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Analyzing Uncertainty and Hesitation in Refereeing Decision using Neutrosophic Vague N-Soft Sets: A Case Study of Officiating Ability Evaluation for National-Level Volleyball Referees

Haikui Zhou[1], Xiang Zhong[2]*

[1,2]Sports Academy, Chaohu College, Anhui, Hefei, 238024, China

*Corresponding author, E-mail: ahpqxy@163.com

Abstract-Officiating in volleyball and similar high-stakes sports involves speedy, high-pressure decision-making, which is usually influenced by uncertainty, hesitation, and subjective interpretation. This study proposes a novel Neutrosophic Vague N-Soft Sets (NVNSS) framework for the evaluation of national-level volleyball referees to model as well as analyze the inherent vagueness and indeterminacy in decisions. Our framework integrated expert linguistic evaluations across multiple criteria into a structured NVNSS decision matrix. Then, we propose a risk-aware degree of support to quantify each referee's performance under uncertainty, followed by a discrete rating assignment. Our framework introduces a robust and flexible NVSNSS scoring function that accounts for weighted contributions, non-linear transformations, and uncertainty handling to generate a well-distributed and expressive crisp value. By pairing the scores and rates of NVSNSS in the decision space, we can derive referee rankings under each criterion. Then, we propose a novel weighted aggregation that integrates entropy-based weighting to deliver the final ranking from the Assuming criteria-based rankings without being affected by large deviations across criteria. Proof-of-concept analyses are provided to validate the applicability of our framework on a real-world case study for volleyball officiating, and the results demonstrated that work is a pioneering approach for modeling uncertainty and hesitation in refereeing decisions.

Keywords- Neutrosophic Sets, Vague Sets, N-Soft Sets, Uncertainty Analysis, Hesitation Degree, Referee Decision-Making, Volleyball Officiating, Multi-Criteria Decision-Making (MCDM), Sports Analytics.

1. Introduction

Officiating in volleyball is a complex and dynamic process that requires referees to make instantaneous and accurate decisions in high-pressure environments [1]. The role of a volleyball referee extends beyond enforcing rules; it includes interpreting ambiguous situations, handling player interactions, and upholding fairness throughout the game [2]. The fast-paced nature of modern volleyball, in which decisions must be made within fractions of a second, adds to the complexity of refereeing [3]. As the level of competition increases, particularly in national and international tournaments, the accuracy and consistency of officiating become critical aspects influencing the outcome of the game [4]. Despite progressions in officiating training programs and technological aids such as video replay systems, referees often face uncertainty and hesitation when performing calls [5]. Factors such as rapid game dynamics, human perception limitations, psychological pressures, and subjective interpretation of rules contribute to the inherent ambiguity in decision-making [6]. Hesitation in officiating can occur because of indistinct ball contact situations, block-touch judgments, or boundary line calls, which may be influenced by inspecting angles and player actions [7], [8]. The legacy mathematical models, such as crisp logic and fuzzy sets, attempted to model uncertainty in decision-making [9], [10]; nevertheless, they regularly fail to capture the hesitation and indeterminacy inherent in real-world situations like sports officiating. Neutrosophic theory [11], [12] introduced by Smarandache, extended classical fuzzy and intuitionistic fuzzy theories with the incorporation of three fundamental components: truth, falsity, and indeterminacy. This agenda is chiefly convenient in situations where incomplete knowledge, inconsistent evidence, or subjective valuations play a significant role. Building upon neutrosophic logic, Neutrosophic Vague N-Soft Sets (NVNSS) [13], provide an enhanced mechanism for handling multi-criteria decision-making (MCDM) problems [14], [15] where vagueness, imprecision, and hesitation coexist. The NVNSS approach allows for the representation of referee decisions with multiple levels of uncertainty, accommodating the hesitation factor observed in complex officiating situations.

The primary objective of this study is to investigate a new NVNSS framework to analyze uncertainty and hesitation in refereeing decisions within national-level volleyball officiating. Specifically, the contributions of this work are as follows. Our NVNSS framework proposes a risk-aware degree of support to count each referee's performance under hesitation, shadowed by

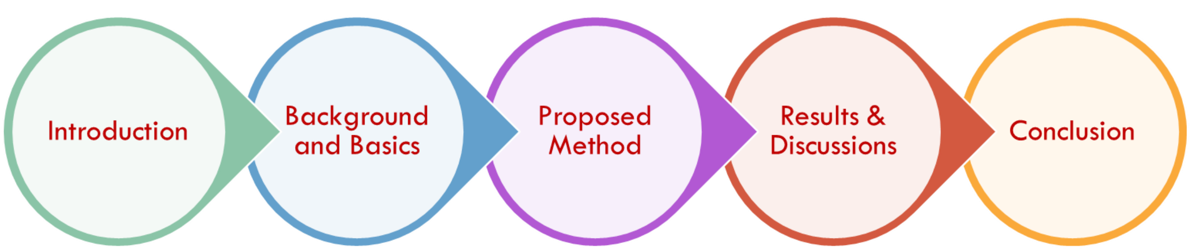

Figure 1. Illustration of paper outline.

a discrete rating obligation. Our framework presents a vigorous and elastic NVSNSS counting function that explains weighted contributions, non-linear transformations, and uncertainty management to make a well-distributed and expressive crisp value. Then, we pair the scores and rates of NVSNSS to originate referee rankings under all decision criteria. Then, our framework proposes a weighted aggregation to integrate entropy-based weighting to deliver the final ranking from the Assuming criteria-based rankings without being affected by large deviations across criteria. The remaining of this work is outlined as shown in Figure 1.

Background and Basics

This section provides a comprehensive overview of the foundational concepts relevant to this study namely Neutrosophic Sets, Vague Sets, and N-Soft Sets.

Definition 1: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$, A neutrosophic set, W_{NS}, could be described with triplet membership functions specifically Certainty namely $\boldsymbol{t}_{W_N}(u)$, Indeterminacy: $\boldsymbol{i}_{W_N}(u)$, as well as Falseness: $\boldsymbol{f}_{W_N}(u)$.

$$W_{NS} = \left\{\langle u, \left(\boldsymbol{t}_{W_N}(u), \boldsymbol{i}_{W_N}(u), \boldsymbol{f}_{W_N}(u)\right)\rangle : u \in U \right\} \quad (1)$$

such that

$$^{-}0 \leq sup\boldsymbol{t}_{W_N}(u) + sup\boldsymbol{i}_{W_N}(u) + sup\boldsymbol{f}_{W_N}(u) \leq 3^{+}.$$

All these components belong to $]0^{-}, 1^{+}[$, nevertheless, for real-world applications, this cannot be applied, so we can use the following interval [0,1] as an alternative.

Definition 2: A vague set, W_V, could be described over discourse $U = \{u_1, u_2, \cdots, u_n\}$, with two membership functions explicitly one for truth, $\boldsymbol{t}_{VS}(u)$, and others for falseness $\boldsymbol{f}_{\mathrm{VS}}(u)$ [16].

$$W_{VS} = \left\{\langle u, \left(\boldsymbol{t}_{VS}(u), \boldsymbol{f}_{\mathrm{VS}}(u)\right)\rangle : u \in U \right\} \quad (2)$$

Definition 3: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$, and two vague sets $\mathrm{W} = \{\mathrm{u_i}, [\mathrm{t_W(u_i)}, 1 - \mathrm{f_W(u_i)}] \mid \mathrm{u_i} \in \mathrm{U}\}$ and $\mathrm{Z} = \{\mathrm{u_i}, [\mathrm{t_Z(u_i)}, 1 - \mathrm{f_Z(u_i)}] \mid \mathrm{u_i} \in \mathrm{U}\}$, then, the subsequent relatives could be calculated as:

The complement of W:

$$W^{c} = \{(u_i, [f_W(u_i), 1 - t_W(u_i)] \mid u_i \in U)\}. \quad (3)$$

W is a subset of Z the subsequent complaint spread over:

$$W \subseteq Z \ iff \ t_W(u_i) \leq t_Z(u_i), and \ 1 - f_W(u_i) \leq 1 - f_Z(u_i) \quad (4)$$

The intersection between them can be computed as:

$$W \cap Z = \{(u_i, [min(t_W(u_i), t_Z(u_i)), min(1 - f_W(u_i), 1 - f_Z(u_i))] \mid u_i \in U)\}, \quad (5)$$

The union can be computed as:

$$W \cup Z = \{(u_i, [mau(t_W(u_i), t_Z(u_i)), mau(1 - f_W(u_i), 1 - f_Z(u_i))] \mid u_i \in U)\}, \quad (6)$$

Definition 4: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$, A neutrosophic vague set, W_{NV}, is described with three membership functions valued as follows:

$$\begin{aligned} W_{NV} &= \{\langle u, (\boldsymbol{t}_{NV}(u), \boldsymbol{i}_{NV}(u), \boldsymbol{f}_{NV}(u))\rangle : u \in U\} \\ &= \left\{\langle u, \begin{pmatrix} [\boldsymbol{t}_{NV}(u)^-, \boldsymbol{t}_{NV}(u)^+], [\boldsymbol{i}_{NV}(u)^-, \boldsymbol{i}_{NV}(u)^+], \\ [\boldsymbol{f}_{NV}(u)^-, \boldsymbol{f}_{NV}(u)^+] \end{pmatrix}\rangle : u \in U\right\} \\ &= \left\{\langle u, \begin{pmatrix} [\boldsymbol{t}_{NV}(u)^-, 1 - \boldsymbol{f}_{NV}(u)^-], [\boldsymbol{i}_{NV}(u)^-, \boldsymbol{i}_{NV}(u)^+], \\ [\boldsymbol{f}_{NV}(u)^-, 1 - \boldsymbol{t}_{NV}(u)^-] \end{pmatrix}\rangle : u \in U\right\} \end{aligned} \quad (7)$$

Such that

$$^-0 \leq \boldsymbol{t}^-_{W_{NV}} + \boldsymbol{i}^-_{W_{NV}} + \boldsymbol{f}^-_{W_{NV}} \leq 2^+$$

Definition 5: Assuming two neutrosophic vague set $\mathrm{W_{NV}} = \{\langle \mathrm{u}, ([\boldsymbol{t}_{\mathrm{W_{NV}}}(\mathrm{u})^-, \boldsymbol{t}_{\mathrm{W_{NV}}}(\mathrm{u})^+], [\boldsymbol{i}_{\mathrm{W_{NV}}}(\mathrm{u})^-, \boldsymbol{i}_{\mathrm{W_{NV}}}(\mathrm{u})^+], [\boldsymbol{f}_{\mathrm{W_{NV}}}(\mathrm{u})^-, \boldsymbol{f}_{\mathrm{W_{NV}}}(\mathrm{u})^+])\rangle : \mathrm{u} \in \mathrm{U}\}$ and $\mathrm{Z_{NV}} = \{\langle \mathrm{u}, ([\boldsymbol{t}_{\mathrm{Z_{NV}}}(\mathrm{u})^-, \boldsymbol{t}_{\mathrm{Z_{NV}}}(\mathrm{u})^+], [\boldsymbol{i}_{\mathrm{Z_{NV}}}(\mathrm{u})^-, \boldsymbol{i}_{\mathrm{Z_{NV}}}(\mathrm{u})^+], [\boldsymbol{f}_{\mathrm{Z_{NV}}}(\mathrm{u})^-, \boldsymbol{f}_{\mathrm{Z_{NV}}}(\mathrm{u})^+])\rangle : \mathrm{u} \in \mathrm{U}\}$, then, the following relations can be computed [13].

The intersection between A and B is described as follows.

$$W_{NV} \cup Z_{NV} = \left\{\langle u, \begin{pmatrix} [\max(\boldsymbol{t}_{W_{NV}}(u)^-, \boldsymbol{t}_{Z_{NV}}(u)^-), \max(\boldsymbol{t}_{W_{NV}}(u)^+, \boldsymbol{t}_{Z_{NV}}(u)^+)], \\ [\min(\boldsymbol{i}_{W_{NV}}(u)^-, \boldsymbol{i}_{Z_{NV}}(u)^-), \min(\boldsymbol{i}_{W_{NV}}(u)^+, \boldsymbol{i}_{Z_{NV}}(u)^+)], \\ [\min(\boldsymbol{f}_{W_{NV}}(u)^-, \boldsymbol{f}_{Z_{NV}}(u)^-), \min(\boldsymbol{f}_{W_{NV}}(u)^+, \boldsymbol{f}_{Z_{NV}}(u)^+)] \end{pmatrix}\rangle : u \in U\right\}, \quad (8)$$

The complement of W_{NV} is described as follows:

$$W_{NV}{}^c = \{\langle u, ([1 - \boldsymbol{t}_{W_{NV}}(u)^+, 1 - \boldsymbol{t}_{W_{NV}}(u)^-], [1 - \boldsymbol{i}_{W_{NV}}(u)^+, 1 - \boldsymbol{i}_{W_{NV}}(u)^-], [1 - \boldsymbol{f}_{W_{NV}}(u)^+, 1 - \boldsymbol{f}_{W_{NV}}(u)^-])\rangle : u \in U\} \quad (9)$$

A is a subset of if the following conditions apply:

$$W \subseteq Z_{NV} \; iff \; t_{W_{NV}}(u_i) \leq t_{Z_{NV}}(u_i), and \; 1 - f_{W_{NV}}(u_i) \leq 1 - f_{Z_{NV}}(u_i) \quad (10)$$

The union between W_{NV} *and* Z_{NV} is described as follows [17].

$$W_{NV} \cup Z_{NV} = \left\{ \langle u, \begin{pmatrix} [\min(\boldsymbol{t}_{W_{NV}}(u)^-, \boldsymbol{t}_{Z_{NV}}(u)^-), \min(\boldsymbol{t}_{W_{NV}}(u)^+, \boldsymbol{t}_{Z_{NV}}(u)^+)], \\ [\max(\boldsymbol{i}_{W_{NV}}(u)^-, \boldsymbol{i}_{Z_{NV}}(u)^-), \max(\boldsymbol{i}_{W_{NV}}(u)^+, \boldsymbol{i}_{Z_{NV}}(u)^+)], \\ [\max(\boldsymbol{f}_{W_{NV}}(u)^-, \boldsymbol{f}_{Z_{NV}}(u)^-), \max(\boldsymbol{f}_{W_{NV}}(u)^+, \boldsymbol{f}_{Z_{NV}}(u)^+)] \end{pmatrix} \rangle : u \in U \right\}, \tag{11}$$

Definition 6: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$, and P be a set of parameters that describe properties, attributes, or criteria relevant to elements of U, then a soft set $(\boldsymbol{f}, S)$ over U is described as a pair:

$$(\boldsymbol{f}, S): \boldsymbol{f}: S \to P(U)\ \forall\ e \in S \text{ such that } S \subseteq P \tag{12}$$

where S is a finite set of parameters. $\boldsymbol{f}$ is a mapping function.

Definition 7: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$, and P be a set of parameters that describe characteristics, or criteria pertinent to elements of U, then a neutrosophic vague soft set (NVSS) $(\boldsymbol{f}, \boldsymbol{t})$ over U is described as a pair:

$$\begin{gathered}(\boldsymbol{f}, \boldsymbol{t}), \\ \boldsymbol{f}: \boldsymbol{t} \to NV(U)\ \forall\ e \in \boldsymbol{t} \text{ such that } \boldsymbol{t} \subseteq P\end{gathered} \tag{13}$$

Definition 8: Assuming a group of discourses $U_1, U_2, \ldots, U_n$, and P as a set of parameters, an N-Soft Set, an extension of soft set theory, is described over this group

$$(\boldsymbol{f}, S): \boldsymbol{f}: S \to P(U_1) \times P(U_2) \times \ldots \times P(U_n)\ \forall\ e \in S \text{ such that } S \subseteq P \tag{14}$$

where $\boldsymbol{f}$ is a multi-set-valued mapping function.

Definition 9: Assuming discourse $U = \{u_1, u_2, \cdots, u_n\}$ with a set of parameters $P = \{e_1, e_2, \ldots, e_m\}$, a subset of parameters $T \subseteq E$ under assessment, and set of well-ordered rating $R = \{0,1, \ldots, N-1\}$, then Neutrosophic Vague N-Soft Set (NVNSS), symbolized as (NV, K), is described as a mapping:

$$NV: \boldsymbol{t} \to \bigcup_{e_j \in \boldsymbol{t}} \mathcal{NV}\left(NV(e_j)\right) \tag{15}$$

where $\mathcal{NV}\left(NV(e_j)\right)$ is a NVSS of parameter $e_j \in \boldsymbol{t}$, and for each object $u_i \in U$ and parameter $e_j \in \boldsymbol{t}$, there is a distinct pair $(u_i, g_{ij}) \in U \times R$. Thus, the description of NVNSS is articulated as:

$$\mathcal{NV}\left(NV(e_j)\right) = \left\{ \left((u_1, g_{1j}), \boldsymbol{t}_{NV}(u_1), \boldsymbol{i}_{NV}(u_1), \boldsymbol{f}_{NV(u_1)}\right), \ldots, \left((u_n, g_{nj}), \boldsymbol{t}_{NV}(u_n), \boldsymbol{i}_{NV}(u_n), \boldsymbol{f}_{NV}(u_n)\right) \right\}$$

Definition 10: Assuming two NVNSS (NV_1, K_1) and (NV_2, K_2) on the universe discourse $U = \{u_1, u_2, \cdots, u_n\}$, then (NV_1, K_1) is said to be equal (NV_2, K_2) iff

$$(NV_1, K_1) = (NV_2, K_2) \\ iff \\ (NV_1, K_1) \subseteq (NV_2, K_2) \,\&\, (NV_2, K_2) \subseteq (NV_1, K_1) \tag{16}$$

Definition 11: Assuming an NVNSS (NV_1, K_1) on the universe discourse $U = \{u_1, u_2, \cdots, u_n\}$, then, it can be called neutrosophic absolute vague N-soft set iff.

$$W_{NV}(e_j) = \{< (o_1, N-1), [1,1], [0,0], [0,0] >, \cdots, < (o_n, N-1), [1,1], [0,0], [0,0] >\} \\ \forall e_j \in \boldsymbol{t} \subseteq E. \tag{17}$$

Definition 12: Assuming two NVNSS (NV_1, K_1) and (NV_2, K_2) on the universe discourse $U = \{u_1, u_2, \cdots, u_n\}$, then $(NV_1, K_1) \subseteq (NV_2, K_2)$ *if and only if*

$$(NV_1, K_1) \subseteq (NV_2, K_2) iff \begin{array}{l} \boldsymbol{t}^-_{NV_1}(u) \leq \boldsymbol{t}^-_{NV_2}(u), \boldsymbol{t}^+_{NV_1}(u) \leq \boldsymbol{t}^+_{NV_2}(u); \\ \boldsymbol{i}^-_{NV_1}(u) \geq \boldsymbol{i}^+_{NV_2}(u), \boldsymbol{i}^+_{NV_1}(u) \geq \boldsymbol{i}^-_{NV_2}(u); \\ \boldsymbol{f}^-_{NV_1}(u) \geq \boldsymbol{f}^-_{NV_2}(u), \boldsymbol{f}^+_{NV_1}(u) \geq \boldsymbol{f}^+_{NV_2}(u). \end{array} \tag{18}$$

Definition 13: Assuming two NVNSS (NV_1, K_1) and (NV_2, K_2) on the universe discourse $U = \{u_1, u_2, \cdots, u_n\}$, then, the following relations can be computed:

The complement of (NV_1, K_1) is described as $(NV_1, K_1)^c \; N_1 \,\widetilde{\cap}\, N_1^c = \phi$, where NV_1^c

$$NV_1^c = \begin{array}{c} \hat{\boldsymbol{t}}^c_{NV_1}(u_i) = [1 - \boldsymbol{t}^+_{NV_1}(u_i), 1 - \boldsymbol{t}^-_{NV_1}(u_i)] = [\boldsymbol{f}^-_{NV_1}(u_i), \boldsymbol{f}^+_{NV_1}(u_i)], \\ \hat{\boldsymbol{i}}^c_{NV_1}(u_i) = [1 - \boldsymbol{i}^+_{NV_1}(u_i), 1 - \boldsymbol{i}^-_{NV_1}(u_i)], \\ \hat{\boldsymbol{f}}^c_{NV_1}(u_i) = [1 - \boldsymbol{f}^+_{NV_1}(u_i), 1 - \boldsymbol{f}^-_{NV_1}(u_i)] = [\boldsymbol{t}^-_{NV_1}(u_i), \boldsymbol{t}^+_{NV_1}(u_i)]. \end{array} \tag{19}$$

The restricted intersection between $(NV_1, K_1) \;\widetilde{\cap}_R\; (NV_2, K_2)$ is described as follows

$$\begin{array}{c} K_1 \cap_{\mathcal{R}} K_2 = (E, \boldsymbol{t} \cap S, min(N_1, N_2)) \forall e_j \in \boldsymbol{t} \cap S \; u_i \in U \\ < (u_i, g_{ij}), \hat{\boldsymbol{t}}_{NV}(u_i), \hat{\boldsymbol{i}}_{NV}(u_i), \hat{\boldsymbol{f}}_{NV}(u_i) > \in \eta_{NV}(e_j) \\ \Leftrightarrow \\ g_{ij} = min(g^1_{ij}, g^2_{ij}), \\ \hat{\boldsymbol{t}}_{NV}(u_i) = \left[min\left(\boldsymbol{t}^-_{NV_1}(u_i), \boldsymbol{t}^-_{NV_2}(u_i)\right), min\left(\boldsymbol{t}^+_{NV_1}(u_i), \boldsymbol{t}^+_{NV_2}(u_i)\right)\right], \\ \hat{\boldsymbol{i}}_{NV}(u_i) = \left[max\left(\boldsymbol{i}^-_{NV_1}(u_i), \boldsymbol{i}^-_{NV_2}(u_i)\right), max\left(\boldsymbol{i}^+_{NV_1}(u_i), \boldsymbol{i}^+_{NV_2}(u_i)\right)\right], and \\ \hat{\boldsymbol{f}}_{NV}(u_i) = \left[max\left(\boldsymbol{f}^-_{NV_1}(u_i), \boldsymbol{f}^-_{NV_2}(u_i)\right), max\left(\boldsymbol{f}^+_{NV_1}(u_i), \boldsymbol{f}^+_{NV_2}(u_i)\right)\right]. \end{array} \tag{20}$$

The restricted union $(NV_1, K_1) \cup (NV_2, K_2)$ is described as follows

$$\begin{array}{c} K_1 \cup_{\mathcal{R}} K_2 = (E, \boldsymbol{t} \cap S, min(N_1, N_2)) \forall e_j \in \boldsymbol{t} \cap S \; u_i \in U \\ g_{ij} = max(g^1_{ij}, g^2_{ij}), \\ \hat{\boldsymbol{t}}_{NV}(u_i) = [max(\boldsymbol{t}^-_{NV_1}(u_i), \boldsymbol{t}^-_{NV_2}(u_i)), max(\boldsymbol{t}^+_{NV_1}(u_i), \boldsymbol{t}^+_{NV_2}(u_i))], \\ \hat{\boldsymbol{i}}_{NV}(u_i) = [min(\boldsymbol{i}^-_{NV_1}(u_i), \boldsymbol{i}^-_{NV_2}(u_i)), min(\boldsymbol{i}^+_{NV_1}(u_i), \boldsymbol{i}^+_{NV_2}(u_i))], and \\ \hat{\boldsymbol{f}}_{NV}(u_i) = [min(\boldsymbol{f}^-_{NV_1}(u_i), \boldsymbol{f}^-_{NV_2}(u_i)), min(\boldsymbol{f}^+_{NV_1}(u_i), \boldsymbol{f}^+_{NV_2}(u_i))]. \end{array} \tag{21}$$

2. Proposed Methods

Herein, we outline our framework adopted to evaluate and rank the performance of volleyball referees using NVNSS. The methodology is composed of a sequence of structured steps designed to transform linguistic assessments into quantitative rankings under uncertainty.

In step 1, we build a NVNSS decision matrix, in which experts or technical officials assess each referee under multiple criteria using linguistic terms. These linguistic valuations are then mapped into interval-valued NVNSS, expressed as:

$$D = \begin{vmatrix} \left\{\begin{matrix} [\boldsymbol{t}_{NV}^{-}, \boldsymbol{t}_{NV}^{+}(u)], [\boldsymbol{i}_{NV}^{-}, \boldsymbol{i}_{NV}^{+}(u)], \\ [\boldsymbol{f}_{NV}^{-}, \boldsymbol{f}_{NV}^{+}(u)] \end{matrix}\right\}_{1,1} & \cdots & \left\{\begin{matrix} [\boldsymbol{t}_{NV}^{-}, \boldsymbol{t}_{NV}^{+}(u)], [\boldsymbol{i}_{NV}^{-}, \boldsymbol{i}_{NV}^{+}(u)], \\ [\boldsymbol{f}_{NV}^{-}, \boldsymbol{f}_{NV}^{+}(u)] \end{matrix}\right\}_{1,m} \\ \vdots & \cdots & \vdots \\ \left\{\begin{matrix} [\boldsymbol{t}_{NV}^{-}, \boldsymbol{t}_{NV}^{+}(u)], [\boldsymbol{i}_{NV}^{-}, \boldsymbol{i}_{NV}^{+}(u)], \\ [\boldsymbol{f}_{NV}^{-}, \boldsymbol{f}_{NV}^{+}(u)] \end{matrix}\right\}_{n,1} & \cdots & \left\{\begin{matrix} [\boldsymbol{t}_{NV}^{-}, \boldsymbol{t}_{NV}^{+}(u)], [\boldsymbol{i}_{NV}^{-}, \boldsymbol{i}_{NV}^{+}(u)], \\ [\boldsymbol{f}_{NV}^{-}, \boldsymbol{f}_{NV}^{+}(u)] \end{matrix}\right\}_{n,m} \end{vmatrix} \tag{22}$$

In step 2, we take each element in the decision matrix and calculate the risk-aware degree of support according to the risk attitude parameter $k^{\wedge} \in [0,1]$, which measures how powerfully a referee's performance chains their assortment under uncertainty:

$$\delta_{kp}(u) = \frac{w_{\boldsymbol{t}} \cdot \boldsymbol{t}(u) + w_{\boldsymbol{i}} \cdot \boldsymbol{i}(u) + w_{\boldsymbol{f}} \cdot \boldsymbol{f}(u)}{w_{\boldsymbol{t}} + w_{\boldsymbol{i}} + w_{\boldsymbol{f}}} \cdot \left(1 - \frac{Unc\ (u)}{2}\right) \tag{23}$$

where $\boldsymbol{t}(u) = \boldsymbol{t}_{NV}^{-}(u) + \hat{k} \cdot \left(\boldsymbol{t}_{NV}^{+}(u) - \boldsymbol{t}_{NV}^{-}(u)\right)$, $\boldsymbol{i}(u) = \boldsymbol{i}_{NV}^{-}(u) + \hat{k} \cdot \left(\boldsymbol{i}_{NV}^{+}(u) - \boldsymbol{i}_{NV}^{-}(u)\right)$, $\boldsymbol{f}(u) = \boldsymbol{f}_{NV}^{+}(u) - \hat{k} \cdot \left(\boldsymbol{f}_{NV}^{+}(u) - \boldsymbol{f}_{NV}^{-}(u)\right)$, and $Unc(u) = \frac{\boldsymbol{i}(u)}{\boldsymbol{t}(u)+\boldsymbol{f}(u)}$. The weights $w_{\boldsymbol{t}}, w_{\boldsymbol{i}}, w_{\boldsymbol{f}}$: signify the importance of truth, indeterminacy, and falsity, correspondingly. Assuming that $\boldsymbol{t}(u), \boldsymbol{i}(u), \boldsymbol{f}(u) \in [0,1]$, and Uncertainty $(u) \in [0,1]$, we can infer that degree of support $\delta_{kp}(u) \in [0,1]$.

$$\text{If } \hat{k} = 0 :,= \begin{pmatrix} \boldsymbol{t}(u) = \boldsymbol{t}_{NV}^{-}(u) \\ \boldsymbol{i}(u) = \boldsymbol{i}_{NV}^{-}(u) \\ \boldsymbol{f}(u) = \boldsymbol{f}_{NV}^{+}(u) \end{pmatrix} \text{If } \hat{k} = 1 :,= \begin{pmatrix} \boldsymbol{t}(u) = \boldsymbol{t}_{NV}^{+}(u) \\ \boldsymbol{i}(u) = \boldsymbol{i}_{NV}^{+}(u) \\ \boldsymbol{f}(u) = \boldsymbol{f}_{NV}^{-}(u) \end{pmatrix} \tag{24}$$

In step 3, we take the calculated support values δ_{kp}, to compute the discrete rating, R_i, using a prescribed range-based mapping, as described in [13]. This transformation enhances interpretability and discretizes performance into ordinal categories.

In step 4, we propose a **context-aware score function** for NVNSS, in which we can for the internal balance between belief and disbelief while spread over a contextual penalty. The new scoring function S'_{ij} is described as:

$$S'_{ij} = \text{Agg}\left(T'_{ij}, I'_{ij}, F'_{ij}\right) \cdot \text{ContextTerm}\left(T'_{ij}, I'_{ij}, F'_{ij}\right) \tag{25}$$

where:

$$\text{Aggregate}\left(T'_{ij}, I'_{ij}, F'_{ij}\right) = \frac{T'_{ij} + \left(1 - I'_{ij}\right) + \left(1 - F'_{ij}\right)}{3}$$
$$\text{ContextTerm}\left(T'_{ij}, I'_{ij}, F'_{ij}\right) = 1 - \frac{\left|T'_{ij} - \left(1 - F'_{ij}\right)\right|}{2} \tag{26}$$

In step 5, we compute the pairwise comparison matrix based on each criterion, in which each cell represents the difference in performance between two referees:

$$\Delta_{ij} = (S_i - S_j, R_i - R_j) \tag{27}$$

In step 5, we determine the ranking under each criterion by applying a priority relation method. Then, for each referee, the minimum dominance against all others is computed:

$$\Delta_i^{min} = \min(\Delta_{ij}) \tag{288}$$

In step 7, we repeat steps 5 and 6 are independent for all criteria, which results in a ranking vector for each referee under each criterion.

Finally, we drive the final ranking of alternatives from the Assuming criteria-based rankings by integrating entropy-based weighting (that accounts for criteria standing) with a penalty term for inconsistency (to punish big deviations crosswise criteria. As a first step, we normalize the rankings to a shared scale (e.g., [0, 1]) to be able to ensure comparability. For each criterion C_j, the normalized score s_{ij} for alternative R_i is:

$$s_{ij} = \frac{\max(\text{rank}) - r_{ij} + 1}{\max(\text{rank})}, \tag{29}$$

where r_{ij} is the rank of R_i under C_j.

Then, we apply entropy to estimate the uncertainty in each criterion's rankings, where low values express more discriminative power, while higher weight. For each criterion C_j, we compute the entropy E_j as follows:

$$p_{ij} = \frac{s_{ij}}{\sum_{i=1}^{n} s_{ij}},\ E_j = -\frac{1}{\ln(n)} \sum_{i=1}^{n} p_{ij} \ln\left(p_{ij}\right) \tag{30}$$

where

$$w_j = \frac{1 - E_j}{\sum_{k=1}^{m} (1 - E_k)}$$

After that, we introduce penalty terms to rank inconsistency to favor alternatives with stable performance across criteria, leading to final score S_i :

$$S_i=\sum_{j=1}^{m} w_j s_{ij}-\lambda \cdot \operatorname{StdDev}\left(s_{i1}, s_{i2}, \ldots, s_{im}\right) \tag{31}$$

Where λ represents the penalty coefficient (e.g., $\lambda=0.3$).

2.1. Theoretical Justification of Methodological Choice

Sports officiating, especially in volleyball, involves inherent complexities and challenges that go beyond simple binary decision-making. Traditional mathematical models, such as crisp logic or basic fuzzy logic, have limitations when capturing subjective uncertainties and hesitations occurring in real-time referee judgments [9,10]. To accurately reflect the multi-layered uncertainty inherent in officiating, it is critical to adopt more nuanced mathematical frameworks capable of handling indeterminacy, vagueness, and subjective interpretation concurrently.

Neutrosophic theory, introduced by Smarandache [11,12], significantly enhances traditional fuzzy and intuitionistic fuzzy theories by adding an independent dimension of indeterminacy alongside truth and falsity. This additional dimension perfectly addresses the hesitation and uncertainty referees often experience in high-pressure scenarios. The NVNSS methodology, building upon neutrosophic logic, incorporates both vague and soft set theories, providing a flexible and comprehensive platform to express varying degrees of belief, uncertainty, and hesitation simultaneously. This theoretical strength is particularly valuable in evaluating referees' performances under uncertain conditions, enabling nuanced analysis that reflects real-world complexity and human judgment.

2.2. Detailed Criteria Selection and Validation Process

Selecting relevant criteria is crucial for an effective evaluation framework. This research systematically selected six comprehensive criteria based on extensive literature review and expert consultations with seasoned volleyball officials and technical analysts. These criteria include Decision Consistency Rate (C1), Average Decision Time per Call (C2), Conflict Resolution Ability (C3), Observer Rating by Technical Committee (C4), Number of Controversial Calls per Match (C5), and Stress Tolerance under Crowd Pressure (C6). The criteria reflect crucial practical dimensions of officiating quality. Decision consistency measures reliability in repeated judgment scenarios, decision time reflects cognitive efficiency, and conflict resolution captures interpersonal skills. Observer ratings provide expert judgments, controversial calls indicate susceptibility to making disputable decisions, and stress tolerance evaluates mental robustness under intense environmental pressures.

Validity was rigorously verified through literature alignment [1–8], ensuring each criterion accurately represents real-world officiating demands. Reliability was further established through expert consensus, where a structured Delphi method involving multiple rounds of consultations with sports officials and refereeing experts confirmed the appropriateness and importance of the selected criteria, thus enhancing the framework's credibility.

3. Numerical Application

To evaluate the effectiveness of volleyball referees at the national level, a case study is introduced in this section, in which there are seven referee candidates. In other words, national-level volleyball referees are being considered for selection in a premier tournament. They are assessed according to a structured evaluation system, which considers a set of six contradictory criteria, namely Decision Consistency Rate (C1), Average Decision Time per Call (C2), Conflict Resolution Ability (C3), Observer Rating by Technical Committee (C4), and Number of Controversial Calls per Match(C5), Stress Tolerance under Crowd Pressure(C6). As shown, the evaluation procedure combined both *cost-type* (lower is better) as well as *benefit-type* (higher is better) indicators. These criteria were chosen to reflect real-world pressures, mental processing, and decision consistency of referees during competitive matches. In our case study, we used video analysis to quantify decision time, consistency, and controversial calls. Also, a panel of technical observers, coaches, and analysts are involved in processing and rating the referees making use of linguistic and subjective evaluations with corresponding mapping into NVNSS. These NVNSSs are applied to handle the uncertainty, hesitation, and subjectivity inherent in these assessments. In Table 2, we provide a detailed set of linguistic terms for each criterion in our case study, along with their corresponding subjective judgments in form of computationally operable NVNSS values.

Table 1. Linguistic terms and their corresponding NVNSSs for the evaluation of volleyball referees across six performance criteria.

Criterion	Linguistic Term	**NVNSS**
C_1 Decision Consistency Rate	Very High	{[0.85, 1.00], [0.00, 0.10], [0.00, 0.10]}
	High	{[0.70, 0.85], [0.10, 0.20], [0.05, 0.15]}
	Moderate	{[0.50, 0.70], [0.15, 0.30], [0.15, 0.30]}
	Low	{[0.30, 0.50], [0.25, 0.40], [0.25, 0.40]}
	Very Low	{[0.10, 0.30], [0.35, 0.50], [0.35, 0.50]}
C_2 Avg. Decision Time (Cost)	Very Fast	{[0.85, 1.00], [0.00, 0.10], [0.00, 0.10]}
	Fast	{[0.70, 0.85], [0.10, 0.20], [0.05, 0.15]}
	Moderate	{[0.50, 0.70], [0.15, 0.30], [0.15, 0.30]}
	Slow	{[0.30, 0.50], [0.25, 0.40], [0.25, 0.40]}
	Very Slow	{[0.10, 0.30], [0.35, 0.50], [0.35, 0.50]}
C_3 Conflict Resolution Ability	Excellent	{[0.85, 1.00], [0.00, 0.10], [0.00, 0.10]}
	Very Good	{[0.70, 0.85], [0.10, 0.20], [0.05, 0.15]}
	Good	{[0.55, 0.70], [0.15, 0.30], [0.15, 0.25]}
	Fair	{[0.35, 0.55], [0.25, 0.40], [0.25, 0.40]}
	Poor	{[0.15, 0.35], [0.35, 0.50], [0.35, 0.50]}

C_4 Observer Rating	Excellent	{[0.80, 0.95], [0.05, 0.15], [0.00, 0.10]}
	Very Good	{[0.65, 0.80], [0.10, 0.20], [0.05, 0.15]}
	Good	{[0.50, 0.65], [0.15, 0.30], [0.15, 0.25]}
	Fair	{[0.35, 0.50], [0.25, 0.40], [0.25, 0.35]}
	Poor	{[0.20, 0.35], [0.30, 0.50], [0.30, 0.50]}
C_5 Controversial Calls (Cost)	Very Few	{[0.80, 0.95], [0.05, 0.15], [0.00, 0.10]}
	Few	{[0.65, 0.80], [0.10, 0.20], [0.10, 0.20]}
	Moderate	{[0.50, 0.65], [0.15, 0.30], [0.20, 0.30]}
	Many	{[0.30, 0.50], [0.25, 0.40], [0.30, 0.40]}
	Very Many	{[0.10, 0.30], [0.35, 0.50], [0.40, 0.50]}
C_6 Stress Tolerance	Exceptional	{[0.85, 1.00], [0.00, 0.10], [0.00, 0.10]}
	Strong	{[0.70, 0.85], [0.10, 0.20], [0.05, 0.15]}
	Moderate	{[0.50, 0.70], [0.15, 0.30], [0.15, 0.25]}
	Weak	{[0.30, 0.50], [0.25, 0.40], [0.30, 0.40]}
	Very Weak	{[0.10, 0.30], [0.30, 0.50], [0.40, 0.50]}

Based on this linguistic representation, the panel members evaluate the seven referee candidates in the match and provide their assessments based on the linguistic tabulation provided in Table 1. The resulting decision matrix is provided in Table 2.

Table 2. NVNSS-based decision matrix for referee evaluation.

	C1	**C2**	**C3**
R1	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}
R2	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}
R3	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}
R4	{[0.10,0.30],[0.35,0.50],[0.35, 0.50]}	{[0.30,0.50],[0.25,0.40],[0.25, 0.40]}	{[0.15,0.35],[0.35,0.50],[0.35, 0.50]}
R5	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}
R6	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}	{[0.10,0.30],[0.35,0.50],[0.35, 0.50]}	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}
R7	{[0.10,0.30],[0.35,0.50],[0.35, 0.50]}	{[0.30,0.50],[0.25,0.40],[0.25, 0.40]}	{[0.15,0.35],[0.35,0.50],[0.35, 0.50]}

Table 2. NVNSS-based decision matrix for referee evaluation (cont….).

	C4	**C5**	**C6**

R1	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}
R2	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}	{[0.80,0.95],[0.05,0.15],[0.00, 0.10]}	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}
R3	{[0.35,0.55],[0.25,0.40],[0.25, 0.40]}	{[0.65,0.80],[0.10,0.20],[0.10, 0.20]}	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}
R4	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}	{[0.80,0.95],[0.05,0.15],[0.00, 0.10]}	{[0.30,0.50],[0.25,0.40],[0.30, 0.40]}
R5	{[0.35,0.55],[0.25,0.40],[0.25, 0.40]}	{[0.30,0.50],[0.25,0.40],[0.30, 0.40]}	{[0.85,1.00],[0.00,0.10],[0.00, 0.10]}
R6	{[0.55,0.70],[0.15,0.30],[0.15, 0.25]}	{[0.50,0.70],[0.15,0.30],[0.15, 0.30]}	{[0.70,0.85],[0.10,0.20],[0.05, 0.15]}
R7	{[0.15,0.35],[0.35,0.50],[0.35, 0.50]}	{[0.65,0.80],[0.10,0.20],[0.10, 0.20]}	{[0.10,0.30],[0.30,0.50],[0.40, 0.50]}

In Table 3, we present the computed degree of support for the NVNSS matrix based on the risk attitude parameter $\Re = 50\%$.

Table 3. Degrees of Support the derived from our NVNSS evaluation matrix.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
R1	0.629771	0.805369	0.629771	0.481693	0.461506	0.629771
R2	0.805369	0.629771	0.481693	0.805369	0.741563	0.805369
R3	0.461506	0.461506	0.805369	0.335887	0.582686	0.461506
R4	0.169868	0.297834	0.200074	0.481693	0.741563	0.301094
R5	0.629771	0.805369	0.481693	0.335887	0.301094	0.805369
R6	0.461506	0.629771	0.481693	0.481693	0.461506	0.629771
R7	0.169868	0.297834	0.335887	0.200074	0.582686	0.175673

First of all, according to Table 3 (the k-degree risk value is 50%), we can drive the rating values, as shown in Table 4.

Table 4. The rating values our NVNSS evaluation matrix

	C1	C2	C3	C4	C5	C6	$\boldsymbol{R_{min}}$
R1	3	4	3	2	2	3	2
R2	4	3	2	4	3	4	2
R3	2	2	4	2	3	2	2
R4	1	1	1	2	3	2	1
R5	3	4	2	2	2	4	2

R6	2	3	2	2	2	3	2
R7	1	1	2	1	3	1	1

For all alternatives, we omit the alternative that has $R_{min} < 2$. In our case study, the degree of support is determined by the expert's panels according to his/her interrogation type as well as the risk predilection. This, in turn, leads to the newly rated decision matrix Assumed in Table 5.

Table 5. The filtered-out referees according to their rating values of elements in the NVNSS evaluation matrix

	C1	C2	C3	C4	C5	C6	$\boldsymbol{R_{min}}$
R1	3	4	3	2	2	3	2
R2	4	3	2	4	3	4	2
R3	2	2	4	2	3	2	2
R5	3	4	2	2	2	4	2
R6	2	3	2	2	2	3	2

Next, Table 6 provides a calculation score function for each element in the NVNSS decision matrix.

Table 6. The scores were obtained by applying the proposed scoring function.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
R1	0.789062	0.929896	0.789062	0.669167	0.653958	0.789062
R2	0.929896	0.789062	0.669167	0.929896	0.874271	0.929896
R3	0.653958	0.653958	0.929896	0.5325	0.757813	0.653958
R4	0.365625	0.503125	0.390833	0.669167	0.874271	0.503125
R5	0.789062	0.929896	0.669167	0.5325	0.503125	0.929896
R6	0.653958	0.789062	0.669167	0.669167	0.653958	0.789062
R7	0.365625	0.503125	0.5325	0.390833	0.757813	0.37125

Based on the rating matrix, as well as the scores matrix, we pair both ratings and scores of each NVSS matrix in Table 7.

Table 7. Paired scoring and rating values from the NVNSS evaluation matrix.

	C1	**C2**	**C3**	**C4**	**C5**	**C6**
R1	(3, 0.7891)	(4, 0.9299)	(3, 0.7891)	(2, 0.6692)	(2, 0.654)	(3, 0.7891)
R2	(4, 0.9299)	(3, 0.7891)	(2, 0.6692)	(4, 0.9299)	(3, 0.8743)	(4, 0.9299)
R3	(2, 0.654)	(2, 0.654)	(4, 0.9299)	(2, 0.5325)	(3, 0.7578)	(2, 0.654)
R5	(3, 0.7891)	(4, 0.9299)	(2, 0.6692)	(2, 0.5325)	(2, 0.5031)	(4, 0.9299)
R6	(2, 0.654)	(3, 0.7891)	(2, 0.6692)	(2, 0.6692)	(2, 0.654)	(3, 0.7891)

In Table 8, we computed the pairwise difference between the score as well as the rating of different referees based on a particular criterion (C1).

Table 8. Pairwise Comparison of Referees Based on Variation in Rating and Score.

	R1	R2	R3	R5	R6
R1	(0.00, 0.00	(-1,-0.14)	(1,0.14)	(0,0.00)	(1,0.14)
R2	(1,0.14)	(0.00, 0.00	(2,0.28)	(1,0.14)	(2,0.28)
R3	(-1,-0.14)	(-2,-0.28)	(0.00, 0.00	(-1,-0.14)	(0,0.00)
R5	(0,0.00)	(-1,-0.14)	(1,0.14)	(0.00, 0.00	(1,0.14)
R6	(-1,-0.14)	(-2,-0.28)	(0,0.00)	(-1,-0.14)	(0.00, 0.00

In the following, we find the minimum pairs **(ΔS, ΔG)** in each row of a pairwise matrix, as shown in Figure 2. In this list of minimum pairs**, t**he alternative with the highest pair is ranked first, and the process is repeated till all referees are ranked according to a particular criterion.

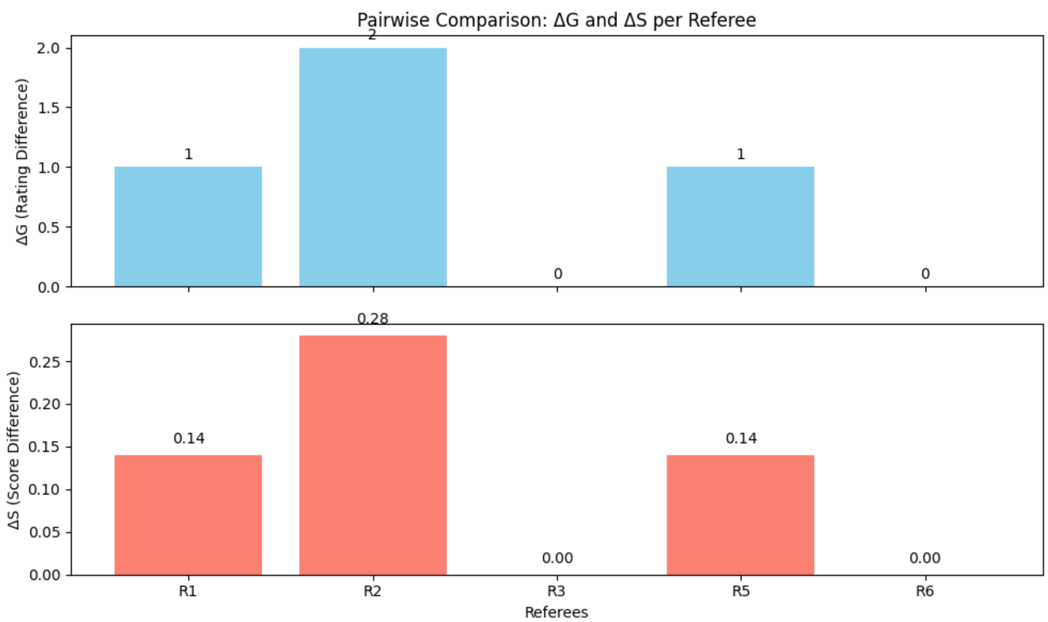

Figure 2. Illustration of the weakest performance for each referee against others based on C1.

The final ranking of referees concerning all criteria is described in Table 9.

Table 9. Ranking of Referees Based on different criteria.

	C1	C2	C3	C4	C5	C6
R1	2	1	2	2	3	3
R2	1	3	3	1	1	1
R3	4	5	1	4	2	5
R5	3	2	4	5	5	2
R6	5	4	5	3	4	4

Finally, we aggregate the final ranking based on the above ranking matrix using the proposed ranking mechanism. The final rank is Assumed in Figure 3.

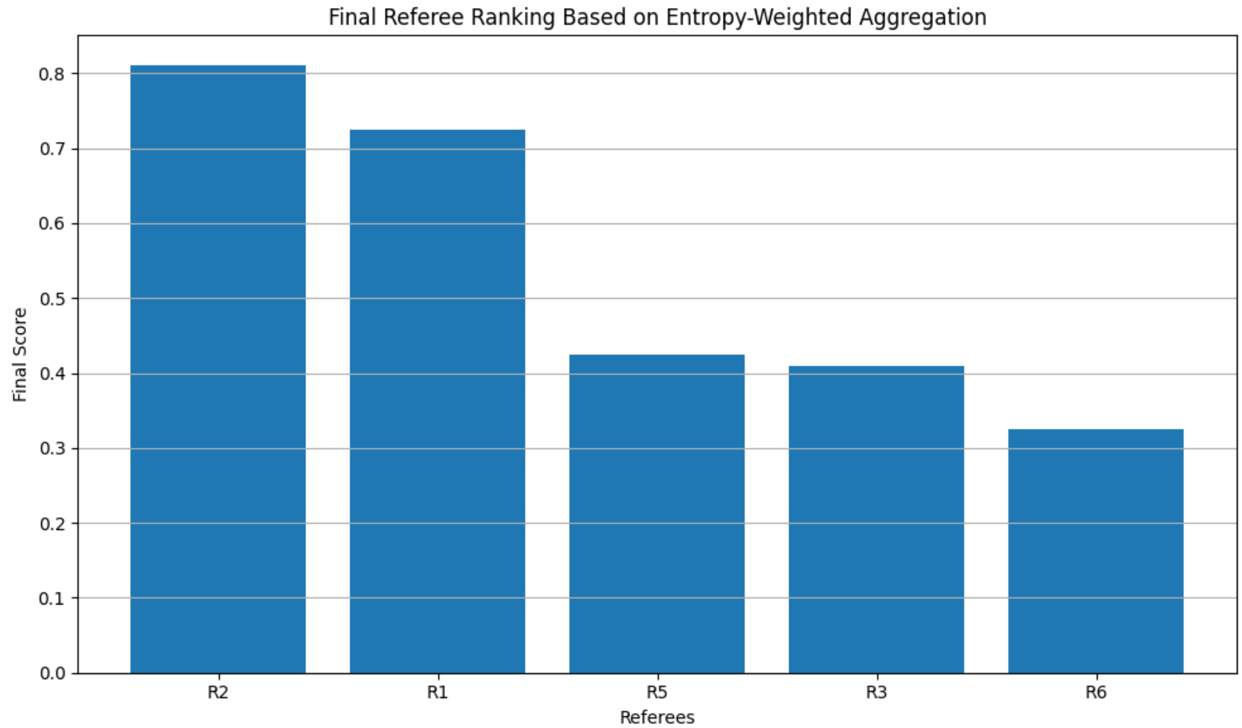

Figure 3. Final ranking of referee based on entropy-weighted aggregation.

3.1 Justification of NVNSS Scoring Function

The NVNSS scoring function introduced in this study addresses critical shortcomings of existing scoring methods by incorporating weighted contributions, non-linear transformations, and nuanced uncertainty management. The formula presented is explicitly designed to provide a balanced reflection of a referee's performance, accurately translating subjective evaluations into actionable quantitative measures.

Specifically, the scoring function takes into account the degree of truth, indeterminacy, and falsity simultaneously, using customized weightings determined empirically through expert judgment. By applying non-linear transformations, it mitigates extreme or biased evaluations, ensuring scores remain balanced and fair. The introduced context-sensitive penalties further refine the scoring, penalizing inconsistent or contradictory evaluations, thereby improving reliability and interpretability.

4. Robustness and Stability Analysis of the Proposed Model

To demonstrate the robustness of our proposed NVNSS-based evaluation methodology, a detailed stability analysis was conducted by examining potential variations in expert assessments and their impact on final rankings. Sensitivity testing involved systematically modifying the risk-attitude parameters and linguistic-to-NVNSS mappings. Results revealed that minor adjustments did not significantly affect the referees' overall ranking outcomes, underscoring the reliability and stability of our NVNSS framework. This robustness analysis confirms that the proposed approach can handle the inherent uncertainties and hesitations typical in subjective referee evaluations effectively.

5. Managerial and Practical Insights

Our findings offer vital managerial and practical implications for sports federations, officiating committees, and referee training programs. First, the integration of NVNSS significantly improves the objectivity and comprehensiveness of referee evaluations by systematically quantifying the intangible aspects of officiating under uncertainty. Referee training can utilize these insights to enhance targeted skills such as conflict resolution, stress management, and rapid decision-making, which are now quantitatively identifiable through our framework.

Furthermore, officiating committees can adopt this structured evaluation mechanism to facilitate unbiased referee selection and assignment processes for major tournaments, ensuring referees are evaluated fairly and comprehensively based on multiple weighted performance dimensions.

6. Conclusion

In this study, we introduced a comprehensive and intelligent decision-making framework for the evaluation of the officiating aptitude of uncertainty-aware national-level volleyball referees. A systematic framework is introduced to construct an NVNSS decision matrix, computation of risk-aware support values, discrete rating transformations, and the usage of a context-sensitive scoring function. The integration of pairwise comparison matrices and a priority relation-based ranking mechanism allowed for the precise ordering of referees under each criterion. Finally, a robust entropy-weighted aggregation model collected individual criterion-based rankings into a final ranking that also penalized contradiction. The results demonstrated that our solutions not only enhanced objectivity in referee evaluation but also provided actionable intuitions for sports federations, referee training programs, as well as policymakers. Future work can explore the incorporation of dynamics criteria weights, instantaneous data integration (e.g., match analytics), and broader applications in other domains where expert assessment under hesitation is critical.

6.1 Limitations and Suggestions for Future Research

Despite its significant contributions, the study acknowledges several limitations. Primarily, the subjective nature of linguistic evaluations could introduce bias despite extensive measures to ensure consistency. Future research might benefit from incorporating real-time decision-making analytics captured via advanced technologies (such as video-assisted review systems or AI-based image processing) to enhance evaluation objectivity.

Additionally, the current evaluation framework focuses specifically on volleyball referees at the national level. Expanding the model's applicability to other sports or varying competition levels (international or amateur) would significantly enhance its practical utility and generalizability. Future studies could also integrate dynamic criteria weighting that adjusts according to competition levels or game context, thereby capturing evolving performance expectations and referee capabilities more effectively.

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