NSS

Neutrosophic Sets and Systems, Vol. 83, 2025

University of New Mexico

Decomposition of Neutrosophic Zero-divisor graph

Balakrishnan A[1], Kanchana M[2], Said Broumi[3,4], Thirugnanasambandam K*,

[1]Muthurangam Government Arts College(Autonomous), Vellore, India;balajayamaha@gmail.com

[2]VIT, Vellore, India; mkanchana378@gmail.com

3Department of Mathematics SIMATS Engineering ,Saveetha Institute of Medical and Technical Sciences Thandalam, Chennai, Tamilnadu India. broumisaid78@gmail.com

4Laboratory of Information Processing, Faculty of Science Ben MSik, University of Hassan II, Casablanca, Morocco

*Correspondence:Muthurangam Government Arts College(Autonomous), Vellore, India.
kthirugnanasambandam@gmail.com;

Abstract. Let $\langle\mathscr{R}\cup\mathscr{I}\rangle$ be a commutative ring and let $\overline{\Gamma}(\hat{Z}_n)$ be the neutrosophic zero-divisor graph of R, where the vertex set of $\hat{Z}_n$ are non-zero zero divisors with $(\mathscr{T},\mathscr{I},\mathscr{F})$ truth, indeterminacy, and falsity membership functions such that the two vertices u, v are adjacent if n divides uv. In this article, we introduce decomposition of the neutrosophic zero-divisor graph of a commutative ring and also discuss some special neutrosophic zero-divisor graphs of $\overline{\Gamma}(\hat{Z}_n)$ where n is a prime number, such as $\overline{\Gamma}(\hat{Z}_{2^2p^2})$, $\overline{\Gamma}(\hat{Z}_{3^2p^2})$, $\overline{\Gamma}(\hat{Z}_{5^2p^2})$, and $\overline{\Gamma}(\hat{Z}_{p^2q^2})$.

Keywords: Neutrosophic Zero divisor graph; decomposition; complete neutrosophic bipartite graph; Neutrosophic cycle.

AMS Subject Classification: 05C75, 05C76, 05C38, 03E72, 03B52.

---

1. **Introduction**

To reveal the inherent algebraic structures and characteristics within ring, the zero divisor graph proves as a valuable instrument. Beck introduced the concept of zero divisor graph for a commutative ring R and constructing graph for it containing vertex set and edge set [1]. Selvi et al., discussed cyclic path that covers in zero divisor graphs through theorems [2]. Kuppan et al., discussed the zero divisor concept in a commutative ring in the field of fuzzy [3]. We extent, the zero divisor graph concept in the field of neutrosophic graph by using the following references. Smarandache presented the logic of neutrosophic sets for the basic understanding [4]. Jun et al., presented the algebraic structures which includes neutrosophic zero divisors with examples [5]. Smarandache et al., presented a editorial volume contains algebraic structures like, ring, commutative ring, modules with their application [6]. Kandasamy et al., gave the basics definitions and terminologies for fuzzy and neutrosophic graphs in uncertainty through modelling graph structures [7]. Vasantha Kandasamy et al., gave a new dimension to graph theory as a neutrosophic graph theory through basic definition of neutrosophic graph with examples [8]. Vasantha Kandasamy et al., presented a book containing the basic concepts of neutrosophic ring with example and theorem [9]. Rozina ali presented a review related on the study of neutrosophic groups and their generalizations [10]. Ali et al., discussed the fuzzy zero divisor graph in multiset dimension associated with commutative ring [11]. Panda et al., gave the overview of neutrosophic sets in graph theory to understanding the concept of neutrosophic set logic [12]. [13] refers tothe structure and properties of L A $-\Gamma-$ semigroups and its generalization by incorporating $\Gamma-$ semigroup theory. the models established algebraic operations and homomorphism properties implementing computational applications. [14] resolves the starphene structure and graphene structure through resolvability theory, the mathemaical models and its applications in electronics were discussed. [16] introduced neutrosophic zero divisor rings by incorporating neutrosophic logic with rign by addressing uncertainty in algebraic structures, the basic definitions and properties were discussed with neutrosophic mathematics. [15] represents the hyper ring framework by extending the zero divisor by addressing new structural properties. The results are given for general classical non-commutative and hyperstructures. [17] discussed the automorphism of connectivity, diameter and domination properties in algebraic graph theory by zero divisor graph of a ring. [18] re-evaluated the neutrosophic graphs includes zero divisor graph, layered graphs and weak graphs with its application in artificial intelliegence, decision making, chemistry. These concepts and ideas motivated to extend the neutrosophic zero divisor graph for decomposition of zero divisor graph into cycle, star and bipartite for specifically prime powers. So, initially the article discuses for prime numbers 2, 3, 5, 7. Also, the evaluation of its examples were taken into MATLAB to visualize the graph of it to apply future scope. In this article, we introduce the concept of neutrosophic zero divisor graph for decomposing into neutrosophic cycles and complete neutrosophic bipartite graph through theorem with examples.

1.1. *Preliminaries*

The following definitions [1.1 – 1.7] we refer[[4], [9], [10], [12]] are used to formulate the theorems in section 3.

Definition 1.1. A graph $\overline{\mathscr{G}}(\mathscr{V}, \mathscr{E}, \mathscr{T}, \mathscr{I}, \mathscr{F})$ is said to be a neutrosophic graph if:

(1) $\mathscr{T} : \mathscr{V} \to [0,1]$, $\mathscr{I} : \mathscr{V} \to [0,1]$, and $\mathscr{F} : \mathscr{V} \to [0,1]$ such that $0 \leq \mathscr{T}_v + \mathscr{I}_v + \mathscr{F}_v \leq 3 \quad \forall v \in \mathscr{V}$.

(2) For any edge $e = (u, v) \in \mathscr{E}$, $\mathscr{T} : \mathscr{E} \to [0, 1]$, $\mathscr{I} : \mathscr{E} \to [0, 1]$, and $\mathscr{F} : \mathscr{E} \to [0, 1]$ such that $0 \leq \mathscr{T}_e + \mathscr{I}_e + \mathscr{F}_e \leq 3$.

Definition 1.2. Let $\mathscr{R}$ be any ring. Then, $\langle \mathscr{R} \cup \mathscr{I} \rangle$ is a neutrosophic ring generated by $\mathscr{R}$ and $\mathscr{I}$ under the operations of $\mathscr{R}$.

Example 1.1: Consider $\mathscr{R}$ as the ring of real numbers. Then, $\langle \mathscr{R} \cup \mathscr{I} \rangle$ is the neutrosophic ring of real numbers.

Definition 1.3. Consider $\langle \mathscr{R} \cup \mathscr{I} \rangle$ as a neutrosophic ring. If $\forall x, y \in \langle \mathscr{R} \cup \mathscr{I} \rangle$, $xy = yx$, then $\langle \mathscr{R} \cup \mathscr{I} \rangle$ is a commutative neutrosophic ring.

If $xy \neq yx$, then $\langle \mathscr{R} \cup \mathscr{I} \rangle$ is a *non-commutative neutrosophic ring.*

Definition 1.4. Consider $\langle \mathscr{R} \cup \mathscr{I} \rangle$ as a neutrosophic ring. An element $x \in \langle \mathscr{R} \cup \mathscr{I} \rangle$, where $x = c + d\mathscr{I}$ and $c \neq d$ or $-d$, is said to be a neutrosophic zero divisor if there exists $y = a + b\mathscr{I}$ with $a \neq b$ or $-b$, such that $xy = yx = 0$.

We associate $\overline{\mathscr{G}}(\mathscr{R})$, a simple graph to the ring $\mathscr{R}$, with vertices $\hat{Z}(\mathscr{R})^* = \hat{Z}(\mathscr{R}) \setminus \{0\}$, the set of all non-zero zero divisors of $\mathscr{R}$. For distinct $u, v \in \hat{Z}(\mathscr{R})^*$, vertices u and v are adjacent if and only if $uv = 0$. $\overline{\mathscr{G}}(\mathscr{R})$ is an empty graph if and only if $\mathscr{R}$ is an integral domain.

The following figure 1 represents the neutrosophic zero divisor graph associated with truth, indeterminacy and falsity mambership values on it.

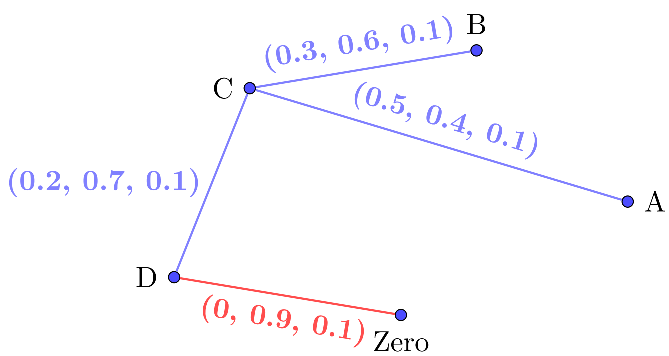


FIGURE 1. Neutrosophic zero divisor graph

By the theorem [10], let $\langle \mathscr{R} \cup \mathscr{I} \rangle$ be a neutrosophic ring. Then, $\langle \mathscr{R} \cup \mathscr{I} \rangle$ is a ring. We use $\mathscr{R}$ as the ring of reals in the following theorems.

Definition 1.5. Let the neutrosophic graph $\overline{\mathscr{G}}$ be partitioned into a set of neutrosophic subgraphs $\{\overline{\mathscr{G}}_1, \overline{\mathscr{G}}_2, \overline{\mathscr{G}}_3, \ldots, \overline{\mathscr{G}}_k\}$ such that:

(1) Each $\overline{\mathscr{G}}_i = (\mathscr{V}_i, \mathscr{E}_i, \mathscr{T}_i, \mathscr{I}_i, \mathscr{F}_i)$ satisfies the bipartite or cycle conditions.

(2) The union of the $\overline{\mathscr{G}}_i$ covers the entire neutrosophic graph, i.e., $\overline{\mathscr{G}} = \bigcup_{i=1}^{k} \overline{\mathscr{G}}_i$, where $\mathscr{E} = \bigcup_{i=1}^{k} \mathscr{E}_i$ and $\mathscr{E}_i \cap \mathscr{E}_j = \emptyset \quad \forall\, i \neq j$.

Definition 1.6. Let $\overline{\mathscr{G}}(\mathscr{V},\mathscr{E})$ be a neutrosophic graph with vertex set $\mathscr{V}$ and edge set $\mathscr{E}$. A neutrosophic cycle in $\overline{\mathscr{G}}$ is defined as a closed circuit in the neutrosophic graph if it satisfies the following:

(1) Let $v_1, v_2, \ldots, v_n, v_1$ form a circuit where $v_i \in \mathscr{V}$ for $1 \leq i \leq n$.
(2) $(v_i, v_{i+1}) \in \mathscr{E}$ for $1 \leq i \leq n-1$ and $(v_n, v_1) \in \mathscr{E}$.
(3) For each edge $e = (u, v)$ in the cycle, the neutrosophic values of the edge are $(\mathscr{T}_e, \mathscr{I}_e, \mathscr{F}_e)$.
(4) Except for the initial vertex at the starting and ending points, no vertex is repeated in the cycle.

The cycle is represented as:

$$\overline{C} = v_1 \to v_2 \to \ldots \to v_n \to v_1$$

Consider $\overline{C}$ as a cycle with edges $e_1, e_2, \ldots, e_n$. Then, the cycle is represented as:

$$\overline{C} = \{(e_1, \mathscr{T}_{e_1}, \mathscr{I}_{e_1}, \mathscr{F}_{e_1}), (e_2, \mathscr{T}_{e_2}, \mathscr{I}_{e_2}, \mathscr{F}_{e_2}), \ldots, (e_n, \mathscr{T}_{e_n}, \mathscr{I}_{e_n}, \mathscr{F}_{e_n})\}.$$

The characteristics of truth, indeterminacy, and falsity of the cycle $\overline{C}$ are given by:

$$\mathscr{T}_{\overline{C}} = \min\{\mathscr{T}_{e_1}, \mathscr{T}_{e_2}, \ldots, \mathscr{T}_{e_n}\},$$

$$\mathscr{I}_{\overline{C}} = \min\{\mathscr{I}_{e_1}, \mathscr{I}_{e_2}, \ldots, \mathscr{I}_{e_n}\},$$

$$\mathscr{F}_{\overline{C}} = \min\{\mathscr{F}_{e_1}, \mathscr{F}_{e_2}, \ldots, \mathscr{F}_{e_n}\}.$$

Example 1.2: The following figure 2 is an neutrosophic cycle, edges associated with truth, indeterminacy and falsity values.

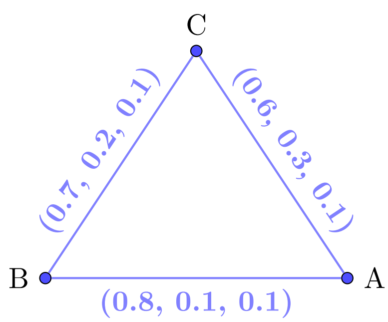


FIGURE 2. Neutrosophic cycle of neutrosophic zero divisor graph

Definition 1.7. The neutrosophic bipartite graph $\overline{\mathscr{G}}(\mathscr{U},\mathscr{V},\mathscr{E},\mathscr{T},\mathscr{I},\mathscr{F})$ is defined as follows:

(1) $\mathscr{U} \cap \mathscr{V} = \emptyset$ for two disjoint vertex sets $\mathscr{U}$ and $\mathscr{V}$.
(2) $\mathscr{E} = \{e \in \mathscr{E} \mid e = (u, v) \text{ where } u \in \mathscr{U}, v \in \mathscr{V}\}$.

(3) $(\mathscr{T}, \mathscr{I}, \mathscr{F}) \in [0,1]$, where $\mathscr{T}$, $\mathscr{I}$, and $\mathscr{F}$ represent the truth, indeterminacy, and falsity degree associated with each edge in $\mathscr{E}$.

A neutrosophic bipartite graph $\overline{\mathscr{G}}(\mathscr{U}, \mathscr{V}, \mathscr{E}, \mathscr{T}, \mathscr{I}, \mathscr{F})$ is complete if for every pair (u, v) where $u \in \mathscr{U}$ and $v \in \mathscr{V}$, there exists an edge $e = (u, v) \in \mathscr{E}$. Therefore, the total number of edges is $|\mathscr{U}| = m$ and $|\mathscr{V}| = n$, and is denoted by $\overline{K}_{m,n(\mathscr{T},\mathscr{I},\mathscr{F})}$.

Example 1.3: The figure 3 neutrosophic graph is an example of a complete neutrosophic bipartite graph with $\mathscr{U} = 4$ distinct vertices and $\mathscr{V} = 4$ distinct vertices. The $e = (\mathscr{T}, \mathscr{I}, \mathscr{F})$ values of each edge are given as a label for each edge.

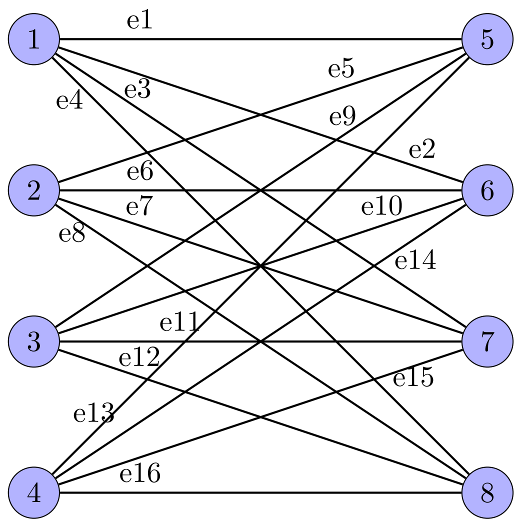


Figure 3. A complete neutrosophic bipartite graph

e1: (0.5, 0.4, 0.6)	e2: (0.7, 0.2, 0.5)	e3: (0.3, 0.2, 0.3)
e4: (0.4, 0.4, 0.4)	e5: (0.8, 0.4, 0.3)	e6: (0.2, 0.2, 0.4)
e7: (0.8, 0.5, 0.1)	e8: (0.5, 0.3, 0.6)	e9: (0.4, 0.2, 0.8)
e10: (0.1, 0.3, 0.3)	e11: (0.3, 0.3, 0.2)	e12: (0.7, 0.2, 0.8)
e13: (0.5, 0.2, 0.7)	e14: (0.6, 0.4, 0.2)	e15: (0.7, 0.5, 0.1)
e16: (0.8, 0.3, 0.3)		

Theorem 1.8. *[2] For any distinct prime numbers p and q, $\overline{\Gamma}(\hat{Z}_{pq})$ can be decomposed into $(q-1)\overline{C}_{p-1}$, where $q > p$.*

2. Neutrosophic zero-divisor graphs

The following theorems (2.1 – 2.4) discusses special cases of $\overline{\Gamma}(\hat{Z}_n)$ where n is a prime number, such as $\overline{\Gamma}(\hat{Z}_{2^2p^2})$, $\overline{\Gamma}(\hat{Z}_{3^2p^2})$, $\overline{\Gamma}(\hat{Z}_{5^2p^2})$, and $\overline{\Gamma}(\hat{Z}_{p^2q^2})$.

Theorem 2.1. *Let p be a prime number greater than 2. The neutrosophic zero divisor graph $\overline{K}_{1,2(p-1)}$ is decomposed into:*

(1) *a copy of* $\overline{K}_{1,2(p-1)}$ *neutrosophic star graph,*

(2) *a copy of* $\overline{K}_{2(p-1)}$ *complete neutrosophic graph, and*

(3) $p(p-1)$ *copies of neutrosophic cycle* $\overline{C}_4$.

Proof. Consider p, $p > 2$ be any prime number, and $\overline{\Gamma}(\hat{Z}_{(2^2p^2)})$ be a neutrosophic zero-divisor graph. Its vertex set and edge set are defined as follows:

The vertex set $\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ is given by:

$$\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \{(v_i, \mathscr{T}_i, \mathscr{I}_i, \mathscr{F}_i) \mid v_i \in \mathscr{V}, \mathscr{T}_i, \mathscr{I}_i, \mathscr{F}_i \in [0,1], \text{ and } 0 \leq \mathscr{T}_i + \mathscr{I}_i + \mathscr{F}_i \leq 3\}$$

Similarly, the edge set $\mathscr{E}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ is defined as:

$$\mathscr{E}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \{(e_k, \mathscr{T}_k, \mathscr{I}_k, \mathscr{F}_k) \mid e_k \in \mathscr{E}, \mathscr{T}_k, \mathscr{I}_k, \mathscr{F}_k \in [0,1], \text{ and } 0 \leq \mathscr{T}_k + \mathscr{I}_k + \mathscr{F}_k \leq 3\}$$

of $\overline{\Gamma}(\hat{Z}_{(2^2p^2)})$, where $\mathscr{T}, \mathscr{I}, \mathscr{F}$ are truth, indeterminacy, and falsity membership values of the vertex and edge sets of $\hat{Z}_{(2^2p^2)}$. The vertex set $\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} \in (\overline{\Gamma}(\hat{Z}_{(2^2p^2)}))$ is given by:

$$\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}(\overline{\Gamma}(\hat{Z}_{(2^2p^2)})) = \{2, 4, \ldots, 2^2p^2 - 2, p, 2p, 3p, \ldots, 2^2p^2 - p\}$$

Case (i): Let $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})} \in \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ where

$$\begin{aligned} \mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{2p^2\}, \\ \mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{4, 8, 12, \ldots, 4p(p-1)\}, \\ \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{2, 6, 10, \ldots, 2(2p^2-1)\} \setminus \mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}. \end{aligned}$$

So,

$$|\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}| = 1, \quad |\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}| = p(p-1), \quad |\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}| = p(p-1).$$

The vertex set $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ is the middle of a neutrosophic star graph, and $v_1 \in \mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ is adjacent to all vertices of vertex sets $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ and $\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}$. It clearly states that there exist two neutrosophic star graphs such as $\overline{K}_{(1,p(p-1))}$ and $\overline{K}_{(2p(p-1))}$, respectively. Hence, $\overline{K}_{(1,2p(p-1))}$ with $2(p-1)$ edges.

Case (ii): Let $\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} \in \bar{\Gamma}(\hat{Z}_{2^2p^2})$ be a subset, and

$$\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})} = \{2p, 4p, 6p, \ldots, 2p(2p-1)\}.$$

If for any vertices u and v, where $u, v \in \mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}$, u is adjacent to v, then there exists $e \in \mathscr{E}_{\mathscr{T},\mathscr{I},\mathscr{F}}$, an edge between u and v.

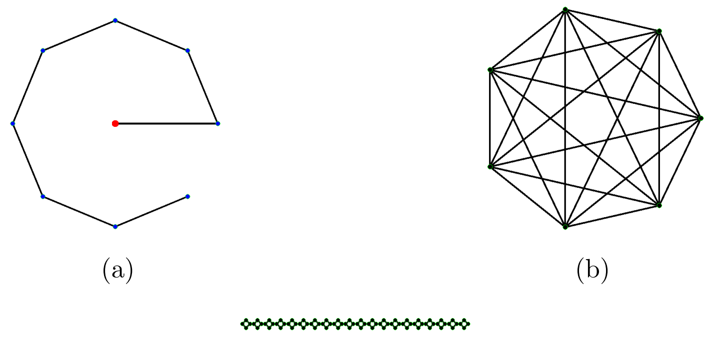
Thus, it is clear that the vertex set $\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}$ forms a complete neutrosophic graph $\overline{K}_{2p-1}$ with $2(p-1)$ vertices.

Case (iii): The neutrosophic zero-divisor graph $\bar{\Gamma}(\hat{Z}_{2^2p^2})$ decomposes into three types of complete neutrosophic bipartite graphs:

$$\overline{K}_{2(p-1),(p-1)}, \quad \overline{K}_{2(p-1)}, \quad \text{and} \quad \overline{K}_{2,p(p-1)}.$$

By Theorem 2.1, these three complete neutrosophic bipartite graphs are covered in $p(p-1)$ copies of $\overline{C_4}$. Hence, the theorem. $\square$

Example: Let $p = 5 > 2$, then the following Figure 4 is the decompositions of $\bar{\Gamma}(\hat{Z}_{2^2 5^2})$ into a neutrosophic star graph with 8 vertices, a complete graph with 8 vertices and 48 edges and 20 copies of neutrosophic cycle.



(b)

FIGURE 4. a) A neutrosophic star graph b) complete neutrosophic graph c) neutrosophic cycle

Theorem 2.2. *If $p, p > 3$, is any prime number, then $\bar{\Gamma}(\hat{Z}_{3^2 p^2})$ is decomposed into:*

(1) *1-copy of a complete neutrosophic graph $\overline{K}_{3p-1}$:*
This graph has $3p-1$ vertices associated with $\mathscr{T}, \mathscr{I}, \mathscr{F}$ and

$$\frac{(3p-1)(3p-2)}{2}$$

edges described by $(\mathscr{T}_{ij}, \mathscr{I}_{ij}, \mathscr{F}_{ij})$.

(2) $\frac{9p(p-1)}{2}$*-copies of $\overline{C}_4$, the neutrosophic cycle:*
Each cycle has the degree of truth membership defined as:

$$\mathscr{T}_{\overline{C}_4} = \min\left\{\mathscr{T}_{e_1}, \mathscr{T}_{e_2}, \mathscr{T}_{e_3}, \mathscr{T}_{e_4}\right\},$$

where $e_k \in \mathscr{E}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$, $k = 1, 2, 3, 4$.

Then,

$$\mathscr{E}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \sum_{e \in \mathscr{E}_{(\mathscr{T},\mathscr{I},\mathscr{F})}} \mathscr{T}_e = \frac{9p(p-1)}{2}.$$

Proof. Let p be any prime number and $p > 3$. Let $\bar{\Gamma}(\hat{Z}_{3^2 p^2})$ be a non-zero neutrosophic zero-divisor graph. The vertex set of $\bar{\Gamma}(\hat{Z}_{3^2 p^2})$ is defined as:

$$\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \left\{3, 6, 9, \ldots, 3(3p^2-1),\, p, 2p, 3p, \ldots, p(9p-1)\right\}.$$

Case (i): Let $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$, where

$$\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} = \{3p, 6p, 9p, \ldots, 3p(3p-1)\}.$$

Therefore, the cardinality of $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ is $3p-1$. The total number of edges in $\overline{K}_{3p-1}$ is:

$$\binom{3p-1}{2} = \frac{(3p-1)(3p-2)}{2},$$

and each edge contributes to the degree of the truth function $\mathscr{T}_{ij}$. The total contribution is given by:

$$\mathscr{E}_{\overline{K}_{3p-1}(\mathscr{T},\mathscr{I},\mathscr{F})} = \sum_{(i,j)\in\overline{K}_{3p-1}} \mathscr{T}_{ij}.$$

If any two vertices $v_1, v_2 \in \mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ are adjacent, then the vertex set $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ forms a complete neutrosophic graph $\overline{K}_{3p-1}$ with $3p-1$ vertices.

Case (ii): Let $\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}\big(\bar{\Gamma}(\hat{Z}_{3^2p^2})\big)$ be vertex subsets, where:

$$\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})} = \mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})} \setminus \mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} = \{p, 2p, 3p, \ldots, 8p\}, \quad \left|\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = 6(p-1),$$

$$\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})} = \{9p, 18p, 27p, \ldots, 9p(p-1)\}, \quad \left|\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = (p-1),$$

$$\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})} = \{p^2, 2p^2, 3p^2, \ldots, 8p^2\}, \quad \left|\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = 6,$$

$$\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})} = \{9, 18, 27, \ldots, 9(p^2-1)\}, \quad \left|\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = p(p-1),$$

$$\mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})} = \mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})} \setminus \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})} = \{3, 6, 9, \ldots, 3(3p^2-1)\}, \quad \left|\mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = 2p(p-1),$$

$$\mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})} = \{3p^2, 6p^2\}, \quad \left|\mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})}\right| = 2.$$

If

$$(\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}), (\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}), (\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}),$$
$$(\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})}) \text{ and } (\mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})})$$

are pairs of adjacent vertex sets, then there exist the following complete neutrosophic bipartite graphs in $\bar{\Gamma}(\hat{Z}_{3^2p^2})$:

$$\overline{K}_{6(p-1),(p-1)(\mathscr{T},\mathscr{I},\mathscr{F})}, \quad \overline{K}_{(p-1),6(\mathscr{T},\mathscr{I},\mathscr{F})}, \quad \overline{K}_{6,p(p-1)(\mathscr{T},\mathscr{I},\mathscr{F})}, \quad \overline{K}_{p(p-1),2(\mathscr{T},\mathscr{I},\mathscr{F})}, \quad \text{and}$$
$$\overline{K}_{2,2p(p-1)(\mathscr{T},\mathscr{I},\mathscr{F})}.$$

The complete neutrosophic bipartite graphs in $\bar{\Gamma}(\hat{Z}_{3^2p^2})$ are given as follows:

$$\overline{K}_{(6(p-1),(p-1))(\mathscr{T},\mathscr{I},\mathscr{F})} = \frac{6(p-1)(p-1)}{4}, \quad \overline{K}_{(p-1),6(\mathscr{T},\mathscr{I},\mathscr{F})} = \frac{6(p-1)}{4}, \quad \overline{K}_{(6,p(p-1))(\mathscr{T},\mathscr{I},\mathscr{F})} = \frac{6p(p-1)}{4},$$
$$\overline{K}_{(p(p-1),2)(\mathscr{T},\mathscr{I},\mathscr{F})} = \frac{2p(p-1)}{4}, \quad \overline{K}_{(2,2p(p-1))(\mathscr{T},\mathscr{I},\mathscr{F})} = \frac{4p(p-1)}{4}.$$

Each of these bipartite graphs is a copy of the complete neutrosophic cycle $\overline{C}_4$ in $\bar{\Gamma}(\hat{Z}_{3^2p^2})$. The sum of the complete neutrosophic bipartite graphs is given by:

$$\overline{K}_{6(p-1),(p-1)} + \overline{K}_{(p-1),6} + \overline{K}_{6,p(p-1)} + \overline{K}_{p(p-1),2} + \overline{K}_{2,2p(p-1)}$$
$$= \frac{6(p-1)(p-1)}{4} + \frac{6(p-1)}{4} + \frac{6p(p-1)}{4} + \frac{2p(p-1)}{4} + \frac{4p(p-1)}{4} = \frac{9p(p-1)}{2}.$$

The values of the edges of the cycle $\overline{C}_4$ are $(\mathscr{T}, \mathscr{J}, \mathscr{F})$. The truth values are defined by:

$$\mathscr{T}_{\overline{C}_4} = \min\{\mathscr{T}(e_1), \mathscr{T}(e_2), \mathscr{T}(e_3), \mathscr{T}(e_4)\}, \quad e_k \in E_{(\mathscr{T},\mathscr{J},\mathscr{F})}(\overline{\Gamma}(\hat{Z}_{(3^2p^2)})),\ k = 1,2,3,4.$$

The total sum of the truth values is:

$$E_{(\mathscr{T},\mathscr{J},\mathscr{F})} = \sum_{e \in E_{(\mathscr{T},\mathscr{J},\mathscr{F})}} \mathscr{T}_e$$

By Theorem 2.1, since $\overline{C}_4$ has $\frac{9p(p-1)}{2}$ copies of complete neutrosophic bipartite graphs, we have:

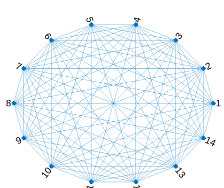
$$E_{(\mathscr{T},\mathscr{J},\mathscr{F})} = \frac{9p(p-1)}{2}.$$

□

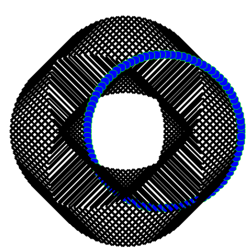
Example: Let $p = 5 > 3$. Then, the following figure 5 neutrosophic graph occurs for the decomposition of $\bar{\Gamma}(\hat{Z}_{3^25^2})$:

$$\overline{K}_{3p-1} = \overline{K}_{14},$$

with 91 edges and 90 copies of $\bar{C}_4$.



(a)



(b)

FIGURE 5. a) A neutrosophic complete graph $\overline{K}_{14}$ b) neutrosophic cycle

Theorem 2.3. *If $p > 5$ is any prime number, then the complement of the neutrosophic graph $\bar{\Gamma}(\hat{Z}_{5^2p^2})$ is decomposed into:*

(1) 1-*copy of the complete neutrosophic graph $\bar{K}_{5p-1}$ with $5p-1$ vertices associated with $(\mathscr{T}, \mathscr{J}, \mathscr{F})$, and*

(2) $15p(p-1)$-*copies of the neutrosophic cycle $\bar{C}_4$.*

Proof. Let p be any prime number and $p > 5$. Let $\bar{\Gamma}(\hat{Z}_{5^2p^2})$ be a non-zero neutrosophic zero divisor graph with vertex set defined as:

$$\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \{5, 10, 15, \ldots, 5(5p^2-1);\, p, 2p, 3p, \ldots, p(25p-1)\} \in \bar{\Gamma}(\hat{Z}_{5^2p^2}),$$

where for every $v \in \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$, $(\mathscr{T}, \mathscr{I}, \mathscr{F}) \in [0,1]$.

Case (i): Let $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$, where: $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} = \{5p, 10p, 15p, \ldots, 5p(5p-1)\}$. Therefore, the cardinality of $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ is: $|\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}| = 5p-1$. If $v_1 \cdot v_2 \equiv 0 \,(\mathrm{mod}\, 5^2p^2)$, then v_1 is adjacent to v_2 in $\bar{\Gamma}(\hat{Z}_{5^2p^2})$, and this forms a complete neutrosophic graph $\bar{K}_{5p-1}$.

Case (ii): Let $\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} \in (\bar{\Gamma}(\hat{Z}_{3^2p^2}))$, where the cardinalities are defined as:

$$\begin{aligned}
&\mathscr{V}_{2(\mathscr{T},\mathscr{J},\mathscr{F})} = \{p, 2p, 3p, \ldots, 24p\}, && |\mathscr{V}_{2(\mathscr{T},\mathscr{J},\mathscr{F})}| = 20(p-1),\\
&\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})} = \{25p, 50p, 75p, \ldots, 25p(p-1)\}, && |\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}| = (p-1),\\
&\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})} = \{p^2, 2p^2, 3p^2, \ldots, 24p^2\}, && |\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}| = 20,\\
&\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})} = \{25, 50, 75, \ldots, 25(p^2-1)\}, && |\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}| = p(p-1),\\
&\mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})} = \{5, 10, 15, \ldots, 5(5p^2-1)\} \setminus \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}, && |\mathscr{V}_{6(\mathscr{T},\mathscr{I},\mathscr{F})}| = 4p(p-1),\\
&\mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})} = \{5p^2, 10p^2, 15p^2, 20p^2\}, && |\mathscr{V}_{7(\mathscr{T},\mathscr{I},\mathscr{F})}| = 4.
\end{aligned}$$

If the pairs $(\mathscr{V}_{2(\mathscr{T},\mathscr{J},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{J},\mathscr{F})})$, $(\mathscr{V}_{3(\mathscr{T},\mathscr{J},\mathscr{F})}, \mathscr{V}_{4(\mathscr{T},\mathscr{J},\mathscr{F})})$, $(\mathscr{V}_{4(\mathscr{T},\mathscr{J},\mathscr{F})}, \mathscr{V}_{5(\mathscr{T},\mathscr{J},\mathscr{F})})$, $(\mathscr{V}_{5(\mathscr{T},\mathscr{J},\mathscr{F})}, \mathscr{V}_{7(\mathscr{T},\mathscr{J},\mathscr{F})})$, and $(\mathscr{V}_{7(\mathscr{T},\mathscr{J},\mathscr{F})}, \mathscr{V}_{6(\mathscr{T},\mathscr{J},\mathscr{F})})$ are pairs of adjacent vertex subsets, then there exist the following complete neutrosophic bipartite graphs: $\bar{K}_{20(p-1),(p-1)}$, $\bar{K}_{(p-1),20}$, $\bar{K}_{20,p(p-1)}$, $\bar{K}_{p(p-1),4}$, $\bar{K}_{4,4p(p-1)}$. By Theorem 2.1, each $\bar{K}_{\hat{Z}_{5^2p^2}}$ contains cycles $\bar{C}_4$. The total number of such cycles is:

$$\frac{20(p-1)(p-1)}{4} + \frac{20(p-1)}{4} + \frac{20p(p-1)}{4} + \frac{4p(p-1)}{4} + \frac{16p(p-1)}{4} = 15p(p-1) \text{ copies of } \bar{C}_4.$$

□

Example: Taking $p = 7 > 5$, we get the following complete neutrosophic graph in figure 6: $\bar{K}_{(5p-1)} = \bar{K}_{34}$, where $5p-1 = 34$ vertices.

The number of edges in a complete graph $\bar{K}_n$ is given by:

$$\mathscr{E} = \frac{n(n-1)}{2}.$$

Substituting $n = 34$, we get:

$$\mathscr{E} = \frac{34(34-1)}{2} = \frac{34 \cdot 33}{2} = 561.$$

Thus, $\bar{K}_{34}$ has 561 edges.

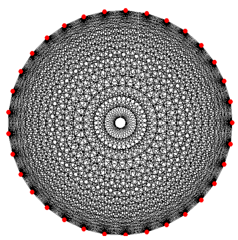
The total number of $\bar{C}_4$ cycles in a complete graph $\bar{K}_n$ is given by:

$$\text{Number of } \bar{C}_4 = \binom{n}{4}.$$

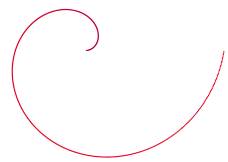
Substituting $n = 34$, we calculate:

$$\binom{34}{4} = \frac{34 \cdot 33 \cdot 32 \cdot 31}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{34 \cdot 33 \cdot 32 \cdot 31}{24} = 46,376 \div 24 = 630.$$

Thus, $\bar{K}_{34}$ forms 630 copies of $\bar{C}_4$.



(a)



(b)

FIGURE 6. a) A neutrosophic complete graph b) Neutrosophic cycle

Theorem 2.4. *Let p and q, with $p < q$, be any distinct prime numbers. Then the neutrosophic zero divisor graph $\bar{\Gamma}(\hat{Z}_{p^2q^2})$ is decomposed as follows:*

(1) *A copy of the complete neutrosophic graph $\bar{K}_{pq-1}$, where $pq-1$ is the number of vertices.*
(2) *$\frac{3pq(p-1)(q-1)}{4}$ copies of the neutrosophic cycle $\bar{C}_4$.*

Proof. Let p and q, with $p < q$, be distinct prime numbers. The vertex set of the neutrosophic zero divisor graph $\bar{\Gamma}(\hat{Z}_{p^2q^2})$ is defined as:

$$\mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})} = \{p, 2p, 3p, \ldots, p(pq^2-1);\ q, 2q, 3q, \ldots, q(p^2q-1)\},$$

where each $v \in \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ is associated with neutrosophic components $(\mathscr{T}, \mathscr{I}, \mathscr{F}) \in [0, 1]$.

Case (i): Let $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ be a subset defined as:

$$\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})} = \{pq, 2pq, 3pq, \ldots, pq(pq-1)\}.$$

The cardinality of $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ is $|\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}| = pq-1$. If $v_1, v_2 \in \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$, then $v_1 \cdot v_2 \bmod p^2q^2 = 0$, hence v_1 is adjacent to v_2 in $\bar{\Gamma}(\hat{Z}_{p^2q^2})$. Therefore, $\mathscr{V}_{1(\mathscr{T},\mathscr{I},\mathscr{F})}$ forms a complete neutrosophic graph $\bar{K}_{pq-1}$.

Case (ii): Let $\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}, \mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})} \subset \mathscr{V}_{(\mathscr{T},\mathscr{I},\mathscr{F})}$ be the remaining vertex subsets with their cardinalities defined as follows:

$$\begin{aligned}
\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{p, 2p, 3p, \ldots, p(pq-1)\}, & |\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}| &= q(p-1),\\
\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{q, 2q, 3q, \ldots, q(p^2-1)\}, & |\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}| &= p(q-1),\\
\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{p^2, 2p^2, 3p^2, \ldots, p^2(q-1)\}, & |\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}| &= q-1,\\
\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})} &= \{q^2, 2q^2, 3q^2, \ldots, q^2(p-1)\}, & |\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}| &= p-1.
\end{aligned}$$

The subsets $\mathscr{V}_{2(\mathscr{T},\mathscr{I},\mathscr{F})}$ and $\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}$, $\mathscr{V}_{3(\mathscr{T},\mathscr{I},\mathscr{F})}$ and $\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}$, and $\mathscr{V}_{4(\mathscr{T},\mathscr{I},\mathscr{F})}$ and $\mathscr{V}_{5(\mathscr{T},\mathscr{I},\mathscr{F})}$ are pairwise adjacent and form complete neutrosophic bipartite graphs as follows: $\bar{K}_{q(p-1),p(q-1)}$, $\bar{K}_{p(q-1),(q-1)}$, $\bar{K}_{(q-1),(p-1)}$. The edge weights are associated with $(\mathscr{T}, \mathscr{J}, \mathscr{F}) = (1, 0, 0)$.

By Theorem 2.1, the number of neutrosophic cycles $\bar{C}_4$ contributed by each bipartite graph is given by:

$$\begin{aligned}
\bar{K}_{q(p-1),p(q-1)} &= \frac{q(p-1)\cdot p(q-1)}{4},\\
\bar{K}_{p(q-1),(q-1)} &= \frac{p(q-1)\cdot (q-1)}{4},\\
\bar{K}_{(q-1),(p-1)} &= \frac{(q-1)\cdot (p-1)}{4}.
\end{aligned}$$

The total number of neutrosophic cycles $\bar{C}_4$ is:

$$\bar{K}_{q(p-1),p(q-1)} + \bar{K}_{p(q-1),(q-1)} + \bar{K}_{(q-1),(p-1)} = \frac{3pq(p-1)(q-1)}{4}.$$

□

3. Conclusion

In this article, the decomposition of the neutrosophic zero divisor graph $\bar{\Gamma}(\hat{Z}_n)$, where n is a prime number, into cycles and complete neutrosophic graphs is discussed for special cases. Specifically, the following graphs are examined:

$$\bar{\Gamma}(\hat{Z}_{2^2p^2}),\ \bar{\Gamma}(\hat{Z}_{3^2p^2}),\ \bar{\Gamma}(\hat{Z}_{5^2p^2}),\ \text{and}\ \bar{\Gamma}(\hat{Z}_{p^2q^2}).$$

For each case, example graphs are provided, which illustrate the decomposition into complete neutrosophic graphs and neutrosophic cycles. These example graphs were generated using MATLAB. These theorems and its examples contribute to a depth understanding of neutrosophic graph structures and their decomposition properties, potentially benefiting mathematical modeling in uncertain and fuzzy environments.

Future study: Generalization to higher-order prime powers by extending these outcomes to more complex structures, where n is higher powers of prime numbers or mixed prime compositions. The structural properties of the graph structures developed can be explored for cryptographic key generation and secure communication systems, ehancing visualization techniques for large-scale neutrosophic graphs involving advanced computational tools beyond MATLAB, such as Python or TensorFlow and implimenting in applied sciences.

Funding: "This research received no external funding"

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Beck, I. Coloring of commutative rings. *Journal of Algebra* **1988**, *116*, 208–226.
2. Rajakumaran, V.; Selvi, N. Cyclic path covers in zero divisor graph. *International Journal of Mathematical Archive* **2018**, *9*, 163–167.
3. Kuppan, A.; Sankar, J.R. Fuzzy zero divisor graph in a commutative ring. *TWMS Journal of Applied and Engineering Mathematics* **2021**, *11*, 42–50.
4. Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic*; American Research Press: Rehoboth, NM, USA,**1999**.
5. Jun, Y.B.; Smarandache, F. Neutrosophic rings, modules, and linear algebra. *Neutrosophic Sets and Systems* **2018**, *22*, 96–109.
6. Smarandache, F.; Jun, Y.B. *Neutrosophic Algebraic Structures and Their Applications*; Pons Editions: Brussels, Belgium,**2019**.
7. Kandasamy, W.B.V.; Smarandache, F. *Fuzzy, Neutrosophic, and Uncertainty Modeling in Mathematics and Physics*; American Research Press: Rehoboth, NM, USA, **2003**.
8. Kandasamy, W.B.V.; Ilanthendral, K.; Smarandache, F. *Neutrosophic Graphs: A New Dimension to Graph Theory*; United States of America, **2015**; pp. 9–65.
9. Kandasamy, V.; Smarandache, F. Neutrosophic rings. *arXiv Mathematics e-prints* **2006**. Available online: https://arxiv.org/abs/math/0607765.
10. Ali, R. A review study on neutrosophic groups and their generalizations. *HAL Archives* **2021**. Available online: https://hal.archives-ouvertes.fr/hal-03232679.
11. Ali, N.; Siddiqui, H.M.A.; Qureshi, M.I.; Abdalla, M.E.M.; Abd El-Gawaad, N.S.; Tolasa, F.T. On study of multiset dimension in fuzzy zero divisor graphs associated with commutative rings. *International Journal of Computational Intelligence Systems* **2024**, *17*, 1–10.
12. Panda, N.R.; Raut, P.K.; Baral, A.; Sahoo, S.K.; Satapathy, S.S.; Broumi, S. An overview of neutrosophic graphs. *Neutrosophic Sets and Systems* **2025**, *77*, 450–462.
13. Graph, C. R. U. Z. D. $LA - \Gamma -$ Semigroups. *Communications in Mathematics and Applications* **2021**, *12*(3), 545–557.
14. Ahmad, A.; Koam, A. N.; Siddiqui, M. H. F.; Azeem, M. Resolvability of the starphene structure and applications in electronics. *Ain Shams Engineering Journal* **2022**, *13*(2), 101587.
15. Hamidi, M. Zero Divisor Graphs Based on General Hyperrings. *Journal of Algebraic Hyperstructures and Logical Algebras* **2023**, *4*(2), 131–149.
16. Chalapathi, T.; Madhavi, L. A Study on Neutrosophic Zero Rings. *Neutrosophic Sets and Systems Book Series* **2019**, *30*, 191.
17. Patil, A.; Khairnar, A.; Momale, P. S. Zero-divisor graph of a ring with respect to an automorphism. *Soft Computing* **2022**, *26*(5), 2107–2119.

18. Fujita, T.; Smarandache, F. A reconsideration of advanced concepts in neutrosophic graphs: Smart, zero divisor, layered, weak, semi, and chemical graphs. *Infinite Study* **2024**.

Received: Nov. 2, 2024. Accepted: March 31, 2025