Neutrosophic Sets and Systems, Vol. 83, 2025
University of New Mexico

Quadripartitioned Neutrosophic Soft Pre - Open And Pre - Closed Sets

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Abstract: The purpose of this article is to extend elements and salient features of quadripartitioned neutrosophic soft sets. The exploration of multiple traits has led to the introduction of the notion of quadric partitioned neutrosophic soft pre-open and pre-closed sets. Additionally, this article showcases the formulation and emphasis of several correlated theorems.

Key words : quadripartitioned neutrosophic soft set (QNSS), quadripartitioned neutrosophic soft topology (QNST), quadripartitioned neutrosophic soft open (QNSO), quadripartitioned neutrosophic soft closed (QNSC), quadripartitioned neutrosophic soft pre-open (QNSPO), quadripartitioned neutrosophic soft pre-closed (QNSPC)

1. Introduction

Molodtsov [1] introduced the soft sets concept as an innovative mathematical tool to address problems involving uncertainties. He provided a definition for a soft set as an assortment of subsets within a universal set, parameterized in a way that each element signifies an approximate group of elements that pertain to the given soft set. Maji et al. [2003] further explored soft sets, investigating subset and superset relationships, as well as the equality of soft sets in greater detail. In addition, they brought forth operations concerning soft sets, encompassing union, intersection, complement, difference, and other operations, scrutinizing their essential characteristics.

Uncertainties pose significant challenges in various real-life domains, such as Engineering, Economics, Social Sciences, Environment, Business Management, and Medical Sciences. Classical mathematical modeling often struggles with complex and difficult-to-describe uncertain data. In navigating uncertainties, researchers have expanded their toolkit by employing fuzzy sets and intuitionistic fuzzy sets, which differentiate themselves through their membership and non-membership functions. These tools struggle to cope with uncertain and conflicting data when characterizing objects in unclear situations.

Smarandache [2] proposed neutrosophic sets as a generalization of intuitionistic fuzzy sets. Neutrosophic sets assign specific values to membership, non-membership, and indeterminacy, allowing for a more nuanced representation of uncertainty than fuzzy or intuitionistic sets. This additional dimension makes neutrosophic sets a potentially more powerful tool for handling complex situations.

Maji et al. [3] integrated soft sets and neutrosophic sets, creating the concept of neutrosophic soft sets. Chatterjee et al. [6] proposed the extensive structure of quadripartitioned single-valued neutrosophic sets, expanding on Smarandache's four numerical-valued neutrosophic logic and Belnap's four-valued logic. This extension differentiates between the indeterminacy elements, specifying 'unknown' and 'contradiction.' In 2021, the researchers defined quadripartitioned neutrosophic soft sets [12], establishing their operations and properties. They further explored quadripartitioned neutrosophic topological spaces [12]. Currently, researchers are introducing the notions of pre-open and pre-closed sets in the context of quadripartitioned neutrosophic soft sets.

2. Preliminaries

2.1 Definition [7]

Let χ be a universal set and R is a parameter set. A non-empty set A, which is a subset of R. Denote the set of all quadripartitioned neutrosophic sets of χ as R(χ). A quadripartitioned neutrosophic soft set (QNSS) over χ is defined as a pair (K, A) where K is a function mapping element from A to subsets of χ (i.e., K: A $\rightarrow$ P(X)). A is defined as $A = \{< \mu, \mathrm{T_A}(\mu), \mathbb{C}_\mathrm{A}(\mu), \mho_\mathrm{A}(\mu), \mathrm{F_A}(\mu) >: r \in \chi\}$ and $\mathrm{T_A}$, $\mathbb{C}_\mathrm{A}$, $\mho_\mathrm{A}$, $\mathrm{F_A}$ χ *to the interval* [0,1] and $0 \leq \mathrm{T}_A(\mu), \mathbb{C}_A(\mu), \mho_A(\mu), \mathrm{F}_A(\mu) \leq 4$. Here $\mathrm{T_A}(\mu)$ indicates the truth membership, $\mathbb{C}_A(\mu)$ indicates the contradiction membership, $\mho_\mathrm{A}(\mu)$ indicates the ignorance membership and $\mathrm{F_A}(\mu)$ indicates the false membership.

2.2 Definition [11]

(K, A) is a quadripartitioned neutrosophic soft topology if it satisfies the following conditions: Let (K, A) be quadripartitioned neutrosophic set on (χ, R) and $\mathfrak{I}$ be a collection of quadripartitioned neutrosophic soft subsets of (K, A). Then (K, A) is called quadripartitioned neutrosophic soft topology if the following conditions are satisfied

i) $\phi_\mathrm{E}, \chi_E \in \mathfrak{I}$

ii) The union of any arbitrary collection of elements in $\mathfrak{I} \in \mathfrak{I}$.

iii) The intersection of any finite collection of elements in $\mathfrak{I} \in \mathfrak{I}$.

Then (χ, $\mathfrak{I}$, A) is a quadripartitioned neutrosophic soft topological space on χ.

3. Neutrosophic pre - open

This section presents characterizations of the QNS pre-open sets in χ through several theorems.

3.1 Definition

Let P be a QNSS belonging to a QNST on a universe χ. Then P is said to be a QNSPO set of χ if $\exists$ a QNSO set $s.t$ QNSO such that QNSO $\subseteq$ P $\subseteq$ QNSO (QNScl (P)).

3.2 Theorem

A subset S of a QNST χ is a QNSPO set $\Leftrightarrow$S satisfies S $\subseteq$ QNS int (QNScl (S)).

Proof:

Let us assume that S $\subseteq$ QNS int (QNScl (S)).

To Prove: S is QNST set

w.k.t., QNSO = QNS int(S)

Hence QNSO $\subseteq$ S $\subseteq$ QNSO (QNScl(S))

Then S is QNST set.

Converse part :

Let us assume, S is a QNST set in χ.

That is QNSO $\subseteq$ S $\subseteq$ QNSO (QNScl (S)) for some QNSO

at the same time, QNSO $\subseteq$ QNS int (S), thus QNSO (QNScl (S)) $\subseteq$ QNS int (QNScl (S))

Therefore S $\subseteq$ QNSO (QNScl (S)) $\subseteq$ QNS int (QNScl (S))

This implies that S $\subseteq$ QNS int (QNScl (S))

The theorem has been proven

3.3 Theorem

Let $\mathcal{P}$ and $\mathcal{Q}$ be the QNSPO sets in a QNST. Then $\mathcal{P} \cup \mathcal{Q}$ is also a QNSPO set.

Proof:

Consider two QNSPO sets $\mathcal{P}$ and $\mathcal{Q}$, in χ.

$\mathcal{P}$ $\subseteq$QNS int (QNScl ($\mathcal{P}$)) and $\mathcal{Q}$ $\subseteq$ QNS int (QNScl ($\mathcal{Q}$))

Then $\mathcal{P}$ $\cup$ Q $\subseteq$ QNS int (QNScl ($\mathcal{P}$)) $\cup$ QNScl ($\mathcal{Q}$)

$\mathcal{P}$ $\cup$ Q $\subseteq$ QNS int (QNScl ($\mathcal{P}$) $\cup$ QNScl ($\mathcal{Q}$))

$\mathcal{P}$ $\cup$ Q $\subseteq$ QNS int (QNScl ($\mathcal{P}$ $\cup$ $\mathcal{Q}$))

Then immediately we say that by using the definition, union of two QNSPO is a QNSPO set in χ.

The theorem has been proven`

3.4 Theorem

Consider a QNST (χ, τ). If $\{P_\alpha\}_{\alpha\epsilon\Delta}$ is a collection of QNSO sets in a QNST X then $\cup_{\alpha\epsilon\Delta}\{P_\alpha\}$ is QNSPO set in χ.

Proof:

For each $\alpha\epsilon\Delta$,we posses a QNSO set QNSOα such that QNSOα $\subseteq$ Pα $\subseteq$ QNSOα (QNScl (P))

Then$\cup_{\alpha\epsilon\Delta}$ $\mathrm{QNSO}\alpha$ $\subseteq$ QNSO α $\subseteq$$\cup_{\alpha\epsilon\Delta}\{P_\alpha\}$ $\subseteq$$\cup_{\alpha\epsilon\Delta}$ QNSOα(QNScl (P))

$\cup_{\alpha\epsilon\Delta}\{P_\alpha\}$ $\subseteq$$\cup_{\alpha\epsilon\Delta}$ QNSintα(QNScl (P))

Hence the proof.

3.5 Theorem

Let (χ, τ) be a QNST spaces. All QNSO sets within a QNST on the set χ are also QNSPO sets in χ.

Proof:

Consider P be QNSO set in QNST.

$\Rightarrow$ P = QNS int (P)

We know that , P $\subseteq$ QNScl (P)

$\Rightarrow$ QNS int (P) $\subseteq$ QNS int (QNScl (P)),

$\Rightarrow$ P $\subseteq$ QNS int (QNScl (P))

$\therefore$ P is a QNSPO set in χ.

Hench the proof

3.6 Theorem

Let $\mathcal{P}$ be a QNSPO set in χ. and suppose $\mathcal{P} \subseteq \mathcal{Q} \subseteq$ QNScl($\mathcal{P}$) then $\mathcal{Q}$ is QNSPO set in χ.

Proof:

Given $\mathcal{P}$ is a QNSPO set in QNST χ.

$\Rightarrow$ $\mathcal{P}$ = QNS int ($\mathcal{P}$) and $\mathcal{P} \subseteq$ QNScl ($\mathcal{P}$)

$\Rightarrow$QNS int ($\mathcal{P}$) $\subseteq$ QNS int (QNScl ($\mathcal{P}$))

$\Rightarrow$ $\mathcal{P} \subseteq$ QNS int (QNScl ($\mathcal{P}$))

Hence the proof.

3.7 Lemma

Let us consider $\mathcal{M}$ be QNSO set in QNST χ and $\mathcal{N}$ be a QNSPO set in χ then $\ni$ QNSO set $\mathcal{G}$ in χ such that $\mathcal{N} \subseteq \mathcal{G} \subseteq$ QNScl ($\mathcal{N}$) it follows that $\mathcal{M} \cap \mathcal{N} \subseteq \mathcal{M} \cap \mathcal{G} \subseteq \mathcal{M} \cap$ QNScl ($\mathcal{M}$) $\subseteq$ QNScl ($\mathcal{M} \cap \mathcal{N}$))

Since $\mathcal{M} \cap \mathcal{G}$ is open, lemma, $\mathcal{M} \cap \mathcal{N}$ is QNSPO set in χ.

3.8 Proposition

Let χ and ν be two QNST spaces, where χ is a QNs product related to ν. If P is a QNSPO set of χ and Q is a QNSPO set of ν, then the QNS product P x Q is a QNSPO set of the QNS product topological space χ x ν.

Proof:

Consider $\zeta_1 \subseteq$ P $\subseteq$ QNScl (ζ_1) and $\varsigma_2 \subseteq$ Q $\subseteq$ QNScl (ς_2)

Then ζ_1x $\varsigma_2 \subseteq$ P x Q $\subseteq$ QNScl (ζ_1) x QNScl (ς_2)

$\zeta_1 x \varsigma_2 \subseteq$ P x Q $\subseteq$ QNScl (ζ_1 x ς_2)

QNSint (ζ_1 x ς_2) $\subseteq$ QNSint (P x Q) $\subseteq$ QNSint (QNScl (ζ_1 x ς_2))

ζ_1 x $\varsigma_2 \subseteq$ P x Q $\subseteq$ QNS int (QNScl (ζ_1 x ς_2))

Hence P x Q is QNSPO set in χ x ν

4. Quadripartitioned Neutrosophic Soft Pre – Closed (QNSPC):

4.1 Definition

Let us consider P be QNSS of a QNST space χ. Then P is said to be QNSPC sets of χ if there exists a QNSC set such that QNScl (QNSC) $\subseteq$ P $\subseteq$ QNSC.

4.2 Theorem

A subset P in a QNST space χ is QNSC set $\Leftrightarrow$ QNScl (QNSint(P)) $\subseteq$ P

Proof:

Let us assume that QNScl (QNSint (P)) $\subseteq$ P

This implies that QNSC = QNScl (P)

Then definitely QNScl (QNSint (P)) $\subseteq$ A $\subseteq$ QNSC

$\therefore$ P is QNSPC set

Let us assume that Conversely, P be QNSPC set in χ

Then QNSC (QNSint (P)) $\subseteq$ P $\subseteq$ QNSC for some QNS closed set QNSC.

but QNScl (P) $\subseteq$ QNSC

Hence the proof.

4.3 Theorem

Consider χ be QNST space and P be a QNS subset of χ then P is a QNSPC sets $\Leftrightarrow C$ (P) is QNSPO set in χ.

Proof:

Consider a set P that is a QNSPC set subset of χ.

$\Rightarrow$QNScl (QNS int (P)) $\subseteq$ P

$\Rightarrow C$ (P) $\subseteq$ C (QNScl (QNS int (P))) (both sides, taking complement)

$\Rightarrow C$ (P) $\subseteq$ QNS int (QNScl (C (P)))

Hence C (P) is QNSPO set.

Converse part:

Let us assume that C (P) is QNSPOset

That is, C (P) $\subseteq$ QNS int (QNScl (C (P)))

$\Rightarrow$QNScl (QNS int (P)) $\subseteq$ P, a QNSPC set.

The theorem has been proven.

4.4 Theorem

Let (χ, τ) be a QNST spaces. Let M and N be the QNSPC sets in a QNST. Then M $\cap N$ is also a QNSPC set.

Proof:

Consider M and N be two QNSPC sets on (χ, τ)

$\Rightarrow$ QNScl (QNS int (M)) $\subseteq$ M and QNScl (QNS int (N)) $\subseteq$ N

Consider M $\cap$ N $\supseteq$ QNScl (QNS int (M)) $\cap$ QNScl (QNS int (N))

$\supseteq$ QNScl (QNS int(M) $\cap$ QNS int (N))

$\supseteq$ QNScl (QNS int (M $\cap$ N)) QNScl (QNS int (M $\cap$ N)) $\subseteq$ M $\cap$ N

$\therefore$ M $\cap$ N is QNSPC set.

4.5 Remark

The union of any two QNSPC sets need not be a QNSPC set on (χ, τ) .

4.6 Theorem

Let$\{P\}_{\alpha \in \Delta}$be a collection of QNSPC sets on (χ, τ)then $\underset{\alpha \epsilon \Delta}{\cap} P$is QNSPC sets on (χ, τ).

Proof:

Let us take QNS set QNSC_{α} such that QNSC_{α} (QNS int (P)) $\subseteq P_{\alpha} \subseteq$ QNSC_{α} for all $\alpha \in \Delta$

Then $\underset{\alpha \epsilon \Delta}{\cap} P$ QNSC (QNS int (P) $\subseteq \underset{\alpha \epsilon \Delta}{\cap} P \subseteq$ QNSC_{α}

$\underset{\alpha\epsilon\Delta}{\cap} P$ $^{\text{QNSC}_\alpha}$ (QNS int (P)) $\subseteq$ $\underset{\alpha\epsilon\Delta}{\cap} P$

Hence $\underset{\alpha\epsilon\Delta}{\cap} P$ is QNSPC set on (χ, τ)

4.7 Theorem

Every QNSC set in the QNST spaces (χ, τ) is QNSPC set in (χ, τ).

Proof:

Let P be QNSC set means P = QNScl(P) and also QNS int (P) ⊆ P

From that, QNScl (QNS int (P)) ⊆ QNScl (P), QNScl (QNS int (P)) ⊆ , since P = QNScl (P)

∴ P is a QNSPC sets.

4.8 Theorem

Let $\mathcal{P}$ be a QNSC set in QNST spaces $(\chi, \mathfrak{I})$. If QNS int ($\mathcal{P}$)⊆ $\mathcal{Q}$ ⊆ $\mathcal{P}$ then $\mathcal{Q}$ is QNSPC set on $(\chi, \mathfrak{I})$

Proof:

Let $\mathcal{P}$ be a QNSC set in QNST spaces $(\chi, \mathfrak{I})$.

Suppose QNS int ($\mathcal{P}$)⊆ $\mathcal{Q}$ ⊆ $\mathcal{P}$

Then ∃ QNSC set NC, such that QNSC (QNS int ($\mathcal{P}$))⊆ $\mathcal{Q}$ ⊆ $\mathcal{P}$ ⊆ QNSC.

Then QNSC and also QNS int ($\mathcal{Q}$)⊆ $\mathcal{Q}$ ⊆ QNSC

Thus, QNScl (QNS int ($\mathcal{Q}$))⊆ $\mathcal{Q}$

Hence $\mathcal{Q}$ is QNSPC set on $(\chi, \mathfrak{I})$.

4.9 Theorem

If χ and ν are QNST spaces, where χ is a QNS product relative to ν, then the QNS product $\mathcal{P} \times \mathcal{Q}$ is a QNSPC subset of the QNS product topological space $\mathcal{P} \times \mathcal{Q}$, where $\mathcal{P}$ is a QNSPC subset of χ and $\mathcal{Q}$ is a QNSPC subset of ν.

Proof:

Let $\mathcal{P}$ and $\mathcal{Q}$ be two QNSPC set

C1 (QNS int ($\mathcal{P}$))⊆ $\mathcal{P}$ ⊆C1 and C2 (QNS int ($\mathcal{P}$))⊆ $\mathcal{P}$ ⊆ C2

Form the above,

C1 (QNS int ($\mathcal{P}$)) x C2 (QNS int($\mathcal{P}$))⊆ $\mathcal{P}$ x $\mathcal{Q}$ ⊆ C1 x C2

(C1 x C2) (QNS int ($\mathcal{P}$ x Q))⊆($\mathcal{P}$ x $\mathcal{Q}$) ⊆ (C1 x C2)

Hence $\mathcal{P}$ x $\mathcal{Q}$ is QNSPC set in QNST space χ x ν.

5. Conclusion

This paper defines and explores QNS pre-open sets and pre-closed sets in neutrosophic topology and its characterizations are discussed. Also, the researchers have studied QNS pre-closed set in QNST space and its comparisons with other sets.

Acknowledgement

The authors are thankful to the reviewers for their valuable suggestions.

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Received: Nov. 7, 2024. Accepted: April 9, 2025