



A Comparative Review of the Fuzzy, Intuitionistic and Neutrosophic Numbers in Solving Uncertainty Matrix Equations

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Abstract. Matrix equations play a fundamental role in scientific and engineering applications, including linear systems, optimization, and computational modeling. Common types of matrix equations, such as Sylvester, Lyapunov, and Riccati equations, are widely used in these fields. However, classical approaches are less effective to handle uncertainty in real-world problems. To address this, fuzzy set theory, intuitionistic fuzzy set theory, and neutrosophic set theory offer distinct mathematical frameworks for incorporating uncertainty into matrix equations. This paper provides a comparative review of these three approaches in solving matrix equations with uncertain coefficients. It explores their theoretical foundations, including key definitions, theorems, and arithmetic operations. The strengths and advantages of each theory are highlighted, particularly in terms of handling different degrees of uncertainty. Additionally, an analysis of previous studies on the application of these theories to uncertainty matrix equations is presented, identifying their limitations and areas for improvement. The review emphasizes the potential of neutrosophic set theory, which extends fuzziness and intuitionism, offering a more flexible and comprehensive approach. Finally, recommendations are provided to enhance solution quality by refining existing methodologies and leveraging the strengths of neutrosophic sets for better uncertainty modeling in matrix equations.

Keywords: Fuzzy Numbers; Intuitionistic Numbers; Matrix Equations; Neutrosophic Fuzzy Numbers

1. Introduction

A matrix equation is an equation in which matrices represent the mathematical relationships between variables and constants. These equations are widely used to model and solve problems in various fields, such as physics, engineering, computer science, and control systems. Matrix equations often involve operations like matrix addition, multiplication, or inversion [1].

Additionally, they provide a compact and efficient way to handle systems of linear equations, transformations, and other mathematical computations involving arrays of numbers [2].

There are a few types of matrix equations. The most common equations are the linear matrix equation of $AX = B$ and Sylvester matrix equation of $AX + XB = C$. Sylvester matrix equations is written in the form of $AX + XB = C$ where involved the findings the solution matrix X for given matrix A , B and C [3]. Sylvester matrix equations are used in many fields such as control systems, particularly in observer design and model reduction. It can be solved classically using iterative approaches based on Krylov subspaces and the Hessenberg-Schur algorithm.

Furthermore, ranging from linear to nonlinear form such as Lyapunov matrix equation of $A^T X + XA = B$ where A is a square matrix and B is a given positive definite matrix [4]. This equation is important in stability analysis of control systems, for example in verification of a linear time-invariant system. Direct method such as Bartels-Stewart algorithm and iterative method such as Krylov subspace approach are often being employed to solve this equation [6].

There is another type of matrix equation that also important in control system which is Ricatti matrix equation of $AX + XA^T - XBX = C$ [5]. Besides that this equation also plays its role in signal processing, optimization and other computational applications.

However, many practical problems involve uncertainties, vagueness and incomplete informations, making it challenging to obtain precise solutions using classical numerical methods. To address these challenges in 1965, Zadeh et al. [8] introduced fuzzy sets to handle the complexity of problems that cannot be solved using classical technique.

Fuzzy sets theory allows a gradual evaluation to the membership of elements in the set described in the interval $[0,1]$. Wide range of domains are used where the information is incomplete and inaccurate [8]. There are many types of fuzzy numbers were applied including triangular and trapezoidal fuzzy numbers. Zadeh et al. [9] introduced fuzzy sets in 1965 followed by his subsequent work of interval-valued fuzzy sets (IVFS). Then, Jun et al. proposed a concept of combination of fuzzy sets and IVFS that contribute to a more robust and extensible framework for dealing with uncertainty and imprecision. The later work is then expanded into the concept of cubic sets [10], soft sets [11] and cubic soft sets [12]. Basically, fuzzy numbers give a mechanism to communicate probabilities and partial truths instead than merely true or false, which helps to deal with the chaotic and uncertain real life world.

Subsequently, in 1986, Atanassov et al. [13] expended the concept of vagueness and uncertainty with fuzzy set theory to intuitionistic fuzzy sets. In the philosophy of mathematics, intuitionism is a philosophical position that highlights the significance of intuition in comprehending mathematical ideas. Intuitionistic fuzzy sets is a generalization of fuzzy sets where a separate degree of non-membership is introduced. By adding intuitionistic fuzzy sets, which

offer both membership and non-membership values, it overcomes the drawbacks of conventional fuzzy matrices. This is especially helpful for capturing ambiguity and uncertainty in situations involving decision-making.

While intuitionistic fuzzy numbers address some shortcomings of fuzzy frameworks, they do not fully account for indeterminate or contradictory information that frequently arises in complex systems. Therefore, neutrosophic fuzzy numbers was firstly introduced by Smaradache in 1995 is the extension of fuzzy sets and intuitionistic sets that dealing with incomplete, inconsistent and indeterminate information in real life application [14]. The elements of neutrosophic sets are Truth (T), Indeterminate (I) and Falsity (F) membership function. The application in the areas where uncertain indeterminate or contradictor information needs to be presented and analyzed such as decision making, image processing and pattern recognition. The approach of neutrosophic sets could represents the real-life uncertainties where incomplete and inconsistent information are accomodated. The neutrosophic numbers are same like fuzzy numbers where there are triangular neutrosophic numbers, trapezoidal neutrosophic numbers and other types of neutrosophic numbers that had be found by researcher. Figure 1 shows the generalization from fuzzy set into neutrosophic set.

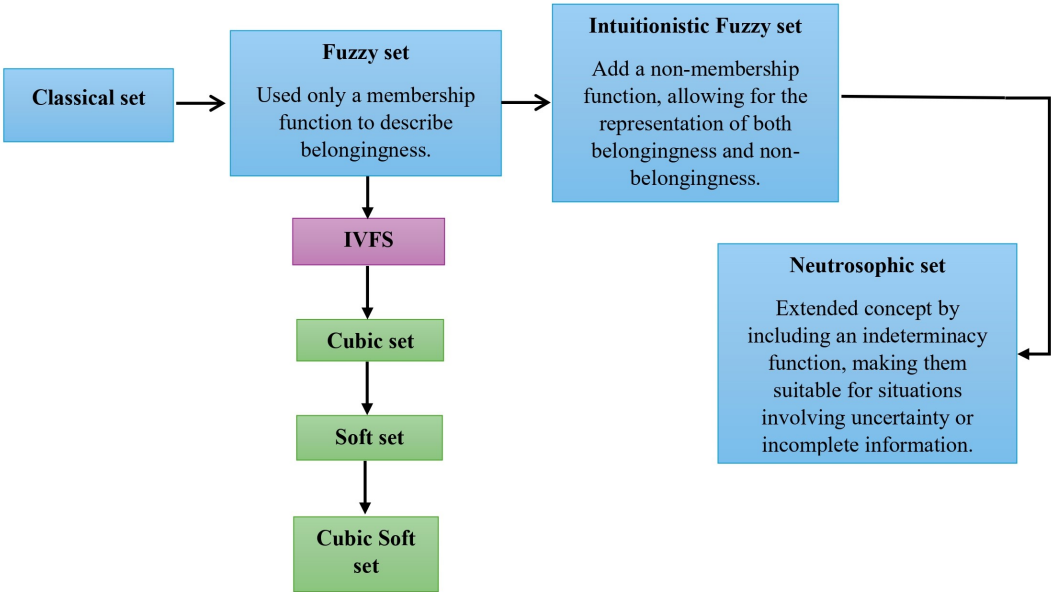


FIGURE 1. Flowchart of the Generalization from Fuzzy set into Neutrosophic set

This study reviews the applications of fuzzy, intuitionistic and neutrosophic numbers in solving uncertainty matrix equations, focusing on their fundamental theories, including definitions and arithmetic operations. In addition to discussing relevant previous studies, this

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review highlights their methodologies, identifies limitations, and compares different theoretical approaches. Since this is a review paper, its main contribution lies in synthesizing and analyzing existing research rather than presenting new findings. By identifying gaps in the literature, evaluating the strengths and weaknesses of current methods, and expanding on the challenges in this field, this paper aims to provide a comprehensive understanding of how these mathematical frameworks have been applied to matrix equations under uncertainty. Towards the end, the potential of neutrosophic numbers is explored, as they offer a more flexible and comprehensive approach. Finally, recommendations are provided to refine existing methodologies and leverage the advantages of neutrosophic sets for better uncertainty modeling, paving the way for future research directions.

This paper is organized as follows. Section 2 contains some preliminaries that discussing the foundational concepts of fuzzy, intuitionistic fuzzy numbers and neutrosophic numbers with their mathematical properties. Subsequently, section 3 examine the role in various types of matrix equations, highlighting the advantages and limitations. Finally the conclusion is drawn in section 4.

2. Preliminaries

This section provides the basic concept of fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets. The general comparison between these three sets are presented at the end of the section.

2.1. Basic Concept of of Fuzzy Numbers

Basically, fuzzy numbers is used to describe the vagueness and uncertainty conditions and information which can be defined as follows.

Definition 2.1. [8] The fuzzy set A in X is characterized by its membership function where X is let to be a nonempty set.

$$\mu_A : X \longrightarrow [0, 1]$$

where $\mu_A(x)$ is the degree of membership of the element x in fuzzy set A for each $x \in X$.

There are many arithmetic operations of fuzzy numbers used in literature. The most common arithmetic operations applied in the L-R fuzzy numbers is given.

Definition 2.2. [15] Let the L-R fuzzy numbers $A = (m, \alpha, \beta)$ and $B = (n, \gamma, \delta)$ which the arithmetic operations are:

$$A \oplus B = (m + n, \alpha + \gamma, \beta + \delta)$$

$$A \ominus B = (m - n, \alpha - \delta, \beta - \gamma)$$

$$A \otimes B = (mn, m\gamma + n\alpha, m\delta + n\beta)$$

Triangular and trapezoidal fuzzy numbers are among the most widely used types of fuzzy numbers due to their simplicity and computational efficiency. They are described in Definition 2.3 and Definition 2.4.

Definition 2.3. [16] Let $M = (l, m, u)$, $I = [0, 1]$, where $0 \leq l \leq m \leq u \leq 1$. M is the triangular fuzzy number (TFN) on I which can be written as follows:

$$\mu_M(x) = \begin{cases} \frac{x-l}{m-l}, & l \leq x \leq m \\ \frac{x-m}{u-m}, & m \leq x \leq u \\ 0, & \text{else,} \end{cases}$$

and graphically represented in Figure 2 below:

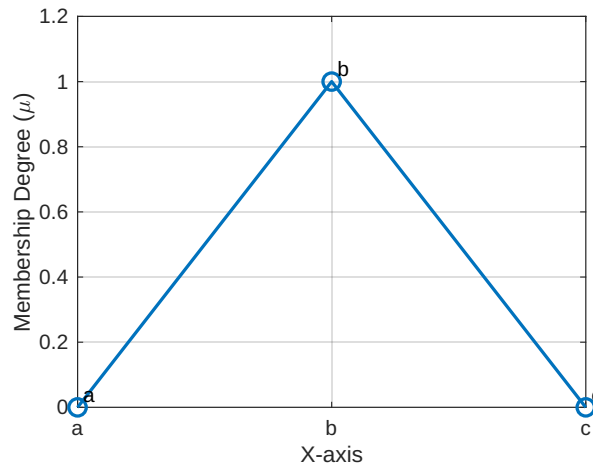


FIGURE 2. Triangular Fuzzy Number

Definition 2.4. [17] Let $M = (l, m, u, v)$, $I = [0, 1]$, where $0 \leq l \leq m \leq u \leq v \leq 1$. M is the trapezoidal fuzzy number (TraFN) on I which can be written as follows:

$$\mu_M(x) = \begin{cases} 0, & x \leq u \\ \frac{x-u}{l-u}, & u \leq x \leq l \\ \frac{v-x}{u-m}, & m \leq x \leq v \\ 0, & \text{else,} \end{cases}$$

which can be represented graphically in the following Figure 3.

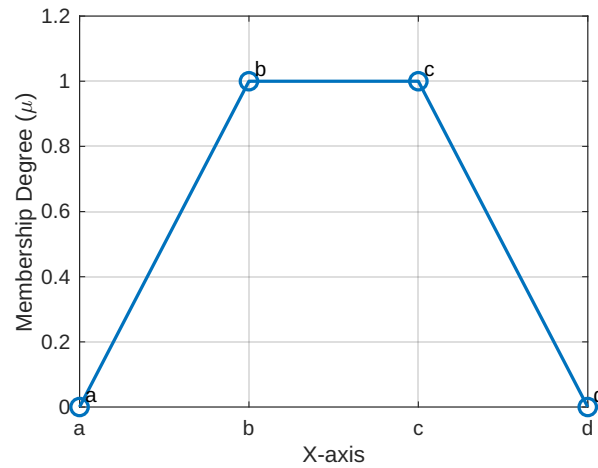


FIGURE 3. Trapezoidal Fuzzy Number

2.2. Basic Concept of Intuitionistic Numbers

Intuitionistic fuzzy sets is a generalization of fuzzy sets to handle situations where available information is insufficient for defining membership and non-membership degrees precisely. The definition is outlined below.

Definition 2.5. [13] An intuitionistic fuzzy set (IFS) A in a universal set X is defined as

$$A = \{x, \mu_A(x), v_A(x) | x \in X\}$$

where $\mu_A(x) : X \rightarrow [0, 1]$ and $v_A(x) : X \rightarrow [0, 1]$ are the membership and non-membership values of x in A that satisfied the condition of $0 \leq \mu_A(x) + v_A(x) \leq 1$.

The general membership of intuitionistic fuzzy numbers is given as follows.

Definition 2.6. [19] Let a be an intuitionistic fuzzy numbers in the set of real numbers, the membership function is defined as below:

$$t_a(x) = \begin{cases} f_a^L(x), & a \leq x < b \\ t_a, & b \leq x \leq c \\ f_a^R(x), & c < x \leq d \\ 0, & \text{otherwise,} \end{cases}$$

and non-membership function is defined as below:

$$f_a(x) = \begin{cases} g_a^L(x), & a_1 \leq x < b \\ t_a, & b \leq x \leq c \\ f_a^R(x), & c < x \leq d_1 \\ 0, & \text{otherwise.} \end{cases}$$

Apart from that, there is also triangular intuitionistic fuzzy numbers which defined as

Definition 2.7. [20] Let $I = (a_1, a_2, a_3; \omega_I, \mu_I)$ be a triangular intuitionistic fuzzy numbers (TrIFN). The membership function is defined as below:

$$\mu_I(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \mu_I, & a_1 \leq x \leq a_2 \\ \omega_I, & x = a_2 \\ \frac{a_3-x}{a_3-a_2} \mu_I, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise,} \end{cases}$$

and non-membership function is defined as:

$$v_I(x) = \begin{cases} \frac{a_2-x+v_a(x-a_1)}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ v_I, & x = a_2 \\ \frac{x-a_2+v_a(a_3-x)}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 1, & \text{otherwise.} \end{cases}$$

The graphical representation of TrIFN is illustrated in Figure 4 below:

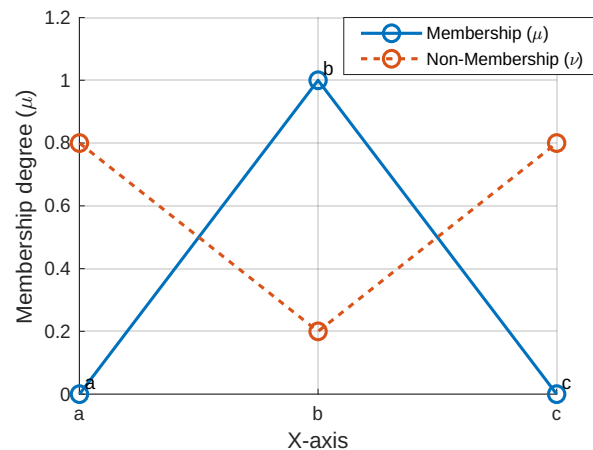


FIGURE 4. Triangular Intuitionistic Number

The arithmetic operations for TrIFN are outlined as in Definition 2.8.

Definition 2.8. [21] Let $X_1 = \langle (a_1, b_1, c_1), (l_1, m_1, n_1) \rangle$ and $X_2 = \langle (a_2, b_2, c_2), (l_2, m_2, n_2) \rangle$ be two triangular intuitionistic fuzzy numbers and $\lambda \leq 0$. The operations involved are as follow:

$$X_1 + X_2 = \langle (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2), (l_1 l_2, m_1 m_2, n_1 n_2) \rangle$$

$$X_1 \times X_2 = \langle (a_1 a_2, b_1 b_2, c_1 c_2), (l_1 + l_2 - l_1 l_2, m_1 + m_2 - m_1 m_2, n_1 + n_2 - n_1 n_2) \rangle$$

$$\lambda X_1 = \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); 1 - (1 - \mu_I)^\lambda, v_I^\lambda \rangle$$

$$X_1^\lambda = \langle (a_1^\lambda, a_2^\lambda, a_3^\lambda, a_4^\lambda); \mu_I^\lambda, 1 - (1 - v_I)^\lambda \rangle$$

Furthermore, for trapezoidal intuitionistic fuzzy numbers (TIFN) with its graphical representation and arithmetic operations are presented in Definition 2.9 and Definition 2.10.

Definition 2.9. [19] Let I be a TIFN. The membership function is defined as below:

$$\mu_I(x) = \begin{cases} \frac{x-a_1}{a_2-a_3} \mu_I, & a_1 \leq x \leq a_2 \\ \mu_I, & a_1 \leq x \leq a_2 \\ \frac{a_4-x}{a_3-a_4} \mu_I, & a_3 \leq x \leq a_4 \\ 1, & \text{otherwise,} \end{cases}$$

and non-membership function is defined as below:

$$v_I(x) = \begin{cases} \frac{b_2-x+v_a(x-b_1)}{b_2-b_3}, & b_1 \leq x \leq b_2 \\ v_I, & b_2 \leq x \leq b_3 \\ \frac{x-b_3+v_a(b_4-x)}{b_4-b_3}, & b_3 \leq x \leq b_4 \\ 1, & \text{otherwise,} \end{cases}$$

where the graph of TIFN is illustrated in the Figure 5 below:

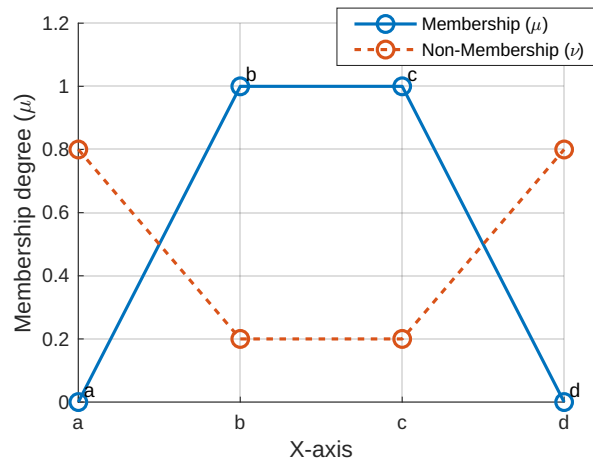


FIGURE 5. Trapezoidal Intuitionistic Number

Definition 2.10. [22] Let $X_1 = \langle (a_1, b_1, c_1, d_1), (p_1, q_1, r_1, s_1) \rangle$ and

$X_2 = \langle (a_2, b_2, c_2, d_2), (p_2, q_2, r_2, s_2) \rangle$ be two trapezoidal intuitionistic fuzzy numbers. The operations are as follows:

$$X_1 + X_2 = \langle (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2), (p_1 p_2, q_1 q_2, r_1 r_2, s_1 s_2) \rangle$$

$$X_1 \times X_2 = \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2), (p_1 + p_2 - p_1 p_2, q_1 + q_2 - q_1 q_2, r_1 + r_2 - r_1 r_2, s_1 + s_2 - s_1 s_2) \rangle$$

$$\lambda X_1 = \langle (1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda, 1 - (1 - c_1)^\lambda, 1 - (1 - d_1)^\lambda), (p_1^\lambda, q_1^\lambda, r_1^\lambda, s_1^\lambda) \rangle, \lambda > 0$$

$$a_1^\lambda = \langle (a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda), (1 - (1 - p_1)^\lambda, 1 - (1 - q_1)^\lambda, 1 - (1 - r_1)^\lambda, 1 - (1 - s_1)^\lambda) \rangle, \lambda \geq 0$$

2.3. Basic Concept of Neutrosophic Numbers

Neutrosophic numbers is an extension of fuzzy numbers and intuitionistic fuzzy numbers that are designed to handle indeterminacy more effectively by incorporating three components. The definition is presented as follows.

Definition 2.11. [23] Let X be a universal set and let $x \in X$. The neutrosophic set A in X is characterized by a Truth (T), Indeterminacy (I) and Falsity (F) membership functions, then neutrosophic numbers is given as:

$$A = \{(x, T(x), I(x), F(x)) : x \in X\}.$$

There is a restriction on the sum of T, I, F which must satisfy $0^+ \leq T + I + F \leq 3^+$. In terms of philosophy, the neutrosophic set is derived from actual standard or non-standard subsets of $]0^-, 1^+[$. However, for technical applications the interval $[0, 1]$ is more appropriate rather than $]0^-, 1^+[$.

Neutrosophic numbers is normally known as single-valued neutrosophic numbers which its membership functions are described as follows.

Definition 2.12. [24] Single-valued neutrosophic numbers, N is defined as $N = \langle [(a_1, a_2, a_3, a_4); T_n], [(b_1, b_2, b_3, b_4); I_n], [(c_1, c_2, c_3, c_4); F_n] \rangle$, then the general T, I and F membership function are outlined as follows:

$$T(x) = \begin{cases} f_n(x), & a_1 \leq x < a_2 \\ T_n, & a_2 \leq x \leq a_3 \\ g_n(x), & a_3 < x \leq a_4 \\ 0, & \text{otherwise,} \end{cases}$$

$$I(x) = \begin{cases} h_n(x), & b_1 \leq x < b_2 \\ I_n, & b_2 \leq x \leq b_3 \\ k_n(x), & b_3 < x \leq b_4 \\ 1, & \text{otherwise,} \end{cases}$$

$$F(x) = \begin{cases} p_n(x), & c_1 \leq x < c_2 \\ F_n, & c_2 \leq x \leq c_3 \\ q_n(x), & c_3 < x \leq c_4 \\ 1, & \text{otherwise.} \end{cases}$$

Neutrosophic numbers also consist of triangular neutrosophic numbers (TriNNs) and trapezoidal neutrosophic numbers (TraNNs). The definitions for both TriNNs and TraNNs are stated in Definition 2.13 and Definition 2.15 respectively.

Definition 2.13. [26] Let $N = \langle (a_1, b_1, c_1); T_N, I_N, F_N \rangle$ is single-valued triangular neutrosophic numbers which consist of truth, indeterminacy and falsity membership function defined as:

$$T_N(x) = \begin{cases} \frac{x-a_1}{b_1-a_1} T_N, & a_1 \leq x < b_1 \\ T_N, & x = b_1 \\ \frac{c_1-x}{c_1-b_1} T_N, & b_1 < x \leq c_1 \\ 0, & \text{otherwise,} \end{cases},$$

$$I_N(x) = \begin{cases} \frac{b_1-x+I_N(x-a_1)}{b_1-a_1}, & a_1 \leq x < b_1 \\ I_N, & x = b_1 \\ \frac{x-b_1+I_N(c_1-x)}{c_1-b_1}, & b_1 < x \leq c_1 \\ 1, & \text{otherwise,} \end{cases},$$

$$F_N(x) = \begin{cases} \frac{b_1-x+F_N(x-a_1)}{b_1-a_1}, & a_1 \leq x < b_1 \\ F_N, & x = b_1 \\ \frac{x-b_1+F_N(c_1-x)}{c_1-b_1}, & b_1 < x \leq c_1 \\ 1, & \text{otherwise.} \end{cases}$$

which graphically represented in the Figure 6 below:

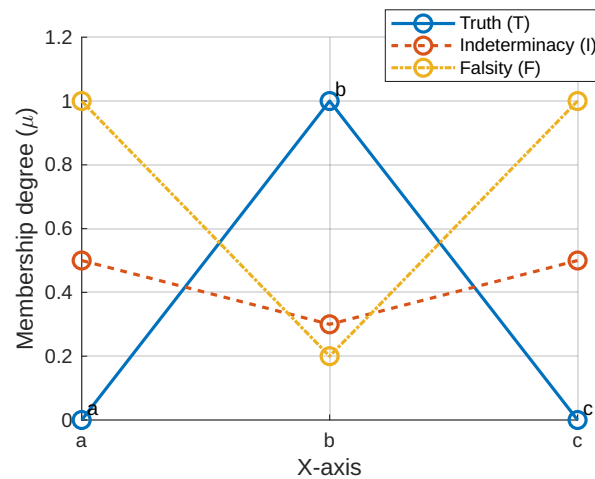


FIGURE 6. Triangular Neutrosophic Number

Next, Definition 2.14 presents the arithmetic operations for TriNNS.

Definition 2.14. [26] Let $X_1 = \langle (a_1, b_1, c_1); T_{X_1}, I_{X_1}, F_{X_1} \rangle$ and

$X_2 = \langle (a_2, b_2, c_2); T_{X_2}, I_{X_2}, F_{X_2} \rangle$ be TriNNs and $\lambda \geq 0$. The operations are as below:

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$$X_1 + X_2 = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2; T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2) \rangle$$

$$X_1 \times X_2 = \langle (a_1 a_2, b_1 b_2, c_1 c_2; T_1 + T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2) \rangle$$

$$\lambda X_1 = \langle (a_1^\lambda, b_1^\lambda, c_1^\lambda); 1 - (1 - T_{x_1})^\lambda, I_{x_1}^\lambda, F_{x_1}^\lambda \rangle$$

$$X_1^\lambda = \langle (a_1^\lambda, b_1^\lambda, c_1^\lambda); 1 - (1 - I_{X_1})^\lambda, 1 - (1 - F_{X_1})^\lambda \rangle$$

Definition 2.15. [25] If N is let to be single-valued trapezoidal neutrosophic numbers (TraNNs), then the truth, indeterminacy and falsity membership function are given as:

$$T_N(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2 \\ T_N, & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_2-a_3}, & a_3 < x \leq a_4 \\ 0, & \text{otherwise,} \end{cases}$$

$$I_N(x) = \begin{cases} \frac{x-b_1}{b_2-b_1}, & b_1 \leq x < b_2 \\ I_N, & b_2 \leq x \leq b_3 \\ \frac{b_4-x}{b_2-b_3}, & b_3 < x \leq b_4 \\ 1, & \text{otherwise,} \end{cases}$$

$$F_N(x) = \begin{cases} \frac{x-c_1}{c_2-c_1}, & c_1 \leq x < c_2 \\ F_N, & c_2 \leq x \leq c_3 \\ \frac{c_4-x}{c_2-c_3}, & c_3 < x \leq c_4 \\ 1, & \text{otherwise.} \end{cases}$$

The true, indeterminacy and falsity membership functions are visually depicted in Figure 7 below:

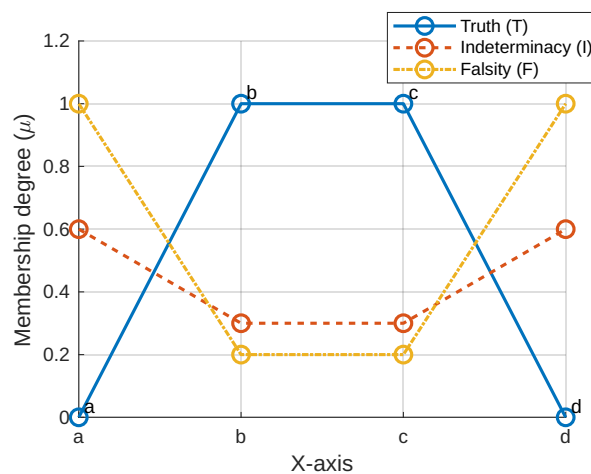


FIGURE 7. Trapezoidal Neutrosophic Number

Definition 2.16 subsequently presents the arithmetic operations for TraNNS.

Definition 2.16. [24] Let $X_1 = \langle (a_1, b_1, c_1, d_1); T_{X_1}, I_{X_1}, F_{X_1} \rangle$ and $X_2 = \langle (a_2, b_2, c_2, d_2); T_{X_2}, I_{X_2}, F_{X_2} \rangle$ be TraNNS and $\lambda \geq 0$. The operations are as below:

$$X_1 + X_2 = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2) \rangle$$

$$X_1 \times X_2 = \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2; T_1 + T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2) \rangle$$

$$\lambda X_1 = \langle (a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda); 1 - (1 - T_{x_1})^\lambda, I_{x_1}^\lambda, F_{x_1}^\lambda \rangle$$

$$X_1^\lambda = \langle (a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda); 1 - (1 - I_{X_1})^\lambda, 1 - (1 - F_{X_1})^\lambda \rangle$$

To summarize the key characteristics and differences between fuzzy numbers, intuitionistic fuzzy numbers, and neutrosophic numbers, the following Table 1 is provided.

TABLE 1. Summary of the Differences between Fuzzy, Intuitionistic and Neutrosophic Numbers

Fuzzy	Intuitionistic	Neutrosophic
Introduced by L.A Zadeh in 1965.	Introduced by Atanassov in 1986.	Introduced by Florentin Smarandache in 1995.
Used to represent uncertainty or vagueness in numerical values.	Extend the concept of fuzzy numbers to handle the hesitation or uncertainty in decision making.	Generalization of fuzzy and intuitionistic fuzzy numbers allowing for the representation of indeterminacy, uncertainty and contradiction simultaneously.
Characterized by a membership function which assigns a degree of membership to each element of the universe of discourse.	Have a non-membership function and a hesitation function: addition to membership function.	Characterized into truth-membership function, the indeterminacy-membership function and falsity-membership function.
The condition: $0 \leq \mu_A(x) \leq 1$	The condition: $0 \leq \mu_A(x) + v_A(x) \leq 1$	The condition: $0 \leq T(x) + I(x) + F(x) \leq 3$
Membership function: Values from 0 to 1, indicating the degree of an element to the fuzzy numbers.	Non-membership function: The degree to which an element does not belong to the fuzzy numbers. Hesitation function: Measures The degree of uncertainty in assigning membership to an element.	Truth-membership function: Degree of truth. Indeterminacy-membership function: The degree of indeterminacy. Falsity-membership function: The degree of falsity.

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Additionally, a graph that combines the membership of the three approaches is illustrated as in Figure 8.

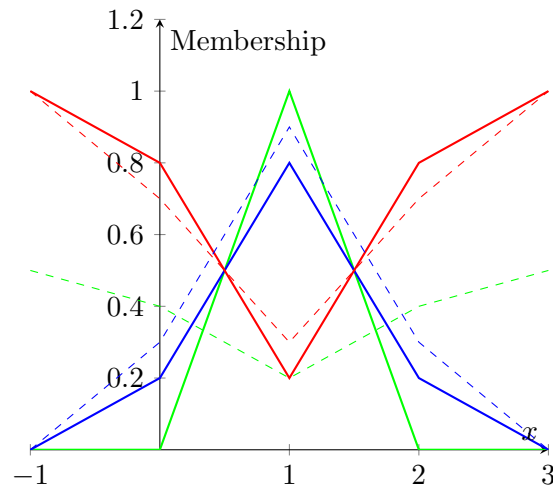


FIGURE 8. Combination of Fuzzy, Intuitionistic Fuzzy, and Neutrosophic membership functions.

The graph illustrates the differences between fuzzy, intuitionistic fuzzy, and neutrosophic membership functions, highlighting how each approach handles uncertainty. The y-axis represents the membership degree, where 1 indicates full membership, and 0 indicates no membership.

From the graph, the green line represents the fuzzy membership function, which assigns a single degree of membership to each element. The intuitionistic fuzzy membership function extends this by introducing a separate degree of non-membership, represented by the blue line for membership and the red line for non-membership. This approach allows for more flexibility in expressing uncertainty compared to traditional fuzzy logic.

In contrast, the neutrosophic membership function further generalizes the concept by incorporating an additional component for indeterminacy. The three components of neutrosophic numbers are represented by blue dotted lines for truth membership, green dotted lines for indeterminate membership, and red dotted lines for falsity membership. This provides a more comprehensive representation of uncertainty, making neutrosophic numbers particularly useful in complex decision-making scenarios.

3. Previous studies on the Solution of Uncertainty Matrix Equation and Its Limitation

In this section, the solution of matrix equations with uncertainty condition that have frequently been addressed in the literature are discussed. The solution are based on Fuzzy numbers, Intuitionistic Fuzzy numbers and Neutrosophic numbers.

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3.1. *Previous studies on the Fuzzy Numbers*

The most common matrix equation which is $AX = B$, has been extended to fuzzy domains where matrices A and B incorporate uncertainty through fuzzy numbers. Researchers have developed methods for solving semi-fuzzy matrix equations [27] and fully fuzzy systems using triangular or trapezoidal fuzzy numbers [15]. Techniques such as the Kronecker product and pseudoinverse methods have been employed to transform these fuzzy equations into solvable linear systems. These advancements have enhanced the applicability of matrix equations in uncertain environments, particularly in control systems and optimization. Although a lot of current methods concentrate on theoretical developments, they still have little practical application in engineering, control systems, and decision-making. Few research validate the efficacy of fuzzy matrix equation solutions with real-world case studies.

The study by Singh et al. [28] demonstrated the consideration of Hukuhara and generalized Hukuhara differences of fuzzy numbers for solving systems of linear equations under fuzzy-ruled uncertainty within an analytical framework. In particular, the criteria for existence and uniqueness were analyzed before solving fuzzy systems. The generalized Hukuhara difference is particularly influential in fuzzy arithmetic and calculus because, under this definition, the fuzzy difference always exists, regardless of the scenario. Moreover, numerous real-world managerial problems, which involve multiple unknowns and constraints within uncertain strategic decision-making processes, are suitable applications for the proposed theory. Additionally, this study could be extended in the future by developing numerical methods for solving arbitrary polynomial and transcendental equations within fuzzy systems using Hukuhara and generalized Hukuhara differences. Nevertheless, the difference between two fuzzy numbers is not a fuzzy number in many instances, which causes issues of non-existence. The computational complexity increases even with the generalized Hukuhara difference that make the method challenging for large-scaled applications.

In a related development, Elsayed et al. [29] constructed analytical and numerical methods for solving a couple of trapezoidal fully fuzzy Sylvester matrix equations (CTrFFSME). This approach aimed to address the limitations of a couple of crisp Sylvester matrix equations, which are less equipped to handle uncertainty. Furthermore, this method can also be applied to other fuzzy systems, such as the Lyapunov matrix equation. However, it is worth nothing that the analytical method is limited to systems of small size, which restricts their use in large-scale problems. Moreover, the efficient numerical techniques for managing larger systems in uncertain environments are still lacking.

The study by Sulaiman et al. [30] presented an algorithm called Shamanskii for solving nonlinear systems. While many studies have been conducted to address this problem, most numerical methods are computationally expensive due to the need for computing and storing

the Jacobian or approximate Jacobian matrix at each iteration, which is both challenging and time-consuming. To mitigate this computational cost, the Shamanskii algorithm begins with a Newton step during the first iteration. For subsequent iterations, Chord steps are computed instead. This approach significantly reduces the computational burden associated with evaluating the Jacobian matrix at every step. As a result, the proposed method is highly competitive compared to classical Newton and Chord methods, achieving a superlinear convergence rate. However, despite addressing this issue, the limitation is the computational cost in nonlinear systems. The method is still computationally demanding even if this eliminates the requirement of recalculating the Jacobian at each iteration. The majority of current numerical techniques for fuzzy nonlinear systems are still costly and demand a large amount of processing and storage capacity.

The study by Behrooz et al. [31] proposed a numerical method to solve algebraic fuzzy complex equations of degree n , where the unknown variables and the right-hand side are fuzzy complex numbers and the coefficients are real crisp numbers. By separating the real and imaginary parts and the parametric representation of fuzzy numbers is used, the algebraic fuzzy complex equation can be converted into a system of nonlinear equations. A parametric form as polynomials of degree n is represented by fuzzy numbers which allow the equation to be expressed in term of polynomial coefficients. The real and imaginary parts of the equation are written as separate systems of equations by using binomial expansion and the assumptions about the signs of the fuzzy components. Then, the resulting non-linear system is solved by Gauss-Newton iterative method where this method is required to solve the Jacobian matrix. However, the fuzzy set framework, limited to a single membership value, often falls short in scenarios requiring a nuanced representation of uncertainty. This approach struggles with the limitations of algebraic Fuzzy complex equations. The additional computing loads result from the conversion of fuzzy complex equations into nonlinear systems. The method's wide applicability may be limited by the assumptions made regarding the signs of fuzzy components during binomial expansion.

3.2. *Previous studies on the Intuitionistic Numbers*

To address this limitation, intuitionistic fuzzy sets extend fuzzy sets by incorporating both membership and non-membership values, thereby offering greater flexibility and precision. This advancement enables a better modeling of uncertainty and ambiguity in real-world applications. For instance, intuitionistic fuzzy matrix equations have been utilized for multi-criteria decision-making problems, where the dual representation of membership and non-membership allows for more comprehensive analyses. Building on this foundation, techniques like intuitionistic fuzzy super matrices and iterative approaches such as Gauss-Seidel and Successive

Over-Relaxation (SOR) have been developed, demonstrating superior performance in applications like electrical circuit analysis and optimization [34]. These advancements illustrate the natural progression from the simplicity of fuzzy sets to the enhanced capabilities of intuitionistic fuzzy sets, paving the way for more complex and accurate analytical tools.

The study of intuitionistic matrix equation by Shah et al. [32] presents a novel framework for multi-criteria decision making (MCDM) using intuitionistic fuzzy super matrices. For the purpose of present a novel class of matrices that can handle both belongingness and non-belongingness criteria in decision-making scenarios with numerous characteristics, intuitionistic fuzzy sets and fuzzy set theory are used. The constraints of conventional fuzzy matrices are intended to be addressed by this expansion, especially in complicated systems that call for evaluations of several criteria. The entries in super matrix is not only scalar values but can be in the form of matrices that allow a more complex representation among variables. The algebraic operations for intuitionistic fuzzy super matrix are defined and different types of intuitionistic fuzzy super matrices are categorized. Apart from that, the practical usefulness of intuitionistic fuzzy methods is limited by a lack of comparative studies evaluating their benefits and drawbacks in comparison to traditional techniques. In this case, a large number of works concentrate on creating intuitionistic fuzzy matrix models without evaluating how well they perform in comparison to more conventional fuzzy techniques.

On the other hand, a general MCDM algorithm based on intuitionistic fuzzy super matrix (IFSUPM) theory are proposed, demonstrating its practicality through a real-world decision-making example. This example showcases the enhanced ability of these matrices to address complex decision-making needs where both membership and hesitation are important [32]. In summary, IFSUPM offers a strong foundation for dealing with MCDM issues, improving the capacity for analyzing complicated situations involving imprecise data. The ideas for new lines of inquiry to investigate the use of IFSUPM in diverse domains are offered. IFSUPM for MCDM are presented in the study from Shah et al. [32] overcome the scalability challenges in IFSUPM. Although the method outperforms traditional fuzzy matrices, handling matrix entries in the form of matrices greatly increases processing complexity. For large-scale decision-making tasks, operations like matrix multiplication and inversion become computationally costly and unworkable, leading to significant scalability issues.

Apart from that, the paper by Saw et al. [34] address an advanced problem in numerical mathematics by relying on iterative techniques for solving intuitionistic fuzzy systems of linear equations including Gauss-Seidal and SOR. The mathematical fundamentals of both approaches are explained in detail, along with demonstrations of convergence under specific circumstances. By reformulating the equations into clear linear systems, the techniques are established to adhere to intuitionistic fuzzy systems. A voltage problem in an electrical circuit

is examined in order to illustrate the effectiveness of the approaches. The result showed that SOR performed better than Gauss-Seidal in terms of converge speed in this situation. On the other hand, while these two approaches improve one aspect, it introduces difficulties in ensuring convergence because they rely on the starting assumptions. This, in turn, leads to further complications where it may diverge or converge slowly in optimization issues, particularly when high-dimensional data is involved.

3.3. *Previous studies on the Neutrosophic Numbers*

Building on the foundations of both fuzzy and intuitionistic fuzzy sets, neutrosophic sets further refine the representation of uncertainty. This tri-part framework provides a more robust approach for handling real-world scenarios that involve contradictory and indeterminate data.

Neutrosophic matrix equations have proven effective in solving eigenvalue problems by expressing uncertainty. The stability and performance can be analyzed in various systems. Abobala et al. [36] presented significant advances in the algebraic theory of neutrosophic matrices. Invertibility, diagonalization, determinant definitions, and eigenvalue computation for neutrosophic matrices are covered in the work and are demonstrated by theorems and examples. The study from this paper shows the practical uses of neutrosophic matrices in areas like graph theory, linear systems, and optimization problems. Despite the paper's thorough discussion of theoretical advancements, it lacks the absence of specific real-world applications in areas like machine learning and decision-making. It is presently unclear how neutrosophic matrices can be incorporated into real-world issues. In addition, the computational complexity of operations, such as determinant calculation, diagonalization, and eigenvalue or eigenvector determination for neutrosophic matrices, is not explored in detail. Developing efficient algorithms for these operations would be valuable. Indeed, investigate real-world applications of neutrosophic matrices in domains characterized by significant uncertainty, such as weather forecasting, medical diagnostics, and economic analysis along with conduct comparative studies to evaluate the effectiveness of neutrosophic matrices against fuzzy and intuitionistic fuzzy matrices, highlighting their advantages and limitations across diverse contexts would be helped for future research directions. Nevertheless, the computation of determinants, diagonalization, and eigenvector determination is still costly. The practical use of neutrosophic matrices is limited by the absence of effective methods for large-scale issues.

On the other hand, in optimization scenarios where objective functions or constraints are uncertain, neutrosophic numbers can be used to define the optimization problem. Neutrosophic matrix techniques can help find solutions that are optimal under varying degrees of uncertainty. To provide further insight, the study by Jdid et al. [37] highlights how operations research, particularly mathematical programming, plays a crucial role in addressing

recurring challenges in transportation systems and optimizing their efficiency. Neutrosophic logic extends traditional models by incorporating indeterminate parameters $(\epsilon, \lambda, \delta)$ into the transportation problem's cost, supply, and demand values and consequently allows the problem to account for real-world uncertainty in production and consumption centers. As the gaps gathered from this study, there is no comparative study with the fuzzy and intuitionistic fuzzy transportation models, which would have clarified the benefits and drawbacks of the neutrosophic method. Additionally the study does not investigate the alternative representations that could offer more detailed models, such as trapezoidal or pentagonal neutrosophic numbers. Therefore, it would be valuable for future research to explore this area in greater depth in employing trapezoidal, pentagonal or others in this field.

Furthermore, Abobala et al. [38] also has carried out about the algebraic study of neutrosophic matrices by utilizing the refined neutrosophic and n-refined neutrosophic matrices. The neutrosophic used is in the form of $M = A + Bi$ where A and B are real or complex numbers and i is the indeterminacy. A few examples are provided to give better understanding on the operations of algebraic neutrosophic matrices. Refined neutrosophic matrices could allow more flexibility by separating truth, indeterminacy and falsity more granularly but the studies in this topic is limited. Therefore, further studies are necessary to figure out how improved neutrosophic matrices could enhance decision-making models and solution accuracy for particular application areas. Nevertheless, neutrosophic matrices continue to be extremely sensitive to uncertain values, which causes instability in practical applications such as economic forecasting and traffic flow modeling.

In more details, the study of neutrosophic matrix equation by using technique such as multi-valued neutrosophic numbers (MVNMs) and decision-making tools like simplified-TOPSIS has been carried out by Mertina et al. [35]. Basically, MVNMs is an extension of conventional matrices that includes membership values for truth, indeterminacy, and falsity in multi-valued systems. This allows better modeling of uncertainty. The TOPSIS method has been utilized to improve the decision-making while working with neutrosophic data by classifying options according to how near the optimal answer. This approach is useful when making decisions based on many MCDM. Nevertheless there are still a few of gaps. First, although the theoretical foundation and illustrative examples are well stated, practical application of the proposed method is limited by the lack of comprehensive real-world verification or case studies. Second, the study does not highlight the advantages and drawbacks of the neutrosophic approach by offering a comparison with other MCDM approaches that are currently used in fuzzy or intuitionistics numbers. Subsequently, the computational complexity of MVNMs is not fully examined, especially when dealing with large-scale issues, which raises questions regarding

scalability. Lastly, the findings are primarily domain-specific, and more study is required to generalize the results to a variety of sectors and fields.

Last but not least, Gbolagade et al. [39] explores a novel application of the Neutrosophic Poisson Distribution Series (NPDS) to analyze uncertainties within the classical univalent harmonic function class. The new features, such as inclusion relations and coefficient inequalities for star-likeness, are produced by applying the NPDS with an analytic univalent harmonic function class and equipping it with a Salagean derivative operator. The study demonstrates the effectiveness of this approach in capturing the complexities associated with harmonic functions. The Python is used to create graphs to visualize these complexities. This integration aims to enhance analytical techniques and open avenues for future research in neutrosophic series and geometric function theory. In summary, the gap is highlighted in applying neutrosophic concepts, specifically the NPDS, to the study of harmonic functions. However, despite the use of Salagean derivative operator in complex analysis, it is not typically applied in matrix equation. Even so, extending this operator to matrix equation could provide new mathematical tools for solving systems of equations and also help in dealing with nonlinearities in matrix equation by offering a framework that can handle complex or non-standard differential forms could be considered.

In summary, while fuzzy, intuitionistic fuzzy, and neutrosophic approaches have progressively enhanced the modeling of uncertainty, each framework introduces its own set of challenges, particularly in terms of computational complexity, real-world applicability, and scalability.

Next, the following Table 2 is presented to critically summarizes the previous studies and identifies limitations in current approaches in solving uncertain matrix equations.

TABLE 2. The Summary of the Studies and Limitation on Solving Uncertain Matrix Equation

Types of set	Author	Methodology	Limitation
Fuzzy	Daud et al. [15]	Solving semi-fuzzy matrix equation and fully fuzzy systems using triangular or trapezoidal numbers.	Concentrate on theoretical developments only.
	Singh et al. [28]	The consideration of Hukuhara and generalized Hukuhara differences of fuzzy numbers for solving systems of linear equations.	The increasing of computational complexity.
	Elsayed et al. [29]	Analytical and numerical methods for solving the CTrFFSME.	The method limited to small-sized systems.
	Sulaiman et al. [30]	Solving nonlinear system using Shamanskii algorithm.	The numerical technique costly and demand a large amount of processing and storage capacity.
	Behrooz et al. [31]	Numerical solution of algebraic fuzzy complex equations using Gauss-Newton iterative method.	Limited to a single membership value.
Intuitionistic	Shah et al. [32]	A novel framework for MCDM using intuitionistic fuzzy super matrices.	Lack of comparative research.
	Saw et al. [34]	Solving IFLS using Gauss-Seidal and SOR method.	Difficulties in ensuring convergence.
Neutrosophic	Abobala et al. [36]	Cover the algebraic theory of neutrosophic matrices.	The absence of real world applications and computation of determinants, diagonalization and eigenvector determination costly.
	Jdid et al. [38]	Find solutions that are optimal under varying degrees of uncertainty.	Lack of studies on detailed models such as pentagonal and trapezoidal neutrosophic numbers.
	Abobala et al. [37]	Utilized the refined neutrosophic and n-refined neutrosophic matrices on algebraic studies.	Sensitive to indeterminate values.
	Mertina et al. [35]	Solving neutrosophic matrix equation using MVNMs and simplified-TOPSIS.	The advantages and drawbacks of the neutrosophic approach does not being highlighted.

4. Conclusions

As the conclusion, this study has reviewed the important role of fuzzy, intuitionistic, and neutrosophic numbers in solving matrix equations under uncertainty. By exploring different forms of these numbers, such as triangular and trapezoidal representations, the improvement of computational mathematics has been shown and make fuzzy systems more flexible in handling uncertain matrix equation. The transition from traditional fuzzy numbers to neutrosophic numbers represents a key step forward in handling more complex and uncertain problems.

Despite these advancements, there are still notable gaps that need to be addressed. Research on higher-order fuzzy, intuitionistic, and neutrosophic numbers especially pentagonal and octagonal forms remains limited. Their potential application in crucial matrix equations, such as Lyapunov and Riccati equations, is yet to be fully explored. Future work in this area may lead to significant advancements in solving matrix equations, particularly in fields like control systems, optimization, and artificial intelligence.

Additionally, while many theoretical developments have been proposed, computational efficiency remains a challenge. Developing more practical numerical methods is essential to ensure that these approaches can be effectively implemented in large-scale problems. In particular, adapting the neutrosophic numbers with machine learning techniques or numerical optimization methods could provide a better tool for solving uncertainty matrix equation.

A comparative evaluation with other uncertainty methods, such as probability theory and rough sets, could also help further establish the strengths and limitations of neutrosophic numbers in different contexts. By addressing these challenges and expanding the scope of applications, future research can bridge the gap between theoretical advancements and real-world implementations. It is expected that this review may provide a foundation for researchers to explore neutrosophic theories further and apply them to any application.

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