NSS
Neutrosophic Sets and Systems, Vol. 83, 2025
University of New Mexico

Properties of quadripartitioned neutrosophic interval-valued set to the reciprocal fraction function via various operators

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Abstract. A novel technique used to generate quadripartitioned neutrosophic interval-valued sets applied to the reciprocal fraction function is presented in this paper. A new extension of neutrosophic interval-valued sets and interval-valued fuzzy sets are quadripartitioned neutrosophic interval-valued sets. Quadripartitioned neutrosophic interval-valued weighted averaging, geometric, and generalized concepts will all be covered in this article. To obtain the weighted average and geometric, we employed an aggregating model. Using algebraic approaches, a number of sets with significant properties will be further examined.

Keywords: weighted averaging, geometric, generalized averaging, generalized geometric.

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1. **Introduction**

A basic idea that arises in many real-world situations, reciprocal functions describe inverse connections that are essential for comprehending and forecasting behavior in a variety of domains. Their adaptability and significance in analytical tasks are demonstrated by their applications in both the scientific and social sciences. Numerous theories have been proposed to explain uncertainty, such as fuzzy sets (FS), which have membership grades (MG) ranging from 0. An intuitionistic FS (IFS) for $\sigma, \rho \in [0, 1]$ was built by Atanassov utilizing two MGs: $0 \leq \sigma + \rho \leq 1$ and positive σ and negative ρ. The Pythagorean FSs (PFS) concept was established by Yager [9] and is characterized by its MG and non-MG (NMG) with $\sigma + \rho \geq 1$ to

$\sigma^2 + \rho^2 \leq 1$. The use of IFSs and PFSs in several domains has been the subject of numerous research. They still have limited information-communication capabilities.

Shahzaib et al. [1] defined the SFS for certain AOs using MADM. SFS requires that $0 \leq \sigma^2 + \rho^2 \leq 1$ rather than $0 \leq \sigma + \rho \leq 1$. Hussain et al. first proposed the concept of an intelligent decision support system for SFS [2]. Rafiq et al. [4] were the first to present SFSs and their uses in DM. For example, the DM problem $\sigma^2 + \rho^2 \geq 1$ has a feature. Fermatean FS (FFS) was developed in 2019 by Senapati et al. [5], with the stipulation that $0 \leq \sigma^3 + \rho^3 \leq 1$. Yager was the first to suggest the idea of generalized orthopair FSs [8]. In the RFF-rung orthogonal pair FS (RFF-ROFS), both the MG and the NMG have power RFF; nevertheless, their aggregate can never be more than one. In order to solve MADM issues (AOs), aggregation operators are necessary. A range of IFS averaging operators can be used to average IFS data, according to Xu et al. [6]. Based on IFSs, Xu et al. [7] developed geometric operators, such as weighted, ordered weighted, and hybrid operators. Li et al. [3] proposed generalized ordered weighted averaging operators (GOWs) in 2002. Zeng et al. [10] explained how to compute ordered weighted distances using AOs and distance measurements. Reciprocal functions may be used to represent the flow rate of liquids via pipes. In systems with laminar or turbulent flow, the relationship between flow rate and pressure drop frequently shows inverse features. Instead of utilizing exponentiation or logarithms for particular decision making problems, we use quadripartitioned neutrosophic interval-valued aggregation operators, which are more accurate.

2. **Operations for RFFIVQNN**

Assuming that ϵ is a fractional part function, the fractional part of ω, where ω is a real number, may be written as follows: $\epsilon = \epsilon\lfloor\omega\rfloor = \langle\omega\rangle = \omega - \{\omega\}$ is also included. A fractional part function can also be used to describe the difference between a real number and its highest integer value, which is established using the greatest integer function. The fractional component of $\omega = 0$ if ω is an integer. Here, provided it exists, $\epsilon\lfloor\omega\rfloor = \frac{1}{\omega}$ is a reciprocal fractional part function. It is commonly known that the fractional portion of ω equals 0 whenever it is an integer. Therefore, for $\epsilon\lfloor\omega\rfloor = \frac{1}{\omega}$ to be defined, ω cannot be an integer. Its domain is $\epsilon\lfloor\omega\rfloor = \frac{1}{\omega}$, which includes all real numbers with the exception of integers.

Definition 2.1. The NS $\beth = \Big\{\varsigma, \Big\langle \Big\lfloor \lfloor\lfloor\mathfrak{T}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{T}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{C}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{C}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{U}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{U}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{F}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{F}^u\rfloor\lfloor\varsigma\rfloor\rfloor \Big\rfloor \Big\rangle \Big| \varsigma \in \mathcal{A}\Big\}$, where $\mathfrak{T}^l, \mathfrak{C}^l, \mathfrak{U}^l, \mathfrak{F}^l : \mathcal{A} \to \lfloor 0, 1\rfloor$ denote the truth MG, contradiction MG, unknown MG and false MG of $\varsigma \in \mathcal{A}$ to $\beth$, respectively and $0 \leq \lfloor\mathfrak{T}^u\rfloor\lfloor\varsigma\rfloor^\epsilon + \lfloor\mathfrak{C}^u\rfloor\lfloor\varsigma\rfloor^\epsilon + \lfloor\mathfrak{U}^u\rfloor\lfloor\varsigma\rfloor^\epsilon + \lfloor\lfloor\mathfrak{F}^l\rfloor\lfloor\varsigma\rfloor^\epsilon \leq 1$. For convenience, $\beth = \Big\langle \Big\lfloor \lfloor\mathfrak{T}^l, \lfloor\mathfrak{T}^u\rfloor\rfloor, \lfloor\mathfrak{C}^l, \lfloor\mathfrak{C}^u\rfloor\rfloor, \lfloor\mathfrak{U}^l, \lfloor\mathfrak{U}^u\rfloor\rfloor, \lfloor\mathfrak{F}^l, \lfloor\mathfrak{F}^u\rfloor\rfloor \Big\rfloor \Big\rangle$ is represent a IVQNSN.

Definition 2.2. Let $\gimel = \Big\{\varsigma, \Big\langle \Big\lfloor \lfloor\lfloor \mathfrak{T}^l \rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{T}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{C}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{C}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{U}^l\rfloor\lfloor\varsigma\rfloor,$
$\lfloor\mathfrak{U}^u\rfloor\lfloor\varsigma\rfloor\rfloor, \lfloor\lfloor\mathfrak{F}^l\rfloor\lfloor\varsigma\rfloor, \lfloor\mathfrak{F}^u\rfloor\lfloor\varsigma\rfloor\rfloor \Big\rfloor \Big\rangle \Big| \varsigma \in \mathcal{A}\Big\},$
$\gimel_1 = \langle\lfloor\lfloor\lfloor\mathfrak{T}_1^l\rfloor, \lfloor\mathfrak{T}_1^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{C}_1^l\rfloor, \lfloor\mathfrak{C}_1^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{U}_1^l\rfloor, \lfloor\mathfrak{U}_1^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{F}_1^l\rfloor, \lfloor\mathfrak{F}_1^u\rfloor\rfloor\rfloor\rangle,$
$\gimel_2 = \langle\lfloor\lfloor\lfloor\mathfrak{T}_2^l\rfloor, \lfloor\mathfrak{T}_2^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{C}_2^l\rfloor, \lfloor\mathfrak{C}_2^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{U}_2^l\rfloor, \lfloor\mathfrak{U}_2^u\rfloor\rfloor, \lfloor\lfloor\mathfrak{F}_2^l\rfloor, \lfloor\mathfrak{F}_2^u\rfloor\rfloor\rfloor\rangle$ be any three IVQNNs. Then

$$(1)\ \gimel_1 \bigsqcup \gimel_2 = \begin{pmatrix} \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{T}_1^l\rfloor^\epsilon + \lfloor\mathfrak{T}_2^l\rfloor^\epsilon \\ -\lfloor\mathfrak{T}_1^l\rfloor^\epsilon \cdot \lfloor\mathfrak{T}_2^l\rfloor^\epsilon\end{matrix}}, \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{T}_1^u\rfloor^\epsilon + \lfloor\mathfrak{T}_2^u\rfloor^\epsilon \\ -\lfloor\mathfrak{T}_1^u\rfloor^\epsilon \cdot \lfloor\mathfrak{T}_2^u\rfloor^\epsilon\end{matrix}} \\ \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{C}_1^l\rfloor^\epsilon + \lfloor\mathfrak{C}_2^l\rfloor^\epsilon \\ -\lfloor\mathfrak{C}_1^l\rfloor^\epsilon \cdot \lfloor\mathfrak{C}_2^l\rfloor^\epsilon\end{matrix}}, \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{C}_1^u\rfloor^\epsilon + \lfloor\mathfrak{C}_2^u\rfloor^\epsilon \\ -\lfloor\mathfrak{C}_1^u\rfloor^\epsilon \cdot \lfloor\mathfrak{C}_2^u\rfloor^\epsilon\end{matrix}} \\ \lfloor\mathfrak{U}_1^l\rfloor^\epsilon\lfloor\mathfrak{U}_2^l\rfloor^\epsilon, \lfloor\mathfrak{U}_1^u\rfloor^\epsilon\lfloor\mathfrak{U}_2^u\rfloor^\epsilon \\ \lfloor\mathfrak{F}_1^l\rfloor^\epsilon\lfloor\mathfrak{F}_2^l\rfloor^\epsilon, \lfloor\mathfrak{F}_1^u\rfloor^\epsilon\lfloor\mathfrak{F}_2^u\rfloor^\epsilon \end{pmatrix},$$

$$(2)\ \gimel_1 \oslash \gimel_2 = \begin{pmatrix} \lfloor\mathfrak{T}_1^l\rfloor^\epsilon\lfloor\mathfrak{T}_2^l\rfloor^\epsilon, \lfloor\mathfrak{T}_1^u\rfloor^\epsilon\lfloor\mathfrak{T}_2^u\rfloor^\epsilon \\ \lfloor\mathfrak{C}_1^l\rfloor^\epsilon\lfloor\mathfrak{C}_2^l\rfloor^\epsilon, \lfloor\mathfrak{C}_1^u\rfloor^\epsilon\lfloor\mathfrak{C}_2^u\rfloor^\epsilon \\ \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{U}_1^l\rfloor^\epsilon + \lfloor\mathfrak{U}_2^l\rfloor^\epsilon \\ -\lfloor\mathfrak{U}_1^l\rfloor^\epsilon \cdot \lfloor\mathfrak{U}_2^l\rfloor^\epsilon\end{matrix}}, \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{U}_1^u\rfloor^\epsilon + \lfloor\mathfrak{U}_2^u\rfloor^\epsilon \\ -\lfloor\mathfrak{U}_1^u\rfloor^\epsilon \cdot \lfloor\mathfrak{U}_2^u\rfloor^\epsilon\end{matrix}} \\ \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{F}_1^l\rfloor^\epsilon + \lfloor\mathfrak{F}_2^l\rfloor^\epsilon \\ -\lfloor\mathfrak{F}_1^l\rfloor^\epsilon \cdot \lfloor\mathfrak{F}_2^l\rfloor^\epsilon\end{matrix}}, \sqrt[\epsilon]{\begin{matrix}\lfloor\mathfrak{F}_1^u\rfloor^\epsilon + \lfloor\mathfrak{F}_2^u\rfloor^\epsilon \\ -\lfloor\mathfrak{F}_1^u\rfloor^\epsilon \cdot \lfloor\mathfrak{F}_2^u\rfloor^\epsilon\end{matrix}} \end{pmatrix}$$

$$(3)\ \propto \cdot \gimel = \begin{pmatrix} \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\mathfrak{T}^{l\epsilon}\rfloor^\propto}, \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\lfloor\mathfrak{T}^{u\epsilon}\rfloor^\propto} \\ \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\mathfrak{C}^{l\epsilon}\rfloor^\propto}, \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\lfloor\mathfrak{C}^{u\epsilon}\rfloor^\propto} \\ \lfloor\mathfrak{U}^{l\epsilon}\rfloor^\propto, \lfloor\lfloor\mathfrak{U}^{u\epsilon}\rfloor^\propto, \lfloor\mathfrak{F}^{l\epsilon}\rfloor^\propto, \lfloor\lfloor\mathfrak{F}^{u\epsilon}\rfloor^\propto \end{pmatrix}$$

$$(4)\ \gimel^\propto = \begin{pmatrix} \lfloor\mathfrak{T}^{l\epsilon}\rfloor^\propto, \lfloor\lfloor\mathfrak{T}^{u\epsilon}\rfloor^\propto, \lfloor\mathfrak{C}^{l\epsilon}\rfloor^\propto, \lfloor\lfloor\mathfrak{C}^{u\epsilon}\rfloor^\propto \\ \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\mathfrak{U}^{l\epsilon}\rfloor^\propto}, \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\lfloor\mathfrak{U}^{u\epsilon}\rfloor^\propto} \\ \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\mathfrak{F}^{l\epsilon}\rfloor^\propto}, \sqrt[\epsilon]{\shortmid - \lfloor \shortmid - \lfloor\lfloor\mathfrak{F}^{u\epsilon}\rfloor^\propto} \end{pmatrix}.$$

3. **Aggregating operators**

We use RFFIVQNWA, RFFIVQNWG, GRFFIVQNWA, and GRFFIVQNWG to describe the AOs.

3.1. *RFFIVQNWA*

Definition 3.1. Let $\gimel_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ be the RFFIVQNNs, $W = \lfloor\upsilon_1, \upsilon_2, ..., \upsilon_n\rfloor$ be the weight of $\gimel_i$, $\upsilon_i \geq 0$ and $\bigsqcup_{i\hookrightarrow 1}^n \upsilon_i = 1$. Then RFFIVQNWA $\lfloor\gimel_1, \gimel_2, ..., \gimel_n\rfloor = \bigsqcup_{i\hookrightarrow 1}^n \upsilon_i\gimel_i$.

Theorem 3.2. *Let* $\gimel_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ *be the RFFIVQNNs. Then RFFIVQNWA* $\lfloor\gimel_1, \gimel_2, ..., \gimel_n\rfloor$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \end{pmatrix}.$$

Proof If $n = 2$, then RFFIVQNWA$\lfloor \gimel_1, \gimel_2 \rfloor = \upsilon_1 \gimel_1 \bigsqcup \upsilon_2 \gimel_2$, where

$$\upsilon_1 \gimel_1 = \begin{pmatrix} \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1}} \\ \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1}} \\ \lfloor \mathfrak{U}_1^l \rfloor^{\epsilon} \rfloor^{\upsilon_1}, \lfloor \mathfrak{U}_1^u \rfloor^{\epsilon} \rfloor^{\upsilon_1} \\ \lfloor \mathfrak{F}_1^l \rfloor^{\epsilon} \rfloor^{\upsilon_1}, \lfloor \mathfrak{F}_1^u \rfloor^{\epsilon} \rfloor^{\upsilon_1} \end{pmatrix}$$

$$\upsilon_2 \gimel_2 = \begin{pmatrix} \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2}} \\ \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2}} \\ \lfloor \mathfrak{U}_2^l \rfloor^{\epsilon} \rfloor^{\upsilon_2}, \lfloor \mathfrak{U}_2^u \rfloor^{\epsilon} \rfloor^{\upsilon_2} \\ \lfloor \mathfrak{F}_2^l \rfloor^{\epsilon} \rfloor^{\upsilon_2}, \lfloor \mathfrak{F}_2^u \rfloor^{\epsilon} \rfloor^{\upsilon_2} \end{pmatrix}$$

Now, $\upsilon_1 \gimel_1 \bigsqcup \upsilon_2 \gimel_2$

$$= \begin{pmatrix} \sqrt[\epsilon]{\begin{matrix} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor + \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \\ - \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor \cdot \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \end{matrix}}, \sqrt[\epsilon]{\begin{matrix} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor + \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \\ - \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor \cdot \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \end{matrix}} \\ \sqrt[\epsilon]{\begin{matrix} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor + \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \\ - \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor \cdot \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \end{matrix}}, \sqrt[\epsilon]{\begin{matrix} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor + \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \\ - \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_1^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_1} \right\rfloor \cdot \\ \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_2^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_2} \right\rfloor \end{matrix}} \\ \lfloor \mathfrak{U}_1^l \rfloor^{\epsilon} \rfloor^{\upsilon_1} \cdot \lfloor \mathfrak{U}_2^l \rfloor^{\epsilon} \rfloor^{\upsilon_2}, \lfloor \mathfrak{U}_1^u \rfloor^{\epsilon} \rfloor^{\upsilon_1} \cdot \lfloor \mathfrak{U}_2^u \rfloor^{\epsilon} \rfloor^{\upsilon_2} \\ \lfloor \mathfrak{F}_1^l \rfloor^{\epsilon} \rfloor^{\upsilon_1} \cdot \lfloor \mathfrak{F}_2^l \rfloor^{\epsilon} \rfloor^{\upsilon_2}, \lfloor \mathfrak{F}_1^u \rfloor^{\epsilon} \rfloor^{\upsilon_1} \cdot \lfloor \mathfrak{F}_2^u \rfloor^{\epsilon} \rfloor^{\upsilon_2} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{2} \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{2} \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{2} \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{2} \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{2} \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{2} \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{2} \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{2} \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \end{pmatrix}.$$

It valid for $n \geq 3$. Thus, RFFIVQNWA$\lfloor \gimel_1, \gimel_2, ..., \gimel_l \rfloor$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \end{pmatrix}.$$

If $n = l+1$, then RFFIVQNWA $\lfloor \gimel_1, \gimel_2, ..., \gimel_l, \gimel_{l+1} \rfloor$

$$= \begin{pmatrix} \sqrt[\epsilon]{\begin{matrix} \bigsqcup_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor + \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_{l+1}^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \\ - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor \cdot \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_{l+1}^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \end{matrix}}, \\ \sqrt[\epsilon]{\begin{matrix} \bigsqcup_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor + \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_{l+1}^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \\ - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor \cdot \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{T}_{l+1}^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \end{matrix}} \\ \sqrt[\epsilon]{\begin{matrix} \bigsqcup_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor + \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_{l+1}^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \\ - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor \cdot \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_{l+1}^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \end{matrix}} \\ \sqrt[\epsilon]{\begin{matrix} \bigsqcup_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor + \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_{l+1}^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \\ - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \right\rfloor \cdot \left\lfloor 1 - \left\lfloor 1 - \lfloor \mathfrak{C}_{l+1}^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_{l+1}} \right\rfloor \end{matrix}} \\ \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i} \cdot \lfloor \mathfrak{U}_{l+1}^l \rfloor^{\epsilon} \rfloor^{\upsilon_{l+1}}, \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \cdot \lfloor \mathfrak{U}_{l+1}^u \rfloor^{\epsilon} \rfloor^{\upsilon_{l+1}} \\ \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i} \cdot \lfloor \mathfrak{F}_{l+1}^l \rfloor^{\epsilon} \rfloor^{\upsilon_{l+1}}, \circledast_{i\hookrightarrow 1}^{l} \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \cdot \lfloor \mathfrak{F}_{l+1}^u \rfloor^{\epsilon} \rfloor^{\upsilon_{l+1}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \\ \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \\ \circledast_{i\hookrightarrow 1}^{l+1} \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l+1} \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{l+1} \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l+1} \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \end{pmatrix}$$

Theorem 3.3. *Let* $\gimel_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ *be the RFFIVQNNs. Then RFFIVQNWA* $\lfloor \gimel_1, \gimel_2, ..., \gimel_n \rfloor = \gimel$.

Proof Since $\lfloor \mathfrak{T}_i^l \rfloor = \lfloor \mathfrak{T}^l \rfloor$, $\lfloor \mathfrak{C}_i^l \rfloor = \lfloor \mathfrak{C}^l \rfloor$, $\lfloor \mathfrak{U}_i^l \rfloor = \lfloor \mathfrak{U}^l \rfloor$, $\lfloor \mathfrak{F}_i^l \rfloor = \lfloor \mathfrak{F}^l \rfloor$ and $\lfloor \mathfrak{T}_i^u \rfloor = \lfloor \mathfrak{T}^u \rfloor$, $\lfloor \mathfrak{C}_i^u \rfloor = \lfloor \mathfrak{C}^u \rfloor$, $\lfloor \mathfrak{U}_i^u \rfloor = \lfloor \mathfrak{U}^u \rfloor$, $\lfloor \mathfrak{F}_i^u \rfloor = \lfloor \mathfrak{F}^u \rfloor$ and $\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i = 1$. Now, RFFIVQNWA $\lfloor \gimel_1, \gimel_2, ..., \gimel_n \rfloor$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^\epsilon \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^\epsilon \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^\epsilon \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^\epsilon \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_i^l \rfloor^\epsilon \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_i^u \rfloor^\epsilon \rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_i^l \rfloor^\epsilon \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_i^u \rfloor^\epsilon \rfloor^{\upsilon_i} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}^{l\epsilon} \right\rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{U}^{u\epsilon} \right\rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}} \\ \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}^{l\epsilon} \right\rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}^{u\epsilon} \right\rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}} \\ \lfloor \mathfrak{U}^{l\epsilon} \rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}, \lfloor \mathfrak{U}^{u\epsilon} \rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i} \\ \lfloor \mathfrak{F}^{l\epsilon} \rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i}, \lfloor \mathfrak{F}^{u\epsilon} \rfloor^{\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}^{l\epsilon} \right\rfloor}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{U}^{u\epsilon} \right\rfloor} \\ \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}^{l\epsilon} \right\rfloor}, \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}^{u\epsilon} \right\rfloor} \\ \mathfrak{U}^{l\epsilon}, \mathfrak{U}^{u\epsilon} \\ \mathfrak{F}^{l\epsilon}, \mathfrak{F}^{u\epsilon} \end{pmatrix}$$

$$= \beth.$$

Theorem 3.4. *Let* $\beth_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ *be the RFFIVQNNs. Then* $RFFIVQNWA\lfloor \beth_1, \beth_2, ..., \beth_n \rfloor$
where $\underbrace{\lfloor \mathfrak{T}^l \rfloor} = \min \lfloor \mathfrak{T}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{T}^l \rfloor} = \max \lfloor \mathfrak{T}_{ij}^l \rfloor$, $\underbrace{\lfloor \mathfrak{C}^l \rfloor} = \min \lfloor \mathfrak{C}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{C}^l \rfloor} = \max \lfloor \mathfrak{C}_{ij}^l \rfloor$, $\underbrace{\lfloor \mathfrak{U}^l \rfloor} = \min \lfloor \mathfrak{U}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{U}^l \rfloor} = \max \lfloor \mathfrak{U}_{ij}^l \rfloor$, $\underbrace{\lfloor \mathfrak{F}^l \rfloor} = \min \lfloor \mathfrak{F}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{F}^l \rfloor} = \max \lfloor \mathfrak{F}_{ij}^l \rfloor$
and $\underbrace{\lfloor \mathfrak{T}^u \rfloor} = \min \lfloor \mathfrak{T}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{T}^u \rfloor} = \max \lfloor \mathfrak{T}_{ij}^u \rfloor$, $\underbrace{\lfloor \mathfrak{C}^u \rfloor} = \min \lfloor \mathfrak{C}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{C}^u \rfloor} = \max \lfloor \mathfrak{C}_{ij}^u \rfloor$, $\underbrace{\lfloor \mathfrak{U}^u \rfloor} = \min \lfloor \mathfrak{U}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{U}^u \rfloor} = \max \lfloor \mathfrak{U}_{ij}^u \rfloor$, $\underbrace{\lfloor \mathfrak{F}^u \rfloor} = \min \lfloor \mathfrak{F}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{F}^u \rfloor} = \max \lfloor \mathfrak{F}_{ij}^u \rfloor$ *and where* $1 \leq i \leq n$, $j = 1, 2, ..., i_j$. *Then,*

$$\left\langle \underbrace{\lfloor \mathfrak{T}^l \rfloor, \lfloor \mathfrak{T}^u \rfloor}, \underbrace{\lfloor \mathfrak{C}^l \rfloor, \lfloor \mathfrak{C}^u \rfloor}, \overbrace{\lfloor \mathfrak{U}^l \rfloor, \lfloor \mathfrak{U}^u \rfloor}, \overbrace{\lfloor \mathfrak{F}^l \rfloor, \lfloor \mathfrak{F}^u \rfloor} \right\rangle$$

$$\leq IVqNWA\lfloor \beth_1, \beth_2, ..., \beth_n \rfloor$$
$$\leq \left\langle \overbrace{\lfloor \mathfrak{T}^l \rfloor, \lfloor \mathfrak{T}^u \rfloor}, \overbrace{\lfloor \mathfrak{C}^l \rfloor, \lfloor \mathfrak{C}^u \rfloor}, \underbrace{\lfloor \mathfrak{U}^l \rfloor, \lfloor \mathfrak{U}^u \rfloor}, \underbrace{\lfloor \mathfrak{F}^l \rfloor, \lfloor \mathfrak{F}^u \rfloor} \right\rangle.$$

Proof Since, $\underbrace{\lfloor \mathfrak{T}^l \rfloor} = \min \lfloor \mathfrak{T}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{T}^l \rfloor} = \max \lfloor \mathfrak{T}_{ij}^l \rfloor$ and $\underbrace{\lfloor \mathfrak{T}^l \rfloor} \leq \lfloor \mathfrak{T}_{ij}^l \rfloor \leq \overbrace{\lfloor \mathfrak{T}^l \rfloor}$ and $\underbrace{\lfloor \mathfrak{T}^u \rfloor} = \min \lfloor \mathfrak{T}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{T}^u \rfloor} = \max \lfloor \mathfrak{T}_{ij}^u \rfloor$ and $\underbrace{\lfloor \mathfrak{T}^u \rfloor} \leq \lfloor \mathfrak{T}_{ij}^u \rfloor \leq \overbrace{\lfloor \mathfrak{T}^u \rfloor}$.

Now $\underbrace{\lfloor \mathfrak{T}^l \rfloor}, \underbrace{\lfloor \mathfrak{T}^u \rfloor}$

$$\begin{aligned}
&= \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{T}^l \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{T}^u \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&\leq \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \mathfrak{T}_{ij}^l \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \mathfrak{T}_{ij}^u \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&\leq \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{T}^l \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{T}^u \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&= \overbrace{\lfloor \mathfrak{T}^l \rfloor}.
\end{aligned}$$

Since, $\underbrace{\lfloor \mathfrak{C}^l \rfloor} = \min \lfloor \mathfrak{C}_{ij}^l \rfloor$, $\overbrace{\lfloor \mathfrak{C}^l \rfloor} = \max \lfloor \mathfrak{C}_{ij}^l \rfloor$ and $\underbrace{\lfloor \mathfrak{C}^l \rfloor} \leq \lfloor \mathfrak{C}_{ij}^l \rfloor \leq \overbrace{\lfloor \mathfrak{C}^l \rfloor}$ and $\underbrace{\lfloor \mathfrak{C}^u \rfloor} = \min \lfloor \mathfrak{C}_{ij}^u \rfloor$, $\overbrace{\lfloor \mathfrak{C}^u \rfloor} = \max \lfloor \mathfrak{C}_{ij}^u \rfloor$ and $\underbrace{\lfloor \mathfrak{C}^u \rfloor} \leq \lfloor \mathfrak{C}_{ij}^u \rfloor \leq \overbrace{\lfloor \mathfrak{C}^u \rfloor}$.

Now, $\underbrace{\lfloor \mathfrak{C}^l \rfloor}, \underbrace{\lfloor \mathfrak{C}^u \rfloor}$

$$\begin{aligned}
&= \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{C}^l \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{C}^u \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&\leq \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \mathfrak{C}_{ij}^l \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \mathfrak{C}_{ij}^u \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&\leq \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{C}^l \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \Big\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{C}^u \rfloor} \rfloor^{\epsilon} \Big\rfloor^{\upsilon_i}} \\
&= \overbrace{\lfloor \mathfrak{C}^l \rfloor}, \overbrace{\lfloor \mathfrak{C}^u \rfloor}.
\end{aligned}$$

Since, $\underbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor} = \min \lfloor \mathfrak{U}_{ij}^l \rfloor^{\epsilon}$, $\overbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor} = \max \lfloor \mathfrak{U}_{ij}^l \rfloor^{\epsilon}$ and $\underbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor} \leq \lfloor \mathfrak{U}_{ij}^l \rfloor^{\epsilon} \leq \overbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor}$ and $\underbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor} = \min \lfloor \mathfrak{U}_{ij}^u \rfloor^{\epsilon}$, $\overbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor} = \max \lfloor \mathfrak{U}_{ij}^u \rfloor^{\epsilon}$ and $\underbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor} \leq \lfloor \mathfrak{U}_{ij}^u \rfloor^{\epsilon} \leq \overbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor}$. We have,

$$\begin{aligned}
\underbrace{\lfloor \mathfrak{U} \rfloor}^{l\epsilon} &= \circledast_{i \hookrightarrow 1}^{n} \underbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor}^{\upsilon_i}, \circledast_{i \hookrightarrow 1}^{n} \underbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor}^{\upsilon_i} \\
&\leq \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}_{ij}^l \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}_{ij}^u \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\
&\leq \circledast_{i \hookrightarrow 1}^{n} \overbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor}^{\upsilon_i}, \circledast_{i \hookrightarrow 1}^{n} \overbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor}^{\upsilon_i} \\
&= \overbrace{\lfloor \mathfrak{U}^{l\epsilon} \rfloor}, \overbrace{\lfloor \mathfrak{U}^{u\epsilon} \rfloor}.
\end{aligned}$$

Since, $\underbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor} = \min \lfloor \mathfrak{F}_{ij}^l \rfloor^{\epsilon}$, $\overbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor} = \max \lfloor \mathfrak{F}_{ij}^l \rfloor^{\epsilon}$ and $\underbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor} \leq \lfloor \mathfrak{F}_{ij}^l \rfloor^{\epsilon} \leq \overbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor}$ and $\underbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor} = \min \lfloor \mathfrak{F}_{ij}^u \rfloor^{\epsilon}$, $\overbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor} = \max \lfloor \mathfrak{F}_{ij}^u \rfloor^{\epsilon}$ and $\underbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor} \leq \lfloor \mathfrak{F}_{ij}^u \rfloor^{\epsilon} \leq \overbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor}$.

We have,

$$\begin{aligned}
\underbrace{\lfloor \mathfrak{F} \rfloor^{l\epsilon}} &= \circledast_{i\hookrightarrow 1}^{n} \underbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor}^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \underbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor}^{\upsilon_i} \\
&\leq \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_{ij}^{l} \rfloor^{\epsilon} \rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_{ij}^{u} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \\
&\leq \circledast_{i\hookrightarrow 1}^{n} \overbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor}^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \overbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor}^{\upsilon_i} \\
&= \overbrace{\lfloor \mathfrak{F}^{l\epsilon} \rfloor}, \overbrace{\lfloor \mathfrak{F}^{u\epsilon} \rfloor}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{T}^{l} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{C}^{l} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ +1 - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \overbrace{\lfloor \mathfrak{U}^{l} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \overbrace{\lfloor \mathfrak{F}^{l} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \\ + \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{T}^{u} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \underbrace{\lfloor \mathfrak{C}^{u} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \overbrace{\lfloor \mathfrak{U}^{u} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \overbrace{\lfloor \mathfrak{F}^{u} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \end{array} \right) \\
\leq\ &\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_{ij}^{l} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_{ij}^{l} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ +1 - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_{ij}^{l} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_{ij}^{l} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \\ + \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{T}_{ij}^{u} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{C}_{ij}^{u} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{U}_{ij}^{u} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \mathfrak{F}_{ij}^{u} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \end{array} \right) \\
\leq\ &\frac{1}{2} \times \left(\begin{array}{c} \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{T}^{l} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{C}^{l} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ +1 - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \underbrace{\lfloor \mathfrak{U}^{l} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \underbrace{\lfloor \mathfrak{F}^{l} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \\ + \left(\begin{array}{c} \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{T}^{u} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} + \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \overbrace{\lfloor \mathfrak{C}^{u} \rfloor} \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{2} \\ - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \underbrace{\lfloor \mathfrak{U}^{u} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \underbrace{\lfloor \mathfrak{F}^{u} \rfloor} \rfloor^{\epsilon} \rfloor^{\upsilon_i} \right\rfloor^{2} \end{array} \right) \end{array} \right).
\end{aligned}$$

Theorem 3.5. *Let* $\beth_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ *and* $\mathcal{W}_i = \langle \lfloor \lfloor \lfloor \mathfrak{T}_{h_{ij}}^l \rfloor, \lfloor \mathfrak{T}_{h_{ij}}^u \rfloor \rfloor, \lfloor \lfloor \mathfrak{C}_{h_{ij}}^l \rfloor, \lfloor \mathfrak{C}_{h_{ij}}^u \rfloor \rfloor, \lfloor \lfloor \mathfrak{U}_{h_{ij}}^l \rfloor, \lfloor \mathfrak{U}_{h_{ij}}^u \rfloor \rfloor, \lfloor \lfloor \mathfrak{F}_{h_{ij}}^l \rfloor, \lfloor \mathfrak{F}_{h_{ij}}^u \rfloor \rfloor \rfloor \rangle$, *be the RF-FIVQNWAs. For any i, if there is* $\lfloor \mathfrak{T}_{t_{ij}}^l \rfloor^2 \leq \lfloor \mathfrak{T}_{h_{ij}}^l \rfloor^2$ *and* $\lfloor \mathfrak{C}_{t_{ij}}^l \rfloor^2 \leq \lfloor \mathfrak{C}_{h_{ij}}^l \rfloor^2$ *and* $\lfloor \mathfrak{U}_{t_{ij}}^l \rfloor^2 \geq \lfloor \mathfrak{U}_{h_{ij}}^l \rfloor^2$ *and* $\lfloor \mathfrak{F}_{t_{ij}}^l \rfloor^2 \geq \lfloor \mathfrak{F}_{h_{ij}}^l \rfloor^2$ *and* $\lfloor \mathfrak{T}_{t_{ij}}^u \rfloor^2 \leq \lfloor \mathfrak{T}_{h_{ij}}^u \rfloor^2$ *and* $\lfloor \mathfrak{C}_{t_{ij}}^u \rfloor^2 \leq \lfloor \mathfrak{C}_{h_{ij}}^u \rfloor^2$ *and* $\lfloor \mathfrak{U}_{t_{ij}}^u \rfloor^2 \geq \lfloor \mathfrak{U}_{h_{ij}}^u \rfloor^2$ *and* $\lfloor \mathfrak{F}_{t_{ij}}^u \rfloor^2 \geq \lfloor \mathfrak{F}_{h_{ij}}^u \rfloor^2$ *or* $\beth_i \leq \mathcal{W}_i$. *Prove that RFFIVQNWA* $\lfloor \beth_1, \beth_2, ..., \beth_n \rfloor \leq RFFIVQNWA \lfloor \mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n \rfloor$.

Proof For any i, $\lfloor \mathfrak{T}^{l}_{t_{ij}} \rfloor^2 \leq \lfloor \mathfrak{T}^{l}_{h_{ij}} \rfloor^2$.
Therefore, $\text{ı} - \lfloor \mathfrak{T}^{l}_{t_{i}} \rfloor^2 \geq \text{ı} - \lfloor \mathfrak{T}^{l}_{h_{i}} \rfloor^2$.
Hence, $\circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{l}_{t_{i}} \rfloor \right\rfloor^2 \right\rfloor^{\upsilon_i} \geq \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{l}_{h_{i}} \rfloor \right\rfloor^2 \right\rfloor^{\upsilon_i}$
and $\sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{l}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \leq \sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{l}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}$.
Similarly, $\lfloor \mathfrak{T}^{u}_{t_{ij}} \rfloor^2 \leq \lfloor \mathfrak{T}^{u}_{h_{ij}} \rfloor^2$.
Therefore, $\text{ı} - \lfloor \mathfrak{T}^{u}_{t_{i}} \rfloor^2 \geq \text{ı} - \lfloor \mathfrak{T}^{u}_{h_{i}} \rfloor^2$.
Hence, $\circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{u}_{t_{i}} \rfloor \right\rfloor^2 \right\rfloor^{\upsilon_i} \geq \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{u}_{h_{i}} \rfloor \right\rfloor^2 \right\rfloor^{\upsilon_i}$
and $\sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{u}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \leq \sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{T}^{u}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}$.
For any i, $\lfloor \mathfrak{C}^{l}_{t_{ij}} \rfloor^{\epsilon} \leq \left\lfloor \mathfrak{C}^{l}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $\text{ı} - \left\lfloor \mathfrak{C}^{l}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \geq \text{ı} - \left\lfloor \mathfrak{C}^{l}_{h_{i}} \rfloor \right\rfloor^{\epsilon}$.
Hence, $\circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{l}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \geq \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{l}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}$.
This implies that $\sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{l}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \leq \sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{l}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}$.
Similarly, for any i, $\lfloor \mathfrak{C}^{u}_{t_{ij}} \rfloor^{\epsilon} \leq \left\lfloor \mathfrak{C}^{u}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $\text{ı} - \left\lfloor \mathfrak{C}^{u}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \geq \text{ı} - \left\lfloor \mathfrak{C}^{u}_{h_{i}} \rfloor \right\rfloor^{\epsilon}$.
Hence, $\circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{u}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i} \geq \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{u}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}$.
This implies that $\sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{u}_{t_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \leq \sqrt[\epsilon]{\text{ı} - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \text{ı} - \left\lfloor \mathfrak{C}^{u}_{h_{i}} \rfloor \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}$.
For any i, $\left\lfloor \mathfrak{U}^{l}_{t_{ij}} \rfloor \right\rfloor^2 \geq \left\lfloor \mathfrak{U}^{l}_{h_{ij}} \rfloor \right\rfloor^2$ and $\left\lfloor \mathfrak{U}^{l}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \geq \left\lfloor \mathfrak{U}^{l}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $\text{ı} - \frac{\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}^{l}_{t_{ij}} \rfloor \right\rfloor^{\epsilon}}{2} \leq \text{ı} - \frac{\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}^{l}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}}{2}$.
Similarly, for any i,
$\left\lfloor \mathfrak{U}^{u}_{t_{ij}} \rfloor \right\rfloor^2 \geq \left\lfloor \mathfrak{U}^{u}_{h_{ij}} \rfloor \right\rfloor^2$ and $\left\lfloor \mathfrak{U}^{u}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \geq \left\lfloor \mathfrak{U}^{u}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $-\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}^{u}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \leq -\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{U}^{u}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
For any i, $\left\lfloor \mathfrak{F}^{l}_{t_{ij}} \rfloor \right\rfloor^2 \geq \left\lfloor \mathfrak{F}^{l}_{h_{ij}} \rfloor \right\rfloor^2$ and $\left\lfloor \mathfrak{F}^{l}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \geq \left\lfloor \mathfrak{F}^{l}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $\text{ı} - \frac{\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{F}^{l}_{t_{ij}} \rfloor \right\rfloor^{\epsilon}}{2} \leq \text{ı} - \frac{\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{F}^{l}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}}{2}$.
Similarly, for any i,
$\left\lfloor \mathfrak{F}^{u}_{t_{ij}} \rfloor \right\rfloor^2 \geq \left\lfloor \mathfrak{F}^{u}_{h_{ij}} \rfloor \right\rfloor^2$ and $\left\lfloor \mathfrak{F}^{u}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \geq \left\lfloor \mathfrak{F}^{u}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.
Therefore, $-\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{F}^{u}_{t_{ij}} \rfloor \right\rfloor^{\epsilon} \leq -\left\lfloor \circledast_{i \hookrightarrow 1}^{n} \lfloor \mathfrak{F}^{u}_{h_{ij}} \rfloor \right\rfloor^{\epsilon}$.

Hence,

$$\frac{1}{2}\times\left(\begin{array}{c}\left(\begin{array}{c}\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{T}_{ti}^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}+\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{C}_{t_i}^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}\\ +1-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{U}_{t_i}^{l}\rfloor^{\epsilon}\right\rfloor^{2}-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{F}_{t_i}^{l}\rfloor^{\epsilon}\right\rfloor^{2}\end{array}\right)\\ +\left(\begin{array}{c}\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{T}_{ti}^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}+\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{C}_{t_i}^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}\\ -\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{U}_{t_i}^{u}\rfloor^{\epsilon}\right\rfloor^{2}-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{F}_{t_i}^{u}\rfloor^{\epsilon}\right\rfloor^{2}\end{array}\right)\end{array}\right)$$

$$\leq\ \frac{1}{2}\times\left(\begin{array}{c}\left(\begin{array}{c}\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{T}_{hi}^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}+\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{C}_{h_i}^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}\\ +1-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{U}_{h_i}^{l}\rfloor^{\epsilon}\right\rfloor^{2}-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{F}_{h_i}^{l}\rfloor^{\epsilon}\right\rfloor^{2}\end{array}\right)\\ +\left(\begin{array}{c}\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{T}_{hi}^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}+\left\lfloor\sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{C}_{h_i}^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\right\rfloor^{2}\\ -\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{U}_{h_i}^{u}\rfloor^{\epsilon}\right\rfloor^{2}-\left\lfloor\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{F}_{h_i}^{u}\rfloor^{\epsilon}\right\rfloor^{2}\end{array}\right)\end{array}\right).$$

Hence, RFFIVQNWA $\lfloor \beth_1, \beth_2, ..., \beth_n\rfloor \leq$ RFFIVQNWA $\lfloor \mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n\rfloor$.

3.2. *RFFIVQNWG*

Definition 3.6. Let $\beth_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ be the RFFIVQNNs. Then RFFNWG $\lfloor \beth_1, \beth_2, ..., \beth_n\rfloor = \circledast_{i\hookrightarrow 1}^{n}\beth_i^{\upsilon_i}$.

Corollary 3.7. *Let* $\beth_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ *be the RFFIVQNNs. Then RFFIVQNWG* $\lfloor \beth_1, \beth_2, ..., \beth_n\rfloor$

$$=\left(\begin{array}{c}\circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{T}_i^{l}\rfloor^{\epsilon}\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{T}_i^{u}\rfloor^{\epsilon}\rfloor^{\upsilon_i}\\ \circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{C}_i^{l}\rfloor^{\epsilon}\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n}\lfloor\mathfrak{C}_i^{u}\rfloor^{\epsilon}\rfloor^{\upsilon_i}\\ \sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{U}_i^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{U}_i^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\\ \sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{F}_i^{l}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{1-\circledast_{i\hookrightarrow 1}^{n}\left\lfloor 1-\lfloor\mathfrak{F}_i^{u}\rfloor^{\epsilon}\right\rfloor^{\upsilon_i}}\end{array}\right).$$

Corollary 3.8. *Let* $\beth_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ *be the RFFIVQNNs and all are equal. Then RFFNWG*$\lfloor \beth_1, \beth_2, ..., \beth_n\rfloor = \beth$.

In addition to possessing RFFNWG, it also possesses boundedness and monotonicity.

3.3. *Generalized RFFIVQNWA (GRFFIVQNWA)*

Definition 3.9. Let $\beth_i = \langle\lfloor\lfloor\mathfrak{T}_i^l, \mathfrak{T}_i^u\rfloor, \lfloor\mathfrak{C}_i^l, \mathfrak{C}_i^u\rfloor, \lfloor\mathfrak{U}_i^l, \mathfrak{U}_i^u\rfloor, \lfloor\mathfrak{F}_i^l, \mathfrak{F}_i^u\rfloor\rfloor\rangle$ be the RFFIVQNN. Then GRFFIVQNWA $\lfloor \beth_1, \beth_2, ..., \beth_n\rfloor = \left\lfloor \bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i \beth_i^{\propto}\right\rfloor^{1/\propto}$.

Theorem 3.10. *Let* $\daleth_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ *be the RFFIVQNNs. Then GRFFIVQNWA* $\lfloor \daleth_1, \daleth_2, ..., \daleth_n \rfloor$

$$= \left(\begin{matrix} \left\lfloor \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{T}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{T}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}} \\ \left\lfloor \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{C}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{C}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}} \\ \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}}, \\ \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}} \\ \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}}, \\ \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}} \end{matrix} \right).$$

Proof To show that,

$$\bigsqcup_{i\hookrightarrow 1}^{n} \upsilon_i \daleth_i^{\epsilon} = \left(\begin{matrix} \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{T}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{T}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{C}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\imath - \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \imath - \left\lfloor \mathfrak{C}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \end{matrix} \right).$$

Put $n = 2$, $v_1 \daleth_1 \bigsqcup v_2 \daleth_2$

$$= \left(\begin{array}{c}
\sqrt[\epsilon]{\begin{array}{c} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} + \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \\ - \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \end{array}} \\
\sqrt[\epsilon]{\begin{array}{c} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} + \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \\ - \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{T}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon}, \end{array}} \\
\sqrt[\epsilon]{\begin{array}{c} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} + \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \\ - \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon}, \end{array}} \\
\sqrt[\epsilon]{\begin{array}{c} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} + \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \\ - \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{C}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_1}} \right\rceil^{\epsilon} \end{array}} \\
\left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1}, \\
\left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1} \\
\left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_2^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1}, \\
\left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1} \cdot \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_2^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_1}
\end{array} \right)$$

$$= \left(\begin{array}{c}
\sqrt[\epsilon]{\imath - \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_i}}, \sqrt[\epsilon]{\imath - \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \imath - \left\lfloor \mathfrak{T}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_i}} \\
\sqrt[\epsilon]{\imath - \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_i}}, \sqrt[\epsilon]{\imath - \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \imath - \left\lfloor \mathfrak{C}_1^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon} \right\rceil^{v_i}} \\
\circledast_{i \hookrightarrow 1}^{2} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_i}, \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{U}_i^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_i} \\
\circledast_{i \hookrightarrow 1}^{2} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^l \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_i}, \circledast_{i \hookrightarrow 1}^{2} \left\lfloor \sqrt[\epsilon]{\imath - \left\lfloor \imath - \left\lfloor \mathfrak{F}_i^u \right\rfloor^{\epsilon} \right\rceil^{\epsilon}} \right\rceil^{v_i}
\end{array} \right).$$

Hence,

$$\bigsqcup_{i\hookrightarrow 1}^{l} \upsilon_i \gimel_i^{\propto} = \begin{pmatrix} \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_1^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_1^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_1^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_1^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \end{pmatrix}.$$

If $n = l+1$, then $\bigsqcup_{i\hookrightarrow 1}^{l} \upsilon_i \gimel_i^{\propto} + \upsilon_{l+1} \gimel_{l+1}^{\propto} = \bigsqcup_{i\hookrightarrow 1}^{l+1} \upsilon_i \gimel_i^{\propto}$.
Now, $\bigsqcup_{i\hookrightarrow 1}^{l} \upsilon_i \gimel_i^{\propto} + \upsilon_{l+1} \gimel_{l+1}^{\propto} = \upsilon_1 \gimel_1^{\propto} \bigsqcup \upsilon_2 \gimel_2^{\propto} \bigsqcup ... \bigsqcup \upsilon_l \gimel_l^{\propto} \bigsqcup \upsilon_{l+1} \gimel_{l+1}^{\propto}$

$$\bigsqcup_{i\hookrightarrow 1}^{l+1} \upsilon_i \gimel_i^{\epsilon} = \begin{pmatrix} \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_1^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_1^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_1^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}}, \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_1^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \\ \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \\ \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i}, \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \end{pmatrix}.$$

$$\left\lfloor \bigsqcup_{i\hookrightarrow 1}^{l+1} \upsilon_i \gimel_i^{\propto} \right\rfloor^{1/\propto} = \begin{pmatrix} \left\lfloor \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{T}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}} \\ \left\lfloor \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_i^l \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{\shortmid - \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \shortmid - \left\lfloor \mathfrak{C}_i^u \right\rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}} \\ \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \left\lfloor \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{2} \right\rfloor^{\epsilon^{l+1}}}, \\ \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \left\lfloor \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{2} \right\rfloor^{\epsilon^{l+1}}} \\ \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \left\lfloor \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{2} \right\rfloor^{\epsilon^{l+1}}}, \\ \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \left\lfloor \circledast_{i\hookrightarrow 1}^{l+1} \left\lfloor \sqrt[\epsilon]{\shortmid - \left\lfloor \shortmid - \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{2} \right\rfloor^{\epsilon^{l+1}}} \end{pmatrix}$$

3.4. *Generalized RFFIVQNWG (GRFFIVQNWG)*

Definition 3.11. $\gimel_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ be the RFFIVQNNs. Then GRFFIVQNWG $\lfloor \gimel_1, \gimel_2, ..., \gimel_n \rfloor = \frac{1}{\propto} \left\lfloor \circledast_{i\hookrightarrow 1}^{n} \lfloor \propto \gimel_i \rfloor^{\upsilon_i} \right\rfloor$.

Corollary 3.12. *Let* $\gimel_i = \langle \lfloor \lfloor \mathfrak{T}_i^l, \mathfrak{T}_i^u \rfloor, \lfloor \mathfrak{C}_i^l, \mathfrak{C}_i^u \rfloor, \lfloor \mathfrak{U}_i^l, \mathfrak{U}_i^u \rfloor, \lfloor \mathfrak{F}_i^l, \mathfrak{F}_i^u \rfloor \rfloor \rangle$ *be the RFFIVQNNs. Then* $GRFFIVQNWG\lfloor \gimel_1, \gimel_2, ..., \gimel_n \rfloor$

$$
= \left(\begin{array}{c}
\sqrt[\epsilon]{1 - \left\lfloor 1 - \left\lfloor \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}}, \\
\sqrt[\epsilon]{1 - \left\lfloor 1 - \left\lfloor \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{T}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}} \\
\sqrt[\epsilon]{1 - \left\lfloor 1 - \left\lfloor \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^l \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}}, \\
\sqrt[\epsilon]{1 - \left\lfloor 1 - \left\lfloor \circledast_{i \hookrightarrow 1}^{n} \left\lfloor \sqrt[\epsilon]{1 - \left\lfloor 1 - \lfloor \mathfrak{C}_i^u \rfloor^{\epsilon} \right\rfloor^{\epsilon}} \right\rfloor^{\upsilon_i} \right\rfloor^{\epsilon} \right\rfloor^{\epsilon^{l+1}}} \\
\left\lfloor \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{U}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{U}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}} \\
\left\lfloor \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{F}_i^l \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}, \left\lfloor \sqrt[\epsilon]{1 - \circledast_{i \hookrightarrow 1}^{n} \left\lfloor 1 - \lfloor \mathfrak{F}_i^u \rfloor^{\epsilon} \right\rfloor^{\upsilon_i}} \right\rfloor^{\epsilon^{l+1}}
\end{array} \right).
$$

4. Conclusion:

This paper introduces new weighted operators, including averaging and geometric operators. These operators are characterized by boundedness, idempotency, commutativity, associativity, and monotonicity. We examined many standard metrics in order to characterize the weighted vector. Numerous criteria for aggregation operators have been examined. A few aggregating methods for these RFFIVQNNs have been studied, and some conclusions have been drawn.

Acknowledgments: The authors would like to thank the Editor-InChief and the anonymous referees for their various suggestions and helpful comments that have led to the improved in the quality and clarity version of the paper.

Conflicts of Interest: The author declares no conflict of interest.

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Received: Nov. 7, 2024. Accepted: April 10, 2025