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# Neutrosophic Aggregation (INPOWA) Operator for Empirical Insights into Teaching Value: A Case Study of College English Instruction in Higher Education

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Abstract: In the evolving educational landscape, the emphasis on quality assurance in English instruction at the higher education level has gained renewed urgency. As English functions as a global lingua franca, institutions must ensure that their teaching strategies are effective, responsive, and capable of equipping students with both linguistic and critical thinking competencies. This study investigates the dimensions influencing teaching quality in English instruction within universities, focusing on pedagogical, institutional, and student-centered variables. Using a multidimensional evaluation framework, we explore real-world instructional practices and identify performance disparities across diverse models. This study uses the decision-making methodology with two methods, such as ITARA and COPRAS to obtain the weights of factors and ranking the options. These methods are used under the Interval valued Neutrosophic Numbers to dela with uncertainty information. We use the Prioritized Ordered Weighted Aggregation (POWA) Operator to combine the different Neutrosophic numbers. The findings offer empirical insights into effective teaching methodologies and highlight pathways for continuous pedagogical improvement and institutional development.

**Keywords**: Prioritized Ordered Weighted Aggregation (POWA) Operator; Interval Valued neutrosophic Numbers; Teaching Quality; College English Instruction in Higher Education.

# 1. Introduction

The role of English language instruction in higher education has grown beyond grammar acquisition or test preparation. In a globalized academic and professional environment, English serves as a conduit for international collaboration, scholarly communication, and employability. Institutions are under increasing pressure to deliver high-quality instruction that not only supports language mastery but also fosters critical engagement, interdisciplinary thinking, and cultural competence. This growing demand raises questions about how English teaching quality

can be measured and improved in a systematic and evidence-based manner[1], [2]. The complexity of teaching quality evaluation in English instruction lies in its multidimensional nature. It is no longer sufficient to assess teaching effectiveness based solely on exam scores or pass rates. Modern pedagogical approaches emphasize interactive methods, technological integration, communicative competence, and the development of autonomous learning skills. As such, institutions must develop comprehensive evaluation models that consider instructional design, teacher proficiency, student engagement, and contextual adaptability[3], [4]. Moreover, diverse classroom environments-ranging from traditional lecture halls to digital platforms propose unique challenges and opportunities for language acquisition. While blended and online models offer flexibility, they also require robust mechanisms for maintaining instructional coherence and student motivation. In such contexts, student feedback, peer observations, and self-reflective teaching practices become critical tools for assessing instructional success[5], [6]. Empirical studies that explore these elements are essential for bridging the gap between theory and practice. By examining real-world teaching scenarios and gathering both qualitative and quantitative data, educational researchers can identify effective strategies that are contextually appropriate and pedagogically sound. This case study contributes to this body of work by offering a grounded evaluation of various instructional models within university English programs[7], [8]. Additionally, policy frameworks and institutional expectations often shape teaching practices. The balance between academic autonomy and curriculum standardization influences how teaching quality is both implemented and evaluated. Faculty development programs, assessment protocols, and feedback loops must be strategically aligned to promote sustainable instructional excellence[9], [10].

Considering these considerations, the present study seeks to answer key questions:

- What criteria best reflect teaching quality in English instruction at the university level?
- How do different instructional approaches perform when evaluated against these criteria?
- And what insights can be drawn to inform future teaching innovations and policy development?

Through a comprehensive evaluation model, this research provides actionable conclusions that can guide educators, administrators, and policymakers alike.

Due to the fuzziness of people's thinking and the complexity of objective objects, which are easily conveyed by fuzzy information, decision makers find it extremely challenging to provide assessment information of the alternatives using actual numbers in real decision making. The fuzzy set (FS) theory, which Zadeh first presented in 1965, has rapidly advanced and been widely used in fuzzy multi-attribute decision making issues, demonstrating its suitability as a tool for handling fuzzy data. It can't, however, convey non-membership; it only has membership.

As a result, Smarandache added an independent indeterminacy-membership based on IFS to introduce the idea of the neutrosophic set (NS)[11], [12]. Decision makers in NS can characterize their assessment of an item using truth-membership TA(x), indeterminacy-membership IA(x),

and falsity-membership FA(x), respectively. Decision makers can benefit from more precise and thorough opinion expressions via NS, which is a generalization of FS and IFS. In NS, truth-membership, indeterminacy-membership, and false-membership are entirely independent, and indeterminacy is quantified directly[13], [14].

## 2. Prioritized Ordered Weighted Aggregation (POWA) Operator

We use the interval valued Neutrosophic numbers (IVNNs) [15], [16] with the POWA operator to combine the decision matrix[17].

$$f_{i} = \begin{pmatrix} [T_{i}^{L}, T_{i}^{U}], \\ [I_{i}^{L}, I_{i}^{U}], \\ [F_{i}^{L}, F_{i}^{U}] \end{pmatrix}$$
(1)  
$$POWA = \prod_{j=1}^{n_{i}} \bigoplus (w_{ij}x_{ij}) = \begin{pmatrix} \left[1 - \prod_{j=1}^{n_{i}} (1 - T_{ij}^{L})^{w_{ij}}, 1 - \prod_{j=1}^{n_{i}} (1 - T_{ij}^{U})^{w_{ij}}\right], \\ \left[\prod_{j=1}^{n_{i}} (I_{ij}^{L})^{w_{ij}}, \prod_{j=1}^{n_{i}} (I_{ij}^{U})^{w_{ij}}\right], \\ \left[\prod_{j=1}^{n_{i}} (F_{ij}^{L})^{w_{ij}}, \prod_{j=1}^{n_{i}} (F_{ij}^{U})^{w_{ij}}\right] \end{pmatrix}$$
(2)  
$$if F: [0,1]^{n} \to [0,1], then$$
$$Z = \bigoplus^{p} (w_{i}f_{\sigma(i)}) = \begin{pmatrix} \left[1 - \prod_{i=1}^{p} (1 - T_{\sigma(i)}^{L})^{w_{ij}}, 1 - \prod_{i=1}^{p} (1 - T_{\sigma(i)}^{U})^{w_{ij}}\right], \\ \left[\prod_{i=1}^{p} (I_{\sigma(i)}^{L})^{w_{ij}}, \prod_{i=1}^{p} (I_{\sigma(i)}^{U})^{w_{ij}}\right], \\ \left[\prod_{i=1}^{p} (F_{\sigma(i)}^{L})^{w_{ij}}, \prod_{i=1}^{p} (F_{\sigma(i)}^{U})^{w_{ij}}\right] \end{pmatrix}$$
(3)

The results of the POWA under IVNNs are still an IVN, even

$$Z = \begin{pmatrix} \left[1 - \prod_{i=1}^{P} (1 - T_{\sigma(i)}^{L})^{w_{ij}}, \prod_{i=1}^{P} (1 - T_{\sigma(i)}^{U})^{w_{ij}}\right], \\ \left[\prod_{i=1}^{P} (I_{\sigma(i)}^{L})^{w_{ij}}, \prod_{i=1}^{P} (I_{\sigma(i)}^{U})^{w_{ij}}\right], \\ \left[\prod_{i=1}^{P} (F_{\sigma(i)}^{L})^{w_{ij}}, \prod_{i=1}^{P} (F_{\sigma(i)}^{U})^{w_{ij}}\right] \end{pmatrix}$$
(4)

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## Where p=2

$$Z = w_1 x_{\sigma(1)} \oplus w_2 x_{\sigma(2)} \tag{5}$$

Where 
$$x_{\sigma(1)} = \max(x_1, x_2)$$
 (6)

$$x_{\sigma(2)} = \min(x_1, x_2)$$
 (7)

$$w_{1}x_{\sigma(1)} = \begin{pmatrix} \left[1 - \left(T_{\sigma(1)}^{L}\right)^{w_{1}}, 1 - \left(T_{\sigma(1)}^{U}\right)^{w_{1}}\right], \\ \left[\left(I_{\sigma(1)}^{L}\right)^{w_{1}}, \left(I_{\sigma(1)}^{U}\right)^{w_{1}}\right], \\ \left[\left(F_{\sigma(1)}^{L}\right)^{w_{1}}, \left(F_{\sigma(1)}^{U}\right)^{w_{1}}\right] \end{pmatrix}$$
(8)

$$w_{1}x_{\sigma(2)} = \begin{pmatrix} \left[1 - \left(T_{\sigma(2)}^{L}\right)^{w_{2}}, 1 - \left(T_{\sigma(2)}^{U}\right)^{w_{2}}\right], \\ \left[\left(I_{\sigma(2)}^{L}\right)^{w_{2}}, \left(I_{\sigma(2)}^{U}\right)^{w_{2}}\right], \\ \left[\left(F_{\sigma(2)}^{L}\right)^{w_{2}}, \left(F_{\sigma(2)}^{U}\right)^{w_{2}}\right] \end{pmatrix}$$
(9)

$$w_{1}x_{\sigma(1)} \oplus w_{2}x_{\sigma(2)} = \begin{pmatrix} \left[1 - (T_{\sigma(1)}^{L})^{W_{1}} 1 - (T_{\sigma(2)}^{L})^{W_{2}}, \\ 1 - (T_{\sigma(1)}^{U})^{W_{1}} 1 - (T_{\sigma(2)}^{U})^{W_{2}}, \\ \left[1 - (T_{\sigma(1)}^{U})^{W_{1}} (I_{\sigma(2)}^{L})^{W_{2}}, \\ (I_{\sigma(1)}^{U})^{W_{1}} (I_{\sigma(2)}^{U})^{W_{2}}, \\ (I_{\sigma(1)}^{U})^{W_{1}} (F_{\sigma(2)}^{U})^{W_{2}}, \\ \left[(F_{\sigma(1)}^{L})^{W_{1}} (F_{\sigma(2)}^{U})^{W_{2}}, \\ (F_{\sigma(1)}^{U})^{W_{1}} (F_{\sigma(2)}^{U})^{W_{2}}, \\ (F_{\sigma(1)}^{U})^{W_{1}} (F_{\sigma(2)}^{U})^{W_{2}} \end{bmatrix} \end{pmatrix}$$
(10)

When p=k+1

$$Z = \bigoplus_{i=1}^{k} (w_i f_{\sigma(i)}) \oplus (w_i f_{\sigma(k+1)}) = \begin{pmatrix} \left[ 1 - \prod_{i=1}^{k+1} (1 - T_{\sigma(i)}^L)^{w_{ij}}, 1 - \prod_{i=1}^{k+1} (1 - T_{\sigma(i)}^U)^{w_{ij}} \right], \\ \left[ \prod_{i=1}^{k+1} (I_{\sigma(i)}^L)^{w_{ij}}, \prod_{i=1}^{k+1} (I_{\sigma(i)}^U)^{w_{ij}} \right], \\ \left[ \prod_{i=1}^{k+1} (F_{\sigma(i)}^L)^{w_{ij}}, \prod_{i=1}^{k+1} (F_{\sigma(i)}^U)^{w_{ij}} \right] \end{pmatrix}$$
(11)

#### 3. ITARA-COPRAS under POWA Operator

Create the decision matrix using the IVNNs between the criteria and alternatives. These numbers are combined using the POWA to obtain one matrix. This matrix is converted into a crisp value.

Compute the indifference threshold  $h_i$  value using the experts and decision makers.

Normalize the decision matrix. After that the elements of the decision matrix is sorted.

$$y_{ij} = \frac{A_{ij}}{\sum_{i=1}^{m} A_{ij}} \tag{12}$$

The dispersion degree is obtained.

$$q_{ij} = y_{i+1,j} - y_{ij} \tag{13}$$

The distance between the  $q_{ij}$  and  $h_j$ 

$$d_{ij} = \begin{cases} q_{ij} - h_j, q_{ij} > h_j \\ 0, \qquad q_{ij} \le h_j \end{cases}$$
(14)

The importance of the criteria is computed.

$$w_j = \frac{\left(\sum_{i=1}^{m-1} d_{ij}^b\right)^{\frac{1}{b}}}{\sum_{i=1}^{n} \left(\sum_{i=1}^{m-1} d_{ij}^b\right)^{\frac{1}{b}}}$$
(15)

After that, the steps of the COPRAS are introduced to rank alternatives.

COPRAS normalize the decision matrix as the weighted method.

The weighted decision matrix is obtained.

$$r_{ij} = y_{ij} w_j \tag{16}$$

For positive and negative criteria, we compute the max and min values.

$$L_{+i} = \sum_{j=1}^{g} r_{ij}$$
(17)

$$L_{-i} = \sum_{j=g+1}^{n} r_{ij} \tag{18}$$

Relative value is obtained

$$E_o = L_{+i} + \frac{\sum_{i=1}^m L_{-i}}{L_{-i} \sum_{i=1}^m \frac{1}{L_{-i}}}$$
(19)

### 4. Implementation

This section shows the implantation of the proposed approach to obtaining the weights of factors and ranking the options. Three experts use the IVNNs to evaluate the factors and options in Fig 1 to build the decision matrix as shown in Tables 1-3.

Curriculum RelevanceFeedback Timeliness Student EngagementTraditional Lecture- Based ModelPedagogical Methods Language Proficiency ImprovementResource AccessibilityTask-Based Language TeachingClarity of Instruction Assessment Fairness Use of Technology in TeachingFaculty Professional DevelopmentContent and Language Integrated DevelopmentCultural Competence PromotionFaculty Professional DevelopmentFlipped Classroom Approach Hybrid/Blended Learning Model	Factors	Factors	options
	Curriculum Relevance Pedagogical Methods Language Proficiency Improvement Clarity of Instruction Assessment Fairness Use of Technology in Teaching Cultural Competence Promotion	Feedback Timeliness Student Engagement Resource Accessibility Practical Application of Knowledge Faculty Professional Development	Traditional Lecture- Based Model Task-Based Language Teaching Content and Language Integrated Learning Flipped Classroom Approach Project-Based English Instruction Hybrid/Blended Learning Model Gamified English Learning

Fig 1. The factors of this study.

We used the POWA to combine the decision matrix. Next, we apply the proposed approach.

We compute the indifference threshold  $h_j$  value using the experts and decision makers.

Eq. (12) is used to normalize the decision matrix as shown in Fig 2.

The dispersion degree is obtained using eq. (13) as shown in Fig 3.

The distance between the  $q_{ij}$  and  $h_j$  is obtained using Eq. (14) as shown in Fig 4.

The importance of the criteria is computed using eq. (15) as shown in Fig 5.

### Table 1. The first IVNNs.

	$A_1$	$A_2$	Аз	$A_4$	$A_5$	$A_6$	$A_7$
С	([0.5,0.6],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
	6],[0.4,0.5])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.5,0.6])	5],[0.6,0.7])
1							
С	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
	5],[0.3,0.4])	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
2							
С	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])	7],[0.4,0.5])	6],[0.4,0.5])	7],[0.4,0.5])
3							
<i>C</i> 4	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.
	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])
С	([0.2,0.3],[0.3,0.	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.
	4],[0.7,0.8])	5],[0.3,0.4])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])	6],[0.5,0.6])
5							

C	([0.3,0.4],[0.4,0.	([0.5,0.6],[0.5,0.	([0.4,0.5],[0.5,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
6	5],[0.6,0.7])	6],[0.4,0.5])	6],[0.5,0.6])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])
C	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.5],[0.6,0.	([0.1,0.2],[0.1,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
7	6],[0.5,0.6])	7],[0.4,0.5])	7],[0.4,0.5])	2],[0.8,0.9])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
C	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.5,0.6],[0.5,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
8	7],[0.4,0.5])	6],[0.5,0.6])	6],[0.4,0.5])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])
C	([0.5,0.6],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
9	6],[0.4,0.5])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
C	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
10	5],[0.3,0.4])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])	6],[0.5,0.6])	5],[0.6,0.7])
C	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
11	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])
C	([0.5,0.6],[0.5,0.	([0.6,0.7],[0.4,0.	([0.6,0.7],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.
12	6],[0.4,0.5])	5],[0.3,0.4])	5],[0.3,0.4])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	2],[0.8,0.9])

## Table 2. The second IVNNs.

	$A_1$	$A_2$	Аз	$A_4$	$A_5$	$A_6$	$A_7$
C	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.
	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.5,0.6])
C	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.5,0.5],[0.6,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.5,0.5],[0.6,0.
	5],[0.3,0.4])	2],[0.8,0.9])	4],[0.7,0.8])	7],[0.4,0.5])	2],[0.8,0.9])	4],[0.7,0.8])	7],[0.4,0.5])
2	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.4,0.5],[0.5,0.
C	7],[0.4,0.5])	6],[0.5,0.6])	5],[0.6,0.7])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	6],[0.5,0.6])
3	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.5],[0.6,0.
C	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	7],[0.4,0.5])
4	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.5,0.5],[0.6,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.
C	5],[0.3,0.4])	2],[0.8,0.9])	4],[0.7,0.8])	7],[0.4,0.5])	4],[0.7,0.8])	2],[0.8,0.9])	4],[0.7,0.8])
с	([0.2,0.3],[0.3,0.	([0.6,0.7],[0.4,0.	([0.3,0.4],[0.4,0.	([0.5,0.5],[0.6,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.6,0.7],[0.4,0.
6	4],[0.7,0.8])	5],[0.3,0.4])	5],[0.6,0.7])	7],[0.4,0.5])	2],[0.8,0.9])	4],[0.7,0.8])	5],[0.3,0.4])
C	([0.4,0.5],[0.5,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
7	6],[0.5,0.6])	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
C	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.4,0.5],[0.5,0.
8	7],[0.4,0.5])	6],[0.5,0.6])	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	6],[0.5,0.6])
С	([0.5,0.6],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
9	6],[0.4,0.5])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.5,0.6])	5],[0.6,0.7])

C	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
	5],[0.3,0.4])	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
10							
C	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])	7],[0.4,0.5])	6],[0.4,0.5])	7],[0.4,0.5])
11							
C	([0.5,0.6],[0.5,0.	([0.6,0.7],[0.4,0.	([0.6,0.7],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.
	6],[0.4,0.5])	5],[0.3,0.4])	5],[0.3,0.4])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	2],[0.8,0.9])
12							

# Table 3. The third IVNNs.

	$A_1$	$A_2$	Аз	$A_4$	$A_5$	$A_6$	$A_7$
С	([0.1,0.2],[0.1,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.
	2],[0.8,0.9])	2],[0.8,0.9])	4],[0.7,0.8])	6],[0.5,0.6])	7],[0.4,0.5])	7],[0.4,0.5])	6],[0.4,0.5])
1							
C	([0.2,0.3],[0.3,0.	([0.6,0.7],[0.4,0.	([0.3,0.4],[0.4,0.	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.
2	4],[0.7,0.8])	5],[0.3,0.4])	5],[0.6,0.7])	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])
C	([0.3,0.4],[0.4,0.	([0.5,0.6],[0.5,0.	([0.4,0.5],[0.5,0.	([0.2,0.3],[0.3,0.	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
3	5],[0.6,0.7])	6],[0.4,0.5])	6],[0.5,0.6])	4],[0.7,0.8])	5],[0.6,0.7])	6],[0.5,0.6])	5],[0.6,0.7])
C	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.5],[0.6,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.
4	6],[0.5,0.6])	7],[0.4,0.5])	7],[0.4,0.5])	2],[0.8,0.9])	4],[0.7,0.8])	7],[0.4,0.5])	6],[0.4,0.5])
C	([0.5,0.5],[0.6,0.	([0.4,0.5],[0.5,0.	([0.5,0.6],[0.5,0.	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.
5	7],[0.4,0.5])	6],[0.5,0.6])	6],[0.4,0.5])	5],[0.3,0.4])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])
С	([0.5,0.6],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
6	6],[0.4,0.5])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])
<i>C</i>	([0.6,0.7],[0.4,0.	([0.1,0.2],[0.1,0.	([0.2,0.3],[0.3,0.	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.
	5],[0.3,0.4])	2],[0.8,0.9])	4],[0.7,0.8])	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])
C	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.5,0.5],[0.6,0.	([0.5,0.6],[0.5,0.	([0.6,0.7],[0.4,0.	([0.5,0.6],[0.5,0.	([0.5,0.5],[0.6,0.
8	5],[0.6,0.7])	6],[0.5,0.6])	7],[0.4,0.5])	6],[0.4,0.5])	5],[0.3,0.4])	6],[0.4,0.5])	7],[0.4,0.5])
С	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.	([0.2,0.3],[0.3,0.	([0.1,0.2],[0.1,0.	([0.6,0.7],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
9	6],[0.5,0.6])	5],[0.6,0.7])	4],[0.7,0.8])	2],[0.8,0.9])	5],[0.3,0.4])	6],[0.5,0.6])	5],[0.6,0.7])
<i>C</i>	([0.2,0.3],[0.3,0.	([0.6,0.7],[0.4,0.	([0.3,0.4],[0.4,0.	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
	4],[0.7,0.8])	5],[0.3,0.4])	5],[0.6,0.7])	5],[0.6,0.7])	6],[0.5,0.6])	6],[0.5,0.6])	5],[0.6,0.7])
10	([0.3,0.4],[0.4,0.	([0.5,0.6],[0.5,0.	([0.4,0.5],[0.5,0.	([0.2,0.3],[0.3,0.	([0.3,0.4],[0.4,0.	([0.4,0.5],[0.5,0.	([0.3,0.4],[0.4,0.
C	5],[0.6,0.7])	6],[0.4,0.5])	6],[0.5,0.6])	4],[0.7,0.8])	5],[0.6,0.7])	6],[0.5,0.6])	5],[0.6,0.7])
11 C 12	([0.4,0.5],[0.5,0. 6],[0.5,0.6])	([0.4,0.5],[0.5,0. 6],[0.5,0.6])	([0.3,0.4],[0.4,0. 5],[0.6,0.7])	([0.2,0.3],[0.3,0. 4],[0.7,0.8])	([0.1,0.2],[0.1,0. 2],[0.8,0.9])	([0.6,0.7],[0.4,0. 5],[0.3,0.4])	([0.5,0.6],[0.5,0. 6],[0.4,0.5])



Fig 2. The normalized decision matrix.



Fig 3. The dispersion degree.



Fig 4. distance between the  $q_{ij}$  and  $h_j$ .



Fig 5. The importance of factors.

After that, the steps of the COPRAS are introduced to rank alternatives. COPRAS normalize the decision matrix as the weighted method.

The weighted decision matrix is obtained using Eq. (16) as shown in Fig 6.

For positive and negative criteria, we compute the max and min values using eq. (17 and 18).

Relative value is obtained using Eq. (19) as shown in Fig 7. The rank of the alternatives are shown in Fig 8.



Fig 6. The weighted decision matrix.



Fig 8. The rank of options.

Ting Li, Neutrosophic Aggregation (INPOWA) Operator for Empirical Insights into Teaching Value: A Case Study of College English Instruction in Higher Education

# 5. Conclusions

This study underscores the multifaceted nature of teaching quality in college English instruction and the importance of using a robust evaluation framework that reflects the realities of higher education today. The findings demonstrate that no single teaching model fits all contexts; rather, effective instruction is shaped by a combination of curriculum relevance, student interaction, technological adaptability, and continuous faculty development. By integrating empirical insights with practical observations, this research offers a valuable roadmap for enhancing instructional quality in English programs. Ultimately, prioritizing evidence-based teaching practices can lead to more meaningful learning experiences and better outcomes for students in an increasingly interconnected world. This study used the decision-making process to obtain the weights of factors and ranking the options by the COPRAS methodology. This study uses the Interval valued Neutrosophic Numbers to deal with uncertainty information. We used the Prioritized Ordered Weighted Aggregation (POWA) Operator to combine the different decision matrix. The proposed approach is validated using a case study with 12 criteria and 7 alternatives.

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#### References

- H. Haerazi, "ICT integration into English language teaching-learning: Insights from some private higher education institutions," *Englisia J. Lang. Educ. Humanit.*, vol. 11, no. 2, pp. 48–66, 2024.
- [2] N. L. Ali and M. O. Hamid, "13 English-medium instruction and teacher agency in higher education: A case study," Un Lang. Plan. a Glob. world Mult. levels Play. Work, pp. 234–250, 2018.
- [3] C. Calvo-Porral, J.-P. Lévy-Mangin, and I. Novo-Corti, "Perceived quality in higher education: an empirical study," *Mark. Intell. Plan.*, vol. 31, no. 6, pp. 601–619, 2013.
- [4] B. Anthony *et al.*, "Exploring the role of blended learning for teaching and learning effectiveness in institutions of higher learning: An empirical investigation," *Educ. Inf. Technol.*, vol. 24, pp. 3433–3466, 2019.

- [5] R. R. Jablonkai and J. Hou, "English medium of instruction in Chinese higher education: A systematic mapping review of empirical research," *Appl. Linguist. Rev.*, vol. 14, no. 6, pp. 1483–1512, 2023.
- [6] J. Cullen\*, S. Richardson, and R. O'Brien, "Exploring the teaching potential of empiricallybased case studies," *Account. Educ.*, vol. 13, no. 2, pp. 251–266, 2004.
- [7] G. Hu, "English-medium instruction in higher education: Lessons from China," J. Asia *TEFL*, vol. 16, no. 1, p. 1, 2019.
- [8] M. Healy and M. McCutcheon, "Teaching with case studies: An empirical investigation of accounting lecturers' experiences," *Account. Educ. an Int. J.*, vol. 19, no. 6, pp. 555–567, 2010.
- [9] G. Prakash, "Quality in higher education institutions: insights from the literature," *TQM J.*, vol. 30, no. 6, pp. 732–748, 2018.
- [10] G. Hu and J. Lei, "English-medium instruction in Chinese higher education: A case study," *High. Educ.*, vol. 67, pp. 551–567, 2014.
- [11] J. Ye and S. Du, "Some distances, similarity and entropy measures for interval-valued neutrosophic sets and their relationship," *Int. J. Mach. Learn. Cybern.*, vol. 10, pp. 347–355, 2019.
- [12] S. Broumi and F. Smarandache, "Cosine similarity measure of interval valued neutrosophic sets," *Infin. Study*, 2014.
- [13] A. Saha and S. Broumi, New operators on interval valued neutrosophic sets. Infinite Study, 2019.
- [14] I. Deli, "Interval-valued neutrosophic soft sets and its decision making," *Int. J. Mach. Learn. Cybern.*, vol. 8, pp. 665–676, 2017.
- [15] H. Zhang, J. Wang, and X. Chen, "An outranking approach for multi-criteria decisionmaking problems with interval-valued neutrosophic sets," *Neural Comput. Appl.*, vol. 27, pp. 615–627, 2016.
- [16] E. Bolturk and C. Kahraman, "A novel interval-valued neutrosophic AHP with cosine similarity measure," *Soft Comput.*, vol. 22, no. 15, pp. 4941–4958, 2018.
- [17] P. Liu and Y. Wang, "Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making," J. Syst. Sci. Complex., vol. 29, no. 3, pp. 681–697, 2016.

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