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Bijective Single Valued Neutrosophic Graph and Its Application in Fraud Detection Analysis in Social Networks

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Abstract: In this paper, a new concept of bijective single valued neutrosophic set and bijective single valued neutrosophic graphs are introduced. Also, the height, depth, bijective single valued neutrosophic bridge (BSVN – Bridge), bijective single valued neutrosophic cut vertex (BSVN – cut vertex) in bijective single valued neutrosophic graphs are defined. Some properties of bijective single valued neutrosophic cycles have been explored. Relation between Connectivity and BSVN-cut vertices, BSVN-Bridges are given. It contains few significant properties like, if $\mu = d_E$, then μ is not a BSVN-bridge. Also, If $\mu = d_E$, being an edge in a BSVN-cycle graph G of length n, then G has $(n-1)$ BSVN-bridges. An application of BSVN graphs by considering the users as vertices and interaction between the users as edged in an instagram social network is used in fraud detection analysis in social networks by using almost all the properties which were explored related to cycles.

Keywords: Bijective single valued neutrosophic sets; Bijective single valued neutrosophic graphs; Bijective single valued neutrosophic bridges; Bijective single valued neutrosophic cut vertex; height and depth.

1. Introduction

Graph theory is widely recognized as a branch of applied mathematics. Graph serves as a rope to climb the root of real-life problems related to optimization and computer science. When uncertainty arises in the set of vertices, edges or both, a graph transforms into a fuzzy graph and intuitionistic fuzzy graphs, capturing the essence of ambiguity in its structure. But, when relations between vertices and edges become indeterminate, fuzzy graph and intuitionistic fuzzy graphs lose their effectiveness. "Greatness" in mathematics is an art; "A great result should contribute substantial new information and it should be unexpected".Ahmed et al [1] studied Domination on bipolar fuzzy operations. Akram et al [2] studied some operations on Single Valued Neutrosophic Graphs (SVN-graphs) in 2017, which improves the robustness of the solution. Driven by curiosity and innovation, mathematicians dived deep into the various aspect of fuzzy mathematics to refine its theoretical foundation. Beaula Thangaraj et al[3-5] work on different fuzzy numbers and ranking methods to solve critical path problems exemplifies the ongoing expansion of fuzzy applications. It is crucial to acknowledge that, Samarandache [6-9]devoted a great deal of attention in bringing out four main categories of Neutrosophic Graphs, namely, I-edge Neutrosophic Graph, I-vertex Neutrosophic Graph, (t,i,f)-edge Neutrosophic graph and (t,i,f)-vertex Neutrosophic graph which strengthens the framework of SVN-graphs. Neutrosophic graphs also spread its wings as NeutrosophicLabeling graphs, since M.Gomathi et al [10] introduced NeutrosophicLabeling Graphs in 2019. Hussan et al[11] studies about Neutrosophic trees. Mini tom et al [12] studied boundaries and interior nodes in fuzzy graph. Vetrivel et al[13] studied Product Perspective from Fuzzy to Neutrosophic Graph Extension. Vijaya et al[14] have shaped the advancements of Neutrosophic graphs in find the solution of Decision making problem and critical path problems by using Pythagorean Fuzzy numbers and NeutrosophicFuzzy numbers. Wang et al[16,17] launched research efforts in SVN-sets in multispacer and multistructure. Ye, J.,[18-19]studied Single-Valued Neutrosophic Minimum Spanning Tree in 2014. Lastly, but noteworthy to mention that introduction to Fuzzy sets by Zadeh, L.,[20]in 1965 is a primary driving force behind all these explorations. Although Neutrosophic graphs lend a helping hand to emphasize indeterminacy, when it comes to structure domains where membership values need to remain within a defined range Neutrosophic graphs falls short, unable to capture the complexity.Consequently, the concept of Single Valued Neutrosophic graphs arose as a natural response to a particular need in the course of the evolution in fuzzy graph theory. As a final remark, "Neutrosophic graphs yield a fruitful mathematical solution".With all these crucial insights in mind, this paper focus on the study of Bijective Single Valued Neutrosophic cycle graphs.

2. Preliminaries:

Definition 2.1.A Graph G is a pair (V, E), where V is a set of points and E is defined as relation on V. V is also called as vertices or nodes and E is called as edges of a graph G.

Definition2.2[12]. A fuzzy set A on a crisp set X is characterized by a mapping $m : X \to [0,1]$ and the mapping m is known as membership function.

Definition 2.3[12].A fuzzy graph G is a triplet (V, σ, μ) where V is set of vertices $\sigma : V \to [0,1]$ and $\mu : V\,X\,V \to [0,1]$ are the membership function for vertices and edges respectively such that for all $x, y \in V, \mu(x, y) \le \sigma(x) \wedge \sigma(y)$ We can call σ as the fuzzy vertex set of G and μ as fuzzy edge set of G.

Definition 2.4[10]. Let X be a space of objects (points) with generic elements in X denoted by x; then the Neutrosophic Set (NS) A is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$, where the function $T, I, F : X \to]^-0,1^+[$ define respectively a truth-membership function, an indeterminacy-membership function and a falsity-membership function of the element $x \in X$ to the set A with the condition $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.5[10]. A Neutrosophic Graph (NS – Graph) is of the form $G = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ and $\mu = (T_2, I_2, F_2)$ such that $\sigma_T : V \to [0,1], \sigma_I : V \to [0,1], \sigma_F : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy-membership function, falsity-membership function of a vertex $v_i \in V$ respectively and $0 \le \sigma_T(v) + \sigma_I(v) + \sigma_F(v) \le 3, \forall v_i \in V (i = 1,2,\ldots,n)$ And, $\mu_T : V\,X\,V \to [0,1], \mu_I : V\,X\,V \to [0,1], \mu_F : V\,X\,V \to [0,1]$ denote the degree of truth-membership function, indeterminacy-membership function, falsity-membership function of an edge (v_i, v_j) respectively such that for every (v_i, v_j), $\mu_T(\{v_i, v_j\}) \le \min[\sigma_T(v_i), \sigma_T(v_j)]$, $\mu_I(\{v_i, v_j\}) \le \min[\sigma_I(v_i), \sigma_I(v_j)]$, $\mu_F(\{v_i, v_j\}) \ge \max[\sigma_F(v_i), \sigma_F(v_j)]$ and $0 \le \mu_T(v) + \mu_I(v) + \mu_F(v) \le 3$.

Definition 2.6[7]. An Intuitionistic fuzzy graph is of the form G = (V, E) where

i. $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1]$ $\gamma_1 : V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively, and $0 \le \mu_1(v_1) + \gamma_1(v_1) \le 1$ for every $v_i \in V, (i = 1,2\ldots,n)$

ii. $E \subseteq V\,X\,V$ where $\mu_2 : V\,X\,V \to [0,1]$ and $\gamma_2 : V\,X\,V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \le \min[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in V$, $i, j = (1,2,\ldots,n)$.

Definition 2.7[7]. Let X be a crisp set. A Single Valued Neutrosophic Set (SVNS) A is characterized by truth membership function $T_V(p)$, an indeterminacy membership function $I_V(p)$ and a falsity membership function $F_V(p)$ for every point $p \in X, T_V(p), I_V(p), F_V(p) \in [0,1]$.

Definition 2.8 [7].A Single Valued Neutrosophic Graph (SVN-graph) is a pair $G = (V, \sigma, \mu)$ such that:

1. The functions $\sigma_T : V \to [0,1], \sigma_I : V \to [0,1]$, $\sigma_F : V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$ respectively, and $0 \le \sigma_T(v_i) + \sigma_I(v_i) + \sigma_F(v_i) \le 3$ for all $v_i \in V$ $(i = 1,2,..n)$

2. The functions $\mu_T : E \subseteq V\,X\,V \to [0,1]$, $\mu_I : E \subseteq V\,X\,V \to [0,1]$, and $\mu_F : E \subseteq V\,X\,V \to [0,1]$ are defined by, $\mu_T(\{v_i, v_j\}) \le \min[\sigma_T(v_i), \sigma_T(v_j)]$, $\mu_I(\{v_i, v_j\}) \ge \max[\sigma_I(v_i), \sigma_I(v_j)]$ $\mu_F(\{v_i, v_j\}) \ge \max[\sigma_F(v_i), \sigma_F(v_j)]$.

Denotes the degree of truth-membership, indeterminacy-membership and falsity-Membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \le \mu_T(v_i, v_j)\mu_I(v_i, v_j) + \mu_F(v_i, v_j) \le 3$,for all $(v_i, v_j) \in V\,X\,V$ $i, j = (1,2,\ldots,n)$.

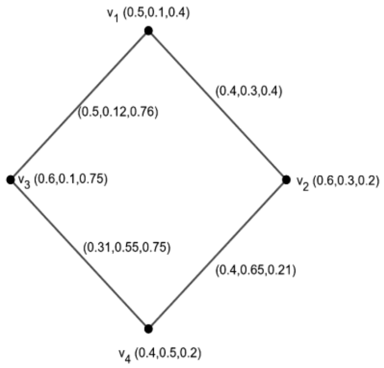

Fig 1:Single Valued Neutrosophic Graph

3. Bijective Single Valued Neutrosophic Graph

Definition 3.1. Let X be a crisp set. A Bijective Single Valued Neutrosophic Set (BSVNS) A is characterized by truth membership function $T_V(p)$, an indeterminacy membership function $I_V(p)$ and a falsity membership function $F_V(p)$, for every point $p \in V$; $T_V(p), I_V(p), F_V(p) \in [0,1]$ such that $T_V(p_i)$ and $T_V(p_j)$, $I_V(p_i)$ and $I_V(p_j)$, $F_V(p_i)$ and $F_V(p_j)$ are distinct respectively, for any i and j.

Definition 3.2. A Bijective Single Valued Neutrosophic Graph (BSVN – Graph) is a triplet $G = (V, \sigma, \mu)$ in which the truth-membership function of vertices and truth-membership function of edges are Bijective .i.e, $\sigma_T : V \to [0,1]$ and $\mu_T : E \subseteq V\,X\,V \to [0,1]$ are bijective. Similarly, Indeterminacy membership function for vertices and edges and falsity-membership function for both vertices and edges are bijective respectively .i.e $\sigma_I : V \to [0,1]$ and $\mu_I : E \subseteq V\,X\,V \to [0,1]$ are bijective, and $\sigma_F : V \to [0,1]$ and $\mu_F : E \subseteq V\,X\,V \to [0,1]$ are Bijective, where $\mu_T(\{v_i, v_j\}) < \min[\sigma_T(v_i), \sigma_T(v_j)]$, $\mu_I(\{v_i, v_j\}) > \max[\sigma_I(v_i), \sigma_I(v_j)]$, $\mu_F(\{v_i, v_j\}) > \max[\sigma_F(v_i), \sigma_F(v_j)]$ for all $\{v_i, v_j\} \in V\,X\,V$ and $v_i \in V$ $i, j = 1,2,\ldots,n$.

The following conditions are trivial in BSVN-Graph as in case of SVN-Graph:

- **i.** $0 \le \sigma_T(v_i) + \sigma_I(v_i) + \sigma_F(v_i) \le 3$ for all $v_i \in V$ $(i = 1,2,..n)$
- **ii.** $0 \le \mu_T(v_i, v_j)\mu_I(v_i, v_j) + \mu_F(v_i, v_j) \le 3$, for all $(v_i, v_j) \in V\,X\,V$ $i, j = (1,2,\ldots,n)$.

Example 3.3: Let G be a graph with vertices v_1, v_2, v_3, v_4 . Let $(\sigma_{T_i}, \sigma_{I_i}, \sigma_{F_i})$ for vertices v_i (where i = 1, 2, 3, 4) are: $(0.3,0.72,0.15)$, $(0.2,0.35,0.5)$, $(0.6,0.8,0.85)$, $(0.12,0.6,0.7)$ respectively. Let $(\mu_{Tij}, \mu_{Iij}, \mu_{Fij})$ for edges (v_i, v_j) . (where 1 ≤ i ≤ 4 and 1 ≤ j ≤ 4): $v_1v_2 =$ $(0.1,0.75,0.55)$, $v_1v_3 = (0.21,0.85,0.9)$, $v_1v_4 = (0.05,0.82,0.8)$, $v_2v_3 = (0.15,0.95,1)$, $v_2v_4 = (0.11,0.7,0.75)$, $v_3v_4 = (0.09,0.81,0.91)$ respectively. With this membership functions, G is BSVN-Graph.

Definition 3.4. A path P in a bijective single valued neutrosophic graph $G = (V, \sigma, \mu)$ is a sequence of distinct vertices $v_1v_2v_3,\ldots,v_m$ such that $\mu_T(v_jv_{j+1}) > 0$, $\mu_I(v_jv_{j+1}) > 0$, $\mu_F(v_jv_{j+1}) > 0$ for $1 \le j \le m$.

Definition 3.5. If there is at least one path between every pair of vertices in bijective single valued neutrosophic graph $G=(V,\sigma,\mu)$ Then G is said to be connected, else G is disconnected.

3.1. Connectivity in Bijective Single Valued Neutrosophic Graphs:

The concept of connectivity plays a important role in path and network problems, so some of the basic concepts like bridges, cut vertex, blocks of bijective single valued neutrosophic graph(BSVN-graph)have been defined to study its properties.

3.1.1. Preliminary Definitions for Bijective Single Valued Neutrosophic Graphs

Definition3.1.1. Let $C=(T_C,I_C,F_C)$ be a BSVNS on X. $h_T(C)$ is defined to be the supremum of all truth-membership values associated with x , where $x\in X$. $h_I(C)$ is defined to be infimum of all indeterminacy-membership values associated with x, where $x\in X$. whereas, $h_F(C)$ is the infimum of all falsity-membership values associated with x , where $x\in X$. The height of C is defined as $h(c)=(h_T(C),h_I(C),h_F(C))$.

In a BSVN-Graph G, h_V denote the height of the vertex and h_E denote the height of the edge.

Example 3.1.2: Consider a BSVN-cycle graph G of length 4. Let $(\sigma_{T_i},\sigma_{I_i},\sigma_{F_i})$ for v_i (i = 1, 2, 3, 4) are: $(0.5,0.6,0.41)$ $(0.2,0.35,0.4)$, $(0.4,0.8,0.85)$, $(0.21,0.3,0.7)$ respectively. Let $(\mu_{Tij},\mu_{Iij},\mu_{Fij})$ for v_1v_2, v_1v_3 v_2v_4, v_3v_4 $(i,j=1,2,3,4)$ be: $(0.1,0.83,0.9)$, $(0.2,0.81,0.87)$, $(0.12,0.84,0.89)$, $(0.85,0.85,0.88)$ respectively. In this scenario, $h_V=(\sigma_{T_1},\sigma_{I_4},\sigma_{F_2})$ and $h_E=(\mu_{T_{34}},\mu_{I_{13}},\mu_{F_{13}})$ where $(\sigma_{T_i},\sigma_{I_i},\sigma_{F_i})$ & $(\mu_{Tij},\mu_{Iij},\mu_{Fij})$ denote the membership function for v_i and v_iv_j respectively.

Definition3.1.3: Let $C=(T_C,I_C,F_C)$ be a BSVNS on X, the depth of C is defined and denoted by $d(C)=(d_T(C),d_I(C),d_F(C))$. Infimum of all truth-membership values associated with x, $x\in X$ gives $d_T(C)$ whereas supremum of all indeterminacy-membership values and falsity-membership values of x, $x\in X$ yields $d_I(C)$ and $d_F(C)$ respectively.

Example 3.1.4: Let's consider $(\sigma_{T_i},\sigma_{I_i},\sigma_{F_i})$ for v_i (i = 1, 2, 3, 4) & $(\mu_{Tij},\mu_{Iij},\mu_{Fij})$ for v_1v_2, v_1v_3, v_2v_4 v_3v_4 (i, j = 1,2,3,4) are : $(0.03,0.72,0.95)$, $(0.2,0.35,0.2)$, $(0.1,0.8,0.85)$, $(0.21,0.6,0.7)$ & $(0.1,0.87,0.89)$, $(0.074,0.81,0.87)$, $(0.02,0.8,0.86)$, $(0.05,0.85,0.88)$ respectively in a BSVN-cycle graph G of length 4. In this example, $d_V=(\sigma_{T_1},\sigma_{I_3},\sigma_{F_1})$ and $d_E=(\mu_{T_{24}},\mu_{I_{12}},\mu_{F_{12}})$, where $(\sigma_{T_i},\sigma_{I_i},\sigma_{F_i})$ & $(\mu_{Tij},\mu_{Iij},\mu_{Fij})$ denote the membership function for v_i and v_iv_j respectively.

One of the uniqueness of BSVN-graph is that there exists only one height $(h_V\ \&\ h_E)$ and depth $(d_V\ \&\ d_E)$ for a graph G.

Definition 3.1.5: Let $G=(V,\sigma,\mu)$ be a Bijective Single Valued Neutrosophic Graph and $P=v_iv_j$ is any path in G, where $v_i,v_j\in V$. The T-strength of P is defined to be $\wedge_{i=1}^{n}\mu_T(v_iv_j)$, whereas the I-strength of P is defined as $\vee_{i=1}^{n}\mu_I(v_iv_j)$ and F-strength as $\vee_{i=1}^{n}\mu_F(v_iv_j)$ We can denote the T-strength of path P as $s_T(P)$, I-strength as $s_I(P)$ and F-strength as $s_F(P)$ and BSVN-strength of path P as $s(P)$, where $s(P)=(s_T(P),s_I(P),s_F(P))$.

The T-strength of connectedness of P is ,

$\mu_T^{\infty}(v_iv_j) = \sup\{\wedge_{i=1}^{n} \mu_T^k(v_iv_j) : k = 1,2,\ldots,n\}$

$i.e) \mu_T^{\infty}(v_iv_j) = \sup\{\mu_T(v_iv_1) \wedge \mu_T(v_1v_2) \wedge \mu_T(v_2v_3) \wedge \ldots \wedge \mu_T(v_{k-1}v_j) : v_i, v_1, v_2 \ldots, v_{k-1}, v_j \in V, k = 1,2,..,n\}$

Similarly, I-strength of connectedness of P can be given as,

$\mu_I^{\infty}(v_iv_j) = \inf\{\vee_{i=1}^{n} \mu_I^k(v_iv_j) : k = 1,2,\ldots,n\}$

$i.e) \mu_I^{\infty}(v_iv_j) = \inf\{\mu_I(v_iv_1) \vee \mu_I(v_1v_2) \vee \mu_I(v_2v_3) \vee \ldots \vee \mu_I(v_{k-1}v_j) : v_i, v_1, v_2 \ldots, v_{k-1}, v_j \in V, k = 1,2,..,n\}$

And F-strength of connectedness of P as,

$\mu_F^{\infty}(v_iv_j) = \inf\{\vee_{i=1}^{n} \mu_F^k(v_iv_j) : k = 1,2,\ldots,n\}$

$i.e) \mu_F^{\infty}(v_iv_j) = \inf\{\mu_F(v_iv_1) \vee \mu_F(v_1v_2) \vee \mu_F(v_2v_3) \vee \ldots \vee \mu_F(v_{k-1}v_j) : v_i, v_1, v_2 \ldots, v_{k-1}, v_j \in V, k = 1,2,..,n\}$

The T-strength of connectedness, I-strength of connectedness and F-strength of connectedness any path P in G is denoted by $\mu_T^{\infty}(v_iv_j)$, $\mu_I^{\infty}(v_iv_j)$, $\mu_F^{\infty}(v_iv_j)$ respectively. And, $\mu'_T{}^{\infty}(v_iv_j)$, $\mu'_I{}^{\infty}(v_iv_j)$, $\mu'_F{}^{\infty}(v_iv_j)$ denote, $\mu_T{}^{\infty}{}_{G-\{v_iv_j\}}$, $\mu_I{}^{\infty}{}_{G-\{v_iv_j\}}$ and $\mu_F{}^{\infty}{}_{G-\{v_iv_j\}}$, where $G-\{v_iv_j\}$ is obtained from G when the edge v_iv_j is removed.

Strength of connectedness can also be denoted as $CONN_G(v_iv_j)$, while removing an edge v_kv_n, strength of connectedness can be marked as $CONN_{G-\{v_kv_n\}}(v_iv_j)$.

Definition 3.1.6. An edge $v_iv_j \in E \subseteq V\ X\ V$ in a Bijective Single Valued Neutrosophic Graph G is said to be a T-bridge, if $\mu_T^{\infty}(v_kv_n) > \mu'_T{}^{\infty}(v_kv_n)$, for some $v_k, v_n \in V$. In a similar manner, An edge $v_iv_j \in E \subseteq V\ X\ V$ in G is a I-bridge, if $\mu_I^{\infty}(v_kv_n) < \mu'_I{}^{\infty}(v_kv_n)$, for some $v_k, v_n \in V$. And, an edge $v_iv_j \in E \subseteq V\ X\ V$ in G is called as F-bridge, if there exists $v_k, v_n \in V$ such that $\mu_F^{\infty}(v_kv_n) < \mu'_F{}^{\infty}(v_kv_n)$. A edge $v_iv_j \in E \subseteq V\ X\ V$ in G is said to be a BSVN-bridge, if $\mu_T^{\infty}(P) > \mu'_T{}^{\infty}(P)$, $\mu_I^{\infty}(P) < \mu'_I{}^{\infty}(P)$ $\mu_F^{\infty}(P) < \mu'_F{}^{\infty}(P)$ and for any path $P = v_{k-1} \ldots v_i v_j \ldots v_n$.

Definition 3.1.7. A vertex $v_i \in V$ in G is said to be a T-cut vertex if it's removal decreases $\mu_T^{\infty}(v_kv_n)$, for some $v_k, v_n \in V$. Suppose, if its removal increases $\mu_I^{\infty}(v_kv_n)$, for some $v_k, v_n \in V$, then it is called as I-cut vertex. Similarly, A vertex $v_i \in V$ in G is defined as F-cut vertex, if $\mu_F^{\infty}(v_kv_n) < \mu'_T{}^{\infty}(v_kv_n)$ for some pair of vertices v_k and v_n . A vertex $v_i \in V$ in G is said to be a **BSVN-cut vertex** , if, $\mu_T^{\infty}(v_kv_n) > \mu'_T{}^{\infty}(v_kv_n)$, $\mu_I^{\infty}(v_kv_n) < \mu'_I{}^{\infty}(v_kv_n)$ and $\mu_F^{\infty}(v_kv_n) < \mu'_F{}^{\infty}(v_kv_n)$ for some v_k, v_n in V.

Definition 3.1.8. Bijective Single Valued Neutrosophic Graph G is called a block if it has no BSVN-Cut vertices.

Definition 3.1.9. Bijective Single Valued Neutrosophic Graph G is called as cycle, if G* is a cycle.

3.2. Relation between Connectivity and BSVN-cut vertices, BSVN-Bridges:

Theorem3.2.1. Every bijective single valued neutrosophic graph G is single valued neutrosophic graph.

Proof: By the definition of BSVN-graph, the result is trivial. But the converse need not be true, since it is not bijective.

Theorem3.2.2: Let G be a cycle of BSVN-Graph of length n. If μ be any edge in G and $\mu = d_E$, where $\mu = v_iv_j$ then v_i and v_j are not BSVN-Cut vertices.

Proof: Let G be a cycle of length n in BSVN- Graph G and if $\mu = d_E$ such that $\mu = v_i v_j$. Consider any two vertices v_k, v_n in G. Since, G* is a cycle, there exist two distinct paths between v_k and v_n. Let $P_1 = v_k v_{k-1} \ldots v_i v_j \ldots v_n$ and $P_2 = v_k v_p \ldots v_m \ldots v_n$,where $m \neq i$. By the definition of strength of Path, $s(P_1) = d_E$ and $s(P) = (T_i, I_i, F_i) \neq d_E$ Again, By the definition of Strength of connectedness of $v_k v_n$, $\mu^{\infty}(v_k v_n) = (T_i, I_i, F_i) \neq d_E$ and $\mu'^{\infty}(v_k v_n) = (T_i, I_i, F_i) \neq d_E$ (since G* is a cycle there exists between v_k and v_n). Therefore, $\mu^{\infty}(v_k v_n) = \mu'^{\infty}(v_k v_n)$ which says that v_i is not a BSVN-Cut vertex. Similar proof holds for v_j.

Theorem3.2.3: If G be a Bijective Single Valued Neutrosophic Graph such that G* is a cycle and μ be an edge in G such that $\mu = d_E$, then G has $(n-2)$ BSVN-Cut vertices.

Proof: By theorem 3.2.2, If $\mu = v_i v_j$ and $\mu = d_E$, then v_i and v_j are not BSVN-Cut vertices. Let v_m be any vertex in G, $v_m \neq v_i$ and $v_m \neq v_j$. Considering Path P of any two vertices v_k and v_n , it is significant that there exists two distinct paths for $v_k v_n$ since G* is a cycle. Let it be P_1 and P_2. $s(P_1) = s(v_k v_{k-1} \ldots v_i \ldots v_n) = d_E$ and $s(P_2) = s(v_k v_p \ldots v_m \ldots v_n) \neq d_E$. By the definition of Strength of Connectedness, we can say,
$\mu^{\infty}(v_k v_n) = (T_i, I_i, F_i) \neq d_E$ and $CONN_{G-\{v_m\}}(v_k v_n) = \mu'^{\infty}(v_k v_n) = d_E$. By the definition of
$\mu^{\infty}{}_T(v_k v_n) > \mu'_T{}^{\infty}(v_k v_n)$, $\mu^{\infty}{}_I(v_k v_n) < \mu'_I{}^{\infty}(v_k v_n)$, $\mu^{\infty}{}_F(v_k v_n) < \mu'_F{}^{\infty}(v_k v_n)$ which ensures that v_m is a BSVN-Cut vertex. Since, v_m is any vertex in G and $v_m \neq v_i$ and $v_m \neq v_j$ the result holds.

Theorem3.2.4: Let G be a BSVN- cycle graph of length 3 and if μ be an edge in G and $\mu = h_E$, then G is a block.

Proof: Since G is a BSVN- cycle graph of length 3 and $\mu = h_E$, where $\mu \in E$ and $\mu = v_i v_j$. Let v_m be a vertex, $v_m \neq v_i$ and $v_m \neq v_j$. Since G* is a cycle, we can say that there exists two distinct paths for $v_i v_j$, say P_1 and P_2 Take P_1 as $v_i v_j$ and P_2 as $v_i v_m v_j$. $s(P_1) = h_E$ and $s(P_2) = (T_i, I_i, F_i) \neq h_E$. By the definition of Strength of connectedness and h_E , we can say that $CONN_G(v_i v_j) = h_E$ and $CONN_{G-\{v_m\}}(v_i v_j) = h_E$. Therefore, $CONN_G(v_i v_j) = CONN_{G-\{v_m\}}(v_i v_j) = h_E$ which states that v_m is not a BSVN-Cut vertex. Now, choose a path P between v_m and v_j , it is trivial that there is only two distinct paths between v_m and v_j (since G* is a cycle). Let $P_1 = v_m v_j$ and $P_2 = v_m v_i v_j$. Therefore, $s(P_1) \neq h_E$ and $s(P_2) \neq h_E$ (by the definition of h_E).i.e) Take
$s(P_1) = (T_{i1}, I_{i1}, F_{i1})$ and $s(P_2) = (T_{i2}, I_{i2}, F_{i2})$. $CONN_G(v_m v_j)$ can be given as
$(\sup\{T_{i1}, T_{i2}\}, \inf\{I_{i1}, I_{i2}\}, \inf\{F_{i1}, F_{i2}\})$ Then, $CONN_{G-\{vi\}}(v_i v_j) = (T_i, I_i, F_i)$, here either
$T_i = T_{i1}(or)T_{i2}(OR)I_i = I_{i1}(or)I_{i2}(OR)F_i = F_{i1}(or)F_{i2}$ which ensures that v_i is not a BSVN-cut vertex. Similar result holds for v_j in the path $v_m v_i v_j$. Therefore, v_i and v_j are not BSVN-cut vertices.

Corollary 3.2.5: If G is a BSVN-cycle of length 3 and μ be any edge with $\mu = h_E$ and $\mu = v_i v_j$, then v_m, (where m≠i,j) is not a BSVN-cut vertex. In addition to that, $CONN_G(v_i v_j) = CONN_{G-\{v_m\}}(v_i v_j) = h_E$

Proof: Consider a path P=$v_i v_j$ in a BSVN-cycle graph of length 3. Given that, $\mu = h_E$ & $\mu = v_i v_j$. It is trivial that there exists two distinct paths between $v_i v_j$. Let $P_1 = v_i v_m v_j$ and $P_2 = v_i v_j$. Strength of the paths $s(P_1)$ and $s(P_2)$ for paths P_1 and P_2 is (T_i, I_i, F_i) and h_E respectively. By the definition of $CONN_G, \mu^{\infty}(v_i v_j) = \mu'^{\infty}(v_i v_j) = h_E$.

Corollary 3.2.5 can also be stated that, A BSVN-cycle graph of length 3 with $\mu = h_E$, where $\mu \in V\ X\ V$ has exactly one vertex $v_i \in V$ (i = 1, 2, 3), which is not a BSVN-cut vertex.

Theorem 3.2.6: If $\mu_1 = h_E$ & $\mu_2 = d_E$ be any two edges in a BSVN-Cycle graph of length $n, n \geq 3$ then it has $(n-2)$ BSVN-cut vertices.

Proof: Let G be a BSVN-Cycle graph of length n and $\mu_1 = h_E$ & $\mu_2 = d_E$. Suppose $\mu_2 = d_E$, then the vertices incident with μ_2 are not BSVN-Cut vertices (By theorem 3.2.1). Let it be v_i and v_j. Let's prove the above statement by induction on the length of the BSVN-Cycle, n. Now Let's take n = 3. Consider a vertex v_m, where m ≠ i, j. Considering the path between v_i and v_j, it is trivial that there exist two distinct paths between v_i and v_j, say P_1 and P_2. Let $P_1 = v_i v_j$ and $P_2 = v_i v_m v_j$. The strength of the path P_1 and P_2 can be given as, $s(P_1) = d_E$ and $s(P_2) = (T_i, I_i, F_i) \neq h_E$. By the definition of strength of connectivity and d_E, $CONN_G(v_i v_j) = (T_i, I_i, F_{i)}$ .And $CONN_G - \{v_m\}(v_i v_j)$ becomes d_E which assures that $\mu_T{}^{\infty}(v_i v_j) > \mu'_T{}^{\infty}(v_i v_j)$, $\mu_I{}^{\infty}(v_i v_j) < \mu'_I{}^{\infty}(v_i v_j)$ & $\mu_F{}^{\infty}(v_i v_j) < \mu'_F{}^{\infty}(v_i v_j)$. From this, v_m is a BSVN-cut vertex can be stated. Hence, the graph of length of 3 has 1 cut vertex, the result is true for n=3. Now, Let's assume that there exists path of length $n-2$ in a BSVN-Cycle graph of length $n-1$ has atmost $(n-3)$ BSVN-cut vertices. One need to show that for a BSVN-Cycle graph of length n with $\mu_1 = h_E$ & $\mu_2 = d_E$, it has $(n-2)$ BSVN-cut vertices. To prove this, consider a path $P = v_i \ldots v_k v_n \ldots v_j$, where $\mu_1 = h_E$, $\mu_2 = d_E$ such that, $\mu_1 = v_k v_n$ & $\mu_2 = v_i v_j$, which is of length n in a BSVN-cycle graph of length $(n-1)$. Such a path exists. Then, in a path $P = v_i \ldots v_k v_n \ldots v_j$,it has either v_k or v_n as cut vertex for any two vertices in the path *P*. Therefore, we cay that the path *P* has has atleast one vertex. Hence, the total number of cutvertices in a path *P* is $n-3+1 = n-2$.(by the assumption, in a BSVN-Cycle graph of length $(n-1)$, there exists a path of length $(n-2)$ which has $(n-3)$ BSVN-cut vertices). Hence, the theorem holds.

Theorem 3.2.7: Let G be a BSVN-cycle graph of length n, $n \geq 4$. If μ be an edge in G such that $\mu = h_E$, where $\mu = v_i v_j$, then v_i or v_j is said to be a BSVN-cut vertex, if it's non-neighbouring vertices on G* is not a BSVN-cut vertex on G.

Proof: Let's verify this statement in a contrapositive way. Consider a path P between v_k and v_n, where k, n ≠ i, j. It is trivial that there exists two distinct paths between v_k and v_n . say P_1 and P_2. Now, Let's take $P_1 = v_k v_{k-1} \ldots v_i v_j \ldots v_n$ & $P_2 = v_k \ldots v_p v_{p-1} \ldots v_n$, where $p \neq i, j, i+1, j+1$. Strength of the paths P_1 and P_2 can be given as,

$$
\begin{aligned}
s(P_1) &= s(v_k v_{k-1} \ldots v_i v_j \ldots v_n) \\
&= (T_l, I_l, F_l), \text{ where } l = k, k-1, k-2, i, j, i+1, j+1, \ldots, n \\
&\neq h_E \\
s(P_2) &= s(v_k \ldots v_p v_{p-1} \ldots v_n) \\
&= (T_m, I_m, F_m), \text{ where } m = k, k-1, k-2, p, p-2 \ldots, n
\end{aligned}
$$

$CONN_G(v_k v_n) = (T, I, F)$, where $T = \sup(T_l, T_m), I = \inf(I_l, I_m), F = \inf(F_l, F_m)$

Assume that , v_i is not a BSVN-cut vertex.

Case – i) $T_m > T_l$ (It is possible since v_i is not a BSVN-cut vertex)

$CONN_G(v_k v_n) = (T_m, I, F)$, since $T = \sup(T_l, T_m) = T_m$

But, $CONN_{G-\{v_p\}}(v_k v_n) = (T_l, I_l, F_l)$

Since $T_l < T_m$, v_p is a T-cut vertex for sure.

Consider $CONN_{G-\{v_i\}}(v_k v_n) = s(P_2)$

Since $T_m > T_l$, definitely v_i is not a T-cut vertex, instead it must be:

i. I-cut vertex.
ii. F-cut vertex.
iii. I & F-cut vertex.
iv. Not I & F- cut vertex.

Case–i a) Suppose $I_l < I_m$, $CONN_G(v_k v_n) = (T_m, I_l, F)$ and,

$CONN_{G-\{v_i\}}(v_k v_n) = s(P_2) = (T_m, I_m, F_m)$

$\Rightarrow \mu_I^{\infty}(v_k v_n) < \mu'_I{}^{\infty}(v_k v_n)$

$\Rightarrow v_i$ is a I-cut vertex for sure.

Case – i b) If $F_l < F_m$, $CONN_G(v_k v_n) = (T_m, I, F_l)$ and,

$CONN_{G-\{v_i\}}(v_k v_n) = s(P_2) = (T_m, I_m, F_m)$

$\Rightarrow \mu_F^{\infty}(v_k v_n) < \mu'_F{}^{\infty}(v_k v_n)$

$\Rightarrow v_i$ is a F-cut vertex for sure.

Case –i c) If $I_l < I_m$ & $F_l < F_m$, $CONN_G(v_k v_n) = (T_m, I_l, F_l)$ and,

$CONN_{G-\{v_i\}}(v_k v_n) = s(P_2) = (T_m, I_m, F_m)$

$\Rightarrow \mu_I^{\infty}(v_k v_n) < \mu'_I{}^{\infty}(v_k v_n)$ & $\mu_F^{\infty}(v_k v_n) < \mu'_F{}^{\infty}(v_k v_n)$

$\Rightarrow v_i$ is I-cut vertex & F-cut vertex.

Case – i d) Suppose v_i is not a BSVN-cut vertex.

$\Rightarrow I_l > I_m$ & $F_l > F_m$ (since, $T_m > T_l$)

Then, $CONN_G(v_k v_n)$ becomes (T_m, I_m, F_m)
And, $CONN_{G-\{v_i\}}(v_k v_n) = s(P_2)$

$$= (T_m, I_m, F_m)$$

$$= CONN_G(v_k v_n)$$

But, if we consider $CONN_{G-\{v_p\}}(v_k v_n)$,

$CONN_{G-\{v_p\}}(v_k v_n) = (T_l, I_l, F_l)$

$\Rightarrow v_p$ is a BSVN-cut vertex. Such a case exists for atleast one $v_k v_n$.

Similar result holds for the following cases:

1. $I_m > I_l$
2. $F_m > F_l$
3. $T_m > T_l$ & $I_m > I_l$
4. $T_m > T_l$ & $F_m > F_l$
5. $I_m > I_l$ & $F_m > F_l$

Similar result holds for vertex v_j.

Observation 3.2.8:

In the above theorem – 3.2.7, it is observed that it's non-neighbouring vertices on G* may be T-cut vertex, I-cut vertex, F-cut vertex or the combination of any two.

The above theorem can used to verify whether any vertex v_i of a BSVN cycle graph of length n is a BSVN-cut vertex or not.

Theorem 3.2.9: If μ be an edge in BSVN-cycle graph G of length $n, n \geq 4$ such that $\mu = h_E$ with any vertex v_i is a BSVN-cut vertex, then exactly one of its non-neighbouring vertices on G* is not a BSVN-cut vertex on G.

Proof: Let G be a BSVN-cycle graph of length $n, n \geq 4$ and $\mu = h_E$ such that $\mu = v_i v_j$. Given that, v_i or v_j is a BSVN-cut vertex. Let's assume that v_i is BSVN-cut vertex. Let's consider path between two vertices v_k and v_n. $P_1 = v_k v_{k-1} .. v_i v_j .. v_n$ & $P_2 = v_k .. v_p .. v_n$. By the definition of strength of path, $s(P_1) = (T_l, I_l, F_l)$

$$s(P_1) = (T_l, I_l, F_l) \quad \text{where } l = k, k-1, k-2, i, j, i+1, j+1, \ldots, n$$
$$\neq h_E$$
$$s(P_2) = s(v_k .. v_p .. v_n)$$
$$= (T_m, I_m, F_m) \quad m = k, k-1, k-2, i, j, i+1, j+1, \ldots, n$$

Since v_i is a cut vertex, $T_m < T_l, I_m > I_l, F_m > F_l$

Now, $CONN_G(v_k v_n) = (T_l, I_l, F_l)$

Consider $CONN_{G-\{v_p\}}(v_k v_n) = (T_l, I_l, F_l)$

$$= CONN_G(v_k v_n)$$

$\Rightarrow v_p$ is not BSVN-cut vertex. Hence, the result holds.

Theorem 3.2.10: Let G be a BSVN-cycle graph of length 3. If $\mu = h_E$, then μ is a BSVN-bridge.

Proof: G be a BSVN-cycle graph of length 3 such that $\mu = h_E$ with $\mu = v_i v_j$. Now, Considering the paths between v_i and v_j, it is trivial that there exists two distinct paths between them, say P_1 and P_2. Let $P_1 = v_i v_j$ and Let $P_2 = v_i v_k v_j$. By the definition of strength of path, $s(P_1) = h_E$ & $s(P_2) = (T_i, I_i, F_i)$. By the definition of strength of connectivity, $\mu^{\infty} = h_E$ and $CONN_{G-\{v_i v_j\}}(v_i v_j) = \mu'^{\infty} = (T_i, I_i, F_i)$, which ensures that $v_i v_j$ is a BSVN-bridge. Hence, the result holds.

Theorem 3.2.11: Let G be a BSVN-cycle graph of length 3. If $\mu = d_E$, then μ is not a BSVN-bridge.

Proof: Consider a path between v_i and v_j in a BSVN-cycle graph G of length 3 such that $\mu = d_E$ with $\mu = v_i v_j$ Let P_1 and P_2 be two distinct paths between v_i and v_j. say, $P_1 = v_i v_j$ and Let $P_2 = v_i v_k v_j$. By the definition of strength of path, $s(P_1) = d_E$ & $s(P_2) = (T_i, I_i, F_i)$. By the definition of strength of connectivity, $\mu^{\infty} = (T_i, I_i, F_i)$ and $CONN_{G-\{v_i v_j\}}(v_i v_j) = \mu'^{\infty} = (T_i, I_i, F_i) = \mu^{\infty}$ .which states that $v_i v_j$ is not a BSVN-bridge. Hence, the result holds.

Theorem 3.2.12: If $\mu = d_E$, being an edge in a BSVN-cycle graph G of length n, then G has $(n-1)$ BSVN-bridges.

Proof: It is significant to recall that if G be a SVNG such that G* is a cycle, then a vertex is a SVN-cut vertex of G if and only if it is a same vertex of two SVN-bridges. Since all SVN-graphs are BSVN-graphs, the above result holds for BSVN-graphs also. Also, by theorem – 3.2, if $\mu = d_E$ in a BSVN-cycle graph G of length n, then G has $(n-2)$ BSVN-cut vertices. Hence the stated result can be obtained.

Theorem 3.2.13: If $\mu_1 = d_E$ and $\mu_2 = h_E$, being any two edges in a BSVN-cycle graph G of length n, then G has $(n-1)$ BSVN-bridges.

Proof: It is significant to remember that if G be a SVNG such that G* is a cycle, then a vertex is a SVN-cut vertex of G if and only if it is a same vertex of two SVN-bridges. Since all SVNG are BSVNG, the above result holds for BSVNG also. In addition to that, if $\mu_2 = d_E$ & $\mu_1 = h_E$ in a BSVN-cycle graph G of length n, then G has $(n-2)$ BSVN-cut vertices. Hence the stated result holds.

4. Application of BSVN-Graphs in Fraud detection analysis in Social Networks:

Detecting fraud on social media platforms requires identifying fake accounts, inauthentic interaction and deceptive groups. As these networks are so large and constantly changing, the analytical methods used must be able to handle uncertainty, process information instantly and adapt to real-time changes. Since, Fraudsters constantly adapt their tactics, including creating networks for fake engagements, automating interactions with bots, to disseminate deceptive content, the need to recalculate interval-based values in evolving networks makes them unsuitable for efficient, continuous fraud monitoring. In this scenario, the essential characteristics of Neutrosophic graphs should be overlooked. As a result, this situation captures the real essence of SVNGs to enable more precise way of monitoring to emerging threats by maintaining up-to-date representation of networks.

Mathematical representation of vertices and edges: A vertex $v_i = (T_v, I_v, F_v)$ represents a user in a social network,where, $T_v \rightarrow$ quantifies the user is legitimate. $I_v \rightarrow$ quantifies the uncertainty in classification & $F_v \rightarrow$ measures the user as fraudulent. In this illustration, it can be benchmarked that the user with h_v indicates a trustworthy user, while d_v signals a potential fraudster. An edge $v_i v_j = (T_{ij}, I_{ij}, F_{ij})$ represents an interaction between the users like friend requests or messages, where $T_{ij} \rightarrow$ measures the strength of genuine connection, $I_{ij} \rightarrow$ measures the uncertainty in interactions (like limited requests) & $F_{ij} \rightarrow$ measures the fraudulent interactions (like bot-generated messages). In this scenario, an interactions with d_E signifies fraudulent connection while an interactions with h_E demonstrates a trusted link. Suppose, in a hypothetical situation,

User	Truth (T)	Indeterminacy(I)	Falsity(F)	Status
A (Real user)	0.8	0.05	0.02	Genuine
B (Scammer)	0.3	0.7	0.85	Fraudulent

Table -1 (a)

Interactions	Truth (T)	Indeterminacy(I)	Falsity(F)	Status
A→B(money transfer)	0.7	0.2	0.1	Likely normal
B→A (phishing link)	0.1	0.35	0.7	Fraudulent

Table -1 (b)

In this use case, user A is real but may be tricked to B's interaction. If $T_{BA} = 0.3$,then it is predicted that the interactions with A is a Phishing attempt, which may not be the actual scenario. Thus, to improve fraud detection accuracy, distinct membership values is needed. Hence, the application of BSVN Graphs arose without artifice, as a effective response to meet this particular needs. Money laundering chains, bot networks often manifest as cycles within social networks. BSVN cycles offers a way to prevent this problem from escalating. Within the framework of cycles, if multiple users with d_V form a cycle, it signals a potential fraud ring whereas a cycle with d_E signifies a suspicious network. And, it suggests a warranting immediate investigation to mitigate scams & security risks. If a cycle is formed with h_E , then it clearly benchmarks a strong, trusted & legitimate interaction. At this juncture, user took down multiple genuine users account may be a central controller in a bot network and the connection which dismantles the trusted network between any two users is a pivotal link in fraudulent network.

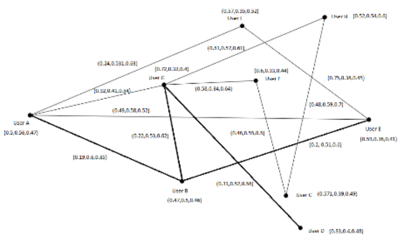

Fig 2: Interaction between the users in Instagram

Here,In a hypothetical case, the interaction between the users A-I has been viewed as a particular piece of Bijective Single valued Neutrosophic Graph to observe which user has to be investigated to disrupt fraudulent networks.

In this particular scenario, user A, user B, user I & user E forms a group , user A, user B & user G have banded together. Similarly, there is another group which consists of user A, user B & user E. And, Finally user G, user F, user C, user H got teamed up. Among four groups, user A & B constitute as a member of three groups. While looking into their membership function,

Users	Truth-membership function	Indeterminacy-membership function	Falsity-membership function
A	0.5	0.56	0.47
B	0.48	0.5	0.46

Table -2

Here, challenge comes in removing the hidden fraud enabler in the system whose removal will effectively disrupt the fraudulent activities in the system. One more crucial point is the interaction between user A and user B is also suspicious. In this context, BSVN-cycles lend us a helping hand to enhance the trustworthy functionality of the network.

Consequences of dispatching user A from the system:

- ✓ It is also significant to observe that the interaction between user I and E is authentic. Applying corollary -3.2.5 in a team formed by user A, I & E, it is evident that removing user A won't affect the credibility between user I and E.
- ✓ Also,if user A is extracted from the network formed by him with user B & E, the trusted network formed by any two users (in group of user A , B and E) won't get dismantled (Applying theorem- 3.2.2). Same applies for user B.
- ✓ In a squad of user A, user G and user B, it is obvious that the user G is the most trusted user when compared to user A & B. Removing the interaction between user A & B will have no effect in breaking the trustworthy interactions in the group whereas eliminating the other two interactions (i.e. the interactions between user A & G and between user G & B) will increase the interaction of fraudulent activities. (Application of theorem 3.2.11 & theorem 3.2.12)

Considering all these crucial insights in mind, if we remove user A from the system, the communication between user B and G will remain as: $(T_{BG}, I_{BG}, F_{BG}) = (0.22, 0.53, 0.62)$. Suppose user B get's dispatched from the network, here the communication between user A and G will remain as $(T_{AG}, I_{AG}, F_{AG}) = (0.32, 0.41, 0.54)$. Since $T_{BG} < T_{AG} \,\&\, F_{BG} > F_{AG}$, it is evident that deactivating B's interaction with other users will build the network validated.

Comprehensive analysis of a cycle formed by user G, H, F & C:

Users	Truth-membership function	Indeterminacy-membership function	Falsity-membership function
G	0.72	0.32	0.4
H	0.52	0.4	0.6
F	0.6	0.33	0.44
C	0.571	0.39	0.49

Table – 3

Clearly, T_G, T_H, T_F and T_C indicates that users G, H, F & C are trusted users. Their network needs to be verified separately to dispatch the suspicious activity.

Interrelation between user G and D:

- ✓ Since user G is genuine, he may be tricked to share fraudulent information with user D. Therefore, instead of removing user G, the other connections of user G can be evaluated to suspect the fraudster. As per the latest data, the communication of user D with user G can be disrupted. (It won't affect any interactions since user G and D aren't in any groups).

Analytical study of the cycle formed by user A, B, I and E:

- ✓ Since the engagement of user A and B is highly suspicious, if the user I and E get's dispatched from the cycle, the risk of increasing the fraudster communication with any two of them will get out of control. Hence, it is better to track the communication status of them to find the efficient solution.

It is essential to note that across all these examined scenarios, the data and graph evaluated here are purely hypothetical to analyse the better outcomes. If needed, more interactions for a particular user can also be considered to get a optimal solution.

5. Conclusion:

In this paper, a new concept of BSVN graph has been explored with some connectivity relations using height and depth. Significance of BSVN graph lies in its Bijective condition. Due to this specific condition it finds its applications in cyber security, especially fraud detection analysis in social networks. It has certain limitations like scalability in large network and need for real world validation of the proposed fraud detection analysis. In future some more properties like order, size, bridges and isomorphism of BSVN graph can be explored which further deepen our work to find its application in many fields.

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Conflicts of Interest

The authors declare no conflict of interest.

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