NSS

Neutrosophic Sets and Systems, Vol. 83, 2025

University of New Mexico

Exploring Neutrosophic Contra Alpha Generalized Semi-Continuous Maps

V. Banu Priya[1], M. Suresh[2], *, S. Chandrasekar[3] and A. Atkinswestley[4]

[1]RMK College of Engineering and Technology; spriya.maths@gmail.com

[2]R.M.D. Engineering College; sureshmaths2209@gmail.com

[3]Sri Meenakshi Government Arts College for Women; chandrumat@gmail.com

[4]K. Ramakrishnan College of Engineering; ats.wesly@gmail.com

*Correspondence: sureshmaths2209@gmail.com

Abstract. In this manuscript, we have introduced the concepts of contra alpha generalized semi-continuous mapping and contra alpha semi-irresolute mapping within the Neutrosophic framework. Furthermore, we have derived certain properties associated with these mappings.

Keywords: Neutrosophic contra α-generalized semi- continuous mapping; Neutrosophic contra α -generalized semi-irresolute mapping

---

1. Introduction

Dontchev [1] initially proposed the notion of contra-continuity, while Jafari and Noiri [3] introduced novel extensions of contra-continuity termed contra-α-continuity in the context of topological spaces. In 2006, Ekici and Etienne Kerre [4] introduced contra-continuous mapping in the domain of fuzzy topological spaces. Additionally, Kresteska and Ekici introduced the concept of intuitionistic fuzzy contra-continuous mapping.

2. Neutrosophic contra α-generalized semi continuous mappings

Definition 2.1. A mapping $\grave{N}^{f_*}_{eut} : (\grave{N}^{\grave{X}}_{eut}, \grave{N}^{t}_{eut}) \to (\grave{N}^{\grave{Y}}_{eut}, \grave{N}^{s}_{eut})$ is called a Neutrosophic contra α-generalized semi continuous ($\grave{N}_{eut}C\alpha GS$ continuous in short) mapping if $\grave{N}^{{f_*}^{-1}}_{eut}(\hat{E}^{*}_{22})$ is a $\grave{N}_{eut}\alpha GSOS$ in $(\grave{N}^{\grave{X}}_{eut}, \grave{N}^{t}_{eut})$ for every $\grave{N}_{eut}CS$ $\hat{E}^{*}_{22}$ of $(\grave{N}^{\grave{Y}}_{eut}, \grave{N}^{s}_{eut})$.

Example 2.2. Let $\grave{N}^{\grave{X}}_{eut} = \{a_1^*, a_2^*\}$, $\grave{N}^{\grave{Y}}_{eut} = \{a_3^*, a_4^*\}$, $\hat{E}^{*}_{11} = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{4}{5}, \frac{1}{2}, \frac{3}{10}) \rangle$ and $\hat{E}^{*}_{22} = \langle y, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{1}{10}, \frac{1}{2}, \frac{9}{10}) \rangle$. Then $\grave{N}^{t}_{eut} = \{0_{\grave{N}_{eut}}, \hat{E}^{*}_{11}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}^{s}_{eut} = \{0_{\grave{N}_{eut}}, \hat{E}^{*}_{22}, 1_{\grave{N}_{eut}}\}$

are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Then $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$continuous mapping.

Theorem 2.3. *Every Neutrosophic contra continuous mapping is a $\grave{N}_{eut}C\alpha GS$ continuous mapping .*

Proof. Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a Neutrosophic contra continuous mapping. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. By hypothesis, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{X}}$. Since every $\grave{N}_{eut}OS$ is a $\grave{N}_{eut}\alpha GSOS$, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Example 2.4. $\grave{N}_{eut}^{\grave{X}}C\alpha GS$ continuous $\not\to$ Neutrosophic contra continuous.
Let $\grave{N}_{eut}^{\grave{X}}= \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{1}{10}, \frac{1}{2}, \frac{9}{10})\rangle$ and $\hat{E_{22}^*} = \langle y, (\frac{4}{5}, \frac{1}{2}, \frac{1}{5}), (\frac{9}{10}, \frac{1}{2}, \frac{1}{10})\rangle$. Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s}= \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}^{\grave{X}}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Then $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}^{\grave{X}}C\alpha GS$ continuous mapping. But $\grave{N}_{eut}^{f_*}$ is not a $\grave{N}_{eut}^{\grave{X}}$ contra continuous mapping. Since ${\hat{E_{22}^*}}^c = \langle y, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{1}{10}, \frac{1}{2}, \frac{9}{10})\rangle$ is a $\grave{N}_{eut}^{\grave{X}}CS$ in $\grave{N}_{eut}^{\grave{Y}}$ but ${\grave{N}_{eut}^{f_*}}^{-1}({\hat{E_{22}^*}}^c) = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{4}{5}), (\frac{1}{10}, \frac{1}{2}, \frac{9}{10})\rangle$ is not a $\grave{N}_{eut}^{\grave{X}}OS$ in $\grave{N}_{eut}^{\grave{X}}$.

Theorem 2.5. *Every $\grave{N}_{eut}C\alpha$ continuous mapping is a $\grave{N}_{eut}C\alpha GS$continuous mapping .*

Proof. Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a $\grave{N}_{eut}C\alpha$ continuous mapping. Let $\hat{E_{11}^*}$be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Then by hypothesis ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha OS$ in $\grave{N}_{eut}^{\grave{X}}$. Since every $\grave{N}_{eut}\alpha OS$ is a $\grave{N}_{eut}\alpha GSOS$, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Example 2.6. $\grave{N}_{eut}C\alpha GS$ continuous mapping $\not\to$ $\grave{N}_{eut}C\alpha$ continuous mapping.
Let $\grave{N}_{eut}^{\grave{X}}= \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{1}{10}, \frac{1}{2}, \frac{9}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$ and $\hat{E_{22}^*} = \langle y, (1, \frac{1}{2}, 0), (\frac{7}{10}, \frac{1}{2}, \frac{3}{10})\rangle$. Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s}= \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on X and Y respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Then $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping. But $\grave{N}_{eut}^{f_*}$ is not a $\grave{N}_{eut}C\alpha$ continuous mapping. Since ${\hat{E_{22}^*}}^c = \langle y, (0, \frac{1}{2}, 1), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10})\rangle$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$ but ${\grave{N}_{eut}^{f_*}}^{-1}({\hat{E_{22}^*}}^c) = \langle x, (0, \frac{1}{2}, 1), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10})\rangle$ is not a $\grave{N}_{eut}\alpha OS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping but not a $\grave{N}_{eut}C\alpha$ continuous mapping.

Remark 2.7. $\grave{N}_{eut}C\gamma$ continuous mapping and $\grave{N}_{eut}C\alpha GS$ continuous mapping are independent of each other.

Example 2.8. $\grave{N}_{eut}C\gamma$ continuous mapping $\not\to$ $\grave{N}_{eut}C\alpha GS$ continuous mapping.
Let $\grave{N}_{eut}^{\grave{X}}= \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{3}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{7}{10})\rangle$ and $\hat{E_{22}^*} = \langle y, (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$. Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s}= \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$

are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Since $\hat{E_{22}^*}^{c} = \langle y, (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5})\rangle$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$ but $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{22}^*}^{c}) = \langle x, (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5})\rangle$ is not a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\gamma$ continuous mapping but not a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Example 2.9. $\grave{N}_{eut}C\alpha GS$ continuous mapping $\not\to$ $\grave{N}_{eut}C\gamma$ continuous mapping.
Let $\grave{N}_{eut}^{\grave{X}} = \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$,
$\hat{E_{22}^*} = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{3}{5}), (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\rangle$ and $\hat{E_{33}^*} = \langle y, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5})\rangle$.
Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s} = \{0_{\grave{N}_{eut}}, \hat{E_{33}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Since $\hat{E_{33}^*}^{c} = \langle y, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$ is a $\grave{N}_{eut}CS$ in Y but $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{33}^*}^{c}) = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$ is not a $\grave{N}_{eut}\gamma OS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping but not a $\grave{N}_{eut}C\gamma$ continuous mapping.

Remark 2.10. $\grave{N}_{eut}CP$ continuous mapping and $\grave{N}_{eut}C\alpha GS$ continuous mapping are independent of each other.

Example 2.11. $\grave{N}_{eut}CP$ continuous mapping $\not\to$ $\grave{N}_{eut}C\alpha GS$ continuous mapping.
Let $\grave{N}_{eut}^{\grave{X}} = \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{7}{10}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5})\rangle$ and
$\hat{E_{22}^*} = \langle y, (\frac{1}{10}, \frac{1}{2}, \frac{4}{5}), (\frac{1}{10}, \frac{1}{2}, \frac{7}{10})\rangle$. Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s} = \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively.
Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Then $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping. But $\grave{N}_{eut}^{f_*}$ is not a $\grave{N}_{eut}C\alpha GS$ continuous mapping. Since $\hat{E_{22}^*}^{c} = \langle y, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{7}{10}, \frac{1}{2}, \frac{1}{10})\rangle$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$ but $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{22}^*}^{c}) = \langle x, (\frac{1}{5}, \frac{1}{2}, \frac{7}{10}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5})\rangle$ is not a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$.

Example 2.12. $\grave{N}_{eut}C\alpha GS$ continuous mapping $\not\to$ $\grave{N}_{eut}CP$ continuous mapping.
Let $\grave{N}_{eut}^{\grave{X}} = \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$,
$\hat{E_{22}^*} = \langle x, (\frac{2}{5}, \frac{1}{2}, \frac{3}{5}), (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\rangle$ and $\hat{E_{33}^*} = \langle y, (\frac{3}{10}, \frac{1}{2}, \frac{7}{10}), (\frac{3}{5}, \frac{1}{2}, \frac{2}{5})\rangle$.
Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s} = \{0_{\grave{N}_{eut}}, \hat{E_{33}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Since $\hat{E_{33}^*}^{c} = \langle y, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$ is a $\grave{N}_{eut}CS$ in Y but $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{33}^*}^{c}) = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{3}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{3}{5})\rangle$ is not a $\grave{N}_{eut}POS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping but not a $\grave{N}_{eut}CP$ continuous mapping.

Theorem 2.13. *A mapping $\grave{N}_{eut}^{f_*} : \grave{N}_{eut}^{\grave{X}} \to \grave{N}_{eut}^{\grave{Y}}$ is a $\grave{N}_{eut}^{f_*}CaGS$ continuous if and only if the inverse image of each $\grave{N}_{eut}^{f_*}OS$ in $\grave{N}_{eut}^{\grave{Y}}$ is a $\grave{N}_{eut}^{f_*}aGSCS$ in $\grave{N}_{eut}^{\grave{X}}$.*

Proof. Necessary Part: Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Y}}$. This implies $\hat{E_{11}^*}^{c}$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}^{c})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$.

Since $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}^c) = (\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}))^c, \grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$.

Sufficient Part: Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. This implies $\hat{E_{11}^*}^c$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Y}}$. By hypothesis, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}^c)$ is $\grave{N}_{eut}G\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Since $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}^c) = (\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}))^c$, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$ is $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Theorem 2.14. *Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a mapping and let $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*}^c)$ be a $\grave{N}_{eut}ROS$ in $\grave{N}_{eut}^{\grave{X}}$ for every $\grave{N}_{eut}CS$ $\hat{E_{11}^*}$ in $\grave{N}_{eut}^{\grave{Y}}$. Then $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Then by hypothesis, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$is a $\grave{N}_{eut}ROS$ in $\grave{N}_{eut}^{\grave{X}}$. Since every $\grave{N}_{eut}ROS$ is a $\grave{N}_{eut}\alpha GSOS$, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Theorem 2.15. *Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a $\grave{N}_{eut}C\alpha GS$ continuous mapping, then $\grave{N}_{eut}^{f_*}$ is a Neutrosophic contra continuous mapping if $\grave{N}_{eut}^{\grave{X}}$is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Then by hypothesis $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Since $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space, $\grave{N}_{eut}^{f_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a Neutrosophic contra continuous mapping.

Theorem 2.16. *Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a $\grave{N}_{eut}C\alpha GS$ continuous mapping and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s}) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^{d})$ is a Neutrosophic contra continuous mapping, then $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*}) : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^{d})$ is a $\grave{N}_{eut}\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then by hypothesis, $\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping, $\grave{N}_{eut}^{f_*^{-1}}(\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*}))$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})$ is a $\grave{N}_{eut}\alpha GS$ continuous mapping.

Theorem 2.17. *Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ be a $\grave{N}_{eut}\alpha GS$ continuous mapping and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s}) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^{d}$ is a Neutrosophic contra continuous mapping, then $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^{d})$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then $\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Y}}$, by hypothesis. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}\alpha GS$ continuous mapping, $\grave{N}_{eut}^{f_*^{-1}}(\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*}))$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Theorem 2.18. *$\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s}) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^{d})$ be mappings. Then the given conditions are equivalent if $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space.*

(1) *$\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.*

(2) $\grave{N}_{eut}$-$cl(\grave{N}_{eut}$-$int(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(B)) \subseteq (\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ *for every* $\grave{N}_{eut}OS$ $\hat{E_{22}^*}$ *in* Z.

Proof. (1)⇒ (2) Let $\hat{E_{22}^*}$ be any $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$, by hypothesis. Since $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}space$, $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}$-$cl((\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})) = (\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$. Now $\grave{N}_{eut}$-$cl(\grave{N}_{eut}$-$int(\grave{N}_{eut}$-$cl(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}))) \subseteq \grave{N}_{eut}$-$cl(\grave{N}_{eut}$-$cl(\grave{N}_{eut}$-$cl(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}))) = \grave{N}_{eut}$-$cl((\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}))$. This implies $\grave{N}_{eut}$-$cl(\grave{N}_{eut}$-$int(\grave{N}_{eut}$-$cl(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}))) \subseteq (\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$.
(2)⇒ (1) Let $\hat{E_{22}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then its complement $\hat{E_{22}^*}^c$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Z}}$. By hypothesis, $\grave{N}_{eut} - cl(\grave{N}_{eut}$-$int(\grave{N}_{eut}$-$cl(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}^c))) \subseteq (\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}^c)$. Hence $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}^c)$ is a $\grave{N}_{eut}\alpha CS$ in $\grave{N}_{eut}^{\grave{X}}$. Since every $\grave{N}_{eut}\alpha CS$ is a $\grave{N}_{eut}\alpha GSCS$, $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*}^c)$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*}$ is $\grave{N}_{eut}C\alpha GS$ continuous mapping.

3. **Neutrosophic contra alpha generalized semi irresolute mappings**

Definition 3.1. A mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ is called a Neutrosophic contra alpha generalized semi irresolute ($\grave{N}_{eut}C\alpha GS$ irresolute in short) mapping if ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $(\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t})$ for every $\grave{N}_{eut}\alpha GSOS$ $\hat{E_{11}^*}$ of $(\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$.

Theorem 3.2. *If* $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ *is a* $\grave{N}_{eut}C\alpha GS$ *irresolute mapping, then* f *is a* $\grave{N}_{eut}C\alpha GS$ *continuous mapping but the converse is not necessarily true.*

Proof. Let $\grave{N}_{eut}^{f_*}$ be a $\grave{N}_{eut}C\alpha GS$ irresolute mapping. Let $\hat{E_{11}^*}$ be any $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since every $\grave{N}_{eut}CS$ is a $\grave{N}_{eut}\alpha GSCS$, $\hat{E_{11}^*}$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{Y}}$. By hypothesis, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Example 3.3. $\grave{N}_{eut}C\alpha GS$ continuous mapping $\not\to$ $\grave{N}_{eut}C\alpha GS$ irresolute mapping.
Let $\grave{N}_{eut}^{\grave{X}} = \{a_1^*, a_2^*\}$, $\grave{N}_{eut}^{\grave{Y}} = \{a_3^*, a_4^*\}$, $\hat{E_{11}^*} = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ and
$\hat{E_{22}^*} = \langle y, (\frac{7}{10}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5})\rangle$. Then $\grave{N}_{eut}^{t} = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^{s} = \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on $\grave{N}_{eut}^{\grave{X}}$ and $\grave{N}_{eut}^{\grave{Y}}$ respectively.
Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ by $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$. Since $\hat{E_{22}^*} = \langle y, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$,
but ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{22}^*}) = (x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2}))$ is not a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Therefore $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping but not a $\grave{N}_{eut}C\alpha GS$ irresolute mapping.

Theorem 3.4. *Let* $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^{t}) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^{s})$ *be a* $\grave{N}_{eut}C\alpha GS$ *irresolute mapping, then* ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ *is a* $\grave{N}_{eut}GSCS$ *in* $\grave{N}_{eut}^{\grave{X}}$ *for every* $\grave{N}_{eut}OS$ $\hat{E_{11}^*}$ *in* $\grave{N}_{eut}^{\grave{Y}}$.

Proof. Let $\grave{N}_{eut}^{f_*}$ be a $\grave{N}_{eut}C\alpha GS$ irresolute mapping. Let $\hat{E_{11}^*}$ be any $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since every $\grave{N}_{eut}OS$ is a $\grave{N}_{eut}\alpha GSOS$, $\hat{E_{11}^*}$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$. By hypothesis, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. This implies ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}GSCS$ in $\grave{N}_{eut}^{\grave{X}}$.

Example 3.5. Let $\grave{N}_{eut}^{f_*}(a_1^*) = a_3^*$ and $\grave{N}_{eut}^{f_*}(a_2^*) = a_4^*$, $\hat{E_{11}^*} = \langle x, (\frac{3}{10}, \frac{1}{2}, \frac{3}{5}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ and $\hat{E_{22}^*} = \langle y, (\frac{7}{10}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5})\rangle$. Then $\grave{N}_{eut}^t = \{0_{\grave{N}_{eut}}, \hat{E_{11}^*}, 1_{\grave{N}_{eut}}\}$ and $\grave{N}_{eut}^s = \{0_{\grave{N}_{eut}}, \hat{E_{22}^*}, 1_{\grave{N}_{eut}}\}$ are $\grave{N}_{eut}Ts$ on X and Y respectively. Define a mapping $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s)$. Then ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}GSCS$ in $\grave{N}_{eut}^{\grave{X}}$ for every $\grave{N}_{eut}OS$ $\hat{E_{11}^*} = \langle x, (\frac{7}{10}, \frac{1}{2}, \frac{1}{5}), (\frac{3}{10}, \frac{1}{2}, \frac{3}{5})\rangle$ in $\grave{N}_{eut}^{\grave{Y}}$. But $\grave{N}_{eut}^{f_*}$ is not a $\grave{N}_{eut}C\alpha GS$ irresolute mapping. Since $\hat{E_{22}^*} = \langle y, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$ but ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{22}^*}) = \langle x, (\frac{4}{5}, \frac{1}{2}, \frac{1}{10}), (\frac{2}{5}, \frac{1}{2}, \frac{1}{2})\rangle$ is not a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$.

Theorem 3.6. *Let $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s)$ is a $\grave{N}_{eut}C\alpha GS$ irresolute mapping, then $\grave{N}_{eut}^{f_*}$ is a Neutrosophic contra continuous mapping if $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. We know that every $\grave{N}_{eut}CS$ is a $\grave{N}_{eut}\alpha GSCS$, $\hat{E_{11}^*}$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{Y}}$. By hypothesis, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Since $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space, ${\grave{N}_{eut}^{f_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{f_*}$ is a Neutrosophic contra continuous mapping.

Theorem 3.7. *If $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s)$ and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ are $\grave{N}_{eut}C\alpha GS$ irresolute mapping, then $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ is a $\grave{N}_{eut}\alpha GS$ irresolute mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then ${\grave{N}_{eut}^{g_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ irresolute mapping, ${\grave{N}_{eut}^{f_*}}^{-1}({\grave{N}_{eut}^{g_*}}^{-1}\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}\alpha GS$ irresolute mapping .

Theorem 3.8. *If $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s)$ is a $\grave{N}_{eut}C\alpha GS$ irresolute mapping and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ is a $\grave{N}_{eut}C\alpha GS$ continuous mappings, then $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ is a $\grave{N}_{eut}\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then by hypothesis, ${\grave{N}_{eut}^{g_*}}^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$is a $\grave{N}_{eut}C\alpha GS$ irresolute mapping, ${\grave{N}_{eut}^{f_*}}^{-1}({\grave{N}_{eut}^{g_*}}^{-1}\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}\alpha GS$continuous mapping.

Theorem 3.9. *If $\grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s)$ is a $N\alpha GS$ irresolute mapping and $\grave{N}_{eut}^{g_*} : (\grave{N}_{eut}^{\grave{Y}}, \grave{N}_{eut}^s) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ is a $NC\alpha GS$ continuous mappings, then $\grave{N}_{eut}^{g_*} \circ \grave{N}_{eut}^{f_*} : (\grave{N}_{eut}^{\grave{X}}, \grave{N}_{eut}^t) \to (\grave{N}_{eut}^{\grave{Z}}, \grave{N}_{eut}^d)$ is a $NC\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then by hypothesis, $\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}\alpha GS$ irresolute mapping, $\grave{N}_{eut}^{f_*^{-1}}(\grave{N}_{eut}^{g_*^{-1}}\hat{E_{11}^*})$is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Theorem 3.10. *Let $\grave{N}_{eut}^{f_*}:(\grave{N}_{eut}^{\grave{X}},\grave{N}_{eut}^{t})\to(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})$ and $\grave{N}_{eut}^{g_*}:(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})\to(\grave{N}_{eut}^{\grave{Z}},\grave{N}_{eut}^{d})$ be any two mappings. If the mapping $\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ irresolute mapping and X is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space,then*

(1) *$(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$ for each $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{Z}}$.*

(2) *$\grave{N}_{eut}$-$cl(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(Neu{-}int(\hat{E_{22}^*}))\subseteq(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ for each Neutrosophic set $\hat{E_{22}^*}$ of $\grave{N}_{eut}^{\grave{Z}}$.*

Proof. (1) Let $\hat{E_{22}^*}$ be a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then $\hat{E_{22}^*}^{c}$ is a $\grave{N}_{eut}\alpha GSOS$in $\grave{N}_{eut}^{\grave{Z}}$. By hypothesis, $\hat{E_{22}^*}^{c}$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. This implies $\hat{E_{22}^*}$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. (2) Let $\hat{E_{22}^*}$ be any $\grave{N}_{eut}S$ in $\grave{N}_{eut}^{\grave{Z}}$ and $\grave{N}_{eut}$-$int(\hat{E_{22}^*})\subseteq\hat{E_{22}^*}$. Then $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}\text{-}int(\hat{E_{22}^*}))\subseteq(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$. Since $\grave{N}_{eut}$-$int(\hat{E_{22}^*})$ is a $\grave{N}_{eut}OS$ in $\grave{N}_{eut}^{\grave{Z}}$, $\grave{N}_{eut}$-$int(\hat{E_{22}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{Z}}$.

Therefore by hypothesis, $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}\text{-}int(\hat{E_{22}^*}))$ is a $\grave{N}_{eut}\alpha GSCS$ in $\grave{N}_{eut}^{\grave{X}}$. Since $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space, $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}\text{-}int(\hat{E_{22}^*}))$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}$-$cl((\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}\text{-}int(B)))=(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}-int(\hat{E_{22}^*}))\subseteq(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$. Therefore $\grave{N}_{eut}$-$cl((\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\grave{N}_{eut}\text{-}int(\hat{E_{22}^*})))\subseteq(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ for each $\grave{N}_{eut}S$ $\hat{E_{22}^*}$ of $\grave{N}_{eut}CS$.

Theorem 3.11. *If $\grave{N}_{eut}^{f_*}:(\grave{N}_{eut}^{\grave{X}},\grave{N}_{eut}^{t})\to(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping and $\grave{N}_{eut}^{g_*}:(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})\to(\grave{N}_{eut}^{\grave{Z}},\grave{N}_{eut}^{d})$ is a Neutrosophic continuous mappings, then $\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*}:(\grave{N}_{eut}^{\grave{X}},\grave{N}_{eut}^{t})\to(\grave{N}_{eut}^{\grave{Z}},\grave{N}_{eut}^{d})$ is a $\grave{N}_{eut}\alpha GS$ continuous mapping.*

Proof. Let $\hat{E_{11}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. Then by hypothesis, $\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Y}}$. Since $\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping, $\grave{N}_{eut}^{f_*^{-1}}(\grave{N}_{eut}^{g_*^{-1}}(\hat{E_{11}^*}))$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. That is $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{11}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Hence $\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

Theorem 3.12. *Let $\grave{N}_{eut}^{f_*}:(\grave{N}_{eut}^{\grave{X}},\grave{N}_{eut}^{t})\to(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})$ and $\grave{N}_{eut}^{g_*}:(\grave{N}_{eut}^{\grave{Y}},\grave{N}_{eut}^{s})\to(\grave{N}_{eut}^{\grave{Z}},\grave{N}_{eut}^{d})$ be any two mappings. Then the given conditions are equivalent if $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space.*

(1) *$\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*}$ is a $\grave{N}_{eut}C\alpha G$ continuous mapping.*

(2) *$(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})\subseteq\grave{N}_{eut}-int(\grave{N}_{eut}-cl(\grave{N}_{eut}-int(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})))$ for each $\grave{N}_{eut}CS$ $\hat{E_{22}^*}$ of $\grave{N}_{eut}^{\grave{Z}}$.*

Proof. (1)⇒(2) Let $\hat{E_{22}^*}$ be a $\grave{N}_{eut}CS$ in $\grave{N}_{eut}^{\grave{Z}}$. By hypothesis, $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}_{eut}^{\grave{X}}$. Since $\grave{N}_{eut}^{\grave{X}}$ is a $\grave{N}_{eut}\alpha gaT_{1/2}$ space, $(\grave{N}_{eut}^{g_*}\circ\grave{N}_{eut}^{f_*})^{-1}(\hat{E_{22}^*})$ is a $\grave{N}_{eut}OS$

in $\grave{N}^{\grave{X}}_{eut}$. Therefore $(\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}) = \grave{N}_{eut}\text{-}int((\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}))$. But $\grave{N}_{eut}\text{-}int((\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}})) \subseteq \grave{N}_{eut}\text{-}int(\grave{N}_{eut}\text{-}cl(\grave{N}_{eut}\text{-}int((\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}))))$. This implies $(\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}) \subseteq \grave{N}_{eut}\text{-}int(\grave{N}_{eut}\text{-}cl(\grave{N}_{eut}\text{-}int((\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}))))$ for every $\grave{N}_{eut}CS\hat{E^*_{22}}$ in $\grave{N}^{\grave{Z}}_{eut}$.

(2)⇒ (1) Let $\hat{E^*_{22}}$ be a $\grave{N}_{eut}CS$ in $\grave{N}^{\grave{Z}}_{eut}$.

By hypothesis, $(\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}) \subseteq \grave{N}_{eut}\text{-}int(\grave{N}_{eut}\text{-}cl(\grave{N}_{eut}\text{-}int((\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}}))))$. This implies $(\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}})$ is a $\grave{N}_{eut}\alpha OS$ in $\grave{N}^{\grave{X}}_{eut}$ and hence $(\grave{N}^{g_*}_{eut} \circ \grave{N}^{f_*}_{eut})^{-1}(\hat{E^*_{22}})$ is a $\grave{N}_{eut}\alpha GSOS$ in $\grave{N}^{\grave{X}}_{eut}$. Therefore $\grave{N}^{f_*}_{eut}$ is a $\grave{N}_{eut}C\alpha GS$ continuous mapping.

References

[1] J. Dontchev, Contra - Continuous Functions and Strongly S - Closed spaces, *Neutrosophic Sets and Systems*, **9 No 2** (1996), 303–310.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On Some New Notions and Functions In Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, **16** (2017), 16–19.

[3] R. Dhavaseelan, S.Jafari and Hanifpage, Neutrosophic generalized α contra-continuity, *Creat. Math. Inform.*, **27(2)**, (2018), 133–139.

[4] Erdal Ekici and Etienne E. Kerre, On fuzzy contra continuities, *Advances in Fuzzy Mathematics*, (2006), 35–44.

[5] V. Banu Priya and S. Chandrasekar, Neutrosophic α generalized semi homeomorphism, *Malaya Journal of Matematik*, **Volume 8, No.4** (2020), 1824–1829.

[6] V. Banu Priya and S. Chandrasekar, Neutrosophic αgs continuity and Neutrosophic αgs irresolute maps, *Neutrosophic Sets and Systems*, **28** (2019), 162–170.

[7] V. Banu Priya and S. Chandrasekar, M. Suresh, Neutrosophic strongly α generalised semi closed set, *Advances in Mathematics:Scientific Journal*, **9(10)** (2020), 8605–8613.

[8] R. Dhavaseelan and S. Jafari, Generalized Neutrosophic Closed Sets, *New Trends in Neutrosophic Theory and Applications*, **2** (2018), 261–273.

[9] Florentin Smarandache, Neutrosophic and Neutrosophic Logic, *Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics*, University of New Mexico-Gallup, (2002).

[10] Floretin Smaradache, Neutrosophic Set, Generalization of Intuitionistic Fuzzy set, *Journal of Defense Resourses Management*, **1** (2010), 107–116.

[11] D. Jayanthi, α Generalized Closed Sets in Neutrosophic Topological Spaces, *International Journal of Mathematics Trends and Technology*, Special Issue-ICRMIT, (2018), 88–91.

[12] C. Maheswari, M. Sathyabama and S. Chandrasekar, Neutrosophic generalized b-closed Sets in Neutrosophic Topological Spaces, *Journal of physics: Conference Series 1139*, (2018).

[13] C. Maheswari and S. Chandrasekar, Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, *Neutrosophic Sets and Systems*, **29** (2019), 89–100.

[14] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, *Computer Science and Engineering*, **2(7)** (2012), 129–132.

[15] A.A. Salama and S.A.Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR Journal of Mathematics*, **3(4)**, (2012), 31–35.

[16] V.K. Shanthi and S.Chandrasekar and K.Safina Begam Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, *International Journal of Research in Advent Technology*, **6(7)** (2018), 1739–1743.

Received: Nov. 5, 2024. Accepted: April 9, 2025