Neutrosophic Sets and Systems, Vol. 83, 2025
University of New Mexico

Introducing Plithogenic Stochastic Processes with an Application to Poisson Process

Abdulrahman Astambli[1], Mohamed Bisher Zeina[2] and Moustafa Mazhar Ranneh[3]

[1] Faculty of Science, Dept. of Mathematical Statistics, University of Aleppo, Aleppo, Syria, e-mail: abdulrahman.est.96@gmail.com.

[2] Faculty of Science, Dept. of Mathematical Statistics, University of Aleppo, Aleppo, Syria, e-mail: bisher.zeina@gmail.com.

[3] Faculty of Science, Dept. of Mathematical Statistics, University of Aleppo, Aleppo, Syria, e-mail: mazhar.ranneh@gmail.com.

Abstract: In this paper, we study and define the mathematical form of plithogenic stochastic processes PSP based on set of three classic stochastic processes. This new definition is a generalization of neutrosophic stochastic process. characteristics of PSP are defined and theorems related to it were well-proved. Also, definition of weakly stationary PSP is introduced and it is proved that a plithogenic stochastic process is weakly stationary if and only if three corresponding crisp stochastic processes are weakly stationary. We also prove that the autocorrelation function of a plithogenic stochastic process is an even bounded function. As an application of this new form of stochastic processes, plithogenic Poisson process is defined and its properties are discussed. Solved example related to plithogenic Poisson process is successfully presented and solved. This new type of stochastic processes opens the road to many future researches in stochastic modelling.

Keywords: Plithogenic Probability, Stochastic Processes, Stationary Stochastic Processes, Characteristics of Stochastic Processes, Ensemble Mean, Covariance Function, Autocorrelation Function, Poisson Process.

1. Introduction

Neutrosophy and Plithogeny are two generalizations of logic and sets of numbers proposed by scientist F. Smarandache [1]–[4]. These two new generalizations raised in the literature and have been applied in many fields of science including pure mathematic, statistics, artificial intelligence, machine learning, engineering, etc. to handle several types of indeterminacy [5]–[9].

In probability theory, interval neutrosophic sets was applied to generalize classic probability distributions and it has been used to model many real-life problems [9]–[14]. Single-valued neutrosophic sets were also applied to study the probability that takes the form (T,I,F) where T is the chance the an event occurs, I is the indeterminacy of occurring and F is the probability that an event will not occur [15]–[19]. Literal neutrosophic sets was used to study random variables dealing with probabilities of the form $p_1 + p_2 I\,; I^2 = I$ and many probability distributions were modelled according to this new definition [20]–[24]. Plithogenic set which is the most general form of any set is also applied to generalize probability theory and its related concepts in [23]–[27].

It's very important to know the mathematical basics of neutrosophic and plithogenic real analysis to study neutrosophic and plithogenic probability theory and its related fields, these basics and basis were well-studied by many researchers in [28]–[32].

Stochastic modelling is the most important application of probability theory grows up in queueing theory, risk theory, financial mathematics, etc. Generalizing theory of stochastic modelling is based on generalizing main concepts of stochastic processes. A stochastic process is a family of random variables follow certain probability distribution. Fuzzy stochastic process is one generalization of classic stochastic processes when random variables are fuzzy random variables, which means that random variables take vague values or parameters of its probability distribution is uncertain [33]–[36]. Neutrosophic stochastic processes is another generalization of stochastic processes which was studied in many cases and first proposed by Zeina et al in [37]. The theoretical and mathematical study of neutrosophic stochastic process was done assuming that it takes the form $N(t) = X_1(t) + X_2(t)I, I^2 = I$ which is also called a literal neutrosophic stochastic process. Characteristics of $N(t)$ including ensemble average function, autocorrelation function, covariance function, concepts of strict and weak stationary are well studied in [37]. Concepts of continuity, integration, differentiation of a literal neutrosophic stochastic process is well studied and many theorems related to it were presented in [38]. Another possible generalization of stochastic processes is studying it in plithogenic case, where we can define a plithogenic stochastic process by $P(t) = X_0(t) + X_1(t)P_1 + X_2(t)P_2 + \cdots + X_n(t)P_n = X_0(t) + \sum_{i=1}^{n} X_i(t)P_i; P_i^2 = P_i, P_iP_j = P_jP_i = P_{max(i,j)}$ and when $n = 1$ we return to a literal neutrosophic stochastic process.

In this paper we will study a special case of this generalization assuming that $n = 2$, so the produced process will take the form: $P(t) = X_0(t) + X_1(t)P_1 + X_2(t)P_2$. We will study the characteristics of this process and the stationary conditions of it and present some theorems related to these important concepts which is the base of number of related topics.

As an application to the new defined stochastic process, we will introduce the plithogenic Poisson stochastic process and study its properties which is the road to define plithogenic queueing theory in future work directions.

Motivation and Research Gap

Literal neutrosophic stochastic processes were introduced in [35], with a discussion of certain properties in [36]. Research on this novel type of stochastic process is currently limited, yet it holds significant importance for modelling both stochastic and dynamic systems. Consequently, the extension of literal neutrosophic stochastic processes to plithogenic stochastic processes represents a crucial advancement in unveiling a new realm of stochastic systems. It can be inferred that the results derived from neutrosophy are a specific instance of those obtained from plithogenic analysis.

Preliminaries

Definition 2.1 [3]

Plithogenic classical numbers are numbers that have the form $a + bP_1 + cP_2$ where $P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_2$ and we call $2 - SP_R = \{a + bP_1 + cP_2; a, b, c \in R; P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_2\}$ the plithogenic field of reals.

Definition 2.2 [23]

Let $a_P = a_0 + a_1P_1 + a_2P_2$ and $b_P = b_0 + b_1P_1 + b_2P_2$ be two plithogenic numbers from $2 - SP_R$, we say that $a_P \geq_p b_P$ iff $a_0 \geq b_0, a_0 + a_1 \geq b_0 + b_1, a_0 + a_1 + a_2 \geq b_0 + b_1 + b_2$

Definition 2.3[25]

Let $a_P = a_0 + a_1P_1 + a_2P_2, b_P = b_0 + b_1P_1 + b_2P_2$ be plithogenic numbers from $2 - SP_R$, then arithmetic operations, square root and absolute value are defined as follows:

1. $a_P \pm b_P = a_0 \pm b_0 \pm P_1(a_1 \pm b_1) + P_2(a_2 \pm b_2)$
2. $a_P \times b_P = (a_0 + a_1P_1 + a_2P_2)(b_0 + b_1P_1 + b_2P_2) = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$
3. $\frac{a_P}{b_P} = \frac{a_0+a_1P_1+a_2P_2}{b_0+b_1P_1+b_2P_2} = \frac{a_0}{b_0} + \frac{a_1b_0-a_0b_1}{b_0(b_0+b_1)}P_1 + \frac{a_2b_1-a_1b_2+a_2b_0-a_0b_2}{(b_0+b_1)(b_0+b_1+b_2)}P_2; b_0 \neq 0, (b_0 + b_1) \neq 0, (b_0 + b_1 + b_2) \neq 0$
4. $\sqrt{a_0 + a_1P_1 + a_2P_2} = \sqrt{a_0} + \left[\sqrt{a_0 + a_1} - \sqrt{a_0}\right]P_1 + \left[\sqrt{a_0 + a_1 + a_2} - \sqrt{a_0 + a_1}\right]P_2$
5. $|a_0 + a_1P_1 + a_2P_2| = |a_0| + [|a_0 + a_1| - |a_0|]P_1 + [|a_0 + a_1 + a_2| - |a_0 + a_1|]P_2$

Definition 2.4[23]

$2 - SP_R$ plithogenic random variables are defined as follows:

$$X_P: \Omega_P \to R(P_1, P_2); \Omega_P = \Omega_0 \times \Omega_1(P_1) \times \Omega_2(P_2)$$

$$X_P = X_0 + X_1P_1 + X_2P_2; P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_2$$

And for properties of random variable X_P see [23].

Definition 2.5[39]

Let X_P be a plithogenic continuous random variable with plithogenic parameter $\theta_P = \theta_0 + \theta_1P_1 + \theta_2P_2$, then neutrosophic probability density function is defined as follows:

$$f(x_P; \theta_P) = f(x_0; \theta_0) + [f(x_0 + x_1; \theta_0 + \theta_1) - f(x_0; \theta_0)]P_1 + [f(x_0 + x_1 + x_2; \theta_0 + \theta_1 + \theta_2) - f(x_0 + x_1; \theta_0 + \theta_1)]P_2$$

Definition 2.6[37]

Literal neutrosophic stochastic process (LNSP) $\{X(t); t \in T\}$ is defined as follows:

$$X: \Omega \times T \to R(I)$$

$$X(t) = X_0(t) + X_1(t)I$$

Where $\{X_0(t); t \in T\}$ and $\{X_1(t); t \in T\}$ are two classic stochastic processes.

Theorem 2.1

Let $X(t) = X_0(t) + X_1(t)I$ be a (LNSP) then:

1. $\mu_X(t) = \mu_{X_0}(t) + \mu_{X_1}(t)I$
2. $R_X(s,t) = R_{X_0}(s,t) + \left[R_{X_1X_0}(s,t) + R_{X_0X_1}(s,t) + R_{X_1}(s,t)\right]I$
3. $C_X(s,t) = R_X(s,t) - \mu_X(s)\mu_X(t)$
4. $Var\left(X(t)\right) = C_X(t,t)$

Proof (see [37]).

2. Results and Discussion

Definition 3.1

We define a $2 - SP_R$ stochastic process $\{X_P(t); t \in T\}$ as follows:

$$X_P: \Omega_P \times T \to 2 - SP_R$$

$$X_P(t) = X_0(t) + X_1(t)P_1 + X_2(t)P_2 \qquad (1)$$

Where $\{X_0(t); t \in T\}$,$\{X_1(t); t \in T\}$ and $\{X_2(t); t \in T\}$ are three classic stochastic processes.

Definition 3.2

Let $\{X_P(t); t \in T\}$ be $(2-SP_R)SP$ we define the ensemble average function, autocorrelation function and covariance function as follows:

1. $\mu_{X_P}(t) = E[X_P(t)]$
2. $R_{X_P}(s,t) = E[X_P(s)X_P(t)]$
3. $C_{X_P}(s,t) = Cov[X_P(s), X_P(t)]$

Theorem 3.1

Let $\{X_P(t); t \in T\}$ be $(2-SP_R)SP$ then:

1. $\mu_{X_P}(t) = \mu_{X_0}(t) + \mu_{X_1}(t)P_1 + \mu_{X_2}(t)P_2$
2. $R_{X_P}(s,t) = R_{X_0}(s,t) + \left[R_{X_0X_1}(s,t) + R_{X_1X_0}(s,t) + R_{X_1}(s,t)\right]P_1 + \left[R_{X_0X_2}(s,t) + R_{X_2X_0}(s,t) + R_{X_2}(s,t) + R_{X_1X_2}(s,t) + R_{X_2X_1}(s,t)\right]P_2$
3. $C_{X_P}(s,t) = C_{X_0}(s,t) + \left[C_{X_0X_1}(s,t) + C_{X_1X_0}(s,t) + C_{X_1}(s,t)\right]P_1 + \left[C_{X_0X_2}(s,t) + C_{X_2X_0}(s,t) + C_{X_2}(s,t) + C_{X_1X_2}(s,t) + C_{X_2X_1}(s,t)\right]P_2$
4. $Var\big(X_P(t)\big) = C_{X_0}(t,t) + \left[2C_{X_0X_1}(t,t) + C_{X_1}(t,t)\right]P_1 + \left[2C_{X_0X_2}(t,t) + 2C_{X_1X_2}(t,t) + C_{X_2}(t,t)\right]P_2$
5. $\sigma[X_P(t)] = \sqrt{C_{X_0}(t,t)} + \left[\sqrt{C_{X_0}(t,t) + 2C_{X_0X_1}(t,t) + C_{X_1X_1}(t,t)} - \sqrt{C_{X_0}(t,t)}\right]P_1 + \left[\sqrt{C_{X_0}(t,t) + 2C_{X_0X_1}(t,t) + C_{X_1}(t,t) + 2C_{X_0X_2}(t,t) + 2C_{X_1X_2}(t,t) + C_{X_2}(t,t)} - \sqrt{C_{X_0}(t,t) + 2C_{X_0X_1}(t,t) + C_{X_1}(t,t)}\right]P_2$

Proof:

1. $\mu_{X_P}(t) = E[X_P(t)] = E[X_0(t) + X_1(t)P_1 + X_2(t)P_2]$

Since $X_0(t), X_1(t), X_2(t)$ become random variables if we fix the value $t \in T$, so:

$$\mu_{X_P}(t) = E[X_0(t)] + E[X_1(t)]P_1 + E[X_2(t)]P_2 = \mu_{X_0}(t) + \mu_{X_1}(t)P_1 + \mu_{X_2}(t)P_2$$

2. $R_{X_P}(s,t) = E[X_P(s)X_P(t)] = E[(X_0(s) + X_1(s)P_1 + X_2(s)P_2)(X_0(t) + X_1(t)P_1 + X_2(t)P_2)]$ (2)

$$= E[X_0(s)X_0(t) + X_0(s)X_1(t)P_1 + X_0(s)X_2(t)P_2 + X_1(s)X_0(t)P_1 + X_1(s)X_1(t)P_1^2 + X_1(s)X_2(t)P_1P_2] + X_2(s)X_0(t)P_2 + X_2(s)X_1(t)P_1P_2 + X_2(s)X_2(t)P_2^2$$

$$= R_{X_0}(s,t) + \left[R_{X_0X_1}(s,t) + R_{X_1X_0}(s,t) + R_{X_1}(s,t)\right]P_1 + \left[R_{X_0X_2}(s,t) + R_{X_2X_0}(s,t) + R_{X_2}(s,t) + R_{X_1X_2}(s,t) + R_{X_2X_1}(s,t)\right]P_2 \quad (3)$$

3. $C_{X_P}(s,t) = Cov[X_P(s), X_P(t)]$

$$= E\{[(X_0(s) + X_1(s)P_1 + X_2(s)P_2) - (\mu_{X_0}(s) + \mu_{X_1}(s)P_1 + \mu_{X_2}(s)P_2)][(X_0(t) + X_1(t)P_1 + X_2(t)P_2) - (\mu_{X_0}(t) + \mu_{X_1}(t)P_1 + \mu_{X_2}(t)P_2)]\}$$

$$= E\left\{\left[\left(X_0(s) - \mu_{X_0}(s)\right) + \left(X_1(s) - \mu_{X_1}(s)\right)P_1 + \left(X_2(s) - \mu_{X_2}(s)\right)P_2\right]\left[\left(X_0(t) - \mu_{X_0}(t)\right) + \left(X_1(t) - \mu_{X_1}(t)\right)P_1 + \left(X_2(t) - \mu_{X_2}(t)\right)P_2\right]\right\}$$

$$= E\Big[\big(X_0(s)-\mu_{X_0}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big)+\big(X_0(s)-\mu_{X_0}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big)P_1 + \big(X_0(s)-\mu_{X_0}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)P_2+\big(X_1(s)-\mu_{X_1}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big)P_1 + \big(X_1(s)-\mu_{X_1}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big)P_1+\big(X_1(s)-\mu_{X_1}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)P_1P_2 + \big(X_2(s)-\mu_{X_2}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big)P_2+\big(X_2(s)-\mu_{X_2}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big)P_2P_1 + \big(X_2(s)-\mu_{X_2}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)P_2P_2\Big]$$

$$= E\Big\{\Big[\big(X_0(s)-\mu_{X_0}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big)\Big] + \Big[\big(X_0(s)-\mu_{X_0}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big)+\big(X_1(s)-\mu_{X_1}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big) + \big(X_1(s)-\mu_{X_1}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big)\Big]P_1 + \Big[\big(X_0(s)-\mu_{X_0}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)+\big(X_2(s)-\mu_{X_2}(s)\big)\big(X_0(t)-\mu_{X_0}(t)\big) + \big(X_1(s)-\mu_{X_1}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)+\big(X_2(s)-\mu_{X_2}(s)\big)\big(X_1(t)-\mu_{X_1}(t)\big) + \big(X_2(s)-\mu_{X_2}(s)\big)\big(X_2(t)-\mu_{X_2}(t)\big)\Big]P_2\Big\}$$

$$= E\{[X_0(s)X_0(t)-X_0(s)\mu_{X_0}(t)-X_0(t)\mu_{X_0}(s)+\mu_{X_0}(s)\mu_{X_0}(t)] + [X_0(s)X_1(t)-X_0(s)\mu_{X_1}(t)-X_1(t)\mu_{X_0}(s)+\mu_{X_0}(s)\mu_{X_1}(t)+X_1(s)X_0(t)-X_1(s)\mu_{X_0}(t) - X_0(t)\mu_{X_1}(s)+\mu_{X_1}(s)\mu_{X_0}(t)+X_1(s)X_1(t)-X_1(s)\mu_{X_1}(t)-X_1(t)\mu_{X_1}(s) + \mu_{X_1}(s)\mu_{X_0}(t)]P_1 + [X_0(s)X_2(t)-X_0(s)\mu_{X_2}(t)-X_2(t)\mu_{X_0}(s)+\mu_{X_0}(s)\mu_{X_2}(t)+X_2(s)X_0(t) - X_2(s)\mu_{X_0}(t)-X_0(t)\mu_{X_2}(s)+\mu_{X_2}(s)\mu_{X_0}(t)+X_1(s)X_2(t)-X_1(s)\mu_{X_2}(t) - X_2(t)\mu_{X_1}(s)+\mu_{X_1}(s)\mu_{X_2}(t)+X_2(s)X_1(t)-X_2(s)\mu_{X_1}(t)-X_1(t)\mu_{X_2}(s) + \mu_{X_2}(s)\mu_{X_1}(t)+X_2(s)X_2(t)-X_2(s)\mu_{X_2}(t)-X_2(t)\mu_{X_2}(s)+\mu_{X_2}(s)\mu_{X_2}(t)]P_2\}$$

$$= [R_{X_0X_0}(s,t)-\mu_{X_0}(s)\mu_{X_0}(t)] + \Big[[R_{X_0X_1}(s,t)-\mu_{X_0}(s)\mu_{X_1}(t)]+[R_{X_1X_0}(s,t)-\mu_{X_1}(s)\mu_{X_0}(t)] + [R_{X_1X_1}(s,t)-\mu_{X_1}(s)\mu_{X_1}(t)]\Big]P_1 + \Big[[R_{X_0X_2}(s,t)-\mu_{X_0}(s)\mu_{X_2}(t)]+[R_{X_2X_0}(s,t)-\mu_{X_2}(s)\mu_{X_0}(t)] + [R_{X_1X_2}(s,t)-\mu_{X_1}(s)\mu_{X_2}(t)]+[R_{X_2X_1}(s,t)-\mu_{X_2}(s)\mu_{X_1}(t)] + [R_{X_2X_2}(s,t)-\mu_{X_2}(s)\mu_{X_2}(t)]\Big]P_2$$

$$= C_{X_0X_0}(s,t)+[C_{X_0X_1}(s,t)+C_{X_1X_0}(s,t)+C_{X_1X_1}(s,t)]P_1+[C_{X_0X_2}(s,t)+C_{X_2X_0}(s,t)+C_{X_1X_2}(s,t)+ C_{X_2X_1}(s,t)+C_{X_2X_2}(s,t)]P_2 \tag{4}$$

4. Substituting $s=t$ in 3 yields to:

$$Var[X_P(t)] = C_{X_P}(t,t) = C_{X_0X_0}(t,t)+[C_{X_0X_1}(t,t)+C_{X_1X_0}(t,t)+C_{X_1X_1}(t,t)]P_1 + [C_{X_0X_2}(t,t)+C_{X_2X_0}(t,t)+C_{X_1X_2}(t,t)+C_{X_2X_1}(t,t)+C_{X_2X_2}(t,t)]P_2$$

$$= C_{X_0}(t,t)+[2C_{X_0X_1}(t,t)+C_{X_1}(t,t)]P_1+[2C_{X_0X_2}(t,t)+2C_{X_1X_2}(t,t)+C_{X_2}(t,t)]P_2 \tag{5}$$

5. $\sigma[X_P(t)] = \sqrt{Var[X_P(t)]}$

$$= \sqrt{C_{X_0}(t,t)+[2C_{X_0X_1}(t,t)+C_{X_1}(t,t)]P_1+[2C_{X_0X_2}(t,t)+2C_{X_1X_2}(t,t)+C_{X_2}(t,t)]P_2}$$

$$= \sqrt{C_{X_0}(t,t)}+\Big[\sqrt{C_{X_0}(t,t)+2C_{X_0X_1}(t,t)+C_{X_1X_1}(t,t)}-\sqrt{C_{X_0}(t,t)}\Big]P_1 + \Big[\sqrt{C_{X_0}(t,t)+2C_{X_0X_1}(t,t)+C_{X_1}(t,t)+2C_{X_0X_2}(t,t)+2C_{X_1X_2}(t,t)+C_{X_2}(t,t)} - \sqrt{C_{X_0}(t,t)+2C_{X_0X_1}(t,t)+C_{X_1}(t,t)}\Big]P_2 \tag{6}$$

Definition 3.3

$2-SP_R$ plithogenic stochastic process is called weakly stationary if it satisfies the following two conditions:

1. $\mu_{X_P}(t)=\mu_{X_P}=\mu_{X_0}+\mu_{X_1}P_1+\mu_{X_1}P_2=constant$
2. $R_{X_P}(s,t)=R_{X_P}(\tau);\tau=|s-t|$

Theorem 3.2

$2-SP_R$ plithogenic stochastic process $X_P(t)=X_0(t)+X_1(t)P_1+X_2(t)P_2$ is weakly stationary if and only if $\{X_0(t);t\in T\}$, $\{X_0(t)+X_1(t);t\in T\}$, $\{X_0(t)+X_1(t)+X_2(t);t\in T\}$ are weakly stationary classic stochastic processes.

Proof:

- We will first suppose that $\{X_0(t);t\in T\}$, $\{X_0(t)+X_1(t);t\in T\}$, $\{X_0(t)+X_1(t)+X_2(t);t\in T\}$ are weakly stationary and prove that $X_P(t)=X_0(t)+X_1(t)P_1+X_2(t)P_2$ is also stationary:

Since $\{X_0(t);t\in T\}$ is weakly stationary then $\mu_{X_0}(t)=\mu_{X_0}=constant$ and $R_{X_0}(t,t-\tau)=R_{X_0}(\tau)$

We also supposed that $\{X_0(t)+X_1(t);t\in T\}$ is weakly stationary then $\mu_{X_0+X_1}(t)=\mu_{X_0+X_1}=constant$ which means that $\mu_{X_1}(t)=\mu_{X_1}=constant$ and

$$R_{X_0+X_1}(t,t-\tau)=E\{[X_0(t)+X_1(t)][X_0(t-\tau)+X_1(t-\tau)]\}=E\{X_0(t)X_0(t-\tau)+X_0(t)X_1(t-\tau)+X_1(t)X_0(t-\tau)+X_1(t)X_1(t-\tau)\}=R_{X_0}(t,t-\tau)+R_{X_0X_1}(t,t-\tau)+R_{X_1X_0}(t,t-\tau)+R_{X_1}(t,t-\tau) \quad (7)$$

Since $\{X_0(t)+X_1(t);t\in T\}$ is weakly stationary then $R_{X_0+X_1}(t,t-\tau)$ must depend only on the difference τ, so the only possible form of it will be:

$$R_{X_0+X_1}(t,t-\tau)=R_{X_0}(\tau)+R_{X_0X_1}(\tau)+R_{X_1X_0}(\tau)+R_{X_1}(\tau)$$

Which means that $R_{X_0X_1}(t,t-\tau)=R_{X_0X_1}(\tau)$, $R_{X_1X_0}(t,t-\tau)=R_{X_1X_0}(\tau)$, $R_{X_1}(t,t-\tau)=R_{X_1}(\tau)$

We also supposed that $\{X_0(t)+X_1(t)+X_2(t);t\in T\}$ is weakly stationary then $\mu_{X_0+X_1+X_2}(t)=\mu_{X_0+X_1+X_2}=constant$ which means that $\mu_{X_2}(t)=\mu_{X_2}=constant$ and

$$R_{X_0+X_1+X_2}(t,t-\tau)=E\{[X_0(t)+X_1(t)+X_2(t)][X_0(t-\tau)+X_1(t-\tau)+X_2(t-\tau)]\}=E[X_0(t)X_0(t-\tau)+X_0(t)X_1(t-\tau)+X_0(t)X_2(t-\tau)+X_1(t)X_0(t-\tau)+X_1(t)X_1(t-\tau)+X_1(t)X_2(t-\tau)+X_2(t)X_0(t-\tau)+X_2(t)X_1(t-\tau)+X_2(t)X_2(t-\tau)]=R_{X_0}(t,t-\tau)+R_{X_0X_1}(t,t-\tau)+R_{X_0X_2}(t,t-\tau)+R_{X_1X_0}(t,t-\tau)+R_{X_1}(t,t-\tau)+R_{X_1X_2}(t,t-\tau)+R_{X_2X_0}(t,t-\tau)+R_{X_2X_1}(t,t-\tau)+R_{X_2}(t,t-\tau) \quad (8)$$

Since $\{X_0(t)+X_1(t)+X_2(t);t\in T\}$ is weakly stationary then $R_{X_0+X_1+X_2}(t,t-\tau)$ must depend only on the difference τ, so the only possible form of it will be:

$$R_{X_0+X_1+X_2}(t,t-\tau)=R_{X_0}(\tau)+R_{X_0X_1}(\tau)+R_{X_0X_2}(\tau)+R_{X_1X_0}(\tau)+R_{X_1}(\tau)+R_{X_1X_2}(\tau)+R_{X_2X_0}(\tau)+R_{X_2X_1}(\tau)+R_{X_2}(\tau)=R_{X_0+X_1+X_2}(\tau) \quad (9)$$

Which means that $R_{X_0X_2}(t,t-\tau)=R_{X_0X_2}(\tau)$, $R_{X_2X_0}(t,t-\tau)=R_{X_2X_0}(\tau)$, $R_{X_2}(t,t-\tau)=R_{X_2}(\tau)$

$$E\big(X_P(t)\big)=E[X_0(t)+X_1(t)P_1+X_2(t)P_2]=\mu_{X_0}(t)+\mu_{X_1}(t)P_1+\mu_{X_2}(t)P_2=\mu_{X_0}+\mu_{X_1}P_1+\mu_{X_2}P_2 = constant$$

$$\begin{aligned}R_{X_P}(t,t-\tau)&=E[X_P(t)X_P(t-\tau)]\\&=R_{X_0}(t,t-\tau)+[R_{X_0X_1}(t,t-\tau)+R_{X_1X_0}(t,t-\tau)+R_{X_1}(t,t-\tau)]P_1\\&\quad+[R_{X_0X_2}(t,t-\tau)+R_{X_2X_0}(t,t-\tau)+R_{X_2}(t,t-\tau)+R_{X_1X_2}(t,t-\tau)+R_{X_2X_1}(t,t-\tau)]P_2\end{aligned}$$

$$=R_{X_0}(\tau)+[R_{X_0X_1}(\tau)+R_{X_1X_0}(\tau)+R_{X_1}(\tau)]P_1+[R_{X_0X_2}(\tau)+R_{X_2X_0}(\tau)+R_{X_2}(\tau)+R_{X_1X_2}(\tau)+R_{X_2X_1}(\tau)]P_2=R_{X_P}(\tau) \quad (10)$$

So, we conclude that $\{X_P(t);t\in T\}$ is weakly stationary.

- Now let's assume that $\{X_P(t); t \in T\}$ is weakly stationary and prove that $\{X_0(t); t \in T\}$, $\{X_0(t) + X_1(t); t \in T\}$, $\{X_0(t) + X_1(t) + X_2(t); t \in T\}$ are weakly stationary.

Since $\{X_P(t); t \in T\}$ is weakly stationary then $E\big(X_P(t)\big) = \mu_{X_P}(t) = \mu_{X_P} = constant$ but $E\big(X_P(t)\big) = \mu_{X_0}(t) + \mu_{X_1}(t)P_1 + \mu_{X_2}(t)P_2 = constant$ so $\mu_{X_0}(t), \mu_{X_1}(t)\ and\ \mu_{X_2}(t)$ must be dependent of time, then

$$\mu_{X_0}(t) = \mu_{X_0} \quad (11)$$

$$\mu_{X_1}(t) = \mu_{X_1} \quad (12)$$

$$\mu_{X_2}(t) = \mu_{X_2} \quad (13)$$

which meant that:

$$\mu_{X_0+X_1}(t) = \mu_{X_0+X_1} \quad (14)$$

$$\mu_{X_0+X_1+X_2}(t) = \mu_{X_0+X_1+X_2} \quad (15)$$

Also, we have:

$$R_{X_P}(t, t-\tau) = R_{X_0}(\tau) + \big[R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\big]P_1 + \big[R_{X_0X_2}(\tau) + R_{X_2X_0}(\tau) + R_{X_2}(\tau) + R_{X_1X_2}(\tau) + R_{X_2X_1}(\tau)\big]P_2 = R_{X_P}(\tau)$$

And since $\{X_P(t); t \in T\}$ is weakly stationary then $R_{X_P}(t, t-\tau)$ must depend only on the difference τ so the following equations must hold:

$$R_{X_0}(t, t-\tau) = R_{X_0}(\tau) \quad (16)$$

$$R_{X_0X_1}(t, t-\tau) = R_{X_0X_1}(\tau) \quad (17)$$

$$R_{X_1X_0}(t, t-\tau) = R_{X_1X_0}(\tau) \quad (18)$$

$$R_{X_1}(t, t-\tau) = R_{X_1}(\tau) \quad (19)$$

$$R_{X_0X_2}(t, t-\tau) = R_{X_0X_2}(\tau) \quad (20)$$

$$R_{X_2X_0}(t, t-\tau) = R_{X_2X_0}(\tau) \quad (21)$$

$$R_{X_2}(t, t-\tau) = R_{X_2}(\tau) \quad (22)$$

From equations (11), (16) we conclude that $\{X_0(t); t \in T\}$ is weakly stationary.

And using equations (14), (16-19) we conclude that $\{X_0(t) + X_1(t); t \in T\}$ is weakly stationary.

Also, using equations (15), (16-22) we conclude that $\{X_0(t) + X_1(t) + X_2(t); t \in T\}$ is weakly stationary.

Theorem 3.3

Suppose that $\{X_P(t); t \in T\}$ is weakly stationary plithogenic stochastic process with autocorrelation function $R_{X_P}(\tau)$ then the following holds:

1. $R_{X_P}(\tau) = R_{X_P}(-\tau)$
2. $\big|R_{X_P}(\tau)\big| \leq R_{X_P}(0)$

Proof:

1. We have

$$R_{X_P}(\tau) = R_{X_0}(\tau) + \big[R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\big]P_1 + \big[R_{X_0X_2}(\tau) + R_{X_2X_0}(\tau) + R_{X_2}(\tau) + R_{X_1X_2}(\tau) + R_{X_2X_1}(\tau)\big]P_2 \quad (23)$$

So:

$$R_{X_P}(-\tau) = R_{X_0}(-\tau) + \left[R_{X_0X_1}(-\tau) + R_{X_1X_0}(-\tau) + R_{X_1}(-\tau)\right]P_1 + \left[R_{X_0X_2}(-\tau) + R_{X_2X_0}(-\tau) + R_{X_2}(-\tau) + R_{X_1X_2}(-\tau) + R_{X_2X_1}(-\tau)\right]P_2 \qquad (24)$$

And using properties of cross-correlation function in classical stationary processes we get:

$$R_{X_P}(-\tau) = R_{X_0}(\tau) + \left[R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\right]P_1 + \left[R_{X_0X_2}(\tau) + R_{X_2X_0}(\tau) + R_{X_2}(\tau) + R_{X_1X_2}(\tau) + R_{X_2X_1}(\tau)\right]P_2 = R_{X_P}(\tau) \qquad (25)$$

2. $\left|R_{X_P}(\tau)\right| = \left|R_{X_0}(\tau) + \left[R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\right]P_1 + \left[R_{X_0X_2}(\tau) + R_{X_2X_0}(\tau) + R_{X_2}(\tau) + R_{X_1X_2}(\tau) + R_{X_2X_1}(\tau)\right]P_2\right|$

$$= \left|R_{X_0}(\tau)\right| + \left[\left|R_{X_0}(\tau) + R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\right| - \left|R_{X_0}(\tau)\right|\right]P_1 + \left[\left|R_{X_0}(\tau) + R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau) + R_{X_0X_2}(\tau) + R_{X_2X_0}(\tau) + R_{X_2}(\tau) + R_{X_1X_2}(\tau) + R_{X_2X_1}(\tau)\right| - \left|R_{X_0}(\tau) + R_{X_0X_1}(\tau) + R_{X_1X_0}(\tau) + R_{X_1}(\tau)\right|\right]P_2$$

$$= \left|R_{X_0}(\tau)\right| + \left[\left|R_{X_0+X_1}(\tau)\right| - \left|R_{X_0}(\tau)\right|\right]P_1 + \left[\left|R_{X_0+X_1+X_2}(\tau)\right| - \left|R_{X_0+X_1}(\tau)\right|\right]P_2 \qquad (26)$$

We have proposed that $X_P(t)$ is a weakly stationary plithogenic stochastic process, which means that $X_0(t), X_0(t) + X_1(t), X_0(t) + X_1(t) + X_2(t)$ are weakly stationary classic stochastic processes according to theorem 3.2, then:

$$\left|R_{X_0}(\tau)\right| \le R_{X_0}(0) \qquad (27)$$

$$\left|R_{X_0+X_1}(\tau)\right| \le R_{X_0+X_1}(0) \qquad (28)$$

$$\left|R_{X_0+X_1+X_2}(\tau)\right| \le R_{X_0+X_1+X_2}(0) \qquad (29)$$

Proving that $\left|R_{X_P}(\tau)\right| \le R_{X_P}(o)$ is equivalent to prove that:

$$\left|R_{X_0}(\tau)\right| + \left[\left|R_{X_0+X_1}(\tau)\right| - \left|R_{X_0}(\tau)\right|\right]P_1 + \left[\left|R_{X_0+X_1+X_2}(\tau)\right| - \left|R_{X_0+X_1}(\tau)\right|\right]P_2 \le R_{X_P}(0) \qquad (30)$$

Which holds according to definitions 2.2.

4. Comparison with neutrosophic stochastic processes

Comparing definitions 2.6 and 3.1 we can see that neutrosophic stochastic process NSP is a special case of plithogenic stochastic process PSP, the following table shows main differences between these two stochastic processes:

Table 1. Comparison between NSP and PSP.

	NSP	**PSP**
Definition	$X_N(t) = X_0(t) + X_1(t)I$	$X_P(t) = X_0(t) + X_1(t)P_1 + X_2(t)P_2$
Weakly stationary condition	Iff $X_0(t), X_0(t) + X_1(t)$ are weakly stationary	Iff$X_0(t), X_0(t) + X_1(t), X_0(t) + X_1(t) + X_2(t)$ are weakly stationary
Ensemble mean function	$\mu_{X_N}(t) = \mu_{X_0}(t) + \mu_{X_1}(t)\mathrm{I}$	$\mu_{X_P}(t) = \mu_{X_0}(t) + \mu_{X_1}(t)P_1 + \mu_{X_2}(t)P_2$

Covariance function	$C_N(s,t) = R_{X_0}(s,t) - \mu_{X0}(s) - \mu_{X0}(t) + I\{R_{X_0X_1}(s,t) + R_{X_1X_0}(s,t) + R_{X_1}(s,t) - \mu_{X_0}(s)\mu_{X_1}(t) - \mu_{X_0}(t)\mu_{X_1}(s) - \mu_{X_1}(s)\mu_{X_1}(t)\}$	$C_{X_P}(s,t) = C_{X_0}(s,t) + \left[C_{X_0X_1}(s,t) + C_{X_1X_0}(s,t) + C_{X_1}(s,t)\right]P_1 + \left[C_{X_0X_2}(s,t) + C_{X_2X_0}(s,t) + C_{X_2}(s,t) + C_{X_1X_2}(s,t) + C_{X_2X_1}(s,t)\right]P_2$
Autocorrelation function	$R_N(s,t) = R_{X_0}(s,t) + I\{R_{X_0X_1}(s,t) + R_{X_1X_0}(s,t) + R_{X_1}(s,t)\}$	$R_{X_P}(s,t) = R_{X_0}(s,t) + \left[R_{X_0X_1}(s,t) + R_{X_1X_0}(s,t) + R_{X_1}(s,t)\right]P_1 + \left[R_{X_0X_2}(s,t) + R_{X_2X_0}(s,t) + R_{X_2}(s,t) + R_{X_1X_2}(s,t) + R_{X_2X_1}(s,t)\right]P_2$

5. Application to plithogenic Poisson process

Lemma 4.1

Let $\{X(t)\},\{Y(t)\}$ be two classic Poisson processes with parameters λ_1, λ_2 respectively, then:

$$R_{XY}(s,t) = E[X(s)Y(t)] = \sum_{x=0}^{\infty}\sum_{y=0}^{\infty} xy\frac{(\lambda_1 s)^x}{x!}e^{-\lambda s}\frac{(\lambda_2 t)^y}{y!}e^{-\lambda t}$$

$$= \sum_{x=0}^{\infty} x\frac{(\lambda_1 s)^x}{x!}e^{-\lambda s}\sum_{y=0}^{\infty} y\frac{(\lambda_2 t)^y}{y!}e^{-\lambda t} = \lambda_1\lambda_2 st \qquad (31)$$

Definition 4.1

Let $\{N_0(t)\},\{N_1(t)\},\{N_2(t)\}$ be three classic Poisson processes with parameters $\lambda_0, \lambda_1, \lambda_2$ respectively, we define plithogenic Poisson process with parameter $\lambda_P = \lambda_0 + \lambda_1 P_1 + \lambda_2 P_2$ as follows:

$$N_P(t) = N_0(t) + N_1(t)P_1 + N_2(t)P_2 \qquad (32)$$

and its probability mass function is:

$$P(N_P(t) = n) = e^{-\lambda_0 t}\frac{(\lambda_0 t)^n}{n!} + \left[e^{-(\lambda_0+\lambda_1)t}\frac{[(\lambda_0+\lambda_1)t]^n}{n!} - e^{-\lambda_0 t}\frac{(\lambda_0 t)^n}{n!}\right]P_1 + \left[e^{-(\lambda_0+\lambda_1+\lambda_2)t}\frac{[(\lambda_0+\lambda_1+\lambda_2)t]^n}{n!} - e^{-(\lambda_0+\lambda_1)t}\frac{[(\lambda_0+\lambda_1)t]^n}{n!}\right]P_2\ ; n = 0,1,2,\ldots \qquad (33)$$

4.1 characteristics of plithogenic Poisson process:

1. $\mu_{N_P}(t) = \mu_{N_0}(t) + \mu_{N_1}(t)P_1 + \mu_{N_2}(t)P_2 = \lambda_0 t + \lambda_1 t P_1 + \lambda_2 t P_2 \qquad (34)$
2. $R_{N_P}(s,t) = R_{N_0}(s,t) + \left[R_{N_0N_1}(s,t) + R_{N_1N_0}(s,t) + R_{N_1}(s,t)\right]P_1 + \left[R_{N_0N_2}(s,t) + R_{N_2N_0}(s,t) + R_{N_1N_2}(s,t) + R_{N_2N_1}(s,t) + R_{N_2}(s,t)\right]P_2$

$$= (\lambda_0 s + \lambda_0^2 st) + [\lambda_0\lambda_1 st + \lambda_1\lambda_0 st + (\lambda_1 s + \lambda_1^2 st)]P_1 + [\lambda_0\lambda_2 st + \lambda_2\lambda_0 st + \lambda_1\lambda_2 st + \lambda_2\lambda_1 st + (\lambda_2 s + \lambda_2^2 st)]P_2; s \le t$$

$$= (\lambda_0 s + \lambda_0^2 st) + [\lambda_0\lambda_1 s^2t^2 + \lambda_1 s + \lambda_1^2 st]P_1 + [\lambda_0\lambda_2 s^2t^2 + \lambda_1\lambda_2 s^2t^2 + \lambda_2 s + \lambda_2^2 st]P_2; s \le t \qquad (35)$$

3. $C_{X_P}(s,t) = C_{X_0}(s,t) + \left[C_{X_0X_1}(s,t) + C_{X_1X_0}(s,t) + C_{X_1}(s,t)\right]P_1 + \left[C_{X_0X_2}(s,t) + C_{X_2X_0}(s,t) + C_{X_1X_2}(s,t) + C_{X_2X_1}(s,t) + C_{X_2}(s,t)\right]P_2$

$$= \left[R_{X_0}(s,t) - \mu_{X_0}(s)\mu_{X_0}(t)\right] + \left\{\left\{\left[R_{X_0X_1}(s,t) - \mu_{X_0}(s)\mu_{X_1}(t)\right] + \left[R_{X_1X_0}(s,t) - \mu_{X_1}(s)\mu_{X_0}(t)\right] + \left[R_{X_1}(s,t) - \mu_{X_1}(s)\mu_{X_1}(t)\right]\right\}P_1\right\} + \left\{\left[R_{X_0X_2}(s,t) - \mu_{X_0}(s)\mu_{X_2}(t)\right] + \left[R_{X_2X_0}(s,t) - \mu_{X_2}(s)\mu_{X_0}(t)\right] + \left[R_{X_1X_2}(s,t) - \mu_{X_1}(s)\mu_{X_2}(t)\right] + \left[R_{X_2X_1}(s,t) - \mu_{X_2}(s)\mu_{X_1}(t)\right] + \left[R_{X_2}(s,t) - \mu_{X_2}(s)\mu_{X_2}(t)\right]\right\}P_2$$

$$= [\lambda_0 s + \lambda_0^2 st - \lambda_0^2 st] + \{[\lambda_0\lambda_1 st - \lambda_0\lambda_1 st] + [\lambda_1\lambda_0 st - \lambda_1\lambda_0 st] + [\lambda_1 s + \lambda_1^2 st - \lambda_1^2 st]\}P_1 + \{[\lambda_0\lambda_2 st - \lambda_0\lambda_2 st] + [\lambda_2\lambda_0 st - \lambda_2\lambda_0 st] + [\lambda_1\lambda_2 st - \lambda_1\lambda_2 st] + [\lambda_2\lambda_1 st - \lambda_2\lambda_1 st] + [\lambda_2 s + \lambda_2^2 st - \lambda_2^2 st]\}P_2; s \le t$$

$$= \lambda_0 s + \lambda_1 s P_1 + \lambda_2 s P_2; s \le t \quad (36)$$

4. $Var\left(X_P(t)\right) = C_{X_P}(t,t) = \lambda_0 t + \lambda_1 t P_1 + \lambda_2 t P_2$ (37)

5. $\sigma[X_P(t)] = \sqrt{\lambda_0 t} + \left[\sqrt{\lambda_0 t + \lambda_1 t} - \sqrt{\lambda_0 t}\right]P_1 + \left[\sqrt{\lambda_0 t + \lambda_1 t + \lambda_2 t} - \sqrt{\lambda_0 t + \lambda_1 t}\right]P_2$ (38)

4.2 Numerical Example

Let $N_P(t) = N_0(t) + N_1(t)P_1 + N_2(t)P_2$ be a plithogenic Poisson process with an intensity:

$$\lambda_P = 6 - 2P_1 - 3P_2$$

Let's calculate the probabilities of: $N_P(2) = n, N_P(2) = 0, N_P(2) = 1, N_P(2) > 1$ and the appropriate characteristics.

Solution:

1. $P(N_P(2) = n) = e^{-12}\frac{12^n}{n!} + \left[e^{-8}\frac{8^n}{n!} - e^{-12}\frac{12^n}{n!}\right]P_1 + \left[e^{-2}\frac{2^n}{n!} - e^{-8}\frac{8^n}{n!}\right]P_2\,; n = 0,1,2,\ldots$

$$= e^{-12}\frac{12^n}{n!} + \left[e^{-8}\frac{8^n}{n!} - e^{-12}\frac{12^n}{n!}\right]P_1 + \left[e^{-2}\frac{2^n}{n!} - e^{-8}\frac{8^n}{n!}\right]P_2\,; n = 0,1,2,\ldots$$

2. $P(N_P(2) = 0) = e^{-12}\frac{12^0}{0!} + \left[e^{-8}\frac{8^0}{0!} - e^{-12}\frac{12^0}{0!}\right]P_1 + \left[e^{-2}\frac{2^0}{0!} - e^{-8}\frac{8^0}{0!}\right]P_2$

$$= e^{-12} + [e^{-8} - e^{-12}]P_1 + [e^{-2} - e^{-8}]P_2 = 0.000006 + 0.000329P_1 + 0.135P_2$$

3. $P(N_P(2) = 1) = e^{-12}\frac{12^1}{1!} + \left[e^{-8}\frac{8^1}{1!} - e^{-12}\frac{12^1}{1!}\right]P_1 + \left[e^{-2}\frac{2^1}{1!} - e^{-8}\frac{8^1}{1!}\right]P_2$

$$= 12e^{-12} + [8e^{-8} - 12e^{-12}]P_1 + [2e^{-2} - 8e^{-8}]P_2$$

$$= 0.0000737 + 0.0026P_1 + 0.27P_2$$

4. $P(N_P(2) > 1) = 1 - P(N_P = 0) = 1 - (0.000006 + 0.000329P_1 + 0.135P_2)$

$$= 0.999994 - 0.000329P_1 - 0.135P_2$$

5. $\mu_{X_P}(2) = \lambda_0 t + \lambda_1 t P_1 + \lambda_2 t P_2 = 6 - 4P_1 - 6P_2$

6. $R_{X_P}(s,t) = (\lambda_0 s + \lambda_0^2 st) + [\lambda_0\lambda_1 s^2t^2 + \lambda_1 s + \lambda_1^2 st]P_1 + [\lambda_0\lambda_2 s^2t^2 + \lambda_1\lambda_2 s^2t^2 + \lambda_2 s + \lambda_2^2 st]P_2; s \le t$

$$= (6s + 36st) + [-12s^2t^2 - 2s + 4st]P_1 + [-18s^2t^2 + 6s^2t^2 - 3s + 9st]P_2; s \le t$$

7. $C_{X_P}(s,t) = \lambda_0 s + \lambda_1 s P_1 + \lambda_2 s P_2; s \le t$

$$= 6s - 2sP_1 - 3sP_2; s \le t$$

8. $Var\left(X_P(2)\right) = \lambda_0 t + \lambda_1 t P_1 + \lambda_2 t P_2 = 6 - 4P_1 - 6P_2$

9. $\sigma\big(X_P(t)\big) = \sqrt{\lambda_0 t + \lambda_1 t P_1 + \lambda_2 t P_2} = \sqrt{\lambda_0 t} + \left[\sqrt{\lambda_0 t + \lambda_1 t} - \sqrt{\lambda_0 t}\right]P_1 + \left[\sqrt{\lambda_0 t + \lambda_1 t + \lambda_2 t} - \sqrt{\lambda_0 t + \lambda_1 t}\right]P_2$

$$= \sqrt{12} + \left[\sqrt{8} - \sqrt{12}\right]P_1 + \left[\sqrt{2} - \sqrt{8}\right]P_2$$

6. Conclusions and future research directions

In this paper, we have defined for the first time the plithogenic stochastic process in the form $X_P(t) = X_0(t) + X_1(t)P_1 + X_2(t)P_2$ and studied its main characteristics including ensemble mean function, autocorrelation function, covariance function and variance function. An important theorem about stationary is proved and results that $X_P(t)$ is stationary iff $X_0(t), X_0(t) + X_1(t), X_0(t) + X_1(t) + X_2(t)$ are stationary in classic sense. An application to Poisson process is presented and a solved example was successfully introduced. This paper is very important to study plithogenic reliability theory, queuing theory and survival analysis.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Acknowledgments: Authors are very thankful to editors-in-chief and to reviewers for the valuable help and recommendations.

References

[1] F. Smarandache *et al.*, "Introduction to neutrosophy and neutrosophic environment," *Neutrosophic Set Med. Image Anal.*, pp. 3–29, 2019, doi: 10.1016/B978-0-12-818148-5.00001-1.

[2] F. Smarandache, "Indeterminacy in neutrosophic theories and their applications," *Int. J. Neutrosophic Sci.*, vol. 15, no. 2, 2021, doi: 10.5281/zenodo.5295819.

[3] F. Smarandache, "Introduction to Plithogenic Logic as generalization of MultiVariate Logic," *Neutrosophic Sets Syst.*, vol. 45, 2021.

[4] F. Smarandache, "Neutrosophic theory and its applications," *Brussels*, vol. I, 2014.

[5] D. Zhang, Y. Ma, H. Zhao, and X. Yang, "Neutrosophic Clustering Algorithm Based on Sparse Regular Term Constraint," *Complexity*, vol. 2021, 2021, doi: 10.1155/2021/6657849.

[6] M. Abdel-Basset, M. El-hoseny, A. Gamal, and F. Smarandache, "A novel model for evaluation Hospital medical care systems based on plithogenic sets," *Artif. Intell. Med.*, vol. 100, Sep. 2019, doi: 10.1016/J.ARTMED.2019.101710.

[7] I. M. Hezam, M. K. Nayeem, A. Foul, and A. F. Alrasheedi, "COVID-19 Vaccine: A neutrosophic MCDM approach for determining the priority groups," *Results Phys.*, vol. 20, 2021, doi: 10.1016/j.rinp.2020.103654.

[8] R. A. K. Sherwani, H. Shakeel, W. B. Awan, M. Faheem, and M. Aslam, "Analysis of COVID-19 data using neutrosophic Kruskal Wallis H test," *BMC Med. Res. Methodol.*, vol. 21, no. 1, 2021, doi: 10.1186/s12874-021-01410-x.

[9] S. Salem, Z. Khan, H. Ayed, A. Brahmia, and A. Amin, "The Neutrosophic Lognormal Model in Lifetime Data Analysis: Properties and Applications," *J. Funct. Spaces*, vol. 2021, 2021, doi: 10.1155/2021/6337759.

[10] F. Shah, M. Aslam, Z. Khan, M. M. A. Almazah, and F. S. Alduais, "On Neutrosophic

Extension of the Maxwell Model: Properties and Applications," *J. Funct. Spaces*, vol. 2022, 2022, doi: 10.1155/2022/4536260.

[11] Z. Khan, M. M. A. Almazah, O. Hamood Odhah, and H. M. Alshanbari, "Generalized Pareto Model: Properties and Applications in Neutrosophic Data Modeling," *Math. Probl. Eng.*, vol. 2022, pp. 1–11, Sep. 2022, doi: 10.1155/2022/3686968.

[12] Z. Khan, M. Gulistan, N. Kausar, and C. Park, "Neutrosophic Rayleigh Model with Some Basic Characteristics and Engineering Applications," *IEEE Access*, vol. 9, pp. 71277–71283, 2021, doi: 10.1109/ACCESS.2021.3078150.

[13] M. B. Zeina, "Neutrosophic Event-Based Queueing Model," *Int. J. Neutrosophic Sci.*, vol. 6, no. 1, 2020, doi: 10.5281/zenodo.3840771.

[14] M. B. Zeina, "Erlang Service Queueing Model with Neutrosophic Parameters," *Int. J. Neutrosophic Sci.*, vol. 6, no. 2, pp. 106–112, 2020, doi: 10.54216/IJNS.060202.

[15] F. Smarandache, "(T, I, F)-Neutrosophic Structures," *Appl. Mech. Mater.*, vol. 811, 2015, doi: 10.4028/www.scientific.net/amm.811.104.

[16] F. Smarandache, "Plithogenic Probability & Statistics are generalizations of MultiVariate Probability & Statistics," *Neutrosophic Sets Syst.*, vol. 43, 2021.

[17] M. Mullai, K. Sangeetha, R. Surya, G. M. Kumar, R. Jeyabalan, and S. Broumi, "A Single Valued neutrosophic Inventory Model with Neutrosophic Random Variable," *Int. J. Neutrosophic Sci.*, vol. 1, no. 2, 2020, doi: 10.5281/zenodo.3679510.

[18] D. Nagarajan and J. Kavikumar, "Single-Valued and Interval-Valued Neutrosophic Hidden Markov Model," *Math. Probl. Eng.*, vol. 2022, 2022, doi: 10.1155/2022/5323530.

[19] M. B. Zeina, "Linguistic Single Valued Neutrosophic M/M/1 Queue," *Res. J. Aleppo Univ.*, vol. 144, 2021, Accessed: Feb. 21, 2023. [Online]. Available: https://www.researchgate.net/publication/348945390_Linguistic_Single_Valued_Neutrosophic_MM1_Queue

[20] M. B. Zeina and M. Abobala, "A novel approach of neutrosophic continuous probability distributions using AH-isometry with applications in medicine," *Cogn. Intell. with Neutrosophic Stat. Bioinforma.*, pp. 267–286, Jan. 2023, doi: 10.1016/B978-0-323-99456-9.00014-3.

[21] N. Sets, S. Massassati, M. B. Zeina, and Y. Karmouta, "Plithogenic and Neutrosophic Markov Chains : Modeling Uncertainty and Ambiguity in Stochastic Processes," *Neutrosophic Sets Syst.*, vol. 59, pp. 119–138, 2023.

[22] D. Dham, M. B. Zeina, and R. K. Al-Hamido, "Symbolic Neutrosophic and Plithogenic Marshall-Olkin Type I Class of Distributions," *Neutrosophic Sets Syst.*, vol. 59, pp. 234–252, 2023, doi: 10.5281/zenodo.10031226.

[23] M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, "Introduction to Symbolic 2-Plithogenic Probability Theory," *Galoitica J. Math. Struct. Appl.*, vol. 7, no. 1, 2023.

[24] N. Sets, N. Altounji, M. M. Ranneh, and M. B. Zeina, "Foundation of 2-Symbolic Plithogenic

Maximum a Posteriori Estimation," *Neutrosophic Sets Syst.*, vol. 59, pp. 59–74, 2023.

[25] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*. Belgium: Pons, 2018. Accessed: Feb. 23, 2023. [Online]. Available: http://arxiv.org/abs/1808.03948

[26] M. B. Zeina, M. Abobala, A. Hatip, S. Broumi, and S. Jalal Mosa, "Algebraic Approach to Literal Neutrosophic Kumaraswamy Probability Distribution," *Neutrosophic Sets Syst.*, vol. 54, pp. 124–138, 2023.

[27] M. B. Zeina and A. Hatip, "Neutrosophic Random Variables," *Neutrosophic Sets Syst.*, vol. 39, 2021, doi: 10.5281/zenodo.4444987.

[28] G. Narzary and B. Basumatary, "A Study on n-th Derivative of Neutrosophic Function and Neutrosophic Differential Equation," *Neutrosophic Sets Syst.*, vol. 75, 2025.

[29] H. Wang, J. Ye, and R. Yong, "Single Valued Neutrosophic Number Ensemble Learning Model for Stability Classification of Open Pit Mine Slopes," *Neutrosophic Sets Syst.*, vol. 75, 2025.

[30] P. Boro and B. Basumatary, "A Study on some Properties of Fourier Integrals Based on Neutrosophic Function," *Neutrosophic Sets Syst.*, vol. 75, 2025.

[31] Y. A. Alhasan, A. Alfahal, and R. A. Abdulfatah, "The 2- refined neutrosophic hyperbolic functions with its differential and integrals," *Neutrosophic Sets Syst.*, vol. 75, no. 2, pp. 1–14, 2025.

[32] H. Sun and Z. Liu, "Enhanced TFNN-GRA Technique for Multiple-Attribute Decision-Making with Triangular Fuzzy Neutrosophic Information and Applications to Performance Evaluation of Internally Cured High-Strength Concrete," *Neutrosophic Sets Syst.*, vol. 75, 2025.

[33] G. Hesamian, M. G. Akbari, and M. Shams, "A Goodness-of-Fit Test Based on Fuzzy Random Variables," *Fuzzy Inf. Eng.*, vol. 15, no. 1, pp. 55–68, Mar. 2023, doi: 10.26599/FIE.2023.9270005.

[34] M. L. PURI, D. A. RALESCU, and L. Zadeh, "Fuzzy Random Variables," *Readings Fuzzy Sets Intell. Syst.*, pp. 265–271, 1993, doi: 10.1016/B978-1-4832-1450-4.50029-8.

[35] D. S. Negi and E. S. Lee, "Analysis and simulation of fuzzy queues," *Fuzzy Sets Syst.*, vol. 46, no. 3, 1992, doi: 10.1016/0165-0114(92)90370-J.

[36] K. Khudr, M. Anan, and M. Zeina, "New Approach to FM/FM/1 Queue and its Performance Measures," *J. King Abdulaziz Univ.*, vol. 30, no. 1, 2018, doi: 10.4197/sci.30-1.7.

[37] M. B. Zeina and Y. Karmouta, "Introduction to Neutrosophic Stochastic Processes," *Neutrosophic Sets Syst.*, vol. 54, 2023.

[38] M. B. Zeina and Y. Karmouta, "MS-Continuity and MS-Differentiation of Neutrosophic Stochastic Processes," *Res. J. Aleppo Univ.*, vol. 167, pp. 1–13, 2023.

[39] M. B. Zeina, S. Hemeda, M. Koreny, and M. Abobala, "On Symbolic n-Plithogenic Random Variables Using a Generalized Isomorphism," vol. 59, 2023, doi: 10.5281/zenodo.10031266.

Received: Nov. 5, 2024. Accepted: April 7, 2025