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Identification of aspects and barriers to implementing branding strategies in female entrepreneurship in the city of Guayaquil based on Plithogenic Neutrosophic Vague Statistics

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Abstract. Branding is one of the marketing strategies that a business must follow to contribute to its prosperity. It is not just about giving a name with a logo; it is about spreading a seal of identity that makes customers identify with the company's products. If creating a successful branding strategy in the market is difficult for any entrepreneur, this is even more difficult for women entrepreneurs. As is known in business, in the labor market, and in everyday life, women face more significant challenges than men. Added to this is the fact that Ecuador is a developing country where there are considerable financial and economic limitations when it comes to boosting a business. This paper aims to identify the aspects and barriers to consider when creating an e-learning platform to help women entrepreneurs in the city of Guayaquil launch their brands. To this end, a survey is designed to identify these aspects. To process the data statistically, we prefer to use Plithogenic Statistics, with data in the form of Neutrosophic Vague Sets. On the one hand, the Plithogenic Statistics allows us to gather probabilities for the various aspects to be considered, including economic, political, social, and other factors. Furthermore, Neutrosophic Vague data allows us to capture greater accuracy in responses and address uncertainty.

Keywords: Branding, Marketing, entrepreneurship feminine, Plithogenic Statistics, Neutrosophic Vague Sets, Plithogeny, Vague Sets, Neutrosophic Sets.

1 Introduction

Branding is the process of creating and managing a brand so that it is recognizable, memorable, and has a lasting impact on people's minds. It goes far beyond a simple logo or name; it involves the visual identity, values, personality, and way a company or product communicates with its audience.

It is a discipline related to the ability of organizations to design names for their products or services that successfully represent their quality and authenticity. Every object is manufactured, but it must have a word, symbol, or sign that represents it and helps it occupy a privileged place in the consumer's mind.

In essence, branding seeks to differentiate a brand from the competition, build trust, and create an emotional connection with customers. Major brands like Apple, Nike, and Coca-Cola have built such strong identities that their colors, fonts, and even advertising messages are easily identifiable.

The branding process must be carried out in a structured and strategic manner. Differentiating its three stages—strategy, creation, and management—allows us to understand that building a brand goes beyond its visual identity; it involves careful planning and ongoing work to position it in the market.

Branding is an integrated process, a dynamic process that requires analysis, creativity, and monitoring to build a strong and competitive brand. It is an analytical process that must go through various stages. The first stage is called strategic; it defines the brand's direction. This is followed by the creation phase, which is the construction of the brand's design. Finally, the management phase corresponds to

the entire process of implementation, control, and improvement. These are steps that must be taken into account when undertaking any endeavor to create a successful company.

One of the challenges facing female entrepreneurship in Latin America is that gender asymmetries still exist in the business world, making it necessary to design development strategies for these businesses [1, 2]. The growth of female entrepreneurship depends primarily on the individual efforts of female entrepreneurs but also depends on the structural transformation of the business ecosystem to ensure equitable conditions for competition and development.

In Ecuador, women-led businesses are seen through various channels. Among these is social media, where advertising and information about various goods and services are very common. Female entrepreneurship has gained momentum in the economic sphere. Thus, within domestic research, progress is being seen in the areas of female empowerment and entrepreneurship.

Some female-owned businesses, as evidenced on social media, lack brand management skills. Therefore, businesses with limited branding strategies are not adequately recognized in the market. It should also be noted that other female-owned businesses employ logos or branded packaging that create awareness in the target market. Furthermore, to create a brand, one must clearly understand the difference between a trade name and a brand. A brand name is the first opportunity a product has to communicate something about itself to the target market. In this regard, there are barriers and challenges in entrepreneurship for women and female-owned businesses, branding, branding strategies, and studies related to brand management for entrepreneurship.

From an entrepreneurial perspective, it can be established that the Latin American market offers an extraordinary opportunity for women and business development. The Inter-American Development Bank demonstrates a profoundly favorable relationship between those economies that provide benefits to improve the situation of women entrepreneurs and national competitiveness. This demonstrates the large number of women who are driving economic growth and contributing to poverty reduction.

On the other hand, according to World Bank statistics, 40% of the female population is economically active in Latin America, yet female participation in entrepreneurial activities does not exceed 15%.

This evidence highlights the structural difficulties women face when trying to enter and establish themselves in the business world. In this regard, one of the main obstacles limiting the growth of women-led businesses is their continued presence as microenterprises or within the informal economy. This phenomenon is due to multiple factors, including:

- Limited access to financing: Female entrepreneurs often face barriers to accessing credit and investment capital, either due to a lack of collateral or biases in the allocation of financial resources.
- Unpaid workload: Many women must balance their role as entrepreneurs with family and domestic responsibilities, which reduces the time and resources available to grow their businesses.
- Lack of business support networks: The absence of mentoring, networking, and business training opportunities limits the opportunities for expansion and consolidation of women's businesses.
- Cultural and gender barriers: Stereotypes and prejudices in the business world can restrict women's opportunities to access more competitive and strategic markets.

This paper aims to identify the aspects that should be considered when designing an e-learning platform dedicated to advising female entrepreneurs on creating their branding strategies in Ecuador. To achieve this objective, we used surveys and interviews with a group of female entrepreneurs nationwide. One of the conditions we imposed on the results we desired was the accuracy of the responses. However, the respondents' opinions contain degrees of imprecision, which are a natural part of how criteria are provided. Neutrosophy is the branch of philosophy that provides us with tools to deal with indeterminacy and uncertainty. To achieve even greater accuracy, we worked with the so-

called Neutrosophic Vague Sets [3-6].

Neutrosophic sets contain three functions: a membership function, an indeterminacy function, and a non-membership function. The three are independent of each other. Vague sets, on the other hand, are composed of two functions: one for minimal membership and one for minimal non-membership, such that the sum of the two is at most equal to one [7-10]. When sets are defined that contain three vague sets—one for membership, one for indeterminacy, and one for non-membership—then we are dealing with Neutrosophic Vague Sets.

Additionally, Plithogeny is the theory that allows us to model the dynamics between several different concepts, their opposites, and their neutrals [11-15]. One of the new theories created by F. Smarandache is Plithogenic Statistics, which extends classical Multivariate Statistics when we are faced with indeterminacy in some of these variables [16-18]. Neutrosophic Statistics is limited only where indeterminacy is in the form of interval-valued data [20, 21].

In this paper, we propose applying the methods of Plithogenic Statistics when data are represented in the form of Neutrosophic Vague Sets. This allows respondents to express their opinions more freely, as they do not need to restrict themselves to a single numerical or linguistic value.

The paper is divided into a Materials and Methods section containing the basic concepts of Plithogenic Statistics and Neutrosophic Vague Sets. In the Results section, we explain how to create the combination of Plithogenic Statistics using data in the form of Neutrosophic Vague Sets, which we are not aware of having been used previously. The final section presents conclusions.

2 Materials and Methods

In this section, we summarize the main concepts and theories that we use in this paper, which are the Plithogenic Statistics and the Neutrosophic Vague Sets.

2.1 Plithogenic Statistics

The main idea of the definition of Plithogenic Probability is the combination of several sub-probabilities of a single event composed of multiple variables [11]. Therefore, the Plithogenic probability of an event occurring is the composition of all the random variables or parameters that model the event. This theory is an extension of the Multivariate Probability theory, where a multi-dimensional study is carried out. In this theory, each of the random variables is represented by its Probability Distribution Function or its Probability Density Function (PDF).

In the foundations of the theory we have that the subclasses of Plithogenic Probability are classified as follows:

(1) Classical MultiVariate Probability: Where all PDFs are the classic ones.
(2) Plithogenic Neutrosophic Probability: In them, the PDFs are represented in the form of $(\mathrm{T}, \mathrm{I}, \mathrm{F})$, such that T is the probability that the event occurs, I is the probability of uncertainty that the event occurs, and F is the probability that the event does not occur. Furthermore, $\mathrm{T}, \mathrm{I}, \mathrm{F} \in [0,1]$, and $0 \leq \mathrm{T} + \mathrm{I} + \mathrm{F} \leq 3$ is fulfilled.
(3) Plithogenic Indeterminate Probability: This is the case when all PDFs have indeterminate data or arguments.
(4) Plithogenic Intuitionistic Fuzzy Probability: If the PDFs are expressed in the form (T, F) where $\mathrm{T}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T} + \mathrm{F} \leq 1$.
(5) Plithogenic Picture Fuzzy Probability: PDFs are taken as $(\mathrm{T}, \mathrm{N}, \mathrm{F})$. $\mathrm{T}, \mathrm{N}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T} + \mathrm{N} + \mathrm{F} \leq 1$; where T is the probability that the event occurs, N is the neutral probability of the event occurring or not, and F is the probability that the event does not occur.
(6) Plithogenic Spherical Fuzzy Probability: If PDFs are expressed as $(\mathrm{T}, \mathrm{H}, \mathrm{F})$. $\mathrm{T}, \mathrm{H}, \mathrm{F} \in [0,1]$, $0 \leq \mathrm{T}^2 + \mathrm{H}^2 + \mathrm{F}^2 \leq 1$; such that T is the probability that the event occurs, H is the neutral probability of it occurring or not, and F is the probability that the event does not occur.

(7) Plithogenic (fuzzy-extension) Probability: If all PDFs are expressed in the form of the (fuzzy-extension set) style.
(8) Plithogenic Hybrid Probability: If we have some PDFs in one of the above styles and others are in other styles.

The Plithogenic Statistics (PS) is the application of Plithogenic Probability theory when data is obtained from real-life events. It includes the analysis and observations of this data.

PS is a generalization of classical Multivariate Statistics, and additionally includes within its methods the analysis of more than one output variable that contains some type of indeterminacy. It is considered to be a multi-indeterminate statistics.

F. Smarandache classifies the Plithogenic Statistics as follows:

- Multivariate Statistics,
- Plithogenic Neutrosophic Statistics,
- Plithogenic Indeterminate Statistics,
- Plithogenic Intuitionistic Fuzzy Statistics,
- Plithogenic Picture Fuzzy Statistics,
- Plithogenic Spherical Fuzzy Statistics,
- and in general: Plithogenic (fuzzy-extension) Statistics,
- and Plithogenic Hybrid Statistics.

PS can be further generalized with the so-called Plithogenic Refined Statistics, which is dedicated to the statistical study based on the Plithogenic Refined Probability.

In a Neutrosophic Population, each element has a triple probability of belonging equal to (T_j, I_j, F_j), such that $0 \leq T_j + I_j + F_j \leq 3$.

When we have data in the form of Single-Valued Neutrosophic Numbers (T_j, I_j, F_j) for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability for all data in the sample is calculated by the following equation:

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \qquad (1)$$

2.2 Neutrosophic Vague Sets

Definition 1 ([6, 7])**.** A Vague Set on U is characterized by a truth-membership function T_V and a false-membership function F_V; $T_V: U \to [0,1]$ and $F_V: U \to [0,1]$, respectively, such that T_V is a lower bound on the degree of membership of u_i inferred from the evidence available for u_i, F_V is a lower bound of the negation of u_i, inferred from the evidence against u_i. Moreover, both must satisfy the condition $T_V(u_i) + F_V(u_i) \leq 1$. Thus, the degree of membership of u_i to the vague set is in the interval $[T_V(u_i), 1 - F_V(u_i)] \subseteq [0,1]$. So, the true value of membership u_i is unknown, but at least it is known to be bounded by the interval $[T_V(u_i), 1 - F_V(u_i)]$.

Definition 2 ([6]). A Single-Valued Neutrosophic Set A over a set U is a set $A = \{\langle u, (T_A(u), I_A(u), F_A(u))\rangle : u \in U, T_A(u), I_A(u), F_A(u) \in [0,1]\}$. Where, $0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$.

Definition 3 ([6]). A Neutrosophic Vague Set A_{NV} (NVS) over a universe of discourse U is a set $A_{NV} = \left\{\langle u, \left(\hat{T}_{A_{NV}}(u), \hat{I}_{A_{NV}}(u), \hat{F}_{A_{NV}}(u)\right)\rangle : u \in U\right\}$. Where, $\hat{T}_{A_{NV}}(u) = [T^-, T^+]$, $\hat{I}_{A_{NV}}(u) = [I^-, I^+]$, and $\hat{F}_{A_{NV}}(u) = [F^-, F^+]$, such that the following holds:

$T^+ = 1 - F^-$, $F^+ = 1 - T^-$, and $0 \leq T^- + I^- + F^- \leq 2$,

If U is continuous, the NVS A_{NV} is written as:

$A_{NV} = \int_U \langle u, \left(\hat{T}_{A_{NV}}(u), \hat{I}_{A_{NV}}(u), \hat{F}_{A_{NV}}(u)\right)\rangle / u, u \in U,$

If U is discrete, the NVS A_{NV} is written as:

$A_{NV} = \sum_{u_i \in U} \langle u_i, \left(\hat{T}_{A_{NV}}(u_i), \hat{I}_{A_{NV}}(u_i), \hat{F}_{A_{NV}}(u_i)\right)\rangle / u_i,\ u_i \in U.$

Some special types of Neutrosophic Vague Sets are the following:

Ψ_{NV} is called an NVS unit, where $\hat{T}_{\Psi_{NV}}(u) = [1,1]$, $\hat{I}_{\Psi_{NV}}(u) = \hat{F}_{\Psi_{NV}}(u) = [0,0]$,

Φ_{NV} is called a zero NVS, where $\hat{T}_{\Phi_{NV}}(u) = [0,0]$, $\hat{I}_{\Phi_{NV}}(u) = \hat{F}_{\Phi_{NV}}(u) = [1,1]$.

Definition 4 ([6])**.** The complement of an NVS A_{NV} is denoted by A_{NV}^C and is defined as follows:

$\hat{T}_{A_{NV}}^C = [1 - T^+, 1 - T^-]$,

$\hat{I}_{A_{NV}}^C = [1 - I^+, 1 - I^-]$,

$\hat{F}_{A_{NV}}^C = [1 - F^+, 1 - F^-]$.

Definition 5 ([6])**.** Let A_{NV} and B_{NV} be two NVSs from the universe of discourse U. When $\forall u \in U$ the conditions are met:

$\hat{T}_{A_{NV}}(u) = \hat{T}_{B_{NV}}(u)$, $\hat{I}_{A_{NV}}(u) = \hat{I}_{B_{NV}}(u)$, and $\hat{F}_{A_{NV}}(u) = \hat{F}_{B_{NV}}(u)$, then it is said that A_{NV} and B_{NV} are equal and are denoted by $A_{NV} = B_{NV}$.

Definition 6 ([6])**.** Let A_{NV} and B_{NV} be two NVSs from the universe of discourse U. When $\forall u \in U$ the conditions are met:

$\hat{T}_{A_{NV}}(u) \leq \hat{T}_{B_{NV}}(u)$, $\hat{I}_{A_{NV}}(u) \geq \hat{I}_{B_{NV}}(u)$, and $\hat{F}_{A_{NV}}(u) \geq \hat{F}_{B_{NV}}(u)$, then it is said that A_{NV} is included in B_{NV} and it is denoted by $A_{NV} \subseteq B_{NV}$.

Definition 7 ([6])**.** Let A_{NV} and B_{NV} be two NVSs of the universe of discourse U. The union C_{NV} of A_{NV} and B_{NV}, is denoted by $C_{NV} = A_{NV} \cup_{NV} B_{NV}$ and it is defined by:

$\hat{T}_{C_{NV}}(u) = \left[\max\left(\hat{T}_{A_{NV}}^-(u), \hat{T}_{B_{NV}}^-(u)\right), \max\left(\hat{T}_{A_{NV}}^+(u), \hat{T}_{B_{NV}}^+(u)\right)\right]$,

$\hat{I}_{C_{NV}}(u) = \left[\min\left(\hat{I}_{A_{NV}}^-(u), \hat{I}_{B_{NV}}^-(u)\right), \min\left(\hat{I}_{A_{NV}}^+(u), \hat{I}_{B_{NV}}^+(u)\right)\right]$,

$\hat{F}_{C_{NV}}(u) = \left[\min\left(\hat{F}_{A_{NV}}^-(u), \hat{F}_{B_{NV}}^-(u)\right), \min\left(\hat{F}_{A_{NV}}^+(u), \hat{F}_{B_{NV}}^+(u)\right)\right]$.

Definition 8 ([6])**.** Let A_{NV} and B_{NV} be two NVSs of the universe of discourse U. The intersection H_{NV} of A_{NV} and B_{NV}, is denoted by $H_{NV} = A_{NV} \cap_{NV} B_{NV}$ and is defined by:

$\hat{T}_{H_{NV}}(u) = \left[\min\left(\hat{T}_{A_{NV}}^-(u), \hat{T}_{B_{NV}}^-(u)\right), \min\left(\hat{T}_{A_{NV}}^+(u), \hat{T}_{B_{NV}}^+(u)\right)\right]$,

$\hat{I}_{H_{NV}}(u) = \left[\max\left(\hat{I}_{A_{NV}}^-(u), \hat{I}_{B_{NV}}^-(u)\right), \max\left(\hat{I}_{A_{NV}}^+(u), \hat{I}_{B_{NV}}^+(u)\right)\right]$,

$\hat{F}_{H_{NV}}(u) = \left[\max\left(\hat{F}_{A_{NV}}^-(u), \hat{F}_{B_{NV}}^-(u)\right), \max\left(\hat{F}_{A_{NV}}^+(u), \hat{F}_{B_{NV}}^+(u)\right)\right]$.

Theorem 1 ([6])**.** Let $\mathcal{P}_{NV}$ be the set of all subsets of NVSs over the universe U. Then, $\langle \mathcal{P}_{NV}, \cup_{NV}, \cap_{NV} \rangle$ is a distributive lattice.

3 Results

This section is dedicated to showing the methods used for the calculations and the results obtained.

Simple random probability sampling was used to select the sample for the survey. This method means that all individuals in a population have the same probability of being considered for a study. The sample is obtained one by one randomly, without replacement of the selected individuals. The order in which the cases are selected has no influence.

In this paper to define the target population to study, the set of female-owned enterprises in the city of Guayaquil were considered likely to be selected. In the city of Guayaquil, 169 registered female-

owned enterprises were identified for surveying. These are considered the study population. To determine the study sample, the following formula was applied with a 10% margin of error and a 95% confidence level, as detailed below for the sample size calculation:

$$n = \frac{k^2 Npq}{e^2(N-1)+k^2 pq} \qquad (2)$$

Where,
n: is the sample size,
N: is the population size,
k: is a constant dependent on the confidence level,
e: is the sampling error,
p: is the proportion of the population that satisfies the characteristic being measured,
q: is 1-p.

The number of cases for simple random sampling, considering the aforementioned criteria, was $61.470 \approx 62$. A structured questionnaire was administered to the sample to obtain relevant information related to the objectives of this project.

On the other hand, in-depth interviews are a qualitative technique that helps delve deeper into each person's behavior, attitudes, reactions, and emotions in response to specific stimuli. The interviewer uses questions that seek to elicit findings related to the research objectives.

The inclusion criterion for selecting individuals to participate in the in-depth interviews was: being the owner of a commercial business located or operating in the city of Guayaquil, where this research will be conducted. It aims to reveal the ways in which they currently develop and apply branding strategies for business management and to gather information on their implementation at points of sale. The objective is also to highlight the specific characteristics of this process in the selected sample.

Table 1 summarizes the barriers that hinder female entrepreneurship in Ecuador, specifically in the city of Guayaquil.

Table 1. Barriers that prevent the development of branding strategies in female entrepreneurship.

Type of barrier	Description
Visual	It involves using logos on the premises, signs describing the service, distinctive colors, and providing a name for the business. However, this marketing requires the business owners to be aware of its necessity. Furthermore, it requires sufficient preparation and hiring a trained specialist, which requires an additional investment.
Social	The value proposition and differentiation are considered important in building close relationships with customers and creating bonds of loyalty based on trust, offering exclusive and innovative products at competitive prices, supported by personalized service. However, the lack of research that allows entrepreneurs to understand their customers' consumption behavior and strengthen the aforementioned bonds of loyalty stands out as a barrier.
Technological	It is the use mainly of social networks or Information and Communication Technologies to promote the business, for example, Instagram, Facebook and WhatsApp. One barrier associated with this aspect for startup clients may be internet access and social media profile management, which are not always ideal for startups.
Economic	The scarcity or reduction of financial resources results from factors beyond the entrepreneur's control, which limit the execution of marketing and branding strategies. While it is true that the government is developing entrepreneurship motivation programs for women, these do not appear to be sufficient, well-known, or widely used.
Educational	It involves the entrepreneurs' academic qualifications or having an advisor with this qualification. Access to marketing and branding courses offered by universities, either in-person or online. Connections to academic departments specializing in the subject.

Type of barrier	Description
	One barrier to address is the lack of training programs or business management resources that would allow female entrepreneurs to consolidate their market position in a highly competitive environment.

Once the barriers faced by female entrepreneurs have been defined, the following outlines the important aspects that should be considered when designing an e-learning platform for female entrepreneurs.

A1. Information about entrepreneurial consumers, that is, data from entrepreneurial consumer behavior studies can be used to design optimal marketing and branding strategies for female entrepreneurship.

A2. Information about financing options or support programs available for the development and empowerment of women in generating significant support for the country's economy.

A3. Construction of a visual identity system, which consists of developing inputs that allow businesses to generate queries about how to define a logo, logotype, or isologo for their business. Guidelines for designing the identity system would be provided, as well as visual input paths they could consult for this process.

A4. Social media and content management for entrepreneurship, which consists of providing female-owned businesses with information on social media management, content development, identifying consumer insights, and strategies that allow them to transform their potential customers into real ones. According to the research results, it is important to provide them with input to learn how to manage social media and design suitable content that will help them achieve significant market positioning.

A5. Branding strategies, which consist of providing them with a foundation of brand management strategies related to the marketing mix variables of promotion, price, product, and location. Similarly, they should consider guidelines so they can design their value proposition based on the functional, physical, and emotional attributes highly valued by the startups' customers. Likewise, personalization and exclusivity should be emphasized as important benefits for potential customers.

Based on these five aspects, the survey was prepared, where customers were asked the following, as indicated in Table 2:

Table 2. Questions asked in the survey to the female owners of the businesses studied in the city of Guayaquil.

#	Question
1	Do you think that your business uses consumer information to develop marketing strategies?
2	Do you have sufficient information about the business's potential for funding from government or non-government agencies?
3	Does the business have an identity that distinguishes it from other businesses and characterizes its own business? For example, the use of logos, signs, uniforms, etc.?
4	Can the use of a unique identity be considered effective and increase customer attraction?
5	Can the business be viewed on social media and mobile apps like WhatsApp?
6	Does the business owner feel qualified to use social media for promotion, or do they have a specialist to support them?
7	Has the use of new technologies been effective in promoting the business?
8	Does the owner know what branding is, use it appropriately, and is she trained to do so?

Additionally, to complete the information, the owners of the 62 enterprises were interviewed.
Let us denote by $U = \{u_1, u_2, \cdots, u_{62}\}$ the set of enterprises to study.
The following evaluation procedure will be followed:

1. The evaluations for each of the questions shown in Table 2 are answered with NVSs.

   It is denoted by E_{NVij} the evaluation of the ith business ($i = 1,2, \ldots ,62$) on the jth aspect ($j = 1,2, \ldots ,8$) that appears in Table 2. E_{NVij} is an NVS.

2. To assist the evaluations in satisfying the conditions that an NVS must meet, she is asked to be guided by Table 3 below:

Table 3. Value of T^- selected by the respondent and from this the maximum value for F^-, on a scale of 0 to 100.

Value of T^-	Maximum value of F^-
0	100
10	90
20	80
30	70
40	60
50	50
60	40
70	30
80	20
90	10
100	0

Respondents are asked to use a scale of 0 to 100 to determine how much T^- and F^- are. These values are then divided by 100 to bring them to a scale of $[0,1]$.

1. From the values T_{ij}^- and F_{ij}^- we have the first two values for E_{NVij}. Evaluators are also asked, if relevant, to set values for I_{ij}^- and I_{ij}^+ independently of the truth and falsity values provided that $I_{ij}^- \leq I_{ij}^+$. These uncertainty values are also scaled by $[0,1]$ divided by 100.

   Then, $T_{ij}^+ = 1 - F_{ij}^-$, $F_{ij}^+ = 1 - T_{ij}^-$ are defined.

   In this way, we have the NVSs:

   $\hat{T}_{E_{NVij}}(u) = \left[T_{ij}^-, T_{ij}^+\right]$, $\hat{I}_{E_{NVij}}(u) = \left[I_{ij}^-, I_{ij}^+\right]$, and $\hat{F}_{E_{NVij}}(u) = \left[F_{ij}^-, F_{ij}^+\right]$. These are understood as subjective probabilities.

2. These evaluations are Plithogenic (fuzzy-extension) Statistics since they are the extension of fuzzy sets to neutrosophic vague sets.
   In this way, we have that the Plithogenic Probabilities are defined as:

   $$PNVP(u_i) = (E_{NVi1}, E_{NVi2}, E_{NVi3}, E_{NVi4}, E_{NVi5}, E_{NVi6}, E_{NVi7}, E_{NVi8}) \quad (3)$$

   Where each E_{NVij} is defined by a triad of intervals of the form:
   $\hat{T}_{E_{NVij}}(u_i) = \left[T_{ij}^-, T_{ij}^+\right]$, $\hat{I}_{E_{NVij}}(u_i) = \left[I_{ij}^-, I_{ij}^+\right]$, and $\hat{F}_{E_{NVij}}(u_i) = \left[F_{ij}^-, F_{ij}^+\right]$.
3. An expected probability is calculated for all enterprises using formula 4, which is defined below:

   $$\overline{PNVP} = (\bar{E}_{NV1}, \bar{E}_{NV2}, \bar{E}_{NV3}, \bar{E}_{NV4}, \bar{E}_{NV5}, \bar{E}_{NV6}, \bar{E}_{NV7}, \bar{E}_{NV8}) \quad (4)$$

Such that:

$$\bar{E}_{NVj} = \frac{\sum_{i=1}^{62} \bar{E}_{NVij}}{62} \qquad (5)$$

The averages are calculated:
$\hat{T}_{\bar{E}_{NVj}} = \left[\frac{\sum_{i=1}^{62} T_{ij}^{-}}{62}, \frac{\sum_{i=1}^{62} T_{ij}^{+}}{62}\right]$, $\hat{I}_{\bar{E}_{NVj}} = \left[\frac{\sum_{i=1}^{62} I_{ij}^{-}}{62}, \frac{\sum_{i=1}^{62} I_{ij}^{+}}{62}\right]$, and $\hat{F}_{\bar{E}_{NVj}} = \left[\frac{\sum_{i=1}^{62} F_{ij}^{-}}{62}, \frac{\sum_{i=1}^{62} F_{ij}^{+}}{62}\right]$.
These are considered the expected probabilities for each of the questions.
These values can be de-neutrosophied with the help of the formula shown below ([21]):

$$\lambda([a,b]) = \frac{a+b}{2} \qquad (6)$$

The three probability values for truth, indeterminacy, and falsity are then converted into a single value using the score function formula below [22, 23]:

$$\mathcal{S}(\langle t,i,f\rangle) = \frac{2+t-i-f}{3} \qquad (7)$$

4. Those values less than 0.5 of each of the elements of the $\overline{PNVP}$ de-neutrosophied with Equation 6 and then converted into a single value with the help of Equation 7, allow us to identify that there is a problem in the aspect measured in the question in general.

After following the procedure above, the results were as follows as shown in Table 4:

Table 4. Results of the NVS average items for each of the questions. The last column contains the de-neutrosophication after applying Equations 6 and 7.

Question	Vague truthfulness	Vague indeterminacy	Vague falseness	De-neutrosophied value
1	[0.328, 0.392]	[0.001, 0.250]	[0.608, 0.672]	0.5315
2	[0.416,0.459]	[0.097, 0.149]	[0.541, 0.584]	0.584
3	[0.613,0.661]	[0.111, 0.223]	[0.339, 0.387]	0.70233333
4	[0.772,0.798]	[0.120, 0.176]	[0.202, 0.228]	0.80733333
5	[0.961,0.976]	[0.076, 0.085]	[0.024, 0.039]	0.95216667
6	[0.545, 0.671]	[0.025, 0.039]	[0.328, 0.454]	0.72833333
7	[0.368, 0.471]	[0.073, 0.202]	[0.529, 0.632]	0.56716667
8	[0.266, 0.338]	[0.108, 0.271]	[0.662, 0.734]	0.4715

4. Conclusion

Research on female entrepreneurship represents an important step in determining the degree of social inequality that exists based on gender. This is a way to vindicate the female sector, which has historically suffered disadvantages imposed by the male population. Furthermore, branding is an important element in running a business. This article studies the behavior of these two elements in the

Ecuadorian city of Guayaquil. To this end, a random sample of 62 female-owned businesses was selected. Potential barriers that may hinder the establishment of branding strategies in these businesses were identified. Essential aspects to measure in the case of a woman wishing to start a business were also identified. A survey was also designed to identify which aspects are successful and which are unsuccessful. Together, this constitutes an evaluative method that serves to create an e-learning tool that can be useful for female entrepreneurs. This same survey was also used to calculate the probabilities of fulfilling the aspects measured in the eight questions. The evaluations regarding having a unique business identity, its effectiveness, the use of social media to promote the business, and the effectiveness of these uses have a high score, with over 70% or 0.7 probability. The remaining aspects had medium scores between 0.4 and less than 0.6; these are the use of customer consumption information to improve the business, understanding potential sources of financing, the effectiveness of new technologies used, and the owner's knowledge of branding. The paper shows how effective the application of Plithogenic Neutrosophic Vague Statistics is in evaluation problems, where accuracy is gained, in addition to taking into account indeterminacy in the calculations.

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