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A Comprehensive Approach for Performance Evaluation in Petrochemical Industry: Integrating Sustainable Balanced Scorecard, Dynamic Network DEA, and DEMATEL in a Neutrosophic Environment

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Abstract: Sustainability is increasingly vital in modern industries, necessitating comprehensive evaluation frameworks that assess both efficiency and sustainability. This paper introduces a novel approach to evaluating the efficiencies of Decision-Making Units (DMUs) using a Sustainable Balanced Scorecard (SBSC), dynamic network DEA, and DEMATEL within a Neutrosophic environment. Through Neutrosophic DEMATEL, we explored complex relationships among SBSC perspectives, gaining valuable insights into performance interactions. Industry experts defined key performance indicators (KPIs) specific to strategic objectives, which were incorporated into a Dynamic Network Slack-Based Measure (DNSBM) model for precise performance assessment. We employed a two-phase approach for evaluation in a Neutrosophic environment. Phase I addressed the dual roles of intermediate measures and carry-over activities using a MILP model. Phase II utilized optimal solutions from Phase I for performance evaluation. To manage uncertainty within the model, we transformed the proposed model into deterministic forms. This enabled computation of efficiency boundaries at varying degrees of variation, providing robust performance analysis. To validate the proposed approach, we analyzed nine oil refineries from the Marun Petrochemical Complex (MPC) across various perspectives for the years 1400 and 1401. The findings reveal that DMU9 consistently exhibits high performance, while DMU3 shows significant declines, particularly in the Customer and Environment perspectives. Other DMUs displayed mixed results, indicating both progress and regress, highlighting the dynamic nature of performance metrics. Our study demonstrates the value of combining expert knowledge with advanced analytical methods for a detailed understanding of organizational performance, **with** potential applications across various industries, including the petrochemical sector.

Keywords: Dynamic network DEA, DNSBM, Sustainable Balanced Scorecard (SBSC), DEMATEL, Neutrosophic.

1. Introduction

In today's competitive world, organizations must measure their performance more accurately and adjust to meet customer and market demands. Data Envelopment Analysis (DEA), a mathematical tool grounded in operational research, assesses the relative efficiency of decision-making units (DMUs) using both qualitative and quantitative metrics. The first DEA model was introduced by Charnes et al. [1]. Following this pioneering study, numerous works have emerged to evaluate DMU efficiency from radial and non-radial perspectives [2], [3]. The radial approach focuses on proportional changes in inputs and/or outputs. However, to more accurately represent production processes, it is crucial to assess nonproportional projections using non-radial models [4]. Consequently, Tone [5] introduced the Slack-Based Measure (SBM) model, integrating the slacks of each input/output into a slacks-based efficiency measure.

In DEA literature, many researchers explore inefficiency sources within DMUs with complex structures, measuring both divisional and overall efficiencies within a unified framework [6]. To achieve this, network DEA methodologies have been developed, which are more adept at identifying inefficiencies compared to traditional DEA models. Since the groundbreaking work of [7], The development of network DEA models has become increasingly common. Network DEA, specifically, is recognized for exploring inefficiency sources within DMUs. However, incorporating dynamic elements into network systems is quite challenging. While network modeling provides intermediate measures for analyzing the internal structure of DMUs, dynamic modeling clarifies the connections between periods through carry-over activities. Dynamic Network DEA (DNDEA) models address the complexity of efficiency evaluation by integrating multiple dynamic stages linked through network structures in each period. This approach involves comparing a series of static models, allowing for a comprehensive analysis. Carry-over activities and intermediate measures are non-separable parts of DNDEA. Intermediate measures and carry-over activities play a dual role as outputs from one stage or period and inputs to the next. This dual function creates a conflict, making it difficult to directly account for their inefficiencies in efficiency measurement [4]. This paper proposes a dynamic DEA model to address this issue.

A key aspect of developing numerical models for performance measurement is accurately describing the production process. To effectively capture this process and its boundaries, it is essential to select relevant variables that represent inputs, outputs, and contextual factors influencing production. Consequently, having performance indicators from various dimensions is crucial for modeling and assessing the efficiency and performance of DMUs. The Balanced Scorecard (BSC) is a managerial tool that organizes indicators into different categories [8]. Over time, the Balanced Scorecard (BSC) has become a well-known and thorough method for evaluating and managing an organization's performance. It covers various aspects, including financial results, customer satisfaction, internal processes, and employee growth. Using the BSC helps organizations execute their strategies and improve performance [8]. For achieving sustainability, it's important to include social and environmental factors in the BSC. The Sustainable Balanced Scorecard (SBSC) was created by adding a sustainability perspective to the BSC, making it a useful tool for managing sustainability effectively [8]. SBSC architectures can be developed in different ways, such as adding a specific sustainability perspective, integrating sustainability indicators into the existing BSC, or expanding the core BSC framework. Although previous research has significantly contributed to our understanding of SBSC, scholars often have different opinions on the best SBSC architecture for achieving sustainability goals [9]. This study examines a particular SBSC model that adds a fifth dimension to the traditional BSC framework. This approach aligns with recommendations from [10]. To ensure these frameworks are applied efficiently in practice, an essential step is the development of strategy maps. These maps provide a clear linkage between strategic goals and the performance criteria used to assess them, as explored in the following.

To effectively use a BSC framework, creating a strategy map is essential [11] . This map shows how the organization's strategic vision connects to its performance goals. It clarifies how different performance criteria are related and linked together [12]. The strategy map explores the key assumptions about cause-and-effect relationships that create value for both customers and shareholders, while achieving strategic goals [13]. Despite the valuable contributions of previous research, the Balanced Scorecard (BSC) has faced significant criticism. Organizations still struggle with managing the connections among different BSC perspectives [14]. Additionally, while the strategy map is a useful tool, it does not fully resolve the challenges of implementing strategies or validating the causal relationships within the BSC, especially in dynamic contexts [15].

The existing literature does not show a clear trend in effectively addressing this gap. Moreover, emphasize the urgent need for a structured approach to incorporate crucial environmental factors into performance measurement systems. Many studies exploring strategy maps have examined how the Decision-Making Trial and Evaluation Laboratory (DEMATEL) can serve as a powerful tool for multi-criteria decision analysis, overcoming challenges found in other analytical methods. The DEMATEL method analyzes the structure of each criterion and evaluates both the direct and indirect relationships among clearly defined elements [16].

The utilization of DEMATEL method has significantly enriched the development of BSC and strategy maps within the academic sphere. DEMATEL's systematic analysis of criteria and its evaluation of both direct and indirect relationships among elements have strengthened and expanded the effectiveness of BSCs. This multidisciplinary approach not only refines performance evaluation frameworks but also offers a flexible tool for aligning strategic goals with the unique needs of various sectors [17]. Combining SBSC with DEMATEL provides a robust framework for performance evaluation, although it is not solely Mathematical-based. However, integration of Data Envelopment Analysis (DEA) with it enhances its effectiveness significantly. Given the complexity of real-world data, a more advanced approach is required, to handle uncertainty and imprecision within the performance evaluation models.

In real-world problems, the available data often lacks clarity and is collected with uncertainty. To address this challenge, many DEA researchers have started incorporating Zadeh's fuzzy sets [18] into their models and have developed various fuzzy DEA methods [19]. Smaranache [20], introduced Neutrosophic sets to manage imprecise, incomplete, and uncertain data, but their practical use has been limited by unclear definitions. Edalatpanah [21] was first to extend DEA models by incorporating Single-Valued Neutrosophic Numbers (SVNNs) to enhance efficiency in private institutions. Rasinojehdehi [22] introduced a Neutrosophic network SBM model for assessing the efficiency of Iranian airlines, addressing the dual role of intermediate measures and their inefficiencies.

The proposed framework in this paper integrates the Sustainable Balanced Scorecard (SBSC), Neutrosophic DEMATEL, and Neutrosophic Dynamic Network Data Envelopment Analysis models to provide a comprehensive performance assessment. It aims to measure efficiency over time in alignment with strategic goals by effectively combining these models. While some studies have combined SBSC and DEA models, they often treat the Key Performance Indicators (KPIs) of SBSC as static inputs and outputs in a traditional DEA model. This approach overlooks the dynamic nature of KPIs. To address this gap, the proposed framework incorporates time, considering both the causal relationships among SBSC perspectives at a single point in time and their dynamic relationships across different periods. The integration of these advanced Neutrosophic and fuzzy sets with performance assessment models lays the foundation for the framework proposed in this paper, which is discussed in the next section. Figure 1. illustrates the methodology steps proposed in this study.

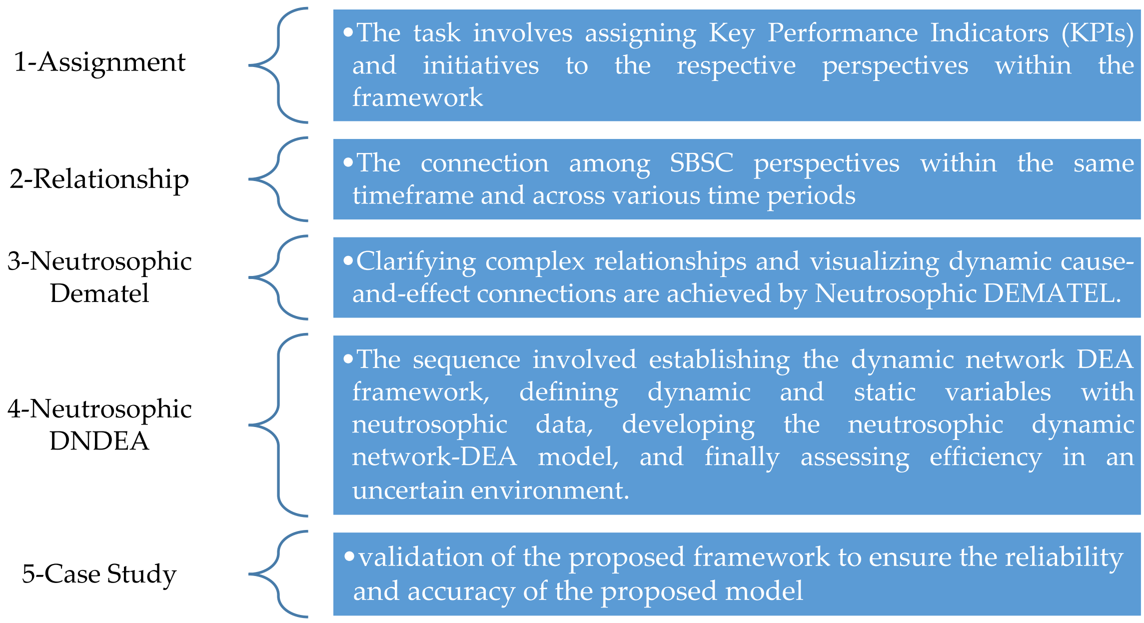


Figure 1. Methodology steps.

The rest of this paper is structured as follows: Section 2 provides the preliminaries and definitions necessary for this study. Section 3 presents the proposed DNDEA model. In Section 4, we develop the network structure of the SBSC. Section 5 offers a case study, and finally, Section 6 concludes the paper.

2. Preliminaries

In this section, we will explore the foundational concepts of Neutrosophic logic and Neutrosophic DEMATEL essential for this paper.

2.1. Neutrosophic logic

Smarandache [20] introduced the theory of Neutrosophic set, which addresses vagueness, uncertainty, and the indeterminacy of values. Neutrosophy logic offers several advantages:

1. It allows for the inclusion of unknown information in models through the indeterminacy degree, enabling experts to express opinions on uncertain preferences.
2. It captures disagreements between decision-makers and experts.
3. It considers all aspects of decision-making situations by simultaneously accounting for truth, indeterminacy, and falsity [23].

Definition 1 ([20]). Let Z be a space of points with a generic element denoted by z. A Neutrosophic set A in Z is indicated by $A = \{< z, T_A(z), I_A(z), F_A(z) | z \in Z >\}$, where $T_A(z)$, $I_A(z)$, $F_A(z) \in]0^-, 1^+[$ signify the truth, indeterminacy and falsity-membership functions, respectively, such that $0^- \leq sup\, T_A\,(z) + sup\, I_A\,(z) + sup\, F_A\,(z) \leq 3^+$.

Wang et al. [24] addressed the challenges of applying Neutrosophic sets to practical problems by simplifying the approach. They achieved this by converting nonstandard interval numbers into Single Valued Neutrosophic Sets (SVNSs) based on standard interval numbers.

Definition 2 ([24]). Suppose Z is a point space with a generic element shown by z., SVNS in A in Z is represented as $A = \{< z, T_A(z), I_A(z), F_A(z) | z \in Z >\}$, where $T_A(z)$, $I_A(z)$, $F_A(z) \in [0,1]$ satisfying $0 \leq sup\, T_A\,(z) + sup\, I_A\,(z) + sup\, F_A\,(z) \leq 3$.

For an SVNS $\{< z, T_A(z), I_A(z), F_A(z) | z \in Z >\}$, the ordered triple components $< T_A(z), I_A(z), F_A(z) >$ are described as an SVNN, and each SVNN can be characterized as $a =< T_a, I_a, F_a >$, where $T_a, I_a, F_a \in [0,1]$ and $0 \leq T_a + I_a + F_a \leq 1$.

Definition 3 ([25]). A Triangular Neutrosophic Number, abbreviated as TNNs, $\tilde{\Gamma} = \prec (m_1, m_2, m_3), (\omega, \theta, \chi) \succ$ have the truth, indeterminacy, and falsehood membership functions of z as indicated in Equations (1) to (3):

$$T_{\tilde{\Gamma}}(z) = \begin{cases} \dfrac{(z - m_1)}{(m_2 - m_1)}\omega & m_1 \le z < m_2, \\ \omega & z = m_2, \\ \dfrac{(m_3 - z)}{(m_3 - m_2)}\omega & m_2 \le z < m_3, \\ 0 & otherwise. \end{cases} \quad (1)$$

$$I_{\tilde{\Gamma}}(z) = \begin{cases} \dfrac{(m_2 - z)}{(m_2 - m_1)}, & m_1 \le z < m_2, \\ \theta, & z = m_2, \\ \dfrac{(z - m_3)}{(m_3 - m_2)}\theta, & m_2 \le z < m_3, \\ 1, & otherwise. \end{cases} \quad (2)$$

$$F_{\tilde{\Gamma}}(z) = \begin{cases} \dfrac{(m_2 - z)}{(m_2 - m_1)}, & r_1 \le z < r_2, \\ z, & z = m_2, \\ \dfrac{(z - m_3)}{(m_3 - m_2)}\chi, & m_2 \le z < m_3, \\ 1, & otherwise. \end{cases} \quad (3)$$

Where, $0 \le T_{\tilde{\Gamma}}(z) + I_{\tilde{\Gamma}}(z) + F_{\tilde{\Gamma}}(z) \le 3, z \in \tilde{\Gamma}$.

Definition 4 ([26]). suppose $\tilde{\Gamma} = \prec (m_1, m_2, m_3), (\omega, \theta, \chi) \succ$ is a triangular Neutrosophic number. Then $\varphi_{(\alpha,\beta,\gamma)}$ represents the aggregate coefficient which is defined as follows:

$$\varphi_{(\alpha,\beta,\gamma)} = [\tilde{\Gamma}^{L}_{(\alpha,\beta,\gamma)}, \Gamma^{U}_{(\alpha,\beta,\gamma)}] = [m_1 + (m_2 - m_3)\hbar, m_3 - (m_3 - m)\hbar]$$

Where, $\alpha \in [0, \eta], \beta \in [\lambda, 1], \gamma \in [\kappa, 1]$, and $\hbar = \frac{1}{4}(\frac{\alpha}{\omega} + 2\frac{(1-\beta)}{1-\theta} + \frac{(1-\gamma)}{1-\chi})$ associated with the triangular Neutrosophic number $\tilde{\Gamma}$.

In the next subsection, we present Neutrosophic DEMATEL, which is a powerful technique used to analyze interdependencies among factors by incorporating Neutrosophic sets to handle uncertain and indeterminate information comprehensively.

2.2. Neutrosophic DEMATEL

The Neutrosophic DEMATEL model is employed to handle internal dependencies among criteria and constructs a causal graph between criteria. A brief discussion of the Neutrosophic DEMATEL method is provided below [16].

Step 1. Identify experts with significant experience in the field.

Step 2. Select the most critical criteria that influence the given problem.

Step 3. Construct the linguistic direct-relation matrix. This matrix displays the degree of influence each criterion has on the others. Collect opinions from each expert and create a pairwise comparison matrix for each, with elements expressed in linguistic terms such as equally influential, slightly influential, strongly influential, very strongly influential, and absolutely influential (See Table 1). This matrix, referred to as the linguistic direct-relation matrix, is an $n \times n$ matrix where each element t_{ij} indicates the degree of effect between criterion i and criterion j.

Step 4. Convert the linguistic terms of the direct-relation matrix into the triangular Neutrosophic scale. The triangular Neutrosophic scale is represented in the form $t_{ij} = \langle (t_{ij}^1, t_{ij}^2, t_{ij}^3); \alpha_{ij}, \beta_{ij}, \theta_{ij} \rangle$, Where t_{ij}^1, t_{ij}^2 *and* t_{ij}^3 denote respectively the lower, median, and upper bounds of the Neutrosophic number of ith criterion over jth criterion, and α_{ij}, β_{ij} *and* θ_{ij} are respectively the truth, indeterminacy and falsity membership functions of ith criterion over jth criterion.

Step 5. Transform the Neutrosophic scales into precise values using the Equations (4):

$$r(t_{ij}) = \left|(t_{ij}^1 \times t_{ij}^2 \times t_{ij}^3)\frac{\alpha_{ij} + \beta_{ij} + \theta_{ij}}{9}\right|. \tag{4}$$

Step 6. integrate the viewpoints of all experts into a unified matrix and measure the aggregated opinions by averaging the experts' assessments for each criterion applying Equation (5):

$$s_{ij} = \frac{\sum_{k=1}^{m} r^k}{n}, \tag{5}$$

where s_{ij} denotes the average opinion value for the ith and jth criteria, and r^k represents the crisp opinion value of the ith and jth criteria provided by the kth DM (DM_k), where $k = 1, \dots, m$.

Step 7. Construct the crisp direct-relation matrix S. This matrix is derived from the preceding step (Step 6), where all averaged opinions of experts are integrated. The initial direct-relation matrix, denoted as S, is an $n \times n$ matrix where each element t_{ij} signifies the degree of influence between criterion i and criterion j (Equation (6)).

$$\begin{bmatrix} 1 & \cdots & s_{1n} \\ \vdots & \ddots & \vdots \\ s_{n1} & \cdots & 1 \end{bmatrix}. \tag{6}$$

Step 8. Normalize the direct-relation matrix using the Equations (7) and (8).

$$U = K \times S. \tag{7}$$

$$K = Min\left(\frac{1}{Max \sum_{i=1}^{n} s_{ij}}, \frac{1}{Max \sum_{j=1}^{n} s_{ij}}\right), i, j \in \{1, \dots, n\}. \tag{8}$$

Step 9. Calculate the total-relation matrix P using the Equation (9).

$$P = U \times (I - U)^{-1}, \tag{9}$$

where I denotes the identity matrix.

Step 10. Calculate the two indices $Q + R$ and $Q - R$ for each criterion and create the causal diagram. Begin by determining the sum of the rows Q and the sum of the columns R for each criterion individually. The vectors Q and R are computed using the Equations (10) and (11), where P= $[z_{ij}]$ and $i, j \in \{1, \dots, n\}$.

$$Q_i = \sum_{j=1}^{n} z_{ij} \quad \forall i, i \in \{1, \dots, n\}. \tag{10}$$

$$R_j = \sum_{i=1}^{n} z_{ij} \quad \forall j, \quad j \in \{1, \dots, n\}. \tag{11}$$

Table 1. The alignment of linguistic terms with Neutrosophic Triangular values.

Explanation	Scale	Neutrosophic triangular scale
equally influential	1	$\langle(1,1,1); 0.5,0.5,0.5)\rangle$
slightly influential	3	$\langle(2,3,4); 0.30,0.75,0.70)\rangle$
strongly influential	5	$\langle(4,5,6); 0.80,0.15,0.20)\rangle$
Very strongly influential	7	$\langle(6,7,8); 0.90,0.10,0.10)\rangle$
Absolutely influential	9	$\langle(9,9,9); 1.00,0.00,0.00)\rangle$

Table 1. Continue.

Explanation	Scale	Neutrosophic triangular scale
sporadic values between two close scales	2	$\langle(1,2,3);0.40,0.60,0.65)\rangle$
	4	$\langle(3,4,5);0.35,0.60,0.40)\rangle$
	6	$\langle(5,6,7);0.70,0.25,0.30)\rangle$
	8	$\langle(7,8,9);0.85,0.10,0.15)\rangle$

In this subsection, we explained how to apply DEMATEL in a Neutrosophic environment. In Section 4, we use this method to construct the relationships between SBSC perspectives in the petrochemical industry. In the next section we present our proposed DNDEA model.

2. Proposed model

In this section we propose a novel DNDEA model in Neutrosophic environment. The proposed model addresses the issue regarding the dual role of intermediate measures and carry-overs and considers their inefficiencies in the efficiency measurement. The proposed model measures the overall and perspective's efficiencies in the SBM framework and categorizes the intermediate measures and carry-overs into two groups of input and output and considers their slacks and surpluses in objective function. The proposed model has two different phases. In the first phase, role determination' the role of intermediate measures and carry over activities is determined considering the importance of SBSC perspective. In the second stage the overall efficiency of the DMUs is computed based on the results obtained from Phase I. This paper employs the symbols for data and variables as detailed in Table 2.

Table 2. Symbols for data and variables.

Role	Symbol	Definition
input	$\tilde{x}_{ip}^{tk}$	Neutrosophic Input i to perspective k (Divk) of DMUj at period t.
output	$\tilde{y}_{rp}^{tk}$	Neutrosophic Output r from perspective k of DMUj at period t.
link	$\tilde{z}_{dp}^{t(k,h)}$	Neutrosophic intermediate product d between interconnected perspectives k and h of DMUp in period t.
Carry-over	$\tilde{z}_{cp}^{\mathrm{k(t,t+1)}}$	Neutrosophic Carry-over c from perspective k in period t to period t+1.
Input surplus	$\tilde{s}_{ip}^{tk-}$	The surplus associated with $\tilde{s}_{ip}^{tk}$.
Output slack	$\tilde{s}_{rp}^{tk+}$	The Slack associated with $\tilde{y}_{rp}^{tk}$.
Link surplus	$\tilde{s}_{dp}^{t(f,k)-}$	The surplus associated with $\tilde{z}_{dp}^{t(k,h)}$.
Carry-over slack	$\tilde{s}_{dp}^{t(k,h)+}$	The slack associated with $\tilde{z}_{dp}^{t(k,h)}$.
intensity	λ_j^{tk}	Intensity of DMUj corresponding to perspective k in period t.
Intermediate classifier	$BIN_d^{t(k,h)}$	a binary variable which determines the role of intermediate $z_d^{t(k,h)}$ as input to h or output from k.
Carry-over classifier	$bin_c^{(t,t+1)}$	a binary variable which determines the role of carry-over $z_c^{(t,t+1)}$ as input to period t+1 or output from period t.
symbol	$h_k^{(\mathrm{t,t+1})}$	The number of all carry overs from perspective k between periods t and t+1
symbol	$l_{t(f,k)}$	The number of all intermediate measure between perspective h and k in period t

3.1. Phase-I: Role determination

As discussed earlier, the role of intermediate measures and carry overs are determined in the first phase of the proposed approach. Equations (12)-(29) present the model for Phase-I, where inputs, outputs, intermediate measures, and carryovers are represented as Neutrosophic numbers.

$$\tilde{\psi}^{*}_{phase-I} = Max \sum_{t=1}^{T} W^{t} \sum_{k=1}^{K} W^{k} \left[\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{tk-}}{\tilde{x}_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(f,k)-}}{\tilde{z}_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{(t,(t+1))-}}{\tilde{z}_{cp}^{k(t,t+1)}} + \sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}}) \right]. \tag{12}$$

$$\text{s.t. } \textstyle\sum_{j=1}^{n} \lambda_j^{tk} \tilde{x}_{ij}^{tk} + \tilde{s}_{ip}^{tk-} = \tilde{x}_{ip}^{tk}, \quad (k = 1, \dots, K), (i = 1, \dots, m_k). \tag{13}$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{y}_{rj}^{tk} - \tilde{s}_{rp}^{tk+} = \tilde{y}_{rp}^{tk}, \quad (k = 1, \dots, K), (r = 1, \dots, r_k). \tag{14}$$

$$\sum_{j=1}^{n} \lambda_j^{th} \tilde{z}_{dj}^{t(k,h)} + \tilde{s}_{dp}^{t(k,h)-} = \tilde{z}_{d}^{t(k,h)}, \quad (d = 1, \dots, l_{(k,h)}), \forall (k,h). \tag{15}$$

$$\tilde{s}_{dp}^{t(k,h)-} \leq MBIN_d^{t(k,h)}, \quad (d = 1, \dots, l_{(k,h)}), \forall (k,h). \tag{16}$$

$$\tilde{z}_{dp}^{t(k,h)} - M(1 - BIN_d^{t(k,h)}) \leq \tilde{z}_{d}^{t(k,h)} \leq \tilde{z}_{dp}^{t(k,h)} + M(1 - BIN_d^{t(k,h)}). \tag{17}$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{dj}^{t(k,h)} - \tilde{s}_{dp}^{t(k,h)+} = \tilde{z}'^{t(k,h)}_{d}. \tag{18}$$

$$\tilde{s}_{dp}^{t(k,h)+} \leq M(1 - BIN_d^{t(k,h)}), \quad (d = 1, \dots, l_{(k,h)}), \forall (k,h). \tag{19}$$

$$\tilde{z}_{dp}^{t(k,h)} - M.BIN_d^{t(k,h)} \leq \tilde{z}_{d}^{t(k,h)} \leq \tilde{z}_{dp}^{t(k,h)} + M.BIN_d^{t(k,h)}, \quad (d = 1, \dots, l_{(k,h)}), \forall (k,h) \tag{20}$$

$$\textstyle\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{dj}^{t(k,h)} = \sum_{j=1}^{n} \lambda_j^{th} \tilde{z}_{dj}^{t(k,h)}, \quad (d = 1, \dots, l_{(k,h)}), \forall (k,h) \tag{21}$$

$$\textstyle\sum_{j=1}^{n} \lambda_j^{kt} \tilde{z}_{cj}^{(t,t+1)} = \sum_{j=1}^{n} \lambda_j^{k(t+1)} \tilde{z}_{cj}^{(t,t+1)}. \tag{22}$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{cj}^{t(t+1)} + \tilde{s}_{cp}^{(t,(t+1))-} = \tilde{z}_{c}^{k(t,t+1)}. \tag{23}$$

$$\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))-} \leq M.bin_c^{(t,t+1)}. \tag{24}$$

$$\tilde{z}_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} - M.\left(1 - bin_c^{(t,t+1)}\right) \leq \tilde{z}_c^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} \leq \tilde{z}_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} + M.(1 - bin_c^{(t,t+1)}). \tag{25}$$

$$\sum_{j=1}^{n} \lambda_j^{k(t+1)} \tilde{z}_{cj}^{\mathrm{t}(\mathrm{t}+1)} - \tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))+} = \tilde{z}_c^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)}. \tag{26}$$

$$\tilde{z}_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} - M.bin_c^{(t,t+1)} \leq \tilde{z}_c^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} \leq \tilde{z}_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)} + M.bin_c^{(t,t+1)}. \tag{27}$$

$$\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))+} \leq M.\left(1 - bin_c^{(t,t+1)}\right). \tag{28}$$

$$BIN_d^{t(k,h)}, bin_c^{(t,t+1)} \in \{0,1\}; z_d^{(k,h)}, z'^{(k,h)}_d: free, \lambda_j^{tk} \geq 0, s_{rp}^{tk+} \geq 0, s_{ip}^{tk-} \geq 0, s_{dp}^{t(k,h)-} \geq 0, s_{dp}^{t(k,h)+} \geq 0, s_{cp}^{k(\mathrm{t},(\mathrm{t}+1))+}, s_{cp}^{k(\mathrm{t},(\mathrm{t}+1))-}; \forall d, \forall t, \forall c, \forall (k,h). \tag{29}$$

The model presented in Phase I is a MILP model. The values of W^k and W^t are the relative weights of perspectives and periods, respectively, which are determined exogenously by the Decision maker. M is a large positive number used to ensure the proper activation or deactivation of constraints based on the values of binary variables. The objective function (12) maximizes the reduction or expansion rate of inputs, outputs, intermediate variables, and carryovers.

Equations (13) and (14) represent input and output constraints, respectively, for perspective k of DMUj in period t. Equations (15) to (21) represent intermediate measure constraints. The binary variable $BIN_d^{t(k,h)}$ determines the role of intermediate measure $z_{dp}^{t(k,h)}$. When $BIN_d^{t(k,h)} = 1$, constraints (15) to (17) become active, and constraints (18) to (20) become redundant, considering the intermediate measure as an input with its reduction rate calculated in the objective function. Conversely, when $BIN_d^{t(k,h)} = 0$, constraints (15) to (17) become redundant, and constraints (18) to (20) become active, considering the intermediate measure as an output with its expansion rate calculated in the objective function. Equation (21) ensures continuity between perspectives.

Equations (22)-(28) represents the carry-overs' constraints. Similarly, $bin_c^{(t,t+1)}$ determines the role of carry-over $z_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)}$. If $bin_c^{(t,t+1)} = 1$, the constraints (23)-(25) become active and the constraints (26)-(28) the carry-over $z_{cp}^{\mathrm{k}(\mathrm{t},\mathrm{t}+1)}$ is considered as input to period t+1 and its reduction rate is included in the objective function. Conversely, if $bin_c^{(t,t+1)} = 0$ the carry over is considered as output from period t and its expansion rate is included in the objective function. The constraint (28) forces $\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))+}$ to be zero when the binary variable $bin_c^{(t,t+1)} = 1$, effectively deactivating it. When $bin_c^{(t,t+1)} = 1$, the value of $\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))+}$ is allowed to be any non-negative value up to M. Similarly, the constraint (24) forces $\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))-}$ to be zero when the binary variable $bin_c^{(t,t+1)} = 0$ and when $bin_c^{(t,t+1)} = 1$, $\tilde{s}_{cp}^{k(\mathrm{t},(\mathrm{t}+1))-}$ can take any non-negative value up to M.

Equation (22) ensures the continuity assumption between two consecutive periods. The optimal solution of the first phase determines the role of intermediate measures and carry-over activities. In the second phase based on the results obtained from the first phase, we measure the perspectives' efficiencies.

3.2. Phase-II: efficiency evaluation

In this phase, considering the role of intermediate measures and carry-overs obtained from the first phase, we solve the following programming presented in Equations (29)-(39).

$$\text{Min } \tilde{\rho}_p = \frac{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l_{(f,k)}^{t}} BIN_d^{*t(f,k)} + \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(f,k)-}}{z_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)-}}{z_{cp}^{k(t,t+1)}}\right)\right]}{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 + \frac{1}{r_k + l_{(f,k)}^{t} + h_k^{(t,t+1)} - \sum_{d=1}^{l_{(f,k)}^{t}} BIN_d^{*t(f,k)} - \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}}\right)\right]}, \quad (29)$$

$$\text{s.t. } \sum_{j=1}^{n} \lambda_j^{tk} \tilde{x}_{ij}^{tk} + \tilde{s}_{ip}^{tk-} = \tilde{x}_{ip}^{tk}, \quad (\mathrm{k}=1,\ldots,\mathrm{K}), (\mathrm{i}=1,\ldots,\mathrm{m_k}). \quad (30)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{y}_{rj}^{tk} - \tilde{s}_{rp}^{tk+} = \tilde{y}_{rp}^{tk}, \quad (\mathrm{k}=1,\ldots,\mathrm{K}), (\mathrm{r}=1,\ldots,\mathrm{r_k}). \quad (31)$$

$$\sum_{j=1}^{n} \lambda_j^{th} \tilde{z}_{dj}^{t(k,h)} + \tilde{s}_{dp}^{t(k,h)-} = \tilde{z}_{dp}^{t(k,h)}, \quad \left\{d \mid \forall (k,h), BIN_d^{*t(k,h)} = 1\right\}. \quad (32)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{dj}^{t(k,h)} - \tilde{s}_{dp}^{t(k,h)+} = \tilde{z}_{dp}^{t(k,h)}, \quad \left\{d \mid \forall (k,h), BIN_d^{*t(k,h)} = 0\right\}. \quad (33)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{dj}^{t(k,h)} = \sum_{j=1}^{n} \lambda_j^{th} \tilde{z}_{dj}^{t(k,h)}, \quad \forall d, d \in \left\{1, \ldots, l_{(k,h)}\right\}, \forall (k,h), \forall t, \quad (34)$$

$$\sum_{j=1}^{n} \lambda_j^{kt} \tilde{z}_{cj}^{(t,t+1)} = \sum_{j=1}^{n} \lambda_j^{k(t+1)} \tilde{z}_{cj}^{(t,t+1)}, \quad \forall c, c \in \left\{1, \ldots, h_k^{(t,t+1)}\right\}, \forall k, \forall t \quad (35)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} \tilde{z}_{cj}^{t(t+1)} + \tilde{s}_{cp}^{(t,(t+1))-} = \tilde{z}_{cp}^{k(t,t+1)}, \quad \left\{c \mid \forall (t,t+1), bin_c^{*(t,t+1)} = 1\right\}. \quad (36)$$

$$\sum_{j=1}^{n} \lambda_j^{k(t+1)} \tilde{z}_{cj}^{t(t+1)} - \tilde{s}_{cp}^{k(t,(t+1))+} = \tilde{z}_{cp}^{k(t,t+1)}, \quad \left\{c \mid \forall (t,t+1), bin_c^{*(t,t+1)} = 0\right\}. \quad (37)$$

$$\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{tk-}}{\tilde{x}_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(f,k)-}}{\tilde{z}_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{(t,(t+1))-}}{\tilde{z}_{cp}^{k(t,t+1)}} + \sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}})\right]. \quad (38)$$

$$\tilde{z}_d^{(k,h)}, \tilde{z}'^{(k,h)}_d: free, \lambda_j^k \geq 0, \tilde{s}_{rp}^{k+} \geq 0, \tilde{s}_{ip}^{k-} \geq 0, \tilde{s}_{dp}^{(k,h)-} \geq 0, \tilde{s}_{dp}^{(k,h)+} \geq 0,\ \tilde{s}_{cp}^{k(t,t+1)+},\ \tilde{s}_{cp}^{(t,(t+1))-}. \quad (39)$$

In the model presented for Phase II, the objective function (29) computes the non-oriented overall efficiency. Equation (32) (Equation (33)) is related to those intermediate measures that are determined as inputs (outputs) in Phase I. $\psi^*_{phase-I}$ indicates the optimal value of the objective function in Phase I, and Constraint (38) ensures that the results obtained from the first phase remain unchanged. To ensure feasibility and enhance the robustness of the model, we can express the equality constraint (38) as an inequality and allow for deviations up to ε from it (see Equation (40)).

$$\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{tk-}}{\tilde{x}_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(f,k)-}}{\tilde{z}_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{(t,(t+1))-}}{\tilde{z}_{cp}^{k(t,t+1)}} + \sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}}) \right] \geq \tilde{\psi}^*_{phase-I} - \tilde{\varepsilon}. \quad (40)$$

Additionally, we can penalize this deviation using the penalty parameter τ in the objective function and minimize $\widetilde{\rho_p} + \tau\tilde{\varepsilon}$ instead of minimizing $\widetilde{\rho_p}$.

In the proposed approach, both Phase I and Phase II models utilize inputs, outputs, intermediate products, and carry-overs with Neutrosophic data. Consequently, the efficiency derived from the model is also expressed as a Neutrosophic number. In the next section, our aim is to transform the uncertain model into a deterministic one.

3.3. Period and perspective's efficiency

To measure the efficiency for perspective k in period t, and the period efficiency of DMUp, we compute θ_p^{*tk} and θ_p^{*t} using the Equations (41) and (42), respectively, with the optimal solution of the variables obtained from solving model (12-33).

$$\theta_p^{*tk} = \frac{1 - \frac{1}{m_k + \sum_{d=1}^{l^t_{(f,k)}} BIN_d^{*t(f,k)} + \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t-1,t)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{*tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{*t(f,k)-}}{z_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{*k(t-1,t)-}}{z_{cp}^{k(t,t+1)}} \right)}{1 + \frac{1}{r_k + l^t_{(k,h)} + h_k^{(t,t+1)} - \sum_{d=1}^{l^t_{(k,h)}} BIN_d^{*t(k,h)} - \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}} \right)} \quad (41)$$

$$\theta_p^{*t} = \frac{\sum_{k=1}^{K} W^k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l^t_{(f,k)}} BIN_d^{*t(f,k)} + \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{*tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{*t(f,k)-}}{z_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{*k(t,t+1)-}}{z_{cp}^{*k(t,t+1)}} \right) \right]}{\sum_{k=1}^{K} W^k \left[1 + \frac{1}{r_k + l^t_{(f,k)} + h_k^{(t,t+1)} - \sum_{d=1}^{l^t_{(f,k)}} BIN_d^{*t(f,k)} - \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{*tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{*t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{*k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}} \right) \right]}. \quad (42)$$

3.4. Managing model uncertainty

Based on Definition 4 presented by Akram et al. [26] for a TNN at any variation degree $\varphi_{(\alpha,\beta,\gamma)}$, the upper and lower limits can be calculated. Therefore, for all the data within the model, both upper and lower bounds can be determined. Suppose ,$[(X_{ij}^{tk})^L_\varphi, (X_{ij}^{tk})^U_\varphi]$, $[(Y_{rj}^{tk})^L_\varphi, (Y_{rj}^{tk})^U_\varphi]$, $[(z_{cj}^{t(t+1)})^L_\varphi, (z_{cj}^{t(t+1)})^U_\varphi]$ and $[(z_{dj}^{t(k,h)})^L_\varphi, (z_{dj}^{t(k,h)})^U_\varphi]$ are the Neutrosophic data at a φ variation degree.

According to the Pareto efficiency concept, the maximum efficiency of a specific DMU at variation degree φ occurs when it minimizes input consumption while maximizing output production, while other DMUs maximize input consumption and minimize output production. Conversely, the minimum efficiency occurs when the DMU under consideration maximizes input consumption while minimizing output production, while other DMUs maximize output production and minimize input consumption. Therefore, the boundaries of $\tilde{\rho}_p^*$ and $\tilde{\psi}_{phase-I}^*$ at φ variation degree can be measured.

For clarity and efficiency, we will focus on presenting the model detailing the lower bound of $\tilde{\rho}_p$ at φ variation degree ($\rho_p^{\varphi L}$). The models for the upper limit of $\tilde{\rho}_p(\rho_p^{\varphi U})$, as well as the lower and upper limits of $\tilde{\psi}_{phase-I}$ at φ variation degree ($i.e.\ \psi_{phase-I}^{\varphi L}$ and $\psi_{phase-I}^{\varphi U}$), can be formulated using the same approach. Equations (43)-(53) present the model for measuring $\rho_p^{\varphi L}$. Note that we use the results of the corresponding boundary in Phase I to solve Phase II. For instance, we utilize the optimal solution of lower limit $\psi_{phase-I}^{\varphi L}$ for classifying the intermediate measure and carry overs and measuring $\rho_p^{\varphi L}$.

$$\text{Min } \rho_p^{\varphi L} = \frac{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l_{(f,k)}^{t}} BIN_d^{*t(f,k)} + \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{ip}^{tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(f,k)-}}{z_{dp}^{t(f,k)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)-}}{z_{cp}^{k(t,t+1)}}\right)\right]}{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 + \frac{1}{r_k + l_{(f,k)}^{t} + h_k^{(t,t+1)} - \sum_{d=1}^{l_{(f,k)}^{t}} BIN_d^{*t(f,k)} - \sum_{c=1}^{h_k^{(t,t+1)}} bin_c^{*(t,t+1)}} \left(\sum_{i=1}^{m_k} \frac{\tilde{s}_{rp}^{tk+}}{\tilde{y}_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{\tilde{s}_{dp}^{t(k,h)+}}{\tilde{z}_{dp}^{t(k,h)}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{\tilde{s}_{cp}^{k(t,t+1)+}}{\tilde{z}_{cp}^{k(t,t+1)}}\right)\right]}. \quad (43)$$

$$\text{s.t. } \sum_{j=1}^{n} \lambda_j^{tk} (x_{ij}^{tk})_\varphi^L + \left(s_{ip}^{tk-}\right)_\varphi^U = (x_{ip}^{tk})_\varphi^U,$$
$$(k = 1,\dots, K), (i = 1,\dots, m_k) \quad (44)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} (y_{ij}^{tk})_\varphi^U - (s_{rp}^{tk+})_\varphi^L = (y_{rp}^{tk})_\varphi^L,$$
$$(k = 1,\dots, K), (r = 1,\dots, r_k) \quad (45)$$

$$\sum_{j=1}^{n} \lambda_j^{th} (z_{dj}^{t(k,h)})_\varphi^L + (s_{dp}^{t(k,h)-})_\varphi^U = (z_{dp}^{t(k,h)})_\varphi^U,$$
$$\left\{d \middle| \forall (k,h), BIN_d^{*t(k,h)} = 1\right\}. \quad (46)$$

$$\sum_{j=1}^{n} \lambda_j^{th} (z_{dj}^{t(k,h)})_\varphi^U - (s_{dp}^{t(k,h)+})_\varphi^L = (z_{dp}^{t(k,h)})_\varphi^L,$$
$$\left\{d \middle| \forall (k,h), BIN_d^{*t(k,h)} = 0\right\}. \quad (47)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} (z_{dj}^{t(k,h)})_\varphi^U = \sum_{j=1}^{n} \lambda_j^{th} (z_{dj}^{t(k,h)})_\varphi^L,$$
$$\forall d, d \in \left\{1, \dots, l_{(k,h)}\right\}, \forall (k,h), \forall t. \quad (48)$$

$$\sum_{j=1}^{n} \lambda_j^{kt} (z_{cj}^{(t,t+1)})_\varphi^U = \sum_{j=1}^{n} \lambda_j^{k(t+1)} (z_{cj}^{(t,t+1)})_\varphi^L,$$
$$\forall c, c \in \left\{1, \dots, h_k^{(t,t+1)}\right\}, \forall k, \forall t. \quad (49)$$

$$\sum_{j=1}^{n} \lambda_j^{k(t+1)} (z_{cj}^{(t,t+1)})_\varphi^L + (s_{cp}^{(t,t+1)-})_\varphi^U = (z_{cj}^{(t,t+1)})_\varphi^U,$$
$$\left\{c \middle| \forall (t, t+1), bin_c^{*(t,t+1)} = 1\right\} \quad (50)$$

$$\sum_{j=1}^{n} \lambda_j^{kt} (z_{cj}^{(t,t+1)})_\varphi^U - (s_{cp}^{(t,t+1)+})_\varphi^L = (z_{cj}^{(t,t+1)})_\varphi^L,$$
$$\left\{c \middle| \forall (t, t+1), bin_c^{*(t,t+1)} = 0\right\}. \quad (51)$$

$$\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[\sum_{i=1}^{m_k} \frac{(s_{ip}^{tk-})_\varphi^U}{(x_{ip}^{tk})_\varphi^U} + \sum_{d=1}^{l_{(f,k)}} \frac{(s_{dp}^{t(k,h)-})_\varphi^U}{(z_{dp}^{t(k,h)})_\varphi^U} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{(s_{cp}^{(t,t+1)-})_\varphi^U}{(z_{cj}^{(t,t+1)})_\varphi^U} + \sum_{i=1}^{m_k} \frac{(s_{rp}^{tk+})_\varphi^L}{(y_{rp}^{tk})_\varphi^L} + \sum_{d=1}^{l_{(f,k)}} \frac{(s_{dp}^{t(k,h)+})_\varphi^L}{(z_{dp}^{t(k,h)})_\varphi^L} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{(s_{cp}^{(t,t+1)+})_\varphi^L}{(z_{cj}^{(t,t+1)})_\varphi^L}) \right]. \quad (52)$$

$$\lambda_j^k \geq 0, (s_{rp}^{tk+})_\varphi^L \geq 0, (s_{ip}^{tk-})_\varphi^U \geq 0, (s_{dp}^{t(k,h)-})_\varphi^U \geq 0, (s_{dp}^{t(k,h)+})_\varphi^L \geq 0,\ (s_{cp}^{(t,t+1)+})_\varphi^L \geq 0, (s_{cp}^{(t,t+1)-})_\varphi^U \geq 0. \quad (53)$$

In this subsection, we managed the uncertainty within the proposed model by converting it to deterministic models to compute the efficiency boundaries at varying degrees of variation. In the following sections, we aim to verify the proposed model by applying it to a case study. To do this, we first apply Neutrosophic DEMATEL to construct the network structure of the SBSC in the petrochemical industry.

3. Constructing the network structure of SBSC in the petrochemical industry

In this section, we first apply Neutrosophic DEMATEL to explore the relationships among SBSC perspectives. Subsequently, we utilize expert insights to define the network structure of SBSC, which serves as the Decision-Making Unit.

4.1. Exploring the relationship among SBSC perspectives

In this subsection, we employ the Neutrosophic DEMATEL approach to identify the cause-and-effect relationships within the Sustainable Balanced Scorecard (SBSC). To achieve this, we conducted structured interviews with five experts who have significant experience in the petrochemical field, with a focus on strategy formulation and performance evaluation. The experts were asked to provide pairwise comparisons of the SBSC perspectives based on their understanding of the interrelationships between various performance dimensions, such as financial, customer, internal processes, learning and growth, and environmental factors. The pairwise comparison matrix in Table 3 represents the evaluation provided by one of these experts.

Table 3. Pairwise comparison matrix for the expert1

		T1					T2				
		F1	C1	I1	L1	E1	F2	C2	I2	L2	E2
T1	F1	1	1	3	1	4	2	5	4	7	1
	C1	9	1	1	1	9	9	4	4	5	5
	I1	5	9	1	1	3	4	9	5	6	8
	L1	7	4	9	1	4	3	1	8	7	4
	E1	4	4	4	3	1	1	4	5	1	2
T2	F2	1	1	1	1	1	1	1	1	1	1
	C2	1	1	1	1	1	9	1	1	1	1
	I2	1	2	1	2	1	5	9	1	1	8
	L2	1	1	1	1	1	1	4	9	1	2
	E2	1	1	1	3	1	3	3	1	1	1

After converting the scales in the direct-relation matrix into triangular Neutrosophic numbers, we represented the inherent uncertainty, hesitation, and possible contradiction in expert judgments. Triangular Neutrosophic numbers allow us to capture three aspects of each expert's evaluation: truth

(T), indicating how true the relationship is; indeterminacy (I), representing uncertainty or ambiguity; and falsity (F), reflecting the degree to which the relationship may not hold.

After gathering all five experts' pairwise comparison matrices, we applied the Neutrosophic aggregation method to integrate their judgments into a single unified matrix. This method aggregates the triangular Neutrosophic numbers by mathematically averaging the values for truth (T), indeterminacy (I), and falsity (F) from each expert, resulting in a consensus matrix that represents the collective viewpoints.

By using this approach, we ensured that not only the true relationships but also any indeterminate or contradictory aspects of the expert evaluations were captured and reflected in the final analysis. Once the aggregation process was complete, the consensus matrix was normalized to ensure comparability across the different perspectives, and the total direct-relation matrix, presented in Table 4, was constructed. A threshold value of 0.45 was then applied, as agreed upon by the expert panel, to filter the most significant cause-and-effect relationships for inclusion in the network structure. Only relationships with values exceeding this threshold were included in the final strategy map, which visually represents the network of interrelated SBSC perspectives.

Table 4. Total relationship matrix.

E2	L2	I2	C2	F2	E1	L1	I1	C1	F1	
0.014	0.025	0.014	0.041	0.025	0.014	0.042	0.842	0.25	0.25	F1
0.025	0.051	0.256	0.256	0.256	0.44	0.042	0.621	0.44	1.089	C1
0.256	0.025	0.025	0.025	0.051	0.051	0.042	0.025	1.089	0.823	I1
0.41	0.025	0.256	0.256	0.025	0.042	0.042	1.089	0.025	0.24	L1
0.355	0.012	0.014	0.025	0.256	0.11	0.542	1.02	0.21	0.11	E1
0.416	0.025	0.025	0.042	0.042	0.025	0.423	0.404	0.725	0.025	F2
0.51	0.042	0.042	0.014	1.089	0.256	0.025	0.256	0.256	0.089	C2
0.014	0.256	0.014	1.089	0.823	0.256	0.042	0.322	0.089	0.823	I2
0.042	0.042	1.089	0.042	0.042	0.025	0.823	0.042	0.426	0.509	L2
0.042	1.142	1.025	0.042	0.042	0.41	0..025	0.042	0.042	0.025	E2

Considering cause-and-effect relationships among the five perspectives—financial, customer, internal business processes, environment, and learning and growth—within the SBSC over a specific time period, network relationships among these perspectives were determined. The time delay factor caused by lagging key performance indicators was taken into account to identify cause-and-effect relationships among the SBSC perspectives at different times, reflecting dynamic network relationships. In Figure 2, The cause-and-effect relationships among the five perspectives of the SBSC for the Petrochemical Company at a specific time are represented by solid lines. Dynamic network relationships over different times are represented by dashed lines.

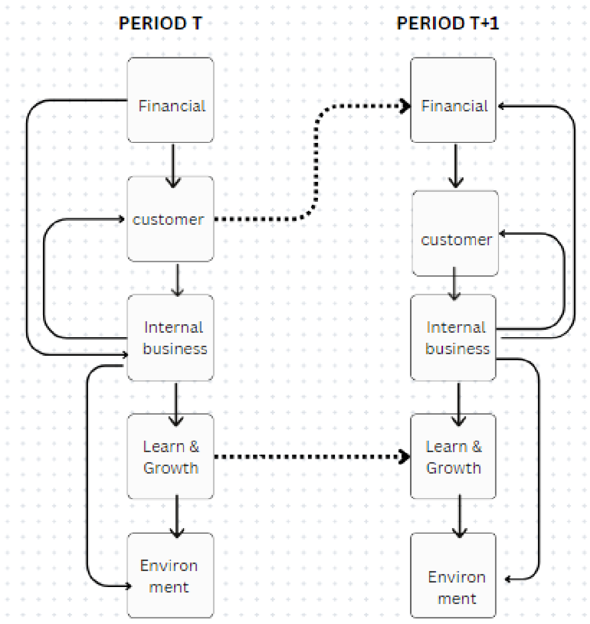


Figure 2. The cause-and-effect relationships among the perspectives of SBSC.

Figure 3 also illustrates the causal diagram based on the results obtained by $Q + R$ and $Q - R$.

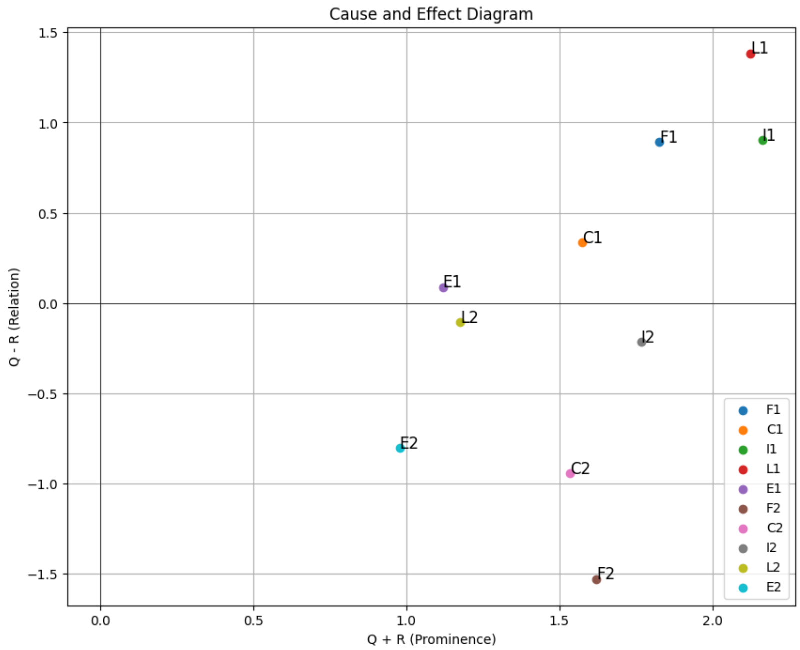


Figure 3. Causal diagram of $Q + R$ vs $Q - R$.

The study findings from the causal diagram (Figure 3) can be summarized as follows: The perspectives of the SBSC in period T—namely (F1), (L1), (I1), (C1), and (E1)—situated above the Q-R axis, act as influential factors on the evaluation criteria. These criteria, in turn, impact the perspectives of the SBSC in period T+1, specifically (F2), (L2), (I2), (C2), and (E2), positioned below the Q-R axis. Its noteworthy that on the Q+R axis, the significance of the criterion increases as we move farther away from zero.

4.2. Defining KPIs and network structure of DMU

Given the relationships identified among the perspectives of SBSC through Neutrosophic DEMATEL, this subsection focuses on defining key performance indicators (KPIs) to illustrate these relationships and construct the final network structure. To establish a comprehensive set of KPIs for the SBSC in the petrochemical industry, we consulted industry professionals. Their collaboration ensured that the KPIs are closely aligned with the sector's specific needs and strategic goals. Table 5. details the defined KPIs, their types, associated strategic initiatives, and corresponding codes.

Table 5. The types of KPIs and their strategic initiatives.

Perspective	Key Performance Indicator (Kpi)	Type	Strategic Initiative	Code	Related References
Financial	Operating Costs	Pure Input to financial perspective	TQM in Budget Planning and Execution to Optimize Operating Costs	OC	[27]
	Investment Capital	Pure Input to financial perspective	Governance Framework with ISO 21500 for Strategic Capital Allocation"	IC	[28], [29]
	Revenue	Pure output from financial perspective	ISO 9001 Certification for Quality Management to Drive Revenue Growth	RE	[29]
	Marketing and Sales Expenses	Output from financial perspective and input to customer perspective	Supplier Quality Management (ISO 9001) to Optimize Marketing and Sales Expenses	MSE	[30]
Customer	Customer Support Costs	Output from financial perspective and input to customer perspective	ISO 10015 for Training to Enhance Customer Support Efficiency	CSC	[31]
	Customer satisfaction rate	Carry over form customer perspective in period t to financial perspective in period t+1	ISO 9001 Certification for Quality Management to Improve Customer Satisfaction"	CSR	[32]

Table 5. Continue.

Perspective	Key Performance Indicator (Kpi)	Type	Strategic Initiative	Code	Related References
Internal Business	R&D Costs	Intermediate from financial perspective to internal business	ISO 14064 for Greenhouse Gas Emissions Management to Optimize R&D Costs	RD	[33]
	Operational Expenditures	Intermediate from financial perspective to internal business	ISO 50001 Energy Management to Reduce Operational Expenditures	OE	[30]
	Product Quality	Intermediate from internal business to customer perspective	ISO 9001 Certification for Quality Management to Ensure Product Excellence	PQ	[34]
Growth & Learning	Innovation Rate	Intermediate from internal process to learning & growth perspective	ISO 9001 Certification for Quality Management to Foster Innovation	IR	[35]
	Training Hours per Employee	Pure input to learning and growth perspective	ISO 10015 for Training	TH	[36]
	Green skills development	Intermediate from learning and growth to environment perspective	ISO 14001 for Resource Efficiency and Environmental Responsibility for Green Skills Development	GS	[37]
	Organizational knowledge	Carry over from learning and growth perspective in period to the same perspective in period t+1	ISO 27001 for Information Security Management	OK	[30]
Environment	CO2 Emissions Reduction Rate	Pure output of environment perspective	ISO 14064 for Greenhouse Gas Emissions Management	CO	[37]
	Waste Reduction Percentage	Carry over from internal business perspective to environment perspective	TQM in Waste Reduction Strategies	WR	[38]

Based on the Table 5 and the results of Neutrosophic DEMATEL, the final structure of SBSC is illustrated in Figure 4.

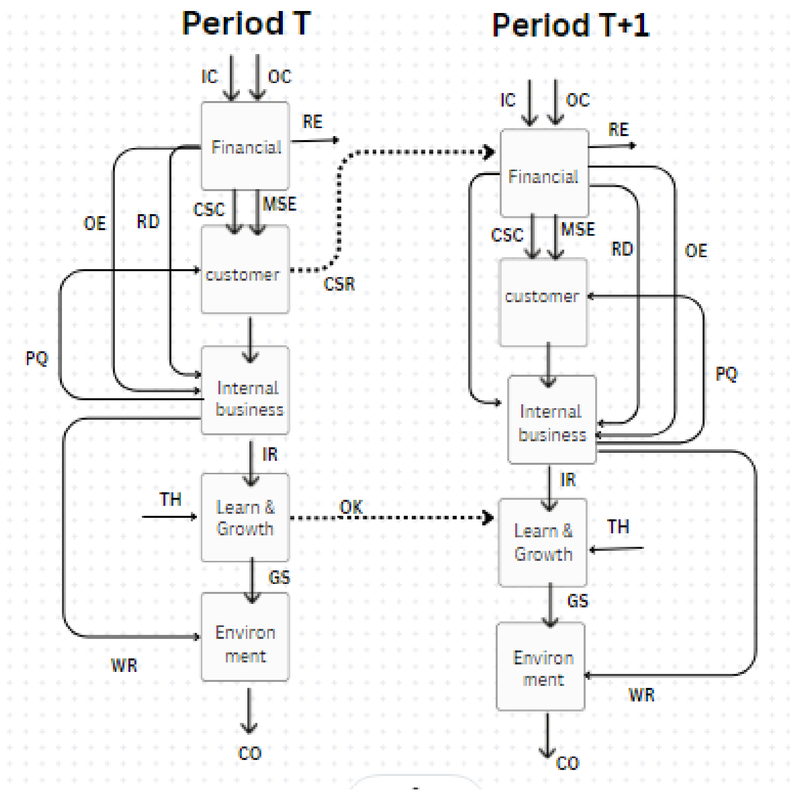


Figure 4. The structure of SBSC as a DMU.

4. Case Study: Marun Petrochemical Complex (MPC)

Marun Petrochemical Complex (MPC) operates on the foundational principle of segregating upstream activities. MPC, consisting of nine oil refineries, assumes the responsibilities previously managed by the National Iranian Oil Company (NIOC). These responsibilities encompass crude oil transportation to refineries and export terminals, processing, production, and distribution of various primary oil products and byproducts across Iran, marketing and exporting surplus specialty products, constructing refineries, marine platforms, pipelines, and communication networks, and facilitating extensive communication links within and beyond the Oil Ministry's industrial and administrative headquarters. This paper focuses on evaluating the efficiency of MPC's nine oil plants as a case study. Furthermore, the related data have been collected from various sources, including insights from experts, online resources, the Codal.ir platform and the Iranian Statistical Center.

To obtain the overall, period, and perspective efficiencies of the DMUs under evaluation, we must determine the roles of carry-overs and intermediate measures by implementing Phase I at the desired degree of variation. For instance, Table 6 represents the roles of carry-overs and intermediate measures obtained from the optimal solution of $\psi^{\varphi L}_{phase-I}$ at $\boldsymbol{\varphi}_{(\mathbf{0.2,0.7,0.9})}$.

Table 6. The roles of carry-overs and intermediate measures for $\psi_{phase-I}^{\varphi L}$ at $\varphi_{(0.2,0.7,0.9)}$.

	Kpi	Intermediate Measure								Carry-Over	
	DMU	MSE	CSC	RD	OE	PQ	IR	GS	WR	CSR	OK
Year 1400	DMU1	INPUT	OUTPUT	INPUT	INPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT
	DMU2	INPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT
	DMU3	INPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT
	DMU4	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT
	DMU5	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT
	DMU6	INPUT	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	INPUT	INPUT	INPUT	OUTPUT
	DMU7	OUTPUT	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	INPUT	INPUT	OUTPUT	OUTPUT
	DMU8	OUTPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	OUTPUT
	DMU9	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	INPUT	OUTPUT	OUTPUT
Year 1401	DMU1	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	INPUT	OUTPUT	INPUT
	DMU2	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	INPUT	INPUT	OUTPUT	OUTPUT	INPUT
	DMU3	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	INPUT	OUTPUT	OUTPUT	INPUT
	DMU4	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT
	DMU5	INPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT
	DMU6	OUTPUT	OUTPUT	INPUT	INPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	OUTPUT
	DMU7	OUTPUT	OUTPUT	INPUT	INPUT	INPUT	OUTPUT	INPUT	OUTPUT	INPUT	INPUT
	DMU8	INPUT	INPUT	OUTPUT	OUTPUT	OUTPUT	INPUT	INPUT	OUTPUT	OUTPUT	INPUT
	DMU9	INPUT	INPUT	OUTPUT	INPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	INPUT	INPUT
	DMU1	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT	INPUT	OUTPUT	OUTPUT	INPUT	OUTPUT
	DMU2	OUTPUT	OUTPUT	OUTPUT	INPUT	INPUT		OUTPUT	OUTPUT	INPUT	OUTPUT

After determining the roles of intermediates and carry-overs for different degrees of variation, we measure the lower and upper bounds of the overall efficiency of the DMUs under evaluation. Table 7 represents the boundaries of the overall score at different degrees of variation.

Table 7. The boundaries of the overall score at different degrees of variation.

DMU	$[\rho_p^{\varphi L}, \rho_p^{\varphi U}]$ at $\varphi_{(0,1.0,1.0)}$	$[\rho_p^{\varphi L}, \rho_p^{\varphi U}]$ at $\varphi_{(0.2,0.7,0.9)}$	$[\rho_p^{\varphi L}, \rho_p^{\varphi U}]$ at $\varphi_{(0.5,0.8,0.9)}$
DMU1	[0.11,0.85]	[0.24,0.85]	[0.31,0.65]
DMU2	[0.44,1.00]	[0.51,1.00]	[0.53,1.00]
DMU3	[0.21,0.69]	[0.25,0.61]	[0.28,0.55]
DMU4	[0.33,1.00]	[0.38,1.00]	[0.41,1.00]
DMU5	[0.19,0.74]	[0.28,0.69]	[0.31,0.69]
DMU6	[0.45,0.98]	[0.45,0.91]	[0.55,0.91]
DMU7	[0.18,0.59]	[0.22,0.51]	[0.36,0.50]
DMU8	[0.29,0.78]	[0.33,0.71]	[0.41,0.71]
DMU9	[0.71,1.00]	[0.74,1.00]	[0.85,1.00]

Figure 5 compares the scores at different degrees of variation. the error bars represent the lower and upper bounds for each degree of variation and the lines for each degree of variation ($\varphi_{(0,1.0,1.0)}$, $\varphi_{(0.2,0.7,0.9)}$, $\varphi_{(0.5,0.8,0.9)}$) with points represent the mid-value of the bounds.

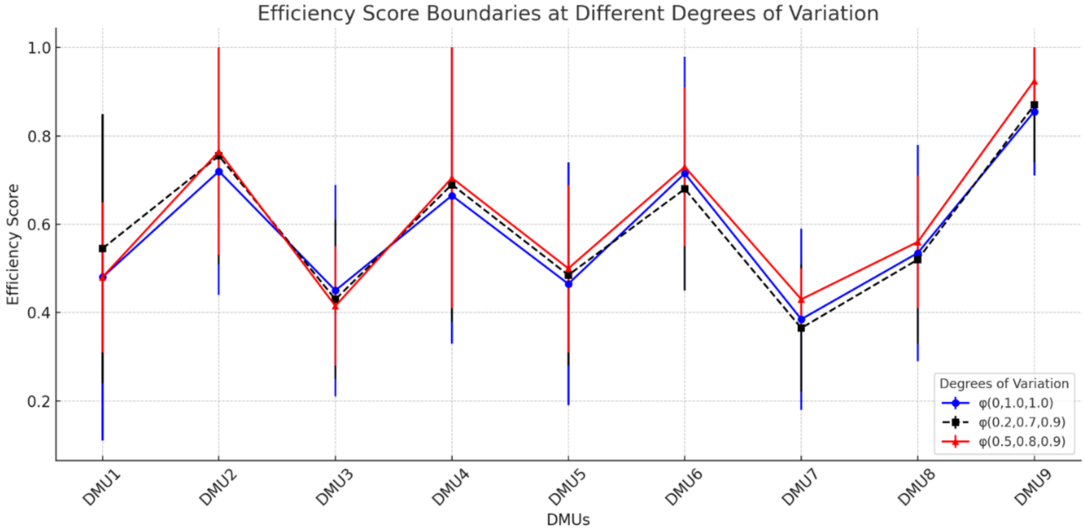


Figure 5. Comparison of the efficiency scores at different degrees of variation.

DMU9 consistently shows the highest lower and upper bound of efficiency across all variation degree, with the lowest score starting at 0.71 and reaching up to 0.85. This indicates that DMU9 performs the best among all DMUs. DMU9's consistently high scores suggest that it is the most robust or adaptable DMU among the DMUs under evaluation. DMU1 and DMU5 have room for significant performance enhancement. Their efficiency is less robust in comparison to higher-performing DMUs like DMU9.

After completing Phase-II and obtaining the optimal solutions for the variables, we can calculate the perspective scores and period scores using the appropriate equations. Specifically, the perspective scores are determined using Equation (46), while the period scores are calculated using Equation (47). Table 8 presents the lower period and perspective efficiencies of the DMUs under evaluation at $\boldsymbol{\varphi}_{(0.5,0.8,0.9)}$.

Table 8. The period efficiency of the DMUs at $\varphi_{(0,1.0,1.0)}$.

	1400	1401
DMU	$[\rho_p^{\varphi L}, \rho_p^{\varphi U}]$	$[\rho_p^{\varphi L}, \rho_p^{\varphi U}]$
DMU1	[0.14,0.74]	[0.09,0.95]
DMU2	[0.25,1.00]	[0.51,1.00]
DMU3	[0.29,0.69]	[0.18,0.61]
DMU4	[0.26,1.00]	[0.38,1.00]
DMU5	[0.11,0.81]	[0.28,0.69]
DMU6	[0.32,1.00]	[0.45,0.91]
DMU7	[0.16,0.65]	[0.22,0.51]
DMU8	[0.21,0.86]	[0.33,0.71]
DMU9	[0.65,1.00]	[0.74,1.00]

In analyzing the general trends of the lower bounds of scores across the two periods at variation degree of $\varphi_{(0,1.0,1.0)}$, several key observations emerge. DMU2 and DMU6 exhibit the most significant increases in their lower bounds, rising from 0.25 to 0.51 and from 0.32 to 0.45, respectively. This substantial improvement suggests that these DMUs have enhanced their lower score over the period. Conversely, DMU1 and DMU3 show a decrease in their lower bounds, from 0.14 to 0.09 and from

0.29 to 0.18, indicating a decline in their lower efficiency. Meanwhile, DMU4, DMU5, DMU7, and DMU8 display moderate changes, with slight increases or decreases in their scores. Notably, DMU9 consistently achieves the highest lower score in both periods, improving from 0.65 to 0.74, thereby demonstrating superior and stable performance. In contrast, DMU1 remains at the lower end of the performance spectrum, with the lowest lower bound in period 1400 and continuing to have one of the lowest in period 1401. Figure 6 compares the period scores between the years 1400 and 1401.

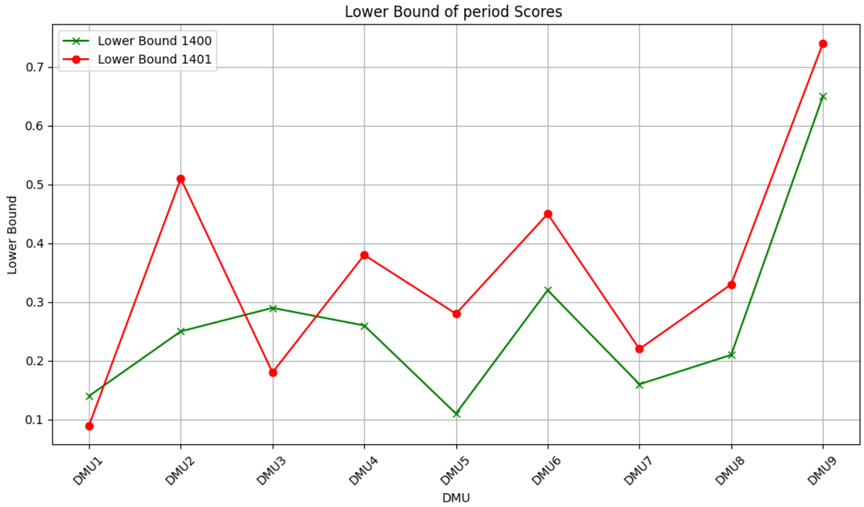


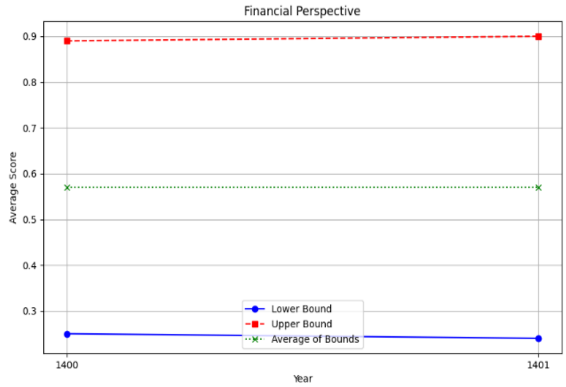
Figure 6. Comparison of the lower period scores between years 1400 and 1401.

In the following, we use the results obtained from Phase-II and apply Equation (46) to compute the perspective scores for the years 1400 and 1401. Table 9 presents the scores for the five perspectives of the SBSC for these years at$\varphi_{(0,1.0,1.0)}$.

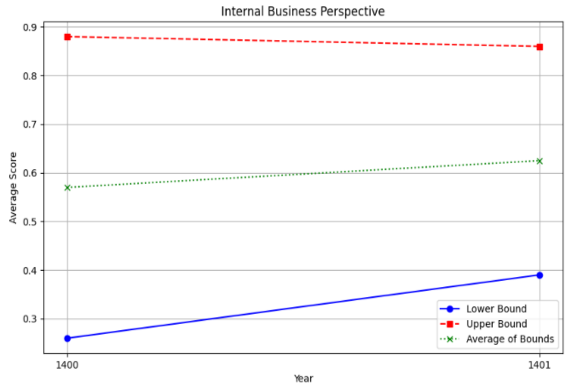
Table 9. The perspectives' scores at $\varphi_{(0,1.0,1.0)}$.

year	1400					1401				
Perspective	Financial	Internal Business	Customer	Learn& Growth	Environment	Financial	Internal Business	Customer	Learn& Growth	Environment
DMU1	[0.08,0.85]	[0.16,0.45]	[0.26,0.1.00]	[0.31,0.79]	[0.07,0.80]	[0.05,1.00	[0.12,0.90]	[0.09,0.95]	[0.15,0.92]	[0.11,0.89]
DMU2	[0.22,1.00]	[0.19,1.00]	[0.25,1.00]	[0.14,1.00]	[0.32,1.00]	[0.45,1.00]	[0.59,1.00]	[0.81,1.00]	[0.40,1.00]	[0.55,1.00]
DMU3	[0.25,0.69]	[0.18,1.00]	[0.31,0.55]	[0.26,0.68]	[0.25,0.75]	[0.05,0.78]	[0.12,0.98]	[0.20,0.69]	[0.21,0.63]	[0.24,0.65]
DMU4	[0.21,1.00]	[0.41,1.00]	[0.14,1.00]	[0.24,1.00]	[0.38,1.00]	[0.45,1.00]	[0.48,1.00]	[0.33,1.00]	[0.41,1.00]	[0.45,1.00]
DMU5	[0.10,0.84]	[0.05,1.00]	[0.15,0.95]	[0.14,0.75]	[0.18,0.89]	[0.18,0.74]	[0.29,0.62]	[0.39,0.87]	[0.21,0.95]	[0.20,0.78]
DMU6	[0.30,1.00]	[0.35,1.00]	[0.41,1.00]	[0.17,1.00]	[0.45,1.00]	[0.32,1.00]	[0.51,0.87]	[0.39,1.00]	[0.41,0.80]	[0.35,0.88]
DMU7	[0.12,0.74]	[0.10,0.65]	[0.09,0.84]	[0.11,0.91]	[0.19,0.68]	[0.12,0.74]	[0.26,0.59]	[0.29,0.65]	[0.14,0.41]	[0.11,0.39]
DMU8	[0.23,0.86]	[0.25,0.80]	[0.19,0.69]	[0.29,0.74]	[0.20,0.95]	[0.21,0.85]	[0.23,0.74]	[0.30,0.72]	[0.25,0.69]	[0.19,0.61]
DMU9	[0.75,1.00]	[0.66,1.00]	[0.55,1.00]	[0.59,1.00]	[0.68,1.00]	[0.71,1.00]	[0.88,1.00]	[0.63,1.00]	[0.85,1.00]	[0.46,1.00]
Average	[0.25,0.89]	[0.26,0.88]	[0.26,89]	[0.25,87]	[0.30,90]	[0.24,0.9]	[0.39,0.86]	[0.38,0.88]	[0.34,0.85]	[0.30,0.81]

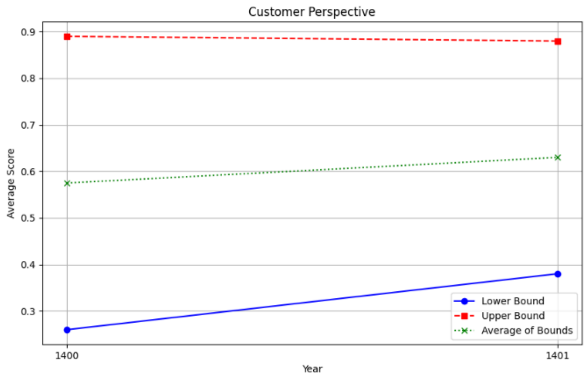
Figure 7 illustrate the trends of the upper, lower, and midpoint averages of perspective scores for the years 1400 and 1401. The results reveal that the DMUs have shown notable improvements in efficiency in the Customer, Learn & Growth, and Internal Business perspectives from 1400 to 1401. In contrast, there has been a decline in efficiency in the Environment perspective. The Financial perspective remains largely unchanged over the same period.


Financial Perspective
Average Score
Year
1400
1401
Lower Bound
Upper Bound
Average of Bounds

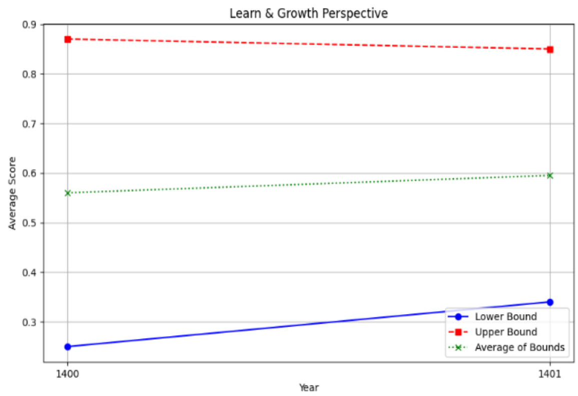
a.


Internal Business Perspective
Average Score
Year
1400
1401
Lower Bound
Upper Bound
Average of Bounds

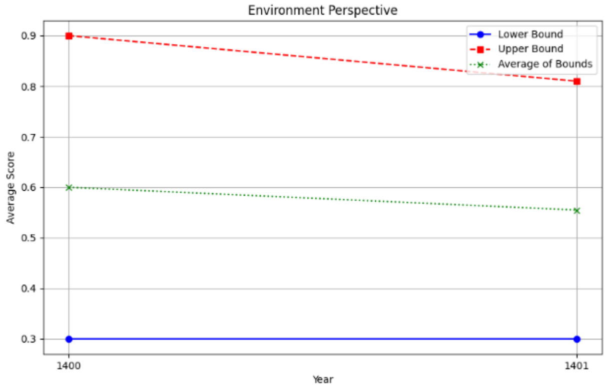
b.


Customer Perspective
Average Score
Year
1400
1401
Lower Bound
Upper Bound
Average of Bounds

c.


Learn & Growth Perspective
Average Score
Year
1400
1401
Lower Bound
Upper Bound
Average of Bounds

d.



e.

Figure 7. The trends of perspectives' scores for the years 1400 and 1401.

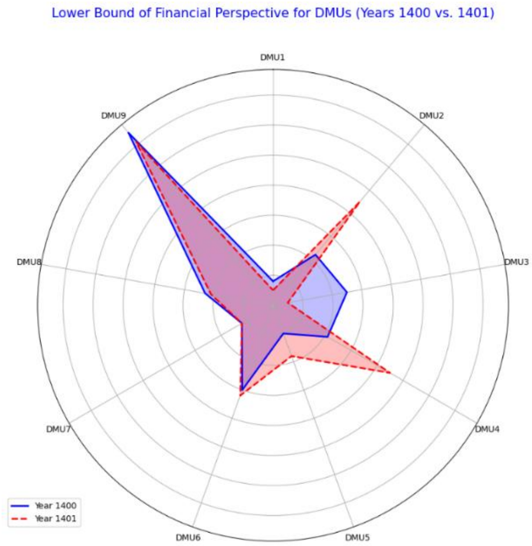
Figures 8 presents a comparative analysis of the lower bounds of the DMUs across various perspectives between the years 1400 and 1401. In the Financial perspective (Figure 8(a)), DMU9 consistently achieves the highest lower bound in both years, reflecting superior performance. Most DMUs exhibit only minor fluctuations, with DMU1 and DMU5 showing slight improvements but still remaining among the lower performers.

In the Internal Business perspective (Figure 8(b)), DMU4 and DMU9 lead with higher lower bounds, indicating strong performance. However, DMU1 demonstrates a decline from 1400 to 1401, while DMU6 and DMU7 show some positive progress.

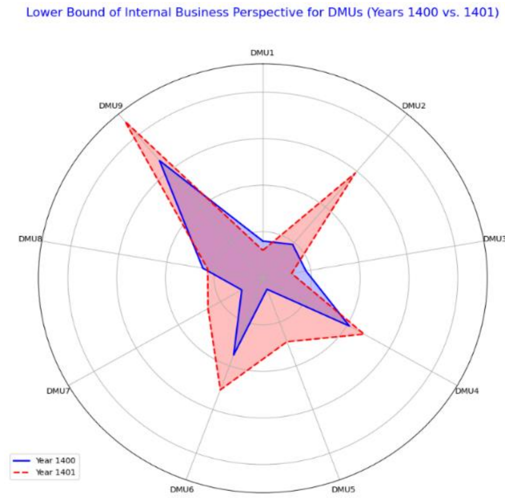
The Customer perspective (Figure 8(c)) highlights DMU2 and DMU9 with high lower bounds in both years, but DMU3 experiences a notable decline, signaling decreased performance. Other DMUs exhibit relative stability with minor variations.

The Learn & Growth perspective (Figure 8(d)) shows DMU9 maintaining high performance, while DMU1 has consistently low lower bounds. DMU5 and DMU7 show improvements, whereas DMU1 and DMU3 experiences a slight decrease.

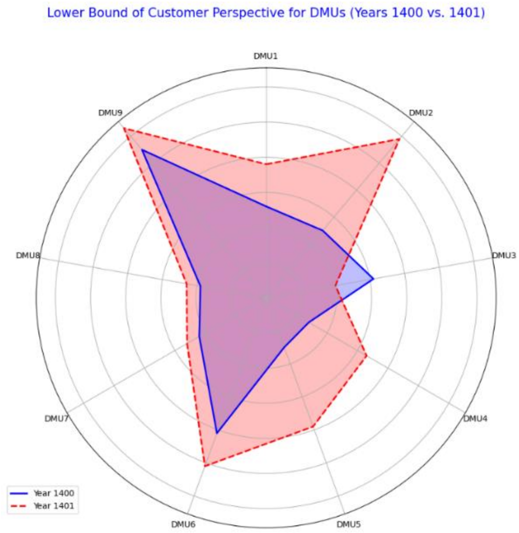
Lastly, in the Environment perspective (Figure 8(e)), DMU9 has the highest lower bounds in both years, but experiences a notable decline. DMU3 and DMU6 also experience declines, indicating reduced efficiency. Overall, DMU9 shows strong performance across most perspectives, while DMU3's decline in several areas, including Customer and Environment, is notable. The remaining DMUs display a mix of minor improvements and declines, reflecting varied changes in performance from 1400 to 1401.



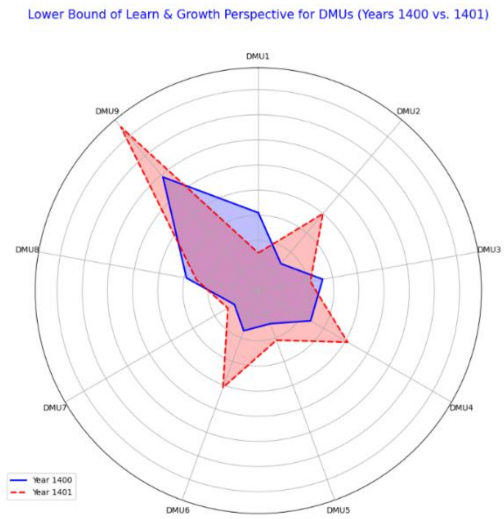
a.


Lower Bound of Internal Business Perspective for DMUs (Years 1400 vs. 1401)
DMU1
DMU2
DMU3
DMU4
DMU5
DMU6
DMU7
DMU8
DMU9
Year 1400
Year 1401

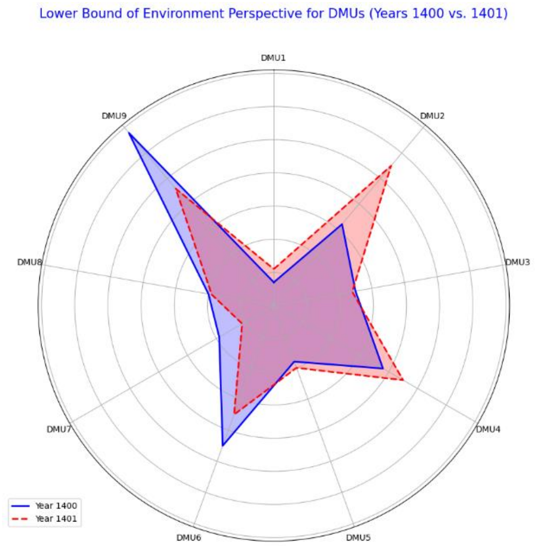
b.


Lower Bound of Customer Perspective for DMUs (Years 1400 vs. 1401)
DMU1
DMU2
DMU3
DMU4
DMU5
DMU6
DMU7
DMU8
DMU9
Year 1400
Year 1401

c.


Lower Bound of Learn & Growth Perspective for DMUs (Years 1400 vs. 1401)
DMU1
DMU2
DMU3
DMU4
DMU5
DMU6
DMU7
DMU8
DMU9
Year 1400
Year 1401

d.



e.

Figure 8. Comparison of the lower perspectives' scores at $\varphi_{(0,1.0,1.0)}$

5. Conclusion

In this paper, we have developed a comprehensive framework for evaluating the efficiencies of DMUs within the petrochemical industry using a Sustainable Balanced Scorecard (SBSC) approach. By employing Neutrosophic DEMATEL, we explored and mapped the intricate relationships among the SBSC perspectives. This analysis provided valuable insights into the interactions between various performance indicators. We consulted with industry experts to define key performance indicators (KPIs) tailored to the specific needs and strategic objectives of the petrochemical sector. The resulting KPIs were integrated into a detailed network structure, allowing for a more nuanced evaluation of DMU performance. After creating the network structure of SBSC and defining the KPIs as inputs, outputs, intermediate measures, and carry-overs, we proposed a dynamic network SBM approach to evaluate the performance of the DMUs. To measure the performance of the DMUs in a Neutrosophic environment, we developed a two-phase approach. In Phase I, named role determination, we addressed the issue of the dual role of intermediate measures and carry-over activities by solving a MILP model. In Phase II, named efficiency evaluation, we used the results obtained from the optimal solution of Phase I to evaluate the performance of the DMUs using the model proposed for Phase II. To handle the inherent uncertainties within the model, with the aid of pareto efficiency concept, we transformed the proposed model into deterministic models, enabling the computation of efficiency boundaries at varying degrees of variation. This approach allowed us to measure the lower and upper bounds of overall efficiency for each DMU, offering a robust analysis of their performance under different degrees of variation. Our findings indicate that integrating expert insights with advanced modeling techniques can significantly enhance the accuracy and relevance of performance evaluations in complex industrial settings. The resulting efficiency scores provide a clear and comprehensive assessment of each DMU, highlighting areas of strength and opportunities for improvement. Overall, this study demonstrates the value of combining expert knowledge with advanced analytical methods to achieve a more detailed and actionable understanding of organizational performance in the petrochemical industry.

To verify our proposed approach, we evaluate nine oil refineries from the Marun Petrochemical Complex (MPC). A comprehensive analysis of the efficiency and performance of the nine DMUs across various perspectives over the years 1400 and 1401 were performed. The results reveal that DMU9 consistently exhibits high performance across multiple perspectives, maintaining the highest lower bounds in the Financial, Internal Business, Customer, and Environment perspectives, indicating robust and reliable efficiency. Conversely, DMU3 shows notable declines in several areas, particularly in the Customer and Environment perspectives, highlighting areas of concern that require further attention. While DMU1 and DMU5 remain among the DMUs with lower performance, they show slight improvements in certain perspectives, suggesting potential for future progress. DMU6 and DMU7, on the other hand, display positive trends in the Internal Business and Learn & Growth perspectives, reflecting successful efforts to enhance their efficiency. The varied changes among the other DMUs indicate a mix of progress and regress, underscoring the dynamic nature of performance metrics.

5.1. Research limitations

Although this study provides valuable insights, there are several limitations that should be acknowledged. First, while the Neutrosophic DEMATEL approach is effective in managing uncertainty, it may still oversimplify certain complex relationships between performance indicators. second, the reliance on expert opinions introduces potential subjectivity, which could affect the consistency of the results.

5.2. Future research suggestion

Future research could address these limitations by applying the proposed framework to a broader range of industries to test its applicability in diverse settings. Additionally, further refinement of the Neutrosophic DEMATEL method could enhance its accuracy, particularly in handling more complex and dynamic relationships between performance indicators. Future studies could also explore the incorporation of external factors—such as market volatility, regulatory changes, or environmental pressures—that may impact DMU performance. Moreover, integrating more objective data sources alongside expert opinions could help mitigate subjectivity and improve the robustness of the model.

References

[1] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *Eur J Oper Res*, vol. 2, no. 6, pp. 429–444, 1978.

[2] H. Bagherzadeh Valami and R. Raeinojehdehi, "Ranking units in data envelopment analysis with fuzzy data," *Journal of intelligent & fuzzy systems*, vol. 30, pp. 2505–2516, 2016, doi: 10.3233/IFS-151756.

[3] R. Rasinojehdehi and S. E. Najafi, "Advancing risk assessment in renewable power plant construction: an integrated DEA-SVM approach," *Big Data and Computing Visions*, vol. 4, no. 1, pp. 1–11, 2024.

[4] C. G. M. de F. Alves and L. A. Meza, "A review of network DEA models based on slacks-based measure: Evolution of literature, applications, and further research direction," *International Transactions in Operational Research*, vol. 30, no. 6, pp. 2729–2760, Nov. 2023, doi: 10.1111/ITOR.13284.

[5] K. Tone, "A slacks-based measure of efficiency in data envelopment analysis," *Eur J Oper Res*, vol. 130, no. 3, pp. 498–509, 2001.

[6] R. Rasi Nojehdehi, H. Bagherzadeh Valami, and S. E. Najafi, "Classifications of linking activities based on their inefficiencies in network DEA," *International journal of research in industrial engineering*, vol. 12, no. 2, pp. 165–176, 2023.

[7] R. Färe and D. Primont, "Efficiency measures for multiplant firms," *Operations Research Letters*, vol. 3, no. 5, pp. 257–260, 1984, doi: 10.1016/0167-6377(84)90057-9.

[8] R. S. Kaplan, D. P. Norton, and others, *Using the balanced scorecard as a strategic management system*. Harvard Business Review, 1996.

[9] S. Jassem, Z. Zakaria, and A. Che Azmi, "Sustainability balanced scorecard architecture and environmental performance outcomes: a systematic review," *International Journal of Productivity and Performance Management*, vol. 71, no. 5, pp. 1728–1760, 2022.

[10] H. Al-Mawali, "Proposing a strategy map based on sustainability balanced scorecard and DEMATEL for manufacturing companies," *Sustainability Accounting, Management and Policy Journal*, vol. 14, no. 3, pp. 565–590, 2023.

[11] R. Lueg, "Strategy maps: the essential link between the balanced scorecard and action," *Journal of Business Strategy*, vol. 36, no. 2, pp. 34–40, 2015.

[12] R. S. Kaplan, D. P. Norton, and others, "Having trouble with your strategy? Then map it," *Focusing Your Organization on Strategy—with the Balanced Scorecard*, vol. 49, no. 5, pp. 167–176, 2000.

[13] C. Valmohammadi and J. Sofiyabadi, "Modeling cause and effect relationships of strategy map using fuzzy DEMATEL and fourth generation of balanced scorecard," *Benchmarking: An International Journal*, vol. 22, no. 6, pp. 1175–1191, 2015.

[14] N. Bénet, A. Deville, and G. Naro, "BSC inside a strategic management control package," *Journal of applied accounting research*, vol. 20, no. 1, pp. 120–132, 2019.

[15] A. T. Tsalis, E. I. Nikolaou, E. Grigoroudis, and P. K. Tsagarakis, "A dynamic sustainability Balanced Scorecard methodology as a navigator for exploring the dynamics and complexity of corporate sustainability strategy," *Civil Engineering and Environmental Systems*, vol. 32, no. 4, pp. 281–300, 2015.

[16] C. Veeramani, V. Gopal, and S. A. Edalatpanah, "Neutrosophic DEMATEL approach for financial ratio performance evaluation of the NASDAQ Exchange," *Neutrosophic Sets and Systems*, vol. 51, pp. 766–782, 2022.

[17] L. E. Quezada, H. A. López-Ospina, C. Ortiz, A. M. Oddershede, P. I. Palominos, and P. A. Jofré, "A DEMATEL-based method for prioritizing strategic projects using the perspectives of the Balanced Scorecard," *Int J Prod Econ*, vol. 249, p. 108518, 2022, doi: 10.1016/j.ijpe.2022.108518.

[18] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[19] H. E. Shermeh, S. E. Najafi, and M. H. Alavidoost, "A novel fuzzy network SBM model for data envelopment analysis: a case study in Iran regional power companies," *Energy*, vol. 112, pp. 686–697, 2016.

[20] F. Smarandache, "A unifying field in logics: neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability," in *American Research Press*, American Research Press, 1999, pp. 1–141.

[21] S. A. Edalatpanah, “Neutrosophic perspective on DEA,” *Journal of applied research on industrial engineering*, vol. 5, no. 4, pp. 339–345, 2018.

[22] R. Rasinojehdehi and H. B. Valami, “A comprehensive neutrosophic model for evaluating the efficiency of airlines based on SBM model of network DEA,” *Decision making: applications in management and engineering*, vol. 6, no. 2, pp. 880–906, 2023.

[23] N. P. Becerra Arévalo, M. F. Calles Carrasco, J. L. Toasa Espinoza, and M. Velasteguí Córdova, “Neutrosophic AHP for the prioritization of requirements for a computerized facial recognition system,” *Neutrosophic Sets and Systems*, vol. 34, no. 1, p. 21, 2020.

[24] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single valued neutrosophic sets*. Infinite study, 2010.

[25] S. A. Edalatpanah and F. Smarandache, *Data envelopment analysis for simplified neutrosophic sets*. Infinite Study, 2019.

[26] M. Akram, S. Naz, and F. Smarandache, “Generalization of maximizing deviation and TOPSIS method for MADM in simplified neutrosophic hesitant fuzzy environment,” *Symmetry (Basel)*, vol. 11, no. 8, p. 1058, 2019.

[27] L. Zhao, J. Gu, J. Abbas, D. Kirikkaleli, and X.-G. Yue, “Does quality management system help organizations in achieving environmental innovation and sustainability goals? A structural analysis,” *Economic research-Ekonomska istraživanja*, vol. 36, no. 1, pp. 2484–2507, 2023.

[28] R. Müller and K. Jugdev, “Critical success factors in projects: Pinto, Slevin, and Prescott–the elucidation of project success,” *International journal of managing projects in business*, vol. 5, no. 4, pp. 757–775, 2012.

[29] D. Zimon, P. Madzík, S. Dellana, R. Sroufe, M. Ikram, and K. Lysenko-Ryba, “Environmental effects of ISO 9001 and ISO 14001 management system implementation in SSCM,” *The TQM Journal*, vol. 34, no. 3, pp. 418–447, 2022.

[30] I. Heras-Saizarbitoria and O. Boiral, “ISO 9001 and ISO 14001: towards a research agenda on management system standards,” *International journal of management reviews*, vol. 15, no. 1, pp. 47–65, 2013.

[31] W. F. Cascio, “Managing human resources,” 2003.

[32] E. L. Psomas and C. V Fotopoulos, “Total quality management practices and results in food companies,” *International Journal of Productivity and Performance Management*, vol. 59, no. 7, pp. 668–687, 2010.

[33] S. Fankhauser and R. S.J. Tol, “On climate change and economic growth,” *Resour Energy Econ*, vol. 27, no. 1, pp. 1–17, 2005, doi: 10.1016/j.reseneeco.2004.03.003.

[34] M. Terziovski and D. Power, “Increasing ISO 9000 certification benefits: a continuous improvement approach,” *International Journal of Quality & Reliability Management*, vol. 24, no. 2, pp. 141–163, 2007.

[35] D. I. Prajogo and A. S. Sohal, “The relationship between organization strategy, total quality management (TQM), and organization performance—the mediating role of

TQM," *Eur J Oper Res*, vol. 168, no. 1, pp. 35–50, 2006, doi: 10.1016/j.ejor.2004.03.033.

[36] P. Massingham and M. Al Holaibi, "ISO 10015 and quality management of professional training: A UAE study," *Journal of Management Development*, vol. 36, no. 6, pp. 835–854, 2017.

[37] O. Boiral, "Corporate greening through ISO 14001: a rational myth?," *organization science*, vol. 18, no. 1, pp. 127–146, 2007.

[38] S. Vinodh and D. Joy, "Structural equation modelling of lean manufacturing practices," *Int J Prod Res*, vol. 50, no. 6, pp. 1598–1607, 2012.

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