ-nwd sets in (X_N, T_N). Then the fuzzy neu. set $\lambda_N = \vee_{i=1}^{\infty}(1 - \lambda_{N_i})$ is a F. neu. σ-first catogory set in (X_N, T_N). Now $I.(\lambda_N) = I.(\vee_{i=1}^{\infty}(1 - \lambda_{N_i})) = I.(1 - [\wedge_{i=1}^{\infty}(\lambda_{N_i})]) = 1 - C.(1 - [\wedge_{i=1}^{\infty}(\lambda_{N_i})])$. Since (X_N, T_N) is a F. neu. σ-baire space, by theorem 3.1, we have $I.(\lambda_N) = 0$. Then $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$ □

Proposition 3.6. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ-B.S, then $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$, where the F. neu. sets $(1 - \lambda_{N_i})$'s ($i = 1$ to ∞) are fuzzy first category sets formed from the fuzzy neu. dense and F. neu. G_δ-set λ_{N_i} in (X_N, T_N).*

Proof. Let the fy. neu. top. space (X_N, T_N) is a F. neu. σ-B.S and the F. neu. sets (λ_{N_i})'s ($i = 1$ *to* ∞) be F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). By prop. 3.5, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$. Then $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$. Also by theorem 3.4, $(1 - \lambda_{N_i})$'s are F. neu. first category sets in (X_N, T_N). Hence $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$, where the fuzzy neu. sets $(1 - \lambda_{N_i})$'s ($i = 1$ *to* ∞) are fuzzy neu. first category sets formed from the F. neu. dense and F. neu. G_δ-set λ_{N_i} in (X_N, T_N). □

Theorem 3.7. *[3] Let (X_N, T_N) be a fy. neu. top. space. Then the following are equivalent.*

(1) *(X_N, T_N) is a F. neu. B.S.*

(2) *$f_n(A_N)^+ = 0_N$ for every F. neu. first category set A_N in (X_N, T_N).*

(3) *$f_n(A_N)^- = 1$, for every F. neu. residual set B_N in (X_N, T_N).*

Proposition 3.8. *If the F. neu. first category sets are formed from the F. neu. dense and F. neu. G_δ-sets in a F. neu. σ-B.S (X_N, T_N), then (X_N, T_N). is a F. neu. B.S.*

Proof. Let the fy. neu. top. space (X_N, T_N) be a fuzzy neu. σ-B.S. By prop.3.6, $I.[\vee_{i=1}^{\infty}(1-\lambda_{N_i})] = 0$, where the F. neu. sets $(1-\lambda_{N_i})$'s $(i = 1 \; to \; \infty)$ are F. neu. first category sets formed from the F. neu. dense and and F. neu. G_δ-sets λ_{N_i} in (X_N, T_N). Now $\vee_{i=1}^{\infty}[I.(1-\lambda_{N_i})] \leq [I.(\vee_{i=1}^{\infty}(1-\lambda_{N_i}))]$. Then we have $\vee_{i=1}^{\infty}[I.(1-\lambda_{N_i})] = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$, where $1-\lambda_{N_i}$ is a fuzzy neu. first category set in (X_N, T_N). By theorem 3.7, (X_N, T_N) is a fu.neu. B.S. □

Theorem 3.9. *[5] A fy. neu. top. space (X_N, T_N) is called F. neu. σ-first category if the F. neu. set 1_{X_N} is a F. neu. σ- first category space in (X_N, T_N). That is $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ Where λ_{N_i}'s are F. neu. σ- nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a F. neu. σ- second category space.*

Proposition 3.10. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ-first category space, then (X_N, T_N) is not a F. neu. σ-Baire space.*

Proof. Let the fy. neu. top. space (X_N, T_N) is a F. neu. σ-first category space. Then $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_{X_N}$, where λ_{N_i}'s are F. neu. σ-nwd sets in (X_N, T_N). Now $I. \vee_{i=1}^{\infty} (\lambda_{N_i}) = I.(1_{X_N}) = 1 \neq 0$. Hence, (X_N, T_N) is not a F. neu.σ-B.S. □

Proposition 3.11. *If $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$, where the F. neu. sets (λ_{N_i})'s are F. neu. dense and F. neu. G_δ-set in a fy. neu. top. space (X_N, T_N), then (X_N, T_N) is a F. neu. σ-second category space.*

Proof. Let (λ_{N_i})'s $(i = 1 \; to \; \infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). By theorem 3.3, $(1-\lambda_{N_i})$'s are F. neu. σ-nwd sets in (X_N, T_N). Now $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$ implies that $1-\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 1$. Then $\vee_{i=1}^{\infty}(1-\lambda_{N_i}) \neq 1$. Hence (X_N, T_N) is not a F. neu. σ-first category space and therefore (X_N, T_N) is a F. neu. σ-second category space. □

Proposition 3.12. *If λ_N is a F. neu. σ-first category set in (X_N, T_N), then there is a F. neu. F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$*

Proof. Let λ_N is a F. neu. σ-first category set in (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ where (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). Now $[1 - C.(\lambda_{N_i})]$'s are F. neu. open sets in (X_N, T_N). Then $\mu_N = \wedge_{i=1}^{\infty}[1 - C.(\lambda_{N_i}]$ is a F. neu. G_δ-set in (X_N, T_N) and $1-\mu_N = 1-[\wedge_{i=1}^{\infty}(1-C.(\lambda_{N_i}))] = \vee_{i=1}^{\infty}(C.(\lambda_{N_i}))$. Now $\lambda = \vee_{i=1}^{\infty}(\lambda_{N_i}) \leq \vee_{i=1}^{\infty}(C.(\lambda_{N_i})) = 1-\mu_N$. That is, $\lambda_N \leq 1-\mu_N$ and $1-\mu_N$ is a F. neu. F_σ-set in (X_N, T_N). Let $\delta_N = 1-\mu_N$. Hence, if λ_N is a F. neu. σ-first category set in (X_N, T_N), then there is a F. neu. F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$. □

Proposition 3.13. *If σ_N is a F. neu. σ-residual set in a fy. neu. top. space (X_N, T_N) such that $\eta_N \leq \delta_N$ where η_N is a F. neu. dense and F. neu. G_δ-set in (X_N, T_N), then (X_N, T_N) is a F. neu. σ-B.S.*

Proof. Let σ_N be a F. neu. σ-residual set in a fy. neu. top. space (X_N, T_N). Then $1 - \delta_N$ is a F. neu. σ-first category set in (X_N, T_N). Now by prop.3.12, their is a F. neu. F_σ-set μ_N in (X_N, T_N) such that $1 - \delta_N \leq \mu_N$. This implies that $1 - \mu_N \leq \sigma_N$. Let $\eta_N = 1 - \mu_N$. Then η_N is a F. neu. G_δ-set in (X_N, T_N) and $\eta_N \leq \delta_N$ implies that $C.(\eta_N) \leq C.(\delta_N)$. If $C.(\eta_N) = 1$, then we have $C.(\delta_N) = 1$. Hence, by theorem3.2, (X_N, T_N) is a F. neu. σ-B.S. □

Proposition 3.14. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ B.S and if $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1$, then there exists atleast one F. neu. F_σ-set λ_{N_i} such that $I.(\lambda_{N_i}) \neq 0$*

Proof. Suppose that $I.(\lambda_{N_i}) = 0$, for $i = 1to\infty$, where (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). Then $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1$. implies that $I.[\vee_{i=1}^{\infty}(\lambda_{N_i})] = I.[1] = 1 \neq 0$, a contradiction to (X_N, T_N) being a F. neu. σ-B.S. Hence $I.(\lambda_{N_i}) \neq 0$, fort atleast one F. neu. F_σ-set λ_{N_i} in (X_N, T_N). □

Proposition 3.15. *If the fy. neu. top. space (X_N, T_N) is F. neu. σ-B.S, then no non-zero F. neu. open set is a F. neu. σ-first category set in (X_N, T_N).*

Proof. Let λ_N be a non zero F. neu. open set in a F. neu. σ-B.S (X_N, T_N). Suppose that $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where the fuzzy neu. sets (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). then $I.(\lambda_{N_i}) = I.(\vee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a F. neu. σ-B.S, $I.(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0$. This implies that $I.(\lambda_N) = 0$. Then we will have $\lambda_N = 0$, which is a contradiction. Since $\lambda_N \in T_N$ implies that $I.(\lambda_N) = \lambda_N \neq 0$. Hence no non-zero F. neu. open set is a F. neu. σ-first category set in (X_N, T_N). □

Theorem 3.16. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu. submaximal space if for each F. neu. set λ_N in (X_N, T_N) such that $C.(\lambda_N) = 1$ then $C.(\lambda_N \in T_N)$ in (X_N, T_N).*

Proposition 3.17. *If the fy. neu. top. space (X_N, T_N) is a F. neu. submaximal space and if λ_N is a F. neu. σ-first category set in (X_N, T_N), then λ_N is a F. neu. first category set in (X_N, T_N)*

Proof. Let $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ be a F. neu. σ-first category set in (X_N, T_N), where the fuzzy sets (λ_{N_i})'s $(i = 1to\infty)$ are F. neu. σ-nwd sets in (X_N, T_N). Then we have $I.(\lambda_{N_i}) = 0$ and (λ_{N_i})'s $(i = 1to\infty)$ are F. neu. F_σ-sets in (X_N, T_N). Now $I.(\lambda_{N_i}) = 0$, implies that $1 - I.(\lambda_N) = 1 - 0 = 1$ and hence $C.(1 - \lambda_N) = 1$. Since (X_N, T_N) is a F. neu. submaximal space, the F. neu. dence sets $(1-\lambda_{N_i})$'s are F. neu. open sets in (X_N, T_N) and hence λ_{N_i}'s are F. neu. closed sets in (X_N, T_N). Then $C.(\lambda_{N_i}) = \lambda_{N_i}$ and $I.(\lambda_{N_i}) = 0$ implies that $I.C.(\lambda_{N_i}) = I.(\lambda_{N_i}) = 0$. That is (λ_{N_i})'s are F. neu. nwd sets (X_N, T_N). Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda_{N_i})$ is a F. neu. first category set in (X_N, T_N). □

Proposition 3.18. *If the F. neu. top space (X_N, T_N) is a F. neu. σ-B.S and F. neu. submaximal space, then (X_N, T_N) is a F. neu. B.S.*

Proof. Let μ_N be a F. neu. σ-first category set in (X_N, T_N). Since (X_N, T_N) is a F. neu. submaximal space, by prop3.17, μ_N is a F. neu.first category set in (X_N, T_N). Since (X_N, T_N) is a F. neu. σ-B.S, by them3.2 $I.(\mu_N) = 0$. Hence, for the F. neu. first category set μ_N in (X_N, T_N), we have $I.(\mu_N) = 0$. Therefore, by thm.3.7, (X_N, T_N) is a F. neu. B.S. □

Theorem 3.19. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu.P-space if countable intersection of F. neu. open sets in (X_N, T_N) is F. neu. open. That is, every non-zero F. neu. G_δ-set in (X_N, T_N) is F. neu. open in (X_N, T_N).*

Proposition 3.20. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ-B.S and F. neu. P-space, then (X_N, T_N) is a F. neu. B.S.*

Proof. Let the fy. neu. top. space (X_N, T_N) be a F. neu. σ-B.S. Then, by prop.3.5, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$, where the F. neu. sets (λ_{N_i})'s $(i = 1 to \infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). Now from, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$, we have $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $I.[\vee_{i=1}^{\infty}(1-\lambda_{N_i})] = 0$. Since the F. neu. sets $(\lambda_{N_i}))$'s are F. neu. dense in (X_N, T_N), $C.(\lambda_{N_i}) = 1$. Then we have $1 - C.(\lambda_{N_i}) = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$. Also, since (X_N, T_N) is a F. neu. P-space, the non-zero F. neu. G_δ-sets (λ_{N_i})'s in (X_N, T_N), are F. neu. open in (X_N, T_N). Then $(1-\lambda_{N_i})$'s are F. neu. closed sets in (X_N, T_N). Then $C.(1-\lambda_{N_i}) = 1-\lambda_{N_i}$ and $I.(1-\lambda_{N_i}) = 0$ implies that $I.C.(1-\lambda_{N_i}) = I.(1-\lambda_{N_i}) = 0$. That is $(1-\lambda_{N_i})$'s are F. neu. nwd sets in (X_N, T_N). Hence, by thm.3.7, (X_N, T_N) is a F. neu. B.S. □

Theorem 3.21. *[4] A F. neu. top space (X_N, T_N) is called a F. neu. almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_{N_i})] = 0$, where the F. neu. sets (λ_{N_i})'s in (X_N, T_N) are such that $I.(\lambda_{N_i}) = 0$. Otherwise (X_N, T_N) is called a fuzzy neu. almost irresolvable space.*

Proposition 3.22. *If the fy. neu. top. space (X_N, T_N) is a F. neu. almost resolvable space, then (X_N, T_N) is a F. neu. σ-second category space.*

Proof. Let (λ_{N_i})'s $(i = 1 to \infty)$ be the F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). Now $C.(\lambda_{N_i}) = 1$ implies that $1 - C.(\lambda_{N_i}) = 0$. That is, $I.(1 - \lambda_{N_i}) = 0$. Since (X_N, T_N) is a F. neu. almost irresolvable space, $\vee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 1$, where the F. neu. sets (λ_{N_i})'s in (X_N, T_N) are such that $I.(1 - \lambda_{N_i}) = 0$. Now $I. \vee_{i=1}^{\infty} (1 - \lambda_{N_i}) \neq 1$ implies that $1 - \vee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 0$. Hence we have $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$, where the F. neu. sets (λ_{N_i})'s are F. neu. dense and F. neu. G_δ-sets in a fy. neu. top. space (X_N, T_N). Thus, by prop.3.11 (X_N, T_N) is a F. neu. σ-second category space. □

Theorem 3.23. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu. hyperconnected space if every F. neu. open set λ_N is F. neu. dense in (X_N, T_N). That is, $C.(\lambda_N) = 1$ for all $0 \neq \lambda_N \in T_N$*

Theorem 3.24. *If $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$, where λ_{N_i}'s are F. neu. dense and F. neu. G_σ-sets in (X_N, T_N). Then (X_N, T_N) is a F. neu. σ-B.S.*

Proposition 3.25. *If $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$, where the F. neu. sets (λ_{N_i})'s are F. neu. G_δ-sets in a F. neu. hyperconnected and F. neu. P-space (X_N, T_N), then (X_N, T_N) is a F. neu. σ-B.S.*

Proof. Let(λ_{N_i})'s $(i = 1 to \infty)$ be the F. neu. G_δ-sets in (X_N, T_N) such that $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$. Since (X_N, T_N) is a F. neu. P-space, the F. neu. G_δ-sets (λ_{N_i})'s in (X_N, T_N), are F. neu. open in (X_N, T_N). Also since (X_N, T_N) is a F. neu. hyperconnected space, the F. neu. open sets (λ_{N_i})'s in (X_N, T_N), are F. neu. dense sets in (X_N, T_N). Hence the F. neu. sets (λ_{N_i})'s $(i = 1 \ to \ \infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N) and $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$. Hence by thm3.24 (X_N, T_N) is a F. neu. σ-B.S. □

4. Conclusion

In this paper several characterization of F. neutrosophic σ-Baire space and other F. neutrosophic spaces are discussed and established.

References

1. Arockiarani I., J.Martina Jency., More on Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Topological Spaces., International Journal of Innovative Research & Studies, Vol 3(5),2014.,643-652.
2. Chang C.L, Fy. Top. Spaces, J.Math.Ana.Appl.24, 1968., 182-190.
3. Poongothai E, Padmavathi E., Baire Spaces on Fuzzy Neutrosophic Topological Spaces., NSS., Vol 51, 2022., 708-723.
4. Poongothai E, Padmavathi E., On Fuzzy Neutrosophic Directed Baire Spaces and Characterization of other fy. neutrosophic space, Mukt Shabd Journal, Vol 12(12), 2023., 570-586.
5. Poongothai E, Kalaivani E, Neutrosophic σ-Baire Spaces on Fuzzy Setting , Journal of Xidian University, page No.253-264.

6. Salama A.A., and S.A.Alblowi, Neutrosophic Sets and Neutrosophic Topological Spaces, IOSR Journal of Mathematics, Vol 3(4), 2012.
7. Smarandache F., Neutrosophic Set, A Generalization of the Intuitionistic Fuzzy Sets, Inter.J.Pure Appl.Math., Vol.24, 2005., 287-297.
8. Thangaraj G. and E. Poongothai, On Fuzzy σ-Baire Spaces, J. Fuzzy Math. And Systems, Vol 3(4), 2013, 275-283.
9. Veereswari V., An Introduction to Fy. Neutrosophic Top. spaces, IJMA, Vol.8(3), 2017., 144-149.
10. Zadeh L.A., Fy. sets, Information and control, Vol.8, 1965, 338-353.

Received: Nov. 6, 2024. Accepted: April 1, 2025