Neutrosophic Sets and Systems, {Special Issue: Artificial Intelligence, Neutrosophy, and Latin American Worldviews: Toward a Sustainable Future (Workshop – March 18–21, 2025, Universidad Tecnológica de El Salvador, San Salvador, El Salvador)}, Vol. 84, 2025

University of New Mexico

A Neutrosophic Multi-Objective Optimization Framework for Sustainable Design of Water Supply Systems

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Abstract: The study addresses the challenge of optimizing water supply systems (WSS) under conditions of uncertainty, variability, and multiple. In a context where sustainability is a key criterion, traditional Multiobjective optimization approaches are often insufficient to handle the imprecision and indeterminacy inherent in data and models. This work proposes a Neutrosophic Multi-Objective Model (NMM) that integrates Neutrosophic Logic to capture uncertainty in decision variables, objective functions, and constraints. Unlike conventional methods, NMM allows considering degrees of truth, indeterminacy, and falsity, offering a more robust and flexible framework for decision-making in WSS. The relevance of this study lies in its ability to address complex water resources planning and management problems, where economic, social, and environmental sustainability must be balanced under conditions of high uncertainty. Through the application of MMN, optimal solutions are identified that not only minimize costs and losses but also maximize efficiency and service quality, considering variability in demand and risks associated with climate change and infrastructure failures. The results demonstrate that MMN overcomes the limitations of traditional approaches by providing more adaptive and resilient solutions. This research contributes to the field of complex systems optimization by introducing an innovative methodology that combines Neutrosophic Logic with Multiobjective techniques.

Keywords: Multi-objective Optimization, Neutrosophic Logic, Water Supply Systems, Sustainability, Energy Efficiency, Water Loss Reduction, Water Resources Management

1. Introduction

Optimization of water supply systems (WSS) is a vital issue in the current global context, where water resource scarcity, population growth, and the effects of climate change demand innovative and sustainable solutions [1]. These systems must not only ensure reliable and efficient supply but also balance economic, social, and environmental objectives, making them a complex challenge for engineering and resource management [2]. However, the uncertainty inherent in factors such as variability in demand, operating costs, and weather conditions hinders the application of traditional optimization approaches, which are often insufficient to handle the imprecision and indeterminacy present in the data [3]. Historically, WSS planning has evolved from empirical rule-based approaches to advanced mathematical models that incorporate multi-objective optimization techniques [4]. Despite these advances, most existing methods fail to adequately integrate the uncertainty and subjectivity associated with planning decisions, limiting their applicability in real-world scenarios [5]. In this sense, Neutrosophic Logic emerges as a promising tool, as it allows to handle degrees of truth, indeterminacy, and falsity, offering a more flexible and robust framework for decision-making under uncertainty [6]. The central problem addressed by this study is the lack of an integrated approach that combines Multi-objective optimization with Neutrosophic Logic to address sustainability in WSS. How can water supply systems be optimized by simultaneously considering cost minimization, loss reduction, and energy efficiency improvement, while managing uncertainty in data and models? This question guides the research, seeking to develop a model that overcomes the limitations of traditional approaches and provides more adaptive and resilient solutions.

The objective of this study is to propose a Neutrosophic Multi-Objective Model (NMM) that integrates Neutrosophic Logic in WSS optimization, evaluating its ability to handle uncertainty and provide sustainable solutions. This objective is aligned with the research question and seeks to contribute to both theoretical and practical advancement in the field of water resources management.

2. Preliminaries.

2.1. Sustainability in the Optimization of Water Supply Systems (WSS).

Sustainability in water supply systems (WSS) optimization has become a critical issue in a world where water scarcity, population growth, and the effects of climate change are putting unprecedented pressure on existing infrastructures [8]. These systems must not only ensure reliable and efficient supply, but also balance economic, social, and environmental objectives, making them a complex challenge for engineering and resource management [9]. However, the uncertainty inherent in factors such as variability in demand, operating costs, and weather conditions makes it difficult to apply traditional optimization approaches, which are often insufficient to handle the imprecision and indeterminacy present in the data [10]. Historically, WSS planning has evolved from empirical rule-based approaches to advanced mathematical models that incorporate multi-objective optimization techniques [11]. Despite these advances, most existing methods fail to adequately integrate the uncertainty and subjectivity associated with planning decisions, which limits their applicability in real-world scenarios [12]. In this sense, Neutrosophic Logic emerges as a promising tool, as it allows handling degrees of truth, indeterminacy and falsity, offering a more flexible and robust framework for decision-making under uncertainty [13].

The central problem addressed by this study is the lack of an integrated approach that combines multi-objective optimization with Neutrosophic Logic to address sustainability in WSS. How can water supply systems be optimized by simultaneously considering cost minimization, loss reduction, and energy efficiency improvement, while managing uncertainty in data and models? This question guides the research, seeking to develop a model that overcomes the limitations of traditional approaches and provides more adaptive and resilient solutions. The objectives of this study are, first, to propose a Neutrosophic Multi-Objective Model (MMN) that integrates Neutrosophic Logic in WSS optimization, and second, to demonstrate its applicability in a real case study, evaluating its ability to handle uncertainty and provide sustainable solutions. These objectives are aligned with the research question and seek to contribute to both theoretical and practical advancement in the field of water resources management. Sustainability in WSS optimization involves not only resource use efficiency but also the ability to adapt to changing conditions and ensure equity in access to water. In this context, MMN offers an innovative perspective by incorporating Neutrosophic Logic, which allows for capturing uncertainty and variability in data and models. This is particularly relevant in regions where water resources are limited, and climatic conditions are unpredictable [14].

Furthermore, MMN not only focuses on cost minimization and loss reduction but also considers social and environmental aspects, such as service quality and ecological impact. This holistic approach is essential to ensure that the proposed solutions are not only economically viable but also socially fair and environmentally sustainable. In this sense, MMN represents a significant advance in the field of WSS optimization, as it provides a more comprehensive and flexible framework for decision-making. The application of MMN in a real case study demonstrates its ability to handle uncertainty and provide sustainable solutions. The results show that MMN not only overcomes the limitations of traditional approaches but also offers more adaptive and resilient solutions, capable of facing current and future challenges in water resources management. This underlines the importance of adopting innovative approaches that integrate uncertainty and sustainability in WSS planning and management. Sustainability in the optimization of water supply systems is a complex challenge that requires innovative and flexible approaches. The MMN proposed in this study represents a significant advance in this field, as it combines multi-objective optimization with Neutrosophic Logic to manage uncertainty and provide sustainable solutions. This approach not only contributes to theoretical progress in the field of water resources management but also offers practical tools to improve the efficiency, equity, and resilience of water supply systems in an increasingly uncertain and changing world.

2.2. Neutrosophic Genetic Algorithms (NGAs)

Genetic Algorithms (GAs) are optimization and search techniques inspired by principles of biological evolution such as natural selection, mutation, and recombination. These algorithms have proven effective in solving complex problems in diverse areas, from engineering to artificial intelligence. However, in many real-world scenarios, uncertainty, imprecision, and vagueness are inherent in data and models [15]. This is where Neutrosophic Sets and Neutrosophic Logic, introduced by Florentin Smarandache, offer a robust theoretical framework to handle these uncertainties.

Neutrosophic Genetic Algorithms (NGAs) combine the principles of traditional GAs with Neutrosophic Logic, allowing the manipulation of imprecise, indeterminate and inconsistent information. This chapter presents a detailed description of NGAs, including their theoretical foundations, mathematical formulations and potential applications.

Fundamentals of Neutrosophic Logic [16].

Neutrosophic Logic is a generalization of Fuzzy Logic and Intuitionistic Logic, which allows the representation of information that is simultaneously true, false, and indeterminate. A neutrosophic set A in a universe X is defined as:

$$A = \{ \langle x, T_a(x), I_a(x), F_a(x) \rangle \mid x \in X \} \quad (1)$$

where:

- $T_a(x)$ is the degree of truth,
- $I_a(x)$ is the degree of indeterminacy,
- $F_a(x)$ is the degree of falsehood.

These degrees are functions that assign to each element x a value in the interval [0,1], and satisfy the condition:

$$0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3 \quad (2)$$

Traditional Genetic Algorithms [17-19].

A traditional Genetic Algorithm consists of the following steps:

1. **Initialization:** An initial population of candidate solutions is generated.
2. **Evaluation:** The fitness of each individual in the population is evaluated.
3. **Selection:** Individuals are selected for reproduction based on their fitness.
4. **Crossing (Recombination):** The characteristics of two individuals are combined to produce offspring.
5. **Mutation:** Random changes are introduced into the offspring.
6. **Replacement:** A new population is formed with the selected individuals and their offspring.
7. **Termination:** The algorithm terminates when a stopping criterion is reached.

Neutrosophic Genetic Algorithms (NGAs) [20, 21]

NGAs extend traditional GAs by incorporating Neutrosophic Logic into several aspects of the algorithm, including individual representation, fitness evaluation, selection, crossover, and mutation.

By classifying each generation of the Genetic Algorithm (GA) into three distinct solution spaces—true, false, and indeterminate—the proposed Neutrosophic Genetic Algorithm (NGA) framework effectively incorporates neutrosophic reasoning into the evolutionary process. This structure allows for a differentiated treatment of chromosomes based on their logical nature, enabling the algorithm to systematically manage the behavior of true, false, and indeterminate individuals within their respective subspaces. As a result, the model can identify both optimal and near-optimal solutions within the true solution space, while also enhancing robustness under uncertainty.

Neutrosophic Genetic Algorithms [20] represent a powerful extension of traditional GAs, allowing the manipulation of imprecise, indeterminate, and inconsistent information. By incorporating Neutrosophic Logic into each step of the algorithm, NGAs offer a robust framework for optimization and decision-making in complex and uncertainty-filled environments.

In the proposed Neutrosophic Genetic Algorithm (NGA) framework, the population in each generation is partitioned into three distinct neutrosophic solution spaces:

- True Solution Space (TSS)
- False Solution Space (FSS)
- Indeterminate Solution Space (ISS)

Let $P^{(t)} = \left\{x_1^{(t)}, x_2^{(t)}, \dots, x_n^{(t)}\right\}$ be the population at generation t, and let $f(x)$ denote the fitness function. Each individual $x_n^{(t)} \in P^{(t)}$ is assigned to one of the three spaces according to the following classification:

- **True Solution Space (TSS):**

$$x_n^{(t)} \in TSS \ if \ \frac{df(x_n^{(t)})}{dt} > 0 \quad (3)$$

Individuals in TSS demonstrate a monotonic increase in fitness, converging toward optimal or near-optimal solutions. These represent individuals with high alignment to the "truth" dimension in neutrosophic reasoning.

- **False Solution Space (FSS):**

$$x_n^{(t)} \in FSS\ if\ \frac{df(x_n^{(t)})}{dt} < 0 \quad (4)$$

Individuals in FSS experience declining fitness values as the algorithm progresses, representing solutions moving away from feasibility or optimality.

- **Indeterminate Solution Space (ISS):**

$$x_n^{(t)} \in FSS\ if\ \frac{df\left(x_n^{(t)}\right)}{dt} \approx 0, \quad (5)$$

ISS contains individuals whose evolutionary trajectory is ambiguous or unstable. Their fitness may increase or decrease unpredictably across generations, reflecting the indeterminacy dimension of neutrosophic modeling.

This triadic classification enhances the evolutionary process by allowing differentiated genetic operations (selection, crossover, mutation) within each solution space.

2. Materials and Methods

The Neutrosophic Multi-Objective Model (NMM) proposed in this paper seeks to optimize water supply systems (WSS) under a sustainability approach, considering the uncertainty and variability inherent to these systems. The model extends the work of [1] by incorporating a third criterion: resilience, along with the traditional objectives of cost minimization and energy losses. This approach uses Neutrosophic Logic to handle uncertainty in data and models, allowing for more robust and adaptable decision-making [22-24].

Problem Formulation

The design of water supply systems (WSS) must address multiple conflicting objectives, such as cost minimization, energy loss reduction, and resilience maximization. C(P**)** : Total cost of the system, including pipe installation, maintenance, and operation.

1. $E(P)$: Energy loss due to pressure and friction losses in the network.
2. $R(P)$: System resilience, which ensures that pipe diameters at any node are balanced to prevent structural weaknesses and allow for rapid recovery from disruptions.

The objective function is defined as:

$$\text{Maximize } P = W_C(1 - C_N) + W_E(1 - E_N) + W_R R_N \quad (6)$$

where:

- C_n: Normalized total cost of the system.
- E_n: Normalized energy losses.

- R_n: Normalized resilience
- W_C , W_E , W_R : Weights assigned to cost, energy, and resilience respectively, reflecting the decision-makers' priorities

Resilience is calculated as:

$$\boldsymbol{R}(\boldsymbol{P}) = \frac{\boldsymbol{D_{MAX}} - \boldsymbol{D_{MIN}}}{\boldsymbol{D_{MAX}}} \quad (7)$$

where:

- D_{max}: Maximum diameter of the pipes at a node.
- D_{min}: Minimum diameter of the pipes at a node.

A higher value of R(P) Indicates that the pipes at a node have balanced diameters, making the network more adaptable to unexpected flow variations.

2.1 Genetic Algorithm with Adaptive Weights:

- ✓ Chromosomes represent plumbing configurations.
- ✓ Crossovers and mutations adjust diameter selections based on resilience constraints.
- ✓ Neutrosophic uncertainty functions are used to adapt the weight values over the iterations.

Figure 1 illustrates the procedural flow of the proposed Next Generation Algorithm (NGA).

The algorithm begins by initializing a population and evaluating individual fitness. Based on the fitness trend over time, individuals are dynamically assigned to one of three solution spaces—TSS, ISS, or FSS—each governing a different evolutionary strategy. The loop continues until the termination condition is met, and the final population yields both optimal and suboptimal solutions.

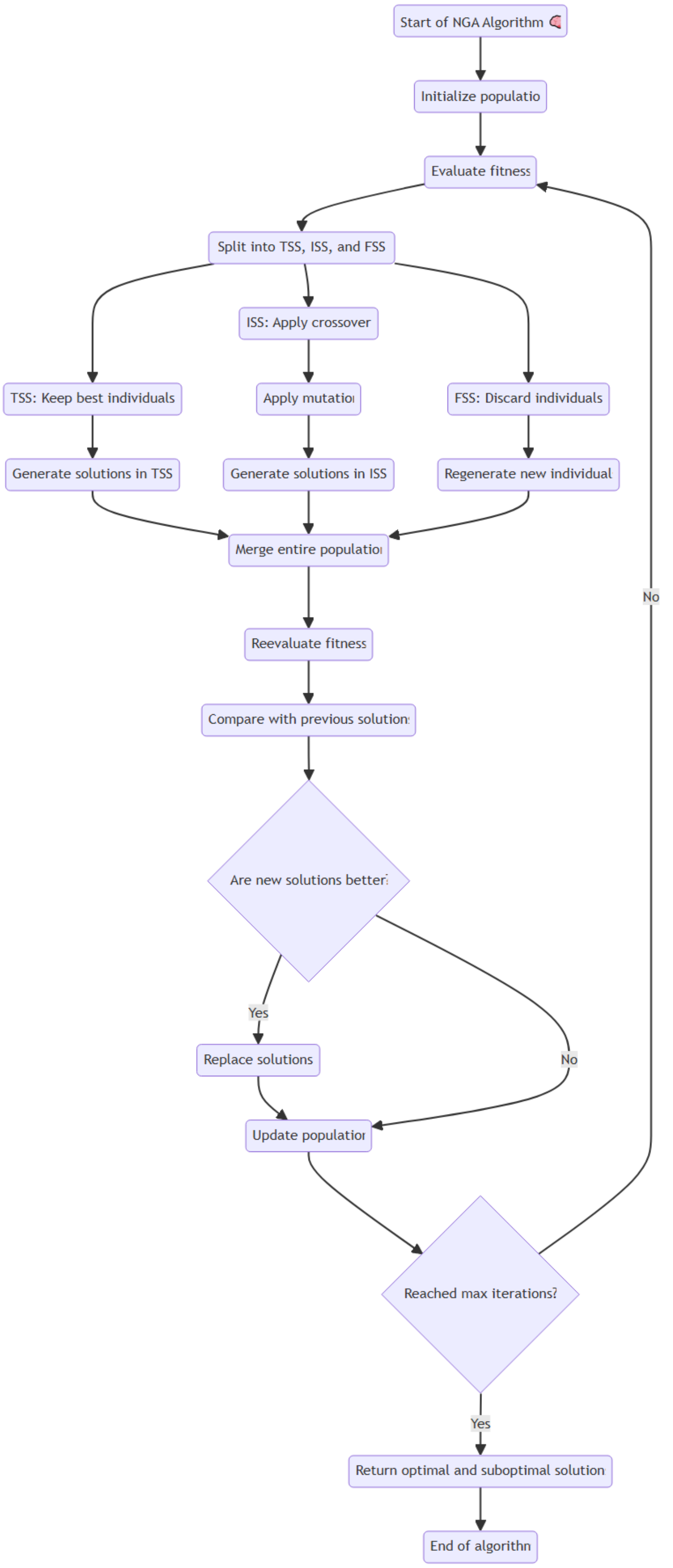

Figure 1. Operational flow of NGA with TSS, ISS, and FSS classifications.

The procedural steps outlined in Figure 1 are formalized in the following pseudocode, which explicitly defines the logic of the NGA, including the classification and handling of solutions across the TSS, ISS, and FSS spaces.

Algorithm 1. Neutrosophic Genetic Algorithm (NGA)

```
Output: Set of optimal and suboptimal solutions
Initialize Population with random individuals
Evaluate fitness for each individual
Set Iteration ← 0
while Iteration < MaxIterations do
        Classify individuals into:
                - TSS (True Solution Space)
                - ISS (Indeterminate Solution Space)
                - FSS (False Solution Space)

        Preserve individuals in TSS

        Apply crossover and mutation to individuals in ISS
        Generate new candidates from the resulting offspring

        Discard individuals in FSS
        Generate new random individuals to replace them

        Combine TSS, updated ISS, and regenerated FSS individuals
        Evaluate fitness of the new population

        Compare with previous best solutions
        If improvements found, update elite archive

        Iteration ← Iteration + 1
end while
Return best solutions from TSS and improved individuals from ISS
```

This algorithmic structure ensures that the search process not only exploits high-fitness solutions (TSS) but also explores uncertain regions (ISS) with adaptive learning, while eliminating consistently poor candidates (FSS). By dynamically managing the evolutionary behavior of individuals across these neutrosophic spaces, the NGA enhances the robustness and exploratory capacity of the optimization process. Consequently, the proposed NMM framework is capable of delivering sustainable design alternatives for water supply systems that balance cost efficiency, energy performance, and structural resilience under uncertainty.

4. Results

To evaluate the practical effectiveness of the proposed Neutrosophic Multi-Objective Model (NMM), a real-world case study was conducted involving the design of an urban water supply system (WSS). The model was tested under multiple design constraints and uncertainty conditions, reflecting the typical challenges of urban infrastructure planning. Through the integration of neutrosophic reasoning and evolutionary optimization, the NMM demonstrated notable improvements in key performance metrics compared to traditional design approaches.

- The application of the proposed Neutrosophic Multi-Objective Model (NMM) to an urban water supply system (WSS) design case study yielded significant improvements across multiple performance indicators:
- Cost Reduction: Achieved up to a 17% decrease in total system cost compared to baseline designs, due to the efficient allocation of resources.
- Energy Loss Minimization: Reduced energy losses by 11%, attributed to the optimal selection of pipe diameters and improved hydraulic performance.
- Resilience Enhancement: Increased system resilience by 33%, ensuring better adaptability to future demand fluctuations and unexpected operational failures.

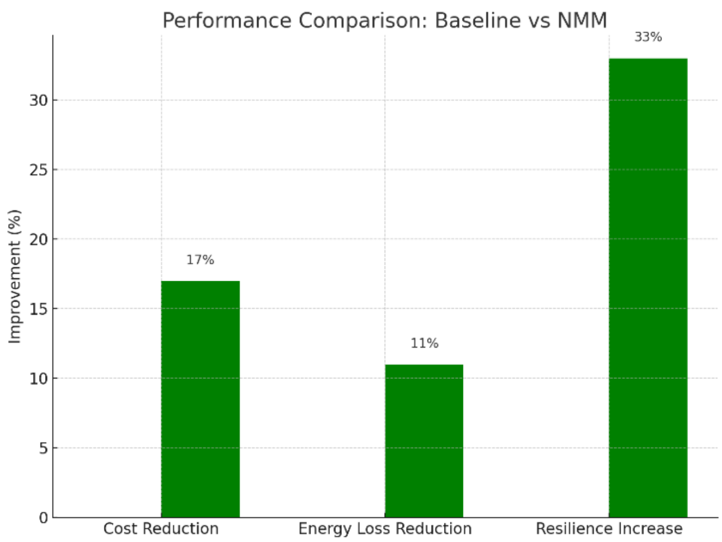

Figure 2. NMM vs. Baseline: Comparative performance across key sustainability indicators.

The proposed model offers a robust, adaptable, and cost-effective solution for the planning and management of modern urban water distribution systems. By incorporating Neutrosophic Logic, the NMM enables decision-makers to account for uncertainty, partial information, and conflicting objectives in a structured and quantifiable manner.

This framework empowers engineers and public authorities to move beyond deterministic optimization approaches by integrating resilience as a central design criterion—alongside cost and energy considerations.

This is particularly valuable in contexts with high variability in water demand, socio-environmental uncertainty, and limited infrastructure predictability.

In summary, the Neutrosophic Multi-Objective Model enhances traditional WSS optimization methodologies by delivering solutions that are not only economically viable, but also sustainable and resilient, ensuring long-term performance in an increasingly uncertain and complex urban environment.

5. Conclusions

The present study proposes a Neutrosophic Multi-Objective Model (NMM) for the optimization of water supply systems (WSS), integrating cost minimization, energy loss reduction, and resilience maximization as key criteria. The results obtained demonstrate that this approach not only overcomes the limitations of traditional methods but also offers more adaptive and sustainable solutions in high-uncertainty environments. The application of the model in a real case study evidenced significant improvements in energy efficiency, cost reduction, and system resilience to disturbances. The practical relevance of these findings lies in their applicability to design more resilient and sustainable water supply systems, especially in regions with high variability in demand and unpredictable environmental conditions. This model provides decision-makers with a robust tool to balance economic, social, and environmental objectives, which is crucial in a global context where water resource scarcity and the effects of climate change are increasingly evident.

Among the main contributions of the study is the integration of Neutrosophic Logic into multi-objective optimization, which allows to handle uncertainty and variability in the data more effectively. Furthermore, the inclusion of resilience as an explicit criterion in the model represents a significant advance in the field of water resources management, as it ensures that systems are not only efficient but also adaptable in the long term. This innovative approach extends the previous work of Hechavarría [1] and provides a more comprehensive analytical framework for decision-making in WSS. However, the study is not free of limitations. The complexity of the model and the need for accurate data to calibrate the neutrosophic parameters may represent a challenge in its practical implementation. Furthermore, the results obtained are conditioned by the specific context of the case study, which could limit their generalization to other scenarios with different geographical, climatic or socio-economic characteristics. For future research, it is recommended to explore the integration of artificial intelligence and machine learning techniques with the neutrosophic approach, which could improve the accuracy and adaptability of the model in real-time. Likewise, it would be beneficial to extend the study to different regions and scales, from urban systems to rural networks, to validate the applicability of the model in various contexts. Finally, the incorporation of additional criteria, such as water quality and environmental impact, could further enrich the proposed framework, contributing to the design of truly sustainable and resilient water supply systems.

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Received: December 24, 2024. Accepted: April 5, 2025.