Neutrosophic Sets and Systems, Vol. 83, 2025
University of New Mexico

Pentapartitioned Neutrosophic Pythagorean Connectedness

R. Radha [1],*, Annapoorna M S[2] , S. Vasundhara[3] , Parul Arora[4], Ankita Tiwari[5] and R.Prema [6]

[1] Assistant Professor; Department of Mathematics, Karpagam College of Engineering, Coimbatore (TN), India. radharmat2020@gmail.com

[2] Assistant Professor, Department of Mathematics, BMS Institute of Technology and Management, Bengaluru, Karnataka, India. anuraga@bmsit.in

[3] Assistant Professor, Department of Humanities and Mathematics, G.Narayanamma Institute of Technology and Science, Shaikpet, Hyderabad, Telangana . vasucall123@gmail.com

[4] Associate Professor, Department of Computer Science and Applications, Bharati Vidyapeeth's Institute of Computer Applications and Management (BVICAM), Paschim Vihar, New Delhi. paruldevsum@gmail.com

[5] Assistant Professor, Department of Engineering Mathematics, College of Engineering, Koneru Lakshmaiah Education Foundation, Guntur, Vaddeswaram, AP, India. tdrankita@gmail.com

[6] Associate Professor & Head, Department of Commerce(IT), KPR College of Arts Science and Research, Coimbatore. rprems6@gmail.com.

* Correspondence: radharmat2020@gmail.com,kathirsubbu1987@gmail.com

Abstract: The aim of this paper is to introduce the concept of Pentapartitioned Neutrosophic Pythagorean [PNP] connectedness is introduced and its properties are also studied. Also we investigate some interrelations between these types of Pentapartitioned Neutrosophic Pythagorean connectedness. We show that the continuous image of Pentapartitioned Neutrosophic Pythagorean connected space is PNP connected.

Keywords: Neutrosophic set, Pentapartitioned Neutrosophic pythagorean set, Pentapartitioned Neutrosophic Pythagorean connected .

1. Introduction

Zadeh [14] introduced the idea of fuzzy sets in 1965 that permits the membership perform valued within the interval[0,1] and set theory its an extension of classical pure mathematics. Intuitionistic Fuzzy set was first introduced by K. T. Atanassov [1] in 1983. After that he introduced, the concept of Intuitionistic sets as generalization of Fuzzy sets. The concept of generalized topological structures in Fuzzy topological spaces using Intuitionistic Fuzzy sets was introduced by D. Coker[3] . D. Coker introduced the concept of Intuitionistic Fuzzy sets, Intuitionistic Fuzzy topological spaces, Intuitionistic topological spaces and Intuitionistic Fuzzy points. R. R. Yager [13] generalized Intuitionistic Fuzzy set and presented a new set called Pythagorean set.

Florentine Smarandache [11] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]−0 1+[. Since neutrosophic set deals the indeterminateness effectively it plays an very important role in several applications areas embrace info technology, decision web, electronic database systems, diagnosis, multicriteria higher cognitive process issues etc., Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [10]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. Further, R. Radha and A. Stanis Arul Mary [5] outlined a brand new hybrid model of Pentapartitioned Neutrosophic Pythagorean sets (PNPS) and Quadripartioned neutrosophic pythagorean sets in 2021. The Pentapartitioned Neutrosophic Pythagorean topological spaces [9] was introduced and its properties are investigated in 2021.A. Haydar introduced the concept of connectedness in Pythagorean fuzzy topological spaces.In this paper, we have applied the concept of connectedness in Pentapartitioned Neutrosophic Pythagorean Topological Spaces.

2 Preliminaries

2.1 Definition [11]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where T_A , $I_A, F_A: U \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

2.2 Definition [6]

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A + F_A \le 1, C_A + U_A \le 1$ and

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \le 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and I_A (x) is an unknown membership.

2.3 Definition [14]

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \le 5$$

2.4 Definition [6]

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

2.5 Definition [6]

Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are pentapartitioned neutrosophic pythagorean sets. Then

A ∪ B = $< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x))$, min $(G_A(x), G_B(x))$,

min $(U_A(x), U_B(x))$, min $(F_A(x), F_B(x)), >$

A ∩ B = $< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x))$, max $(G_A(x), G_B(x))$

$, max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$

2.6 Definition[9]

A PNP topology on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

1) 0,1∈ τ
2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
3) $\bigcup R_i \in \tau$ for any $R_i : i \in I \subseteq \tau$

The complement R* of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R,$\boldsymbol{\tau}$), is called a PNP closed set [PNPCS].

2.7 Definition [9]

Let (X, τ) be an PNPTS and A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ be an PNPS in X. Then the interior and the closure of A are denoted by PNPInt(A) and PNPCl(A) and are defined as follows.

PNPCl(A) = ∩{K∣K is an PNPCS and A ⊆ K} and

PNPInt(A) = ∪{G∣G is an PNPOS and G ⊆ A}

Also, it can be established that PNPCl(A) is an PNPCS and PNPInt(A) is an PNPOS, A is an PNPCS if and only if PNPCl(A) = A and A is an PNPOS if and only if PNPInt(A) = A. We say that A is PNP-dense if PNPCl(A) = X.

3. Pentapartitioned Neutrosophic Pythagorean Connected Spaces.

3.1 Definition

A Pentapartitioned Neutrosophic Pythagorean topological space is said to be PNP disconnected if there exists PNP open set U,V in X ,U≠0, V≠0, such that U∩V = 0 and U∪V = 1. If X is not PNP disconnected then it is said to be PNP connected.

3.2 Example

Let X = {a,b}, τ = {0,1,A,B}, where A ={(a,0.2,0.3,0.6,0.7,0.7),(b,0.1,0.2,0.6,0.5,0.5)} and B = {(a,0.3,0.4,0.6,0.6,0.6), (b,,0.2,0.3,0.6,0.4,0.4)}. Then A and B are PNP open sets in X and A∪B = B ≠ 1, A∩ B = A ≠0. Hence X is PNP connected.

3.3 Example

Let X = {a,b},τ = {0,1,A}, where A ={(a,0.2,0.3,0.6,0.7,0.7),(b,0.1,0.2,0.6,0.5,0.5)} and B = {(a,1,1,0,0,0),(b,,0,0,1,1,1)} and C={(a,0,0,1,1,1),(b,1,1,0,0,0)}. Then A and B are PNP open sets in X and A∪B = B ≠ 1, A∩ B = A ≠0. Hence X is PNP connected.

3.4 Definition

Let N be a PNPTS(X,τ).

a) If there exists a PNP preopen sets U and V in X satisfying the following properties, then N is called PNP CI disconnected(I=1,2,3,4)

C1 : $N \subseteq U \cup V$, $U \cap V \subseteq N^*$, $N \cap U \neq 0$, $N \cap V \neq 0$.

C2 : $N\subseteq U \cup V$, $N \cap U \cap V = 0$, $N \cap U \neq 0$, $N \cap V \neq 0$.

C3 : $N \subseteq U \cup V$, $U \cap V \subseteq N^*$, $U \not\subset N^*$,$V \not\subset N^*$.

C4 : $N \subseteq U \cup V$, $N \cap U \cap V = 0$, $U \not\subset N^*$, $V \not\subset N^*$.

b) N is said to be PNP CI - connected(I=1,2,3,4) if N is not PNPCI -disconnected(I =1,2,3,4).

Obviously, we can obtain the following implicationsbetween several types of PNP CI -connected sets(I = 1,2,3,4).

Figue PNP CI - Connectedness

$$\begin{array}{ccc} 1 & \longrightarrow & 3 \\ \downarrow & & \downarrow \\ 2 & \longrightarrow & 4 \end{array}$$

1) PNP C1 – connectedness
2) PNP C2 – connectedness
3) PNP C3 – connectedness
4) PNP C4 – connectedness

3.5 Example

Let X = {a,b,c} and define the PNP subsets A,B,C and D as follows

A = {(a,0.5,0.4,0.7,0.1,0.2),(b,0.5,0.4,0.7,0.3,0.4),(c,0.4,0.3,0.7,0.3,0.4)}

B = {(a, 0.4,0.3,0.7,0.4.0.5),(b,0.6,0.5.0.7,0.2,0.3),(c,0.2,0.1,0.7,0.2,0.3}

C = {(a,0.5,0.4,0.7,0.1,0.2),(b,0.6,0.5,0.7,0.2,0.3),(c,0.4,0.3,0.7,0.2,0.3)}

D = {(a,0.4,0.3,0.7,0.4,0.5),(b,0.5,0.4,0.7,0.3,0.4),(c,0.2,0.1,0.7,0.3,0.4)}

Then = {0,1, A,B,C,D} is a PNP topological Space on X and consider the PNP set E be

E = {(a,0.6,0.5,0.7,0.1,0.2),(b,0.5,0.4,0.7,0.1,0.2),(c,0.4,0.3,0.7,0.2,0.3)} in X.

Then E is PNP C1- connected , PNP C2 – connected, PNP C3 – connected and PNP C4- connected.

3.6 Example

Consider the PNP topological space (X. τ) given in the Example 3.5 and consider the PNP subset be

F = {(a, 0.2,0.1,0.7,0.3,0.4),(b,0.3,0.2,0.7,0.5,0.6),(c, 0.2,0.1,0.7,0.3,0.4)}

Since $F \cap A \cap B \neq 0$, $F \cap A \cap C \neq 0$, $F \cap A \cap D \neq 0$, $F \cap B \cap C \neq 0$, $F \cap B \cap D \neq 0$ and $F \cap C \cap D \neq 0$, F is PNP C1 – disconnected and hence not PNP C1 – connected.

3.7 Definition

Let (X. τ) be a PNP topological space . If there exists a PNP regular open set A in X such that $0_X \neq$ A$\neq 1_X$, then X is called PNP super disconnected.

3.7 Example

Let X = {a,b,c} and define the PNP subsets A,B,C as follows

A = {(a,0.4,0.3,0.7,0.4,0.5),(b,0.6,0.5,0.7,0.2,0.3),(c,0.2,0.1,0.7,0.2,0.3)}

B = {(a, 0.5,0.4,0.7,0.1.0.2),(b,0.6,0.5.0.7,0.2,0.3),(c,0.4,0.3,0.7,0.2,0.3}

C = {(a,0.4,0.3,0.7,0.4,0.5),(b,0.5,0.4,0.7,0.3,0.4),(c,0.2,0.1,0.7,0.3,0.4)}

Then τ = {0,1, A,B,C,D} is a PNP topological Space on X and (X, τ) is PNP super connected.

3.8 Theorem

In a PNP topolological space(X,τ) , the following conditions are equivalent

1) X is PNP super connected.
2) For each PNP open set A $\neq 0_X$ in X we have PNPCl(A) = 0_X.
3) For each PNP closed set A $\neq 1_X$ in X we have PNPInt(A) = 0_X.
4) There exists no PNP open set A and B in X such that A$\neq 0_X \neq B$, $A \subseteq B^C$.
5) There exists no PNP open set A and B in X such that A$\neq 0_X \neq B$, B = [PNPCl(A)]^C, A = [PNPCl(B)]^C.
6) There exists no PNP closed set A and B in X such that A$\neq 1_X \neq B$, B = [PNPInt(A)]^C, A = [PNPInt(B)]^C.

Proof

(1$\Longrightarrow$ 2)

Assume that there exists a PNP open set A $\neq 0_X$ such that PNPCl(A) $\neq 1_X$. Since B = PNPInt(PNPCl(A)) is a PNP regular open set in X and this is a contradiction with PNP super connectedness of X.

(2$\Longrightarrow$ 3)

Let A$\neq 1_X$ be a PNP closed set in X. If we take B = A^C, then B is a PNP open set in X and AB $\neq 0_X$. Hence PNPCl(B) = $1_X \Rightarrow$ [PNPCl(B)]$^C = 0_X \Rightarrow$ PNPInt(B^C) $= 0_X \Rightarrow PNPInt(A) = 0_X$.

(3$\Longrightarrow$ 4)

Let A and B be PNP sets in X such that A$\neq 0_X \neq B$ and A$\subseteq B^C$. Since B^C is a PNP closed set in X and B $\neq 0_X \Rightarrow B^C \neq 1_X$,we obtain PNPInt($B^C$) = 0_X. Now, we have $0_X \neq A = PNPInt(A) \subseteq$PNPInt($B^C$)= 0_X, which is a contradictoion.

(4$\Longrightarrow$ 5)

It is obvious

(1$\Longrightarrow$ 5)

Let A and B be PNP open sets in X such that A$\neq 0_X \neq B$ and B = [PNPCl(A)]^C, A = [PNPCl(B)]^C.Now, we have PNPInt(PNPCl(A)) = PNPInt(B^C)= [PNPCl(B)]$^C = A$ and A $\neq 0_X$, A $\neq 1_X$. This is a contradiction.

(5⟹ 1)

It is ibvious

(5⟹ 6)

Let A and B be PNP closed set in X such that A$\neq 1_X \neq B$, B = [PNPInt(A)]^C, A = [PNPInt(B)]^C. Taking U = A^C and V = B^C, U and V become PNP open sets and U $\neq 0_X \neq V$, [PNPCl(U)]^C = V and [PNPCl(V)]^C = U. This is a contradiction

(6⟹ 5)

This is similar to (5⟹ 6)

Hence the proof.

3.9 Definition

Let (X, τ) be a PNP topological space

1) X is said to be PNP C5- disconnected if there exists a PNP open and PNP closed set G such that G$\neq 1_X$ and G $\neq 0_X$.
2) X is said to be PNP C5- connected if it is not PNP C5-disconnected.

3.10 Example

Let X = {a,b} and define the PNP subsets A,B,C and D as follows

A = {(a,0.4,0.3,0.7,0.2,0.3),(b,0.2,0.1,0.7,0.6,0.7)}

B = {(a, 0.3,0.2,0.7,0.3.0.4),(b,0.7,0.6.0.7,0.1,0.2)}

C = {(a,0.3,0.2,0.7,0.3,0.4),(b,0.2,0.1,0.7,0.6,0.7)}

D = {(a,0.4,0.3,0.7,0.2,0.3),(b,0.7,0.6,0.7,0.1,0.2)}

Then τ= {0,1, A,B,C,D} is a PNP topological Space on X and (X,τ) is a PNP disconnected, Since A is a non-zero PNP open and PNPclosed set in X.

3.11 Theorem

PNP C5-connectedness implies PNP connectedness.

3.12 Theorem

Let (X,τ) ,(Y,σ) be two PNP topological spaces and f: X⟶Y be a PNP continuous surjection. If (X,τ) is PNP connected, then so is (Y,σ).

Proof:

On the contrary, suppose that is PNP disconnected. Then there exist PNP open sets A$\neq 0_Y$, B$\neq 0_Y$ in Y such that A ∪ B =1_Y,$A \cap B = 0_Y$.Now , we see that U = $f^{-1}(A), V = f^{-1}(B)$ are PNP open sets in X, since f is PNP continuous. From A$\neq 0_Y$, we get U = $f^{-1}(A) \neq 0_X$. Similarly, V$\neq 0_X$. Hence , A$\cup B = 1_Y \Rightarrow f^{-1}(A)\cup f^{-1}(B) = f^{-1}(1_Y) = 1_X \Longrightarrow U \cup V = 1_X, A \cap B = 0_Y \Longrightarrow f^{-1}(A) \cap f^{-1}(B) = f^{-1}(0_Y) = 0_X \Longrightarrow U \cap V = 0_X$. But this is a contradiction to our hypothesis, i.e. Y is PNP connected.

3.13 Corollary

Let (X,τ) ,(Y,σ) be two PNP topological spaces and f: X⟶Y be a PNP continuous surjection. If (X,τ) is PNP C5-connected, then so is (Y,σ).

3.14 Definition

A PNP topological space (X,τ) is said to be PNP strongly connected if there exists nonzero PNP closed sets A and B such that $TR_A + TR_B \leq 1, CR_A + CR_B \leq 1, GR_A + GR_B \geq 1, UR_A + UR_B \geq 1 and\ FR_A + FR_B \geq 1$.

3.15 Example

Let X = {a,b} and define the PNP subsets A and B as follows

A= {(a,0.3,0.2,0.8,0.5,0.6), (b,0.4,0.4,0.8,0.4,0.5)}

B = {(a,0.4,0.3,0.8,0.4, 0.5),(b,0.1,0.1,0.8,0.2,0.3)}.

Then the family τ= {0,1,A,B,A∪B, A∩B} is a PNP topology on X and (X,τ) is a PNP strongly connected.

3.16 Theorem

Let (X,τ) ,(Y,σ) be two PNP topological spaces and f: X⟶Y be a PNP continuous surjection. If (X,τ) is PNP strongly connected, then so is (Y,σ).

Proof

This is analogous to proof of theorem 3.12.

It is clear that in PNP topological spaces, PNP strongly connectedness does not imply PNP C5-connectedness, and the same is true for its converse.

3.17 Example

Let X = {a,b} and define the PNP subsets A,B,C and D as follows

A = {(a,0.4,0.3,0.7,0.2,0.3),(b,0.2,0.1,0.7,0.6,0.7)}

B = {(a, 0.3,0.2,0.7,0.3.0.4),(b,0.7,0.6,0.7,0.1,0.2)}

C = {(a,0.3,0.2,0.7,0.3,0.4),(b,0.2,0.1,0.7,0.6,0.7)}

D = {(a,0.4,0.3,0.7,0.2,0.3),(b,0.7,0.6,0.7,0.1,0.2)}

Then = {0,1, A,B,C,D} is a PNP topological Space on X and (X,τ) is PNP strongly connected, but not PNP C5- connected.

3.18 Example

Let X = {a,b} and define the PNP subsets A and B as follows

A = {(a,0.5,0.4,0.7,0.3,0.4),(b,0.6,0.5,0.7,0.2,0.3)}

B = {(a, 0.5,0.4,0.7,0.1.0.2),(b,0.4,0.3,0.7,0.3,0.4)}

Then = {0,1, A,B,A∪ B, A∩ B} is a PNP topological Space on X and (X,τ) is PNP C5- connected., but not PNP strongly connected.

Conclusion

In this paper, we introduced the notion of PNP connected space which extends the notions of Neutrosophic spaces and Pythagorean topological spaces. The presented concepts in this study are fundamental for further researchers and will open a way to improve more applications on PNP topology.

References

1. K. Atanasov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*.**1986**,volume 20 87-96.
2. Broumi S, Smarandache F (2014) Rough Neutrosophic sets, *Ital J Pure Appl Math* ,***2014,*** *volume* 32:493-502.
3. Coker D., An introduction to Intuitionistic Fuzzy points, submitted to notes on IFS.
4. Haydar Es.A, Connectedness in Pythagorean fuzzy Topological Spaces, IJMTT, volume 65, Issue 7,2019
5. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic pythagorean Soft set, *IRJMETS*, **2021 ,** *Volume 3(2)*,905-914.
6. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, *IRJASH*, **2021**, *volume 3*, 62-82.
7. R. Radha, A. Stanis Arul Mary. Heptapartitioned neutrosophic sets, *IRJCT,* ***2021*** *,volume 2*,222-230.
8. R. Radha, A. Stanis Arul Mary. Neutrosophic Pythagorean soft set, *Neutrosophic sets and systems,* **2021,** ***vol*** *42*,65-78.
9. R. Radha, A.Stanis Arul Mary, Pentapartitioned Neutrosophic Pythagorean Topological Spaces, Journal of XI' AN University of Architecture and Technology,
10. Rama Malik, Surapati Pramanik. Pentapartitioned Neutrosophic set and its properties, *Neutrosophic Sets and Systems*, **2020,** *Vol 36*,184-192.
11. F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, *American Research Press, Rehoboth*.
12. Wang H, Smarandache F, Zhang YQ, Sunderraman R Single valued neutrosophic sets, *Multispace Multistruct* ,***2010*** *,volume* 4:410-413.
13. R.R.Yager, Pythagorean Fuzzy subsets, In, Proc Joint IFSA World Congress and NAFIPS Journal,
14. L.Zadeh , Fuzzy sets, *Information and Control* **1965**, volume 8, 87-96.

Received: Nov. 3, 2024. Accepted: April 16, 2025