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Neutrosophic e-open Sets in a Neutrosophic Topological Spaces

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Abstract. This study presents the concept of a neutrosophic e-open set, defined as the union of neutrosophic $\delta\mathcal{P}$-open sets and $\delta\mathcal{S}$-open sets within the framework of neutrosophic topological spaces. It also explores near open sets, highlighting their characteristics and providing illustrative examples of neutrosophic e-open sets. Furthermore, we examine fundamental properties and offer examples related to the neutrosophic e-interior and e-closure. In addition, we demonstrate the application of the neutrosophic score function and its negative counterpart in solving a mobile phone selection problem. This is achieved by utilizing neutrosophic topological spaces structured around various attributes and alternatives, ultimately determining score values to support decision-making.

Keywords: Neutrosophic set, Neutrosophic e-open set, Neutrosophic e-closed set, Neutrosophic e interior, Neutrosophic e closure, Neutrosophic score function and Neutrosophic negative score function.

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1. Introduction

The concept of fuzzy sets was initially introduced by Zadeh [46] within the framework of logic and set theory. Chang [7] extended this notion to general topology, establishing the foundation of fuzzy topological spaces. Atanassov [4] later proposed intuitionistic fuzzy sets, incorporating both membership and non-membership degrees, which were further developed topologically by Coker [8]. The theory of neutrosophy and neutrosophic sets was initiated by Smarandache [21, 22] in the early 20th century. Subsequently, Salama and Alblowi [18, 19] introduced neutrosophic sets and neutrosophic crisp sets within the framework of neutrosophic topological spaces. Saha [24] defined δ-open sets in fuzzy topological spaces, and Vadivel et al. [10, 29, 30, 32, 33, 40] extended this notion to neutrosophic topological spaces. Ekici [9] introduced e-open sets in general topology, which was further extended to fuzzy and intuitionistic fuzzy contexts by Seenivasan et al. [20] and Chandrasekar et al. [6], respectively. Smarandache [22] also defined single-valued neutrosophic sets characterized by three components: truth (T), indeterminacy (I), and falsehood (F), which were further explored by Wang et al. [43]. Applications of neutrosophic sets in various fields can be found in [1, 2, 5, 11, 12, 16, 17, 25, 26, 28, 31, 36, 44, 45]. In [23], Smarandache introduced the neutrosophic score, accuracy, and certainty functions. Recent contributions by Vadivel et al. [34, 35] involve open set structures in neutrosophic nano topological spaces. Moreover, authors in [27, 37–39, 41, 42] have investigated e-open sets and corresponding classes of continuous and irresolute functions in N-neutrosophic crisp topological spaces. Additional results in nano ideal, neutrosophic, and neutrosophic support soft topological spaces have been studied by Parimala et al. [13–15].

Motivation: In topology, e-open sets is defined and introduced in fuzzy topological spaces and intuitionistic topological spaces but not in neutrosophic topological spaces. By these motivation, we have introduce and discuss neutrosophic e-open sets in neutrosophic topological spaces.

The organization of this paper is as follows: Section 2 presents the fundamental definitions related to neutrosophic sets. In Section 3, we introduce the concept of neutrosophic e-open sets within neutrosophic topological spaces, along with a discussion of their essential properties and illustrative examples. This section also includes an examination of neutrosophic e-interior and e-closure operators. In Section 4, we demonstrate the application of the neutrosophic score function and neutrosophic negative score function to a mobile phone selection problem. The analysis is carried out using neutrosophic topological spaces, structured around specific attributes and alternatives, to compute score values and determine the optimal choice.

2. **Preliminaries**

In this section, we present the fundamental definitions and preliminary concepts of neutrosophic sets, which serve as the foundation for the developments in the subsequent sections.

Definition 2.1. [18] Let Z be a non-empty set. A neutrosophic set (in-short, $N_s s$) H is an object having the form $H = \{\langle q, \mu_H(q), \sigma_H(q), \nu_H(q)\rangle : q \in Z\}$ where $\mu_H,\ \sigma_H,\ \nu_H \to [0,1]$

denotes the degree of membership, indeterminacy and non-membership functions respectively of each element $q \in Z$ to the set H and $0 \leq \mu_H(q) + \sigma_H(q) + \nu_H(q) \leq 3$ for each $q \in Z$.

Definition 2.2. [18] Let Z be a non-empty set & the $N_s s$'s H & $H_\circ$ in the form $H = \{\langle q, \mu_H(q), \sigma_H(q), \nu_H(q)\rangle : q \in Z\}$, $H_\circ = \{\langle q, \mu_{H_\circ}(q), \sigma_{H_\circ}(q), \nu_{H_\circ}(q)\rangle : q \in Z\}$, then

(i) $0_N = \langle q, 0, 0, 1\rangle$ and $1_N = \langle q, 1, 1, 0\rangle$,

(ii) $H \subseteq H_\circ$ iff $\mu_H(q) \leq \mu_{H_\circ}(q)$, $\sigma_H(q) \leq \sigma_{H_\circ}(q)$ & $\nu_H(q) \geq \nu_{H_\circ}(q) : q \in Z$,

(iii) $H = H_\circ$ iff $H \subseteq H_\circ$ and $H_\circ \subseteq H$,

(iv) $1_N - H = \{\langle q, \nu_H(q), 1 - \sigma_H(q), \mu_H(q)\rangle : q \in Z\} = H^c$,

(v) $H \cup H_\circ = \{\langle q, \max(\mu_H(q), \mu_{H_\circ}(q)), \max(\sigma_H(q), \sigma_{H_\circ}(q)), \min(\nu_H(q), \nu_{H_\circ}(q))\rangle : q \in Z\}$,

(vi) $H \cap H_\circ = \{\langle q, \min(\mu_H(q), \mu_{H_\circ}(q)), \min(\sigma_H(q), \sigma_{H_\circ}(q)), \max(\nu_H(q), \nu_{H_\circ}(q))\rangle : q \in Z\}$.

Definition 2.3. [18] A neutrosophic topology (in-short, $N_s t$) on a non-empty set Z is a family Ψ_N of neutrosophic subsets of Z satisfying

(i) $0_N, 1_N \in \Psi_N$.

(ii) $H_\phi \cap H_\varphi \in \Psi_N$ for any $H_\phi, H_\varphi \in \Psi_N$.

(iii) $\bigcup H_\phi \in \Psi_N, \forall\ H_\phi : \phi \in Z \subseteq \Psi_N$.

Then (Z, Ψ_N) is called a neutrosophic topological space (in-short, $N_s ts$) in Z. The Ψ_N elements are called neutrosophic open sets (in-short, $N_s os$) in Z. A $N_s s$ G is called a neutrosophic closed sets (in-short, $N_s cs$) iff its complement G^c is $N_s os$.

Definition 2.4. [18,40] Let (Z, Ψ_N) be $N_s ts$ on Z and G be an $N_s s$ on Z, then the neutrosophic interior (resp. closure, δ interior & δ closure) of G (in-short, $N_s int(G)$ (resp. $N_s cl(G)$, $N_s \delta int(G)$ & $N_s \delta cl(G)$)) are defined as

$$N_s int(G) = \bigcup\{G_\circ : G_\circ \subseteq G \text{ \& } G_\circ \text{ is a } N_s os \text{ in } Z\},$$

$$N_s cl(G) = \bigcap\{G_\circ : G \subseteq G_\circ \text{ \& } G_\circ \text{ is a } N_s cs \text{ in } Z\},$$

$$N_s \delta int(G) = \bigcup\{B_\circ : B_\circ \subseteq G \text{ \& } B_\circ \text{ is a } N_s ros \text{ in } Z\} \text{ \&}$$

$$N_s \delta cl(G) = \bigcap\{A_\circ : G \subseteq A_\circ \text{ \& } A_\circ \text{ is a } N_s rcs \text{ in } Z\}.$$

Definition 2.5. [3] Let (Z, Ψ_N) be $N_s ts$ on Z and H be an $N_s s$ on Z. Then H is said to be a neutrosophic regular (resp. pre, semi, α & β) open set (in-short, $N_s ros$ (resp. $N_s \mathcal{P} os$, $N_s \mathcal{S} os$, $N_s \alpha os$ & $N_s \beta os$)) if $H = N_s int(N_s cl(H))$ (resp. $H \subseteq N_s int(N_s cl(H))$, $H \subseteq N_s cl(N_s int(H))$, $H \subseteq N_s int(N_s cl(N_s int(H)))$ & $H \subseteq N_s cl(N_s int(N_s cl(H)))$).

The complement of an $N_s \mathcal{P} os$ (resp. $N_s \mathcal{S} os$, $N_s \alpha os$, $N_s ros$ & $N_s \beta os$) is called a neutrosophic pre (resp. semi, α, regular & β) closed set (in-short, $N_s \mathcal{P} cs$ (resp. $N_s \mathcal{S} cs$, $N_s \alpha cs$, $N_s rcs$ & $N_s \beta cs$)) in Z.

Definition 2.6. [40] A set H is said to be a neutrosophic δ (resp. δ-pre, δ-semi & e^*) open set (in-short, $N_s\delta os$ (resp. $N_s\delta\mathcal{P}os$, $N_s\delta\mathcal{S}os$ & N_se^*os)) if $H = N_s\delta int(H)$ (resp. $H \subseteq N_sint(N_s\delta cl(H))$, $H \subseteq N_scl(N_s\delta int(H))$ & $H \subseteq N_scl(N_sint(N_s\delta cl(H)))$).

The complement of an $N_s\delta os$ (resp. $N_s\delta\mathcal{P}os$, $N_s\delta\mathcal{S}os$ & N_se^*os) is called a neutrosophic δ (resp. δ-pre, δ-semi & e^*) closed set (in-short, $N_s\delta cs$ (resp. $N_s\delta\mathcal{P}cs$, $N_s\delta\mathcal{S}cs$ & N_se^*cs)) in Z.

Definition 2.7. [23] Let $s : H \to [0, 1]$.

(i) The Neutrosophic Score Function (in short, N_sSF) is

$$s(\mu_H, \sigma_H, \nu_H) = \frac{2 + \mu_H - \sigma_H - \nu_H}{3}$$

(ii) The Neutrosophic Negative Score Function (in short, N_sNSF) is

$$s(\mu_H, \sigma_H, \nu_H) = \frac{1 - \mu_H + \sigma_H + \nu_H}{3}$$

3. Neutrosophic e-open sets in N_sts

In this section, we introduce the definition of a neutrosophic e-open set, explore its fundamental operations, and illustrate the concept through a representative example. Throughout this section, let (Z, Ψ_N) be any N_sts. Let G, $H_\circ$, $I_\circ$ and $K_\circ$ be a N_ss's in N_sts.

Definition 3.1. A set $H_\circ$ is said to be a neutrosophic

(i) e-open set (briefly, N_seos) if $H_\circ \subseteq N_scl(N_s\delta int(H_\circ)) \cup N_sint(N_s\delta cl(H_\circ))$.

(ii) e-closed set (briefly, N_secs) if $H_\circ \supseteq N_scl(N_s\delta int(H_\circ)) \cap N_sint(N_s\delta cl(H_\circ))$.

The family of all N_seos (resp. N_secs) of Z is denoted by $N_seos(Z)$ (resp. $N_secs(Z)$).

Definition 3.2. A set $H_\circ$ is said to be a neutrosophic e interior (resp. e closure) of $H_\circ$ (in-short, $N_seint(H_\circ)$ (resp. $N_secl(H_\circ)$)) are defined by $N_seint(H_\circ) = \bigcup\{P : P \subseteq H_\circ$ & P is a N_seos in $Z\}$ (resp. $N_secl(H_\circ) = \bigcap\{Q : H_\circ \subseteq Q$ & Q is a N_secs in $Z\}$).

Proposition 3.3. The statements hold; however, their converses do not necessarily hold. Every

(i) $N_s\delta os$ (resp. $N_s\delta cs$) is a N_sos (resp. N_scs).

(ii) N_sos (resp. N_scs) is a $N_s\delta\mathcal{S}os$ (resp. $N_s\delta\mathcal{S}cs$).

(iii) N_sos (resp. N_scs) is a $N_s\delta\mathcal{P}os$ (resp. $N_s\delta\mathcal{P}cs$).

(iv) $N_s\delta\mathcal{S}os$ (resp. $N_s\delta\mathcal{S}cs$) is a N_seos (resp. N_secs).

(v) $N_s\delta\mathcal{P}os$ (resp. $N_s\delta\mathcal{P}cs$) is a N_seos (resp. N_secs).

(vi) N_seos (resp. N_secs) is a N_se^*os (resp. N_se^*cs).

Proof. The proof of (i), (ii) & (iii) are studied in [40].

(iv) $H_\circ$ is a $N_s\delta\mathcal{S}os$, then $H_\circ \subseteq N_scl(N_s\delta int(H_\circ)) \subseteq N_scl(N_s\delta int(H_\circ)) \cup N_sint(N_s\delta cl(H_\circ))$. $\therefore H_\circ$ is a N_seos.

(v) $I_\circ$ is a $N_s\delta\mathcal{P}os$, then $I_\circ \subseteq N_sint(N_s\delta cl(I_\circ)) \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ))$. $\therefore$ $I_\circ$ is a N_seos.

(vi) $H_\circ$ is a N_seos then $H_\circ \subseteq N_scl(N_s\delta int(H_\circ)) \cup N_sint(N_s\delta cl(H_\circ))$. So $H_\circ \subseteq N_scl(N_s\delta int(H_\circ)) \cup N_sint(N_s\delta cl(H_\circ)) \subseteq N_scl(N_sint\ (N_s\delta cl(H_\circ)))$. $\therefore H_\circ$ is a N_se^*os.

Similar observations can be made for the corresponding closed sets. □

Example 3.4. Let $Z = \{a_1, a_2, a_3\}$ and define N_ss's Z_1, Z_2 & Z_3 in Z are

$$Z_1 = \langle Z, (\frac{\mu_{a_1}}{0.2}, \frac{\mu_{a_2}}{0.3}, \frac{\mu_{a_3}}{0.4}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.8}, \frac{\nu_{a_2}}{0.7}, \frac{\nu_{a_3}}{0.6})\rangle,$$
$$Z_2 = \langle Z, (\frac{\mu_{a_1}}{0.1}, \frac{\mu_{a_2}}{0.1}, \frac{\mu_{a_3}}{0.4}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.9}, \frac{\nu_{a_2}}{0.9}, \frac{\nu_{a_3}}{0.6})\rangle,$$
$$Z_3 = \langle Z, (\frac{\mu_{a_1}}{0.2}, \frac{\mu_{a_2}}{0.4}, \frac{\mu_{a_3}}{0.4}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.8}, \frac{\nu_{a_2}}{0.6}, \frac{\nu_{a_3}}{0.6})\rangle.$$

Then we have $\Psi_N = \{0_N, Z_1, Z_2, 1_N\}$, then Z_3 is a N_seos but not $N_s\delta\mathcal{P}os$.

Example 3.5. Let $Z = \{a_1, a_2, a_3\}$ and define N_ss's Z_1, Z_2, Z_3 & Z_4 in Z are

$$Z_1 = \langle Z, (\frac{\mu_{a_1}}{0.3}, \frac{\mu_{a_2}}{0.5}, \frac{\mu_{a_3}}{0.5}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.7}, \frac{\nu_{a_2}}{0.5}, \frac{\nu_{a_3}}{0.5})\rangle,$$
$$Z_2 = \langle Z, (\frac{\mu_{a_1}}{0.4}, \frac{\mu_{a_2}}{0.2}, \frac{\mu_{a_3}}{0.6}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.6}, \frac{\nu_{a_2}}{0.8}, \frac{\nu_{a_3}}{0.4})\rangle,$$
$$Z_3 = \langle Z, (\frac{\mu_{a_1}}{0.4}, \frac{\mu_{a_2}}{0.5}, \frac{\mu_{a_3}}{0.6}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.6}, \frac{\nu_{a_2}}{0.5}, \frac{\nu_{a_3}}{0.4})\rangle,$$
$$Z_4 = \langle Z, (\frac{\mu_{a_1}}{0.3}, \frac{\mu_{a_2}}{0.5}, \frac{\mu_{a_3}}{0.4}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}), (\frac{\nu_{a_1}}{0.7}, \frac{\nu_{a_2}}{0.5}, \frac{\nu_{a_3}}{0.6})\rangle.$$

Then we have $\Psi_N = \{0_N, Z_1, Z_2, Z_3, Z_1 \cap Z_2, 1_N\}$, then Z_4 is a N_seos but not $N_s\delta\mathcal{S}os$.

Example 3.6. Let $Z = \{a_1, a_2\}$ and define N_ss's Z_1 & Z_2 in Z are

$$Z_1 = \big\langle Z, (\frac{\mu_{a_1}}{0.3}, \frac{\mu_{a_2}}{0.2}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}), (\frac{\nu_{a_1}}{0.5}, \frac{\nu_{a_2}}{0.5})\big\rangle,$$
$$Z_2 = \big\langle Z, (\frac{\mu_{a_1}}{0.3}, \frac{\mu_{a_2}}{0.5}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}), (\frac{\nu_{a_1}}{0.7}, \frac{\nu_{a_2}}{0.6})\big\rangle.$$

Then we have $\Psi_N = \{0_N, Z_1, 1_N\}$, then Z_2 is a N_se^*os but not N_seos.

The other implications are shown in [40].

Theorem 3.7. The statements are true.

(i) $N_s\delta\mathcal{P}cl(G) \supseteq G \cup N_scl(N_s\delta int(G))$.

(ii) $N_s\delta\mathcal{P}int(G) \subseteq G \cap N_sint(N_s\delta cl(G))$.

(iii) $N_s\delta\mathcal{S}cl(G) \supseteq G \cup N_sint(N_s\delta cl(G))$.

(iv) $N_s\delta\mathcal{S}int(G) \subseteq G \cap N_scl(N_s\delta int(G))$.

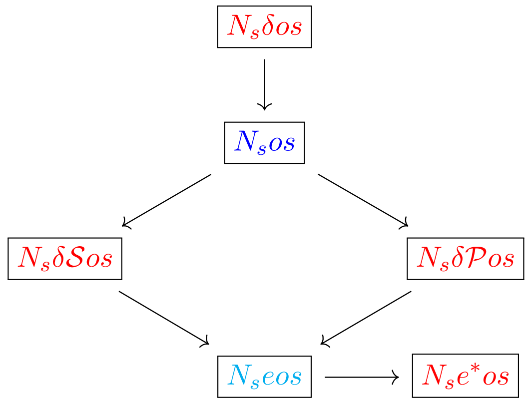

FIGURE 1. $N_s eos$'s in $N_s ts$.

Proof. (i) Since $N_s\delta\mathcal{P}cl(G)$ is $N_s\delta\mathcal{P}cs$, we have

$$N_s cl(N_s\delta int(G)) \subseteq N_s cl(N_s\delta int(N_s\delta\mathcal{P}cl(G))) \subseteq N_s\delta\mathcal{P}cl(G).$$

Thus $G \cup N_s cl(N_s\delta int(G)) \subseteq N_s\delta\mathcal{P}cl(G)$.

The remaining cases follow in a similar manner. □

Theorem 3.8. Let G is a $N_s eos$ iff $G = N_s\delta\mathcal{P}int(G) \cup N_s\delta\mathcal{S}int(G)$.

Proof. Let G is a $N_s eos$. Then $G \subseteq N_s cl(N_s\delta int(G)) \cup N_s int(N_s\delta cl(G))$. By Theorem 3.7, we have

$$\begin{aligned} N_s\delta\mathcal{P}int(G) \cup N_s\delta\mathcal{S}int(G) &= G \cap (N_s int(N_s\delta cl(G))) \cup (G \cap N_s cl(N_s\delta int(G))) \\ &= G \cap (N_s int(N_s\delta cl(G))) \cup N_s cl(N_s\delta int(G)) \\ &= G. \end{aligned}$$

Conversely, if $G = N_s\delta\mathcal{P}int(G) \cup N_s\delta\mathcal{S}int(G)$ then, by Theorem 3.7

$$\begin{aligned} G &= N_s\delta\mathcal{P}int(G) \cup N_s\delta\mathcal{S}int(G) \\ &= (G \cap N_s int(N_s\delta cl(G))) \cup (G \cap N_s cl(N_s\delta int(G))) \\ &= G \cap (N_s int(N_s\delta cl(G)) \cup N_s cl(N_s\delta int(G))) \\ &\subseteq N_s int(N_s\delta cl(G)) \cup N_s cl(N_s\delta int(G)) \end{aligned}$$

and hence G is a $N_s eos$. □

Theorem 3.9. The union (resp. intersection) of any family of $N_s eos(Z)$ (resp. $N_s ecs(Z)$) is a $N_s eos(Z)$ (resp. $N_s ecs(Z)$).

Proof. Let $\{K_\varphi : \varphi \in \Psi_N\}$ be a family of N_seos's. For each $\varphi \in \Psi_N$, $K_\varphi \subseteq N_scl(N_s\delta int(K_\varphi)) \cup N_sint(N_s\delta cl(K_\varphi))$.

$$\bigcup_{\varphi\in\Psi_N} K_\varphi \subseteq \bigcup_{\varphi\in\Psi_N} N_scl(N_s\delta int(K_\varphi)) \cup N_sint(N_s\delta cl(K_\varphi))$$
$$\subseteq N_scl(N_s\delta int(\cup K_\varphi)) \cup N_sint(N_s\delta cl(\cup K_\varphi))$$

The other case is similar. □

Remark 3.10. The intersection of two N_seos's need not be N_seos.

Example 3.11. Let $Z = \{a_1, a_2\}$ and define N_ss's Z_1, Z_2 & Z_3 in Z are

$$Z_1 = \langle Z, (\frac{\mu_{a_1}}{0.2}, \frac{\mu_{a_2}}{0.1}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}), (\frac{\nu_{a_1}}{0.7}, \frac{\nu_{a_2}}{0.5})\rangle,$$
$$Z_2 = \langle Z, (\frac{\mu_{a_1}}{0.3}, \frac{\mu_{a_2}}{0.5}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}), (\frac{\nu_{a_1}}{0.7}, \frac{\nu_{a_2}}{0.2})\rangle,$$
$$Z_3 = \langle Z, (\frac{\mu_{a_1}}{0.1}, \frac{\mu_{a_2}}{0.2}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}), (\frac{\nu_{a_1}}{0.1}, \frac{\nu_{a_2}}{0.1})\rangle.$$

Then we have $\Psi_N = \{0_N, Z_1, 1_N\}$, then Z_2 & Z_3 are N_seos but $Z_2 \cap Z_3$ is not N_seos.

Proposition 3.12. Let $I_\circ$ is a

(i) N_seos and $N_s\delta int(I_\circ) = 0_N$, then $I_\circ$ is a $N_s\delta\mathcal{P}os$.
(ii) N_seos and $N_s\delta cl(I_\circ) = 0_N$, then $I_\circ$ is a $N_s\delta\mathcal{S}os$.
(iii) N_seos and $N_s\delta cs$, then $I_\circ$ is a $N_s\delta\mathcal{S}os$.
(iv) $N_s\delta\mathcal{S}os$ and $N_s\delta cs$, then $I_\circ$ is a N_seos.

Proof. (i) Let $I_\circ$ be a N_seos, that is

$$I_\circ \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ)) = 0_N \cup N_sint(N_s\delta cl(I_\circ)) = N_sint(N_s\delta cl(I_\circ))$$

Hence $I_\circ$ is a $N_s\delta\mathcal{P}os$.

(ii) Let $I_\circ$ be a N_seos, that is

$$I_\circ \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ)) = N_scl(N_s\delta int(I_\circ)) \cup 0_N = N_scl(N_s\delta int(I_\circ))$$

Hence $I_\circ$ is a $N_s\delta\mathcal{S}os$.

(iii) Let $I_\circ$ be a N_seos and $N_s\delta cs$, that is

$$I_\circ \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ))$$
$$= N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ))$$
$$= N_scl(N_s\delta int(I_\circ)).$$

Hence $I_\circ$ is a $N_s\delta\mathcal{S}os$.

(iv) Let $I_\circ$ be a $N_s\delta\mathcal{S}os$ and $N_s\delta cs$, that is

$$I_\circ \subseteq N_scl(N_s\delta int(I_\circ)) \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ)).$$

Hence $I_\circ$ is a N_seos. □

Theorem 3.13. Let $I_\circ$ be a N_secs (resp. N_seos) iff $I_\circ = N_secl(I_\circ)$ (resp. $I_\circ = N_seint(I_\circ)$).

Proof. Suppose $I_\circ = N_secl(I_\circ) = \cap\{H_\circ : I_\circ \subseteq H_\circ$ & $H_\circ$ is a $N_secs\}$. This means $I_\circ \in \cap\{H_\circ : I_\circ \subseteq H_\circ$ & $H_\circ$ is a $N_secs\}$ and hence $I_\circ$ is N_secs.

Conversely, suppose $I_\circ$ be a N_secs in Z. Then, we have $I_\circ \in \cap\{H_\circ : I_\circ \subseteq H_\circ$ & $H_\circ$ is a $N_secs\}$. Hence, $I_\circ \subseteq H_\circ$ implies $I_\circ = \cap\{H_\circ : I_\circ \subseteq H_\circ$ & $H_\circ$ is a $N_secs\} = N_secl(I_\circ)$.

Similarly for $I_\circ = N_seint(I_\circ)$. □

Theorem 3.14. Let $I_\circ$ & $K_\circ$ in Z, then the N_secl sets have

(i) $N_secl(0_N) = 0_N$, $N_secl(1_N) = 1_N$.
(ii) $N_secl(I_\circ)$ is a N_secs in Z.
(iii) $N_secl(I_\circ) \subseteq N_secl(K_\circ)$ if $I_\circ \subseteq K_\circ$.
(iv) $N_secl(N_secl(I_\circ)) = N_secl(I_\circ)$.

Proof. The results derive straightforwardly from the definition of N_secl sets. □

Theorem 3.15. Let $I_\circ$ & $K_\circ$ in Z, then the N_seint sets have

(i) $N_seint(0_N) = 0_N$, $N_seint(1_N) = 1_N$.
(ii) $N_seint(I_\circ)$ is a N_secs in Z.
(iii) $N_seint(I_\circ) \subseteq N_seint(K_\circ)$ if $I_\circ \subseteq K_\circ$.
(iv) $N_seint(N_seint(I_\circ)) = N_seint(I_\circ)$.

Proof. The results derive straightforwardly from the definition of N_seint sets. □

Proposition 3.16. Let $I_\circ$, $H_\circ$ & $L_\circ$ are in Z, then

(i) $N_secl(\overline{I_\circ}) = \overline{N_seint(I_\circ)}$, $N_seint(\overline{I_\circ}) = \overline{N_secl(I_\circ)}$.
(ii) $N_secl(I_\circ \cup H_\circ) \supseteq N_secl(I_\circ) \cup N_secl(H_\circ)$, $N_secl(I_\circ \cap H_\circ) \subseteq N_secl(I_\circ) \cap N_secl(H_\circ)$.
(iii) $N_seint(L_\circ \cup I_\circ) \supseteq N_seint(L_\circ) \cup N_seint(I_\circ)$, $N_seint(L_\circ \cap I_\circ) \subseteq N_seint(L_\circ) \cap N_seint(I_\circ)$.

Proof. (i) The proof is directly from definition.

(ii) $I_\circ \subseteq I_\circ \cup H_\circ$ or $H_\circ \subseteq I_\circ \cup H_\circ$. Hence $N_secl(I_\circ) \subseteq N_secl(I_\circ \cup H_\circ)$ or $N_secl(H_\circ) \subseteq N_secl(I_\circ \cup H_\circ)$. Therefore, $N_secl(I_\circ \cup H_\circ) \supseteq N_secl(I_\circ) \cup N_secl(H_\circ)$. The other one is similar.

(iii) $L_\circ \subseteq L_\circ \cup I_\circ$ or $I_\circ \subseteq L_\circ \cup I_\circ$. Hence $N_seint(L_\circ) \subseteq N_seint(L_\circ \cup I_\circ)$ or $N_seint(I_\circ) \subseteq N_seint(L_\circ \cup I_\circ)$. Therefore, $N_seint(L_\circ \cup I_\circ) \supseteq N_seint(L_\circ) \cup N_seint(I_\circ)$. The other one is similar. □

Remark 3.17. The equality stated in part (ii) of Proposition 3.16 does not hold in the provided example.

Example 3.18. Let $Z = \{a_1, a_2, a_3, a_4\}$ and define N_ss's Z_1, Z_2, Z_3 & Z_4 in Z are

$$Z_1 = \langle Z, (\frac{\mu_{a_1}}{1}, \frac{\mu_{a_2}}{0}, \frac{\mu_{a_3}}{0.2}, \frac{\mu_{a_4}}{0}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}, \frac{\sigma_{a_4}}{0.5}), (\frac{\nu_{a_1}}{0}, \frac{\nu_{a_2}}{1}, \frac{\nu_{a_3}}{0.7}, \frac{\nu_{a_4}}{1})\rangle,$$
$$Z_2 = \langle Z, (\frac{\mu_{a_1}}{0}, \frac{\mu_{a_2}}{1}, \frac{\mu_{a_3}}{0}, \frac{\mu_{a_4}}{0}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}, \frac{\sigma_{a_4}}{0.5}), (\frac{\nu_{a_1}}{1}, \frac{\nu_{a_2}}{0}, \frac{\nu_{a_3}}{1}, \frac{\nu_{a_4}}{0.1})\rangle,$$
$$Z_3 = \langle Z, (\frac{\mu_{a_1}}{1}, \frac{\mu_{a_2}}{0}, \frac{\mu_{a_3}}{0}, \frac{\mu_{a_4}}{1}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}, \frac{\sigma_{a_4}}{0.5}), (\frac{\nu_{a_1}}{0}, \frac{\nu_{a_2}}{0.2}, \frac{\nu_{a_3}}{0}, \frac{\nu_{a_4}}{0})\rangle,$$
$$Z_4 = \langle Z, (\frac{\mu_{a_1}}{0}, \frac{\mu_{a_2}}{0.9}, \frac{\mu_{a_3}}{0.3}, \frac{\mu_{a_4}}{1}), (\frac{\sigma_{a_1}}{0.5}, \frac{\sigma_{a_2}}{0.5}, \frac{\sigma_{a_3}}{0.5}, \frac{\sigma_{a_4}}{0.5}), (\frac{\nu_{a_1}}{1}, \frac{\nu_{a_2}}{0}, \frac{\nu_{a_3}}{0.2}, \frac{\nu_{a_4}}{0})\rangle.$$

Then we have $\Psi_N = \{0_N, Z_1, Z_2, Z_1 \cap Z_2, 1_N\}$ is a N_sts in Z, then $N_secl(Z_3 \cup Z_4) \neq N_secl(Z_3) \cup N_secl(Z_4)$.

Proposition 3.19. If $I_\circ$ is in Z, then

(i) $N_secl(I_\circ) \supseteq N_scl(N_s\delta int(I_\circ)) \cap N_sint(N_s\delta cl(I_\circ))$.
(ii) $N_seint(I_\circ) \subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ))$.

Proof. (i) $N_secl(I_\circ)$ is a N_secs and $I_\circ \subseteq N_secl(I_\circ)$, then

$$\begin{aligned} N_secl(I_\circ) &\supseteq N_scl(N_s\delta int(N_secl(I_\circ))) \cap N_sint(N_s\delta cl(N_secl(I_\circ))) \\ &\supseteq N_scl(N_s\delta int(I_\circ)) \cap N_sint(N_s\delta cl(I_\circ)). \end{aligned}$$

(ii) $N_seint(I_\circ)$ is a N_seos and $I_\circ \supseteq N_seint(I_\circ)$, then

$$\begin{aligned} N_seint(I_\circ) &\subseteq N_scl(N_s\delta int(N_seint(I_\circ))) \cup N_sint(N_s\delta cl(N_seint(I_\circ))) \\ &\subseteq N_scl(N_s\delta int(I_\circ)) \cup N_sint(N_s\delta cl(I_\circ)). \end{aligned}$$

□

Theorem 3.20. Let $H_\circ$ is in Z, then

(i) $N_secl(H_\circ) = N_s\delta\mathcal{P}cl(H_\circ) \cap N_s\delta\mathcal{S}cl(H_\circ)$.
(ii) $N_seint(H_\circ) = N_s\delta\mathcal{P}int(H_\circ) \cap N_s\delta\mathcal{S}int(H_\circ)$.

Proof. (i) It follows directly that, $N_s ecl(H_\circ) \subseteq N_s\delta\mathcal{P}cl(H_\circ) \cap N_s\delta\mathcal{S}cl(H_\circ)$. Conversely, from Definition 3.2, we have

$$\begin{aligned} N_s ecl(H_\circ) &\supseteq N_s cl(N_s\delta int(N_s ecl(H_\circ))) \cap N_s int(N_s\delta cl(N_s ecl(H_\circ))) \\ &\supseteq N_s cl(N_s\delta int(H_\circ)) \cap N_s int(N_s\delta cl(H_\circ)). \end{aligned}$$

Since $N_s ecl(H_\circ)$ is $N_s ecs$, by Theorem 3.7, we have

$$\begin{aligned} N_s\delta\mathcal{P}cl(H_\circ) \cap N_s\delta\mathcal{S}cl(H_\circ) &= (H_\circ \cup N_s cl(N_s\delta int(H_\circ))) \cap (H_\circ \cup N_s int(N_s\delta cl(H_\circ))) \\ &= H_\circ \cup (N_s cl(N_s\delta int(H_\circ)) \cap N_s int(N_s\delta cl(H_\circ))) \\ &= H_\circ \subseteq N_s ecl(H_\circ). \end{aligned}$$

Therefore, $N_s ecl(H_\circ) = N_s\delta\mathcal{P}cl(H_\circ) \cap N_s\delta\mathcal{S}cl(H_\circ)$.

(ii) A similar argument applies as in part (i). □

Theorem 3.21. Let $H_\circ$ is in Z, then

(i) $N_s ecl(1 - H_\circ) = 1 - N_s eint(H_\circ)$.

(ii) $N_s eint(1 - H_\circ) = 1 - N_s ecl(H_\circ)$.

Proof. (i) Let U be $N_s ecs$ in Z and $H_\circ$ be any $N_s s$ in Z. Then $N_s eint(H_\circ) = \cup\{1-U : 1-U \subseteq H_\circ, 1-U$ is a $N_s eos$ in $Z\} = 1 - \cap\{U : U \supseteq 1 - H_\circ, U$ is a $N_s ecs$ in $Z\} = 1 - N_s ecl(H_\circ)$. Thus, $N_s ecl(1 - H_\circ) = 1 - N_s eint(H_\circ)$.

(ii) Let $L_\circ$ be $N_s eos$ in Z and $H_\circ$ be any $N_s s$ in Z. Then $N_s ecl(H_\circ) = \cap\{1 - L_\circ : 1 - L_\circ \supseteq H_\circ, 1 - L_\circ$ is a $N_s ecs$ in $Z\} = 1 - \cup\{L_\circ : L_\circ \subseteq 1 - H_\circ, L_\circ$ is a $N_s eos$ in $Z\} = 1 - N_s eint(H_\circ)$. Thus, $N_s eint(1 - H_\circ) = 1 - N_s ecl(H_\circ)$. □

4. Neutrosophic Function On (μ_M, σ_M, ν_M) and Numerical Example

The definition of neutrosophic score function and neutrosophic negative score function and the algorithm to get the optimum decision-making are discussed in [31]. The usage of mobile phones has significantly increased, becoming an integral part of daily life. With advancements in technology, modern mobile network services operate on cellular network architectures. Contemporary smartphones are equipped with diverse features such as MMS, text messaging, short-range wireless communication (e.g., infrared, Bluetooth), email, internet access, video games, business applications, and digital photography. In this section, we present a practical example illustrating mobile phone selection based on a person's needs, evaluated using the neutrosophic score function and neutrosophic negative score function. This analysis demonstrates the effectiveness and applicability of these functions within the neutrosophic framework.

Step 1: Problem field selection:

Consider the following decision-making scenario involving three individuals, referred to as Person 1, Person 2, and Person 3, each representing an alternative (Per) based on their specific mobile phone requirements. The specifications (Spec) considered for evaluation include Performance, Battery Life, and Camera. Additionally, three mobile phone models Poco X1, Poco X2, and Poco X3 are taken as attributes (Mob), defined in terms of the aforementioned specifications. The objective is to identify the most suitable mobile phone for each person by evaluating the alternatives and attributes based on their neutrosophic score values. The data presented in Table 2 and Table 3 are expressed in terms of the membership, indeterminacy, and non-membership functions for the persons and the mobiles, respectively.

Spec \ Per	Person 1	Person 2	Person 3
Performance	(000.71,000.52,000.29)	(000.64,000.48,000.36)	(000.85,000.49,000.15)
Battery Life	(000.75,000.50,000.25)	(000.71,000.50,000.29)	(000.87,000.52,000.13)
Camera	(000.83,000.45,000.17)	(000.76,000.46,000.24)	(000.86,000.47,000.14)

Table 1. Neutrosophic values for Person 1, Person 2 and Person 3

Mob \ Spec	Performance	Battery Life	Camera
Poco X1	(000.72,000.49,000.28)	(000.76,000.51,000.24)	(000.78,000.48,000.22)
Poco X2	(000.65,000.50,000.35)	(000.70,000.48,000.30)	(000.75,000.47,000.25)
Poco X3	(000.73,000.46,000.27)	(000.76,000.52,000.24)	(000.82,000.48,000.18)

Table 2. Neutrosophic values for mobiles with Performance, Battery Life and Camera

Step 2: Form neutrosophic topologies for (τ_j) and (ν_k):

(i) $\tau_1^* = L_1 \cup L_2 \cup L_3$, where $L_1 = \{(0,0,1),(1,1,0),(0.71,0.52,0.29),(0.75,0.50,0.25),(0.83,0.45,0.17)\}$, $L_2 = \{(0.75,0.52,0.25),(0.83,0.52,0.17),(0.83,0.50,0.17)\}$ and $N = \{(0.71,0.50,0.29),(0.71,0.45,0.29),(0.75,0.45,0.25)\}$.

(ii) $\tau_2^* = L_1 \cup L_2 \cup L_3$, where $L_1 = \{(0,0,1),(1,1,0),(0.64,0.48,0.36),(0.71,0.50,0.29),(0.76,0.46,0.24)\}$, $L_2 = \{(0.76,0.48,0.24),(0.76,0.50,0.24)\}$ and $N = \{(0.64,0.46,0.36),(0.71,0.46,0.29)\}$.

(iii) $\tau_3^* = L_1 \cup L_2 \cup L_3$, where $L_1 = \{(0,0,1),(1,1,0),(0.85,0.49,0.15),(0.87,0.52,0.13),(0.86,0.47,0.14)\}$, $L_2 = \{(0.86,0.49,0.14)\}$ and $N = \{(0.85,0.47,0.15)\}$.

(i)

$\nu_1^* = M_1 \cup M_2 \cup M_3$, where $M_1 = \{(0,0,1),(1,1,0),(0.72,0.49,0.28),(0.76,0.51,0.24),(0.78,$

$0.48, 0.22)\}$, $M_2 = \{(0.78, 0.49, 0.22), (0.78, 0.51, 0.22)\}$ and $M_3 = \{(0.72, 0.48, 0.28), (0.76, 0.48, 0.24)\}$.

(ii) $\nu_2^* = M_1 \cup M_2 \cup M_3$, where $M_1 = \{(0, 0, 1), (1, 1, 0), (0.65, 0.50, 0.35), (0.70, 0.48, 0.30), (0.75, 0.47, 0.25)\}$, $M_2 = \{(0.70, 0.50, 0.30), (0.75, 0.50, 0.25), (0.75, 0.48, 0.25)\}$ and $M_3 = \{(0.65, 0.48, 0.35), (0.65, 0.47, 0.35), (0.70, 0.47, 0.30)\}$.

(iii) $\nu_3^* = M_1 \cup M_2 \cup M_3$, where $M_1 = \{(0, 0, 1), (1, 1, 0), (0.73, 0.46, 0.27), (0.76, 0.52, 0.24), (0.82, 0.48, 0.18)\}$, $M_2 = \{(0.82, 0.52, 0.18)\}$ and $M_3 = \{(0.76, 0.48, 0.24)\}$.

Step 3: Find Neutrosophic Score Values:

1. Neutrosophic score functions:

(i) $N_sSF(L_1) = 0.60733$, $N_sSF(L_2) = 0.69717$ and $N_sSF(L_3) = 0.66$. $N_sSF(\tau_1) = 0.65503$.

(ii) $N_sSF(L_1) = 0.58533$, $N_sSF(L_2) = 0.67666$ and $N_sSF(L_3) = 0.63$. $N_sSF(\tau_2) = 0.63066$.

(iii) $N_sSF(L_1) = 0.64533$, $N_sSF(L_2) = 0.74333$ and $N_sSF(L_3) = 0.74333$. $N_sSF(\tau_3) = 0.71066$.

(i) $N_sSF(M_1) = 0.60266$, $N_sSF(M_2) = 0.68666$ and $N_sSF(M_3) = 0.66666$. $N_sSF(\nu_1) = 0.65199$.

(ii) $N_sSF(M_1) = 0.58333$, $N_sSF(M_2) = 0.65777$ and $N_sSF(M_3) = 0.62$. $N_sSF(\nu_2) = 0.62036$.

(iii) $N_sSF(M_1) = 0.61066$, $N_sSF(M_2) = 0.70666$ and $N_sSF(M_3) = 0.68$. $N_sSF(\nu_3) = 0.66577$.

2. Neutrosophic negative score functions:

(i) $N_sNSF(L_1) = 0.39267$, $N_sNSF(L_2) = 0.30283$ and $N_sNSF(L_3) = 0.34$. $N_sNSF(\tau_1) = 0.34497$.

(ii) $N_sNSF(L_1) = 0.41467$, $N_sNSF(L_2) = 0.32334$ and $N_sNSF(L_3) = 0.37$. $N_sNSF(\tau_2) = 0.36934$.

(iii) $N_sNSF(L_1) = 0.35467$, $N_sNSF(L_2) = 0.25667$ and $N_sNSF(L_3) = 0.25667$. $N_sNSF(\tau_3) = 0.28934$.

(i) $N_sNSF(M_1) = 0.39734$, $N_sNSF(M_2) = 0.31334$ and $N_sNSF(M_3) = 0.33334$. $N_sNSF(\nu_1) = 0.34801$.

(ii) $N_sNSF(M_1) = 0.41667$, $N_sNSF(M_2) = 0.34223$ and $N_sNSF(M_3) = 0.38$. $N_sNSF(\nu_2) = 0.37964$.

(iii) $N_sNSF(M_1) = 0.38934$, $N_sNSF(M_2) = 0.29334$ and $N_sNSF(M_3) = 0.32$. $N_sNSF(\nu_3) = 0.33423$.

Step 4: Final Decision:

1. By arranging the neutrosophic score values in ascending order, we obtain the following sequences $\tau_2 \leq \tau_1 \leq \tau_3$ and $\nu_2 \leq \nu_1 \leq \nu_3$. Based on these rankings, it follows that Person 2 is best suited for Poco X3, Person 1 for Poco X1, and Person 3 for Poco X2.

2. By arranging the neutrosophic negative score values in ascending order, we get the following sequences $\tau_3 \leq \tau_1 \leq \tau_2$ and $\nu_3 \leq \nu_1 \leq \nu_2$. Based on these rankings, it follows that Person 3 is best suited for Poco X2, Person 1 for Poco X1, and Person 2 for Poco X3.

5. Conclusions

In this paper, we studied the notions of neutrosophic e-open sets and neutrosophic e-closed sets within the framework of neutrosophic topological spaces. We also discussed their corresponding interior and closure operators, and examined several fundamental properties along with illustrative examples. Furthermore, a comparative analysis was conducted between neutrosophic e-open sets and near open sets in N_sts. An application of neutrosophic score functions and neutrosophic negative score functions was demonstrated through a mobile phone selection problem, evaluated within a neutrosophic topological space based on attributes and alternatives. For future work, this study can be extended by incorporating the neutrosophic accuracy function and neutrosophic certainty function into real-world applications, providing deeper insights and decision-making support in more complex scenarios.

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