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A Novel Approach in Heptapartitioned Neutrosophic Sets with its Weighted Arithmetic Averaging Operator

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Abstract: The aim of this paper is to introduce the heptapartitioned neutrosophic weighted arithmetic averaging operator and also some of its basic algebraic properties of heptapartitioned neutrosophic set. A score and accuracy function of heptapartitioned neutrosophic set has been established. With the proposed score function, accuracy function and averaging operator a multi-criterion decision making problem has been solved and ranked with their alternatives by using TOPSIS technique with different weightages which has been compared with the MOORA method.

Keywords: Neutrosophic set, Quadripartitioned neutrosophic set, Pentapartitioned set, Heptapartitioned neutrosophic set and MCDM strategy.

1. Introduction

Fuzzy logic, introduced by Lotfi Zadeh [1,2], shifts from traditional binary reasoning to a more nuanced approach that accounts for uncertainty and imprecision. Unlike classical logic, which deals with binary outcomes (true or false), fuzzy logic allows for degrees of truth, making it useful for decision-making in complex or uncertain scenarios. Researchers expanded on this by introducing new models to handle uncertainty in more sophisticated ways.

Krassimir Atanassov's [3] intuitionistic fuzzy sets consider both the degree of membership and non-membership in a set, improving uncertainty management. Building on this, Smarandache [4] introduced neutrosophic sets, which consist of three components—truth, indeterminacy, and falsity—capturing a broader range of possibilities when information is contradictory or uncertain. This led to models like interval-valued neutrosophic sets, neutrosophic soft sets, and bipolar neutrosophic sets, further enhancing uncertainty management across various domains. Salama et al. [5], has done a study on exploring the potential of neutrosophic topological spaces in computer science. Satyanarayana et al. [6] explored on the polarity of generalized neutrosophic ideals in BCK-Algebra.

The quadripartitioned neutrosophic set (QPNS), introduced by Rajashi Chatterjee [7], refined indeterminacy representation, leading to applications like neutrosophic graphs. These graphs, explored by Hussain et al. [8], have real-life uses in complex decision-making. Mohanasundari et al. [9] advanced this by developing weighted aggregation operators for decision-making, facilitating the application of QPNS in more intricate scenarios.

The pentapartitioned neutrosophic set, introduced by Mallik and Pramanik [10], divided indeterminacy into specific categories like contradiction, unknown, and ignorance. This was further refined by Radha and Stanis [11], who added layers of relative truth and falsity. Das et al. [12, 13] extended this by examining pentapartitioned neutrosophic topological spaces, while Pramanik [14] explored interval-valued pentapartitioned neutrosophic sets, enhancing their properties and applications. Broumi et al. [15] advanced the study of pentapartitioned neutrosophic graphs, focusing on their real-time problem-solving potential in dynamic systems. A notable application of these graphs was demonstrated by Quek et al. [16] during the COVID-19 pandemic, where they were used to determine the safest paths and towns, showing the practical relevance of neutrosophic sets in crisis management.

Broumi et al. [17] also examined heptapartitioned neutrosophic sets, further expanding neutrosophic theory. The bipolar single-valued heptapartitioned neutrosophic set, introduced by Ali et al. [18], is used to solve multi-criteria decision-making (MCDM) problems [19, 20, 21], with techniques like TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [22, 23,24] and MOORA (Multi-Objective Optimization by Ratio Analysis) [25, 26] helping decision-makers evaluate alternatives in complex, multi-criteria scenarios. These methods have widespread applications in areas like site selection, supplier evaluation, healthcare decision-making, and environmental management.

This study's goal is to close the gaps by utilizing a comprehensive framework that incorporates efficient sample addresses through the use of heptapartitioned fuzzy logic and neutrosophic techniques. Using the heptapartitioned neutrosophic set (HPNS) and its weighted arithmetic averaging operator, we presented a new method. We thoroughly examined the characteristics of HPNS by putting out and demonstrating two claims as well as a theorem. We use the HPNS framework to solve a multi-criteria decision-making problem using the TOPSIS method to illustrate the usefulness of this approach and provide a novel viewpoint on managing complexity and uncertainty in decision-making processes. The MOORA approach yielded the same outcome as the TOPSIS method, which is contrasted with it. Our goal is to add to the expanding corpus of research on neutrosophic sets and their useful applications in real-world scenarios.

2. Materials and Methods

With the extension of neutrosophic, quadripartitioned, pentapartitioned and heptapartitioned a new approach is proposed on heptapartitioned neutrosophic sets. The physical structure of heptapartitioned neutrosophic sets have been explained with illustrative examples, some of the algebraic properties were discussed with proofs. This work will enhance the future related works in optimizing, neural networks, modeling structures, graphs, probabilistic and statistical methods etc.

The main limitation of heptapartitioned neutrosophic fuzzy sets is their increased complexity compared to standard neutrosophic sets, which can make them computationally expensive to work with, especially when dealing with large datasets or intricate decision-making problems; additionally, the interpretation of the additional membership degrees can be challenging and may require further refinement depending on the application area.

3. Preliminaries

Definition 1. Fuzzy Set [1]

Let U be a non-empty set. A fuzzy set A on U can be defined as follows:

$A = \{(x, \mathcal{T}_A(x): x \in U)\}$, we assume that $\mathcal{T}_A(x) \in [0,1]$. This is so called membership function, whereas $1 - \mathcal{T}_A(x)$ is the non-membership function.

Definition 2. Neutrosophic set [4]

Let U be a non-empty set. A neutrosophic set N on U can be defined as follows:

$N = \{(x, \mathcal{T}_N(x), I_N(x), \mathcal{F}_N(x): x \in U)\}$, whereas $\mathcal{T}_N(x), I_N(x), \mathcal{F}_N(x): U \to [0,1]$ and $0 \le \mathcal{T}_N(x) + I_N(x) + \mathcal{F}_N(x) \le 3$. Here $\mathcal{T}_N(x)$ is truth membership, $I_N(x)$ is indeterminacy and $\mathcal{F}_N(x)$ is falsity membership function.

Definition 3. Quadripartitioned neutrosophic set [7]

Let U be a non-empty set. A quadripartitioned neutrosophic set Q on U can be defined as follows:

$Q = \{(x, \mathcal{T}_Q(x), C_Q(x), U_Q(x), \mathcal{F}_Q(x): x \in U)\}$, whereas

$\mathcal{T}_Q(x), C_Q(x), U_Q(x), \mathcal{F}_Q(x): U \to [0,1]$ and $0 \le \mathcal{T}_Q(x) + C_Q(x) + U_Q(x) + \mathcal{F}_Q(x) \le 4$. Here $\mathcal{T}_Q(x)$ is truth membership, $C_Q(x)$ is contradiction, $U_Q(x)$ is ignorance and $\mathcal{F}_Q(x)$ is false membership function. In quadripartitioned neutrosophic set, an indeterminacy has been split into two parts.

Definition 4. Pentapartitioned neutrosophic set [10]

Let U be a non-empty set. A pentapartitioned neutrosophic set P on U can be defined as follows:

$P = \{(x, \mathcal{T}_P(x), C_P(x), G_P(x), U_P(x), \mathcal{F}_P(x): x \in U)\}$, whereas

$\mathcal{T}_P(x), C_P(x), G_P(x), U_P(x), \mathcal{F}_P(x): U \to [0,1]$ and

$0 \le \mathcal{T}_P(x) + C_P(x) + G_P(x) + U_P(x) + \mathcal{F}_P(x) \le 5.$

Here $\mathcal{T}_P(x)$ is truth membership, $C_P(x)$ is uncertainty, $G_P(x)$ is contradiction, $U_Q(x)$ is unknown membership and $\mathcal{F}_Q(x)$ is false membership function. In pentapartitioned, an indeterminacy has been split into three parts.

Definition 5. Heptapartitioned neutrosophic set [17, 27]

Let U be a non-empty set. A heptapartitioned neutrosophic set $\mathcal{H}$ on U is an object of the form:

$\mathcal{H} = \{\langle x, \mathcal{T}_\mathcal{H}(x), \mathcal{M}_\mathcal{H}(x), \mathcal{C}_\mathcal{H}(x), \mathcal{U}_\mathcal{H}(x), I_\mathcal{H}(x), \mathcal{K}_\mathcal{H}(x), \mathcal{F}_\mathcal{H}(x) : x \in \mathcal{H}\rangle\}$,

were $\mathcal{T}_\mathcal{H}(x), \mathcal{M}_\mathcal{H}(x), \mathcal{C}_\mathcal{H}(x), \mathcal{U}_\mathcal{H}(x), I_\mathcal{H}(x), \mathcal{K}_\mathcal{H}(x), \mathcal{F}_\mathcal{H}(x) \in [0, 1]$.

Moreover,

$0 \le \mathcal{T}_\mathcal{H}(x) + \mathcal{M}_\mathcal{H}(x) + \mathcal{C}_\mathcal{H}(x) + \mathcal{U}_\mathcal{H}(x) + I_\mathcal{H}(x) + \mathcal{K}_\mathcal{H}(x) + \mathcal{F}_\mathcal{H}(x) \le 7$, where $\mathcal{T}_\mathcal{H}(x)$is the absolute membership degree, $\mathcal{M}_\mathcal{H}(x)$is the relative truth membership degree, $\mathcal{C}_\mathcal{H}(x)$is the contradiction membership degree, $\mathcal{U}_\mathcal{H}(x)$is the unknown membership degree, $I_\mathcal{H}(x)$is the ignorance membership degree, $\mathcal{K}_\mathcal{H}(x)$is the relative falsity membership degree, $\mathcal{F}_\mathcal{H}(x)$is the absolute falsity membership degree.

4. Heptapartitioned neutrosophic set

4.1 Definitions

Definition 1. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two HPNSs over the universe U. Then, $\mathcal{H}_1 \subseteq \mathcal{H}_2$ if and only if for any $x \in U$ the following conditions holds: $\mathcal{T}_{\mathcal{H}_1}(x) \le \mathcal{T}_{\mathcal{H}_2}(x)$, $\mathcal{M}_{\mathcal{H}_1}(x) \le \mathcal{M}_{\mathcal{H}_2}(x)$,

$\mathcal{C}_{\mathcal{H}_1}(x) \le \mathcal{C}_{\mathcal{H}_2}(x)$, $\mathcal{U}_{\mathcal{H}_1}(x) \ge \mathcal{U}_{\mathcal{H}_2}(x)$, $I_{\mathcal{H}_1}(x) \ge I_{\mathcal{H}_2}(x), \mathcal{K}_{\mathcal{H}_1}(x) \ge \mathcal{K}_{\mathcal{H}_2}(x)$ and $\mathcal{F}_{\mathcal{H}_1}(x) \ge \mathcal{F}_{\mathcal{H}_2}(x)$.

Definition 2. Let $\mathcal{H}$ be a HPNS over the universe U. The complement of $\mathcal{H}$ is denoted by $\mathcal{H}'$ for any $x \in U$ it obeys the following conditions:

$\mathcal{T}_\mathcal{H}(x) = \mathcal{F}_\mathcal{H}(x)$, $\mathcal{M}_\mathcal{H}(x) = \mathcal{K}_\mathcal{H}(x)$, $\mathcal{C}_\mathcal{H}(x) = I_\mathcal{H}(x)$, $\mathcal{U}_\mathcal{H}(x) = 1 - \mathcal{U}_\mathcal{H}(x)$, $I_\mathcal{H}(x) = \mathcal{C}_\mathcal{H}(x)$, $\mathcal{K}_\mathcal{H}(x) = \mathcal{M}_\mathcal{H}(x)$, $\mathcal{F}_\mathcal{H}(x) = \mathcal{T}_\mathcal{H}(x)$.

Definition 3. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two HPNSs over the universe U. The union of $\mathcal{H}_1$ and $\mathcal{H}_2$ is denoted by $\Omega = \mathcal{H}_1 \cup \mathcal{H}_2$ then, for any $x \in U$ is defined as

$$\Omega = \begin{Bmatrix} \max\left(\mathcal{T}_{\mathcal{H}_1}(x), \mathcal{T}_{\mathcal{H}_2}(x)\right), \max\left(\mathcal{M}_{\mathcal{H}_1}(x), \mathcal{M}_{\mathcal{H}_2}(x)\right), \\ \max\left(\mathcal{C}_{\mathcal{H}_1}(x), \mathcal{C}_{\mathcal{H}_2}(x)\right), \min\left(\mathcal{U}_{\mathcal{H}_1}(x), \mathcal{U}_{\mathcal{H}_2}(x)\right), \\ \min\left(I_{\mathcal{H}_1}(x), I_{\mathcal{H}_2}(x)\right), \min\left(\mathcal{K}_{\mathcal{H}_1}(x), \mathcal{K}_{\mathcal{H}_2}(x)\right), \\ \min\left(\mathcal{F}_{\mathcal{H}_1}(x), \mathcal{F}_{\mathcal{H}_2}(x)\right) \end{Bmatrix}$$

Definition 4. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two HPNSs over the universe U. The intersection of $\mathcal{H}_1$and $\mathcal{H}_2$ is denoted by $\nabla = \mathcal{H}_1 \cap \mathcal{H}_2$ then, for any $x \in U$ is defined as

$$\nabla = \begin{Bmatrix} \min\left(\mathcal{T}_{\mathcal{H}_1}(x), \mathcal{T}_{\mathcal{H}_2}(x)\right), \min\left(\mathcal{M}_{\mathcal{H}_1}(x), \mathcal{M}_{\mathcal{H}_2}(x)\right), \\ \min\left(\mathcal{C}_{\mathcal{H}_1}(x), \mathcal{C}_{\mathcal{H}_2}(x)\right), \max\left(\mathcal{U}_{\mathcal{H}_1}(x), \mathcal{U}_{\mathcal{H}_2}(x)\right), \\ \max\left(I_{\mathcal{H}_1}(x), I_{\mathcal{H}_2}(x)\right), \max\left(\mathcal{K}_{\mathcal{H}_1}(x), \mathcal{K}_{\mathcal{H}_2}(x)\right), \\ \max\left(\mathcal{F}_{\mathcal{H}_1}(x), \mathcal{F}_{\mathcal{H}_2}(x)\right) \end{Bmatrix}$$

Illustrative Example 1.

Let $\mathcal{H}_1 = \{0.327, 0.562, 0.487, 0.789, 0.852, 0.674, 0.921\}$ and

$\mathcal{H}_2 = \{0.632, 0.710, 0.867, 0.452, 0.527, 0.333, 0.274\}$ then

(i) $\mathcal{H}_1 \subseteq \mathcal{H}_2$

(ii) $\mathcal{H}_1{}' = \{0.921, 0.674, 0.852, \ 0.2111, 0.487, 0.562, 0.327\}$

(iii) $\mathcal{H}_1 \cup \mathcal{H}_2 = \{0.632, 0.710, 0.867, 0.452, 0.527, 0.333, 0.274\}$

(iv) $\mathcal{H}_1 \cap \mathcal{H}_2 = \{0.327, 0.562, 0.487, 0.789, 0.852, 0.674, 0.921\}$

Definition 5. A HPNS $\mathcal{H}$ is called an absolute HPNS if and only if, for any $x \in U$, it satisfies the following conditions:

$\mathcal{T}_{\mathcal{H}}(x) = 1$, $\mathcal{M}_{\mathcal{H}}(x) = 1$, $\mathcal{C}_{\mathcal{H}}(x) = 1$, $\mathcal{U}_{\mathcal{H}}(x) = 0$, $I_{\mathcal{H}}(x) = 0$, $\mathcal{K}_{\mathcal{H}}(x) = 0$ and $\mathcal{F}_{\mathcal{H}}(x) = 0$.

Definition 6. A HPNS $\mathcal{H}$ is called an empty HPNS if and only if, for any $x \in U$, it satisfies the following conditions:

$\mathcal{T}_{\mathcal{H}}(x) = 0$, $\mathcal{M}_{\mathcal{H}}(x) = 0$, $\mathcal{C}_{\mathcal{H}}(x) = 0$, $\mathcal{U}_{\mathcal{H}}(x) = 1$, $I_{\mathcal{H}}(x) = 1$, $\mathcal{K}_{\mathcal{H}}(x) = 1$ and $\mathcal{F}_{\mathcal{H}}(x) = 1$.

The arithmetic operations on HPNSs are defined as:

1. $\mathcal{H}_1 + \mathcal{H}_2 = \begin{Bmatrix} \mathcal{T}_{\mathcal{H}_1} + \mathcal{T}_{\mathcal{H}_2} - \mathcal{T}_{\mathcal{H}_1}\mathcal{T}_{\mathcal{H}_2}, \mathcal{M}_{\mathcal{H}_1} + \mathcal{M}_{\mathcal{H}_2} - \mathcal{M}_{\mathcal{H}_1}\mathcal{M}_{\mathcal{H}_2}, \mathcal{C}_{\mathcal{H}_1} + \mathcal{C}_{\mathcal{H}_2} - \mathcal{C}_{\mathcal{H}_1}\mathcal{C}_{\mathcal{H}_2}, \\ \mathcal{U}_{\mathcal{H}_1}\mathcal{U}_{\mathcal{H}_2}, I_{\mathcal{H}_1}I_{\mathcal{H}_2}, \mathcal{K}_{\mathcal{H}_1}\mathcal{K}_{\mathcal{H}_2}, \mathcal{F}_{\mathcal{H}_1}\mathcal{F}_{\mathcal{H}_2} \end{Bmatrix}$

2. $\mathcal{H}_1 \times \mathcal{H}_2 = \begin{Bmatrix} \mathcal{T}_{\mathcal{H}_1}\mathcal{T}_{\mathcal{H}_2}, \mathcal{M}_{\mathcal{H}_1}\mathcal{M}_{\mathcal{H}_2}, \mathcal{C}_{\mathcal{H}_1}\mathcal{C}_{\mathcal{H}_2}, \mathcal{U}_{\mathcal{H}_1} + \mathcal{U}_{\mathcal{H}_2} - \mathcal{U}_{\mathcal{H}_1}\mathcal{U}_{\mathcal{H}_2}, \\ I_{\mathcal{H}_1} + I_{\mathcal{H}_2} - I_{\mathcal{H}_1}I_{\mathcal{H}_2}, \mathcal{K}_{\mathcal{H}_1} + \mathcal{K}_{\mathcal{H}_2} - \mathcal{K}_{\mathcal{H}_1}\mathcal{K}_{\mathcal{H}_2}, \mathcal{F}_{\mathcal{H}_1} + \mathcal{F}_{\mathcal{H}_2} - \mathcal{F}_{\mathcal{H}_1}\mathcal{F}_{\mathcal{H}_2} \end{Bmatrix}$

3. $\alpha\mathcal{H}_1 = \begin{Bmatrix} \left(1 - \left(1 - \mathcal{T}_{\mathcal{H}_1}\right)^{\alpha}\right), \left(1 - \left(1 - \mathcal{M}_{\mathcal{H}_1}\right)^{\alpha}\right), \left(1 - \left(1 - \mathcal{C}_{\mathcal{H}_1}\right)^{\alpha}\right), \\ \left(\mathcal{U}_{\mathcal{H}_1}\right)^{\alpha}, \left(I_{\mathcal{H}_1}\right)^{\alpha}, \left(\mathcal{K}_{\mathcal{H}_1}\right)^{\alpha}, \left(\mathcal{F}_{\mathcal{H}_1}\right)^{\alpha} \end{Bmatrix}$, where $\alpha > 0$.

4. $(\mathcal{H}_1)^{\alpha} = \begin{Bmatrix} \left(\mathcal{T}_{\mathcal{H}_1}\right)^{\alpha}, \left(\mathcal{M}_{\mathcal{H}_1}\right)^{\alpha}, \left(\mathcal{C}_{\mathcal{H}_1}\right)^{\alpha}, \left(1 - \left(1 - \mathcal{U}_{\mathcal{H}_1}\right)^{\alpha}\right), \\ \left(1 - \left(1 - I_{\mathcal{H}_1}\right)^{\alpha}\right), \left(1 - \left(1 - \mathcal{K}_{\mathcal{H}_1}\right)^{\alpha}\right), \left(1 - \left(1 - \mathcal{F}_{\mathcal{H}_1}\right)^{\alpha}\right) \end{Bmatrix}$, where $\alpha > 0$.

Illustrative Example 2.

Let $\mathcal{H}_1 = \{0.327, 0.562, 0.487, 0.789, 0.852, 0.674, 0.921\}$ and

$\mathcal{H}_2 = \{0.632, 0.710, 0.867, 0.452, 0.527, 0.333, 0.274\}$ then

1. $\mathcal{H}_1 + \mathcal{H}_2 = \{0.752, 0.873, 0.932, 0.357, 0.449, 0.224, 0.252\}$
2. $\mathcal{H}_1 \times \mathcal{H}_2 = \{0.207, 0.399, 0.422, 0.884, 0.930, 0.783, 0.943\}$
3. $2\mathcal{H}_1 = \{0.547, 0.808, 0.737, 0.323, 0.726, 0.454, 0.848\}$
4. $(\mathcal{H}_1)^2 = \{0.107, 0.316, 0.237, 0.955, 0.978, 0.894, 0.994\}$

4.2 Score and Accuracy Function

The score function of HPNN $\mathcal{H} = (\mathcal{T}_\mathcal{H}, \mathcal{M}_\mathcal{H}, \mathcal{C}_\mathcal{H}\ \mathcal{U}_\mathcal{H}, I_\mathcal{H}, \mathcal{K}_\mathcal{H}, \mathcal{F}_\mathcal{H})$ is defined as

$\mathcal{S}_\mathcal{H} = \frac{\mathcal{T}_\mathcal{H}+\mathcal{M}_\mathcal{H}+\mathcal{C}_\mathcal{H}}{3} + \frac{\mathcal{U}_\mathcal{H}+ I_\mathcal{H}+ \mathcal{K}_\mathcal{H}+ \mathcal{F}_\mathcal{H}}{4}$, $\mathcal{S}_\mathcal{H}$ lies between $[0, 2]$

The accuracy function of HPNN $\mathcal{H} = (\mathcal{T}_\mathcal{H}, \mathcal{M}_\mathcal{H}, \mathcal{C}_\mathcal{H}\ \mathcal{U}_\mathcal{H}, I_\mathcal{H}, \mathcal{K}_\mathcal{H}, \mathcal{F}_\mathcal{H})$ is defined as

$\mathcal{A}_\mathcal{H} = \frac{\mathcal{T}_\mathcal{H}+\mathcal{M}_\mathcal{H}+\mathcal{C}_\mathcal{H}+\mathcal{U}_\mathcal{H} - I_\mathcal{H} - \mathcal{K}_\mathcal{H} - \mathcal{F}_\mathcal{H}}{3}$, $\mathcal{A}_\mathcal{H}$ lies between $\left[-1, \frac{3}{4}\right]$

Proposition 1. Score value of HPNN lies between $[0, 2]$.

Proof: By the definition of heptapartitioned neutrosophic sets,

$0 \le \mathcal{T}_\mathcal{H} \le 1,\quad 0 \le \mathcal{M}_\mathcal{H} \le 1,\quad 0 \le \mathcal{C}_\mathcal{H} \le 1,\quad 0 \le \mathcal{U}_\mathcal{H} \le 1,\quad 0 \le I_\mathcal{H} \le 1,$

$0 \le \mathcal{K}_\mathcal{H} \le 1,\quad 0 \le \mathcal{F}_\mathcal{H} \le 1.$

$0 \le \mathcal{T}_\mathcal{H} + \mathcal{M}_\mathcal{H} + \mathcal{C}_\mathcal{H} \le 3,\quad 0 \le \mathcal{U}_\mathcal{H} + I_\mathcal{H} + \mathcal{K}_\mathcal{H} + \mathcal{F}_\mathcal{H} \le 4.$

$$0 \le \frac{\mathcal{T}_\mathcal{H}+\mathcal{M}_\mathcal{H}+\mathcal{C}_\mathcal{H}}{3} \le 1; \qquad 0 \le \frac{\mathcal{U}_\mathcal{H}+I_\mathcal{H}+\mathcal{K}_\mathcal{H}+\mathcal{F}_\mathcal{H}}{4} \le 1$$

$$0 \le \frac{\mathcal{T}_\mathcal{H} + \mathcal{M}_\mathcal{H} + \mathcal{C}_\mathcal{H}}{3} + \frac{\mathcal{U}_\mathcal{H} + I_\mathcal{H} + \mathcal{K}_\mathcal{H} + \mathcal{F}_\mathcal{H}}{4} \le 1 + 1$$

$$0 \le \mathcal{S}_\mathcal{H} \le 2.$$

Hence the proof.

Proposition 2. Accuracy function of HPNN lies between $\left[-1, \frac{3}{4}\right]$

Proof: By definition of heptapartitioned neutrosophic sets,

$$-4 \le \mathcal{T}_\mathcal{H} + \mathcal{M}_\mathcal{H} + \mathcal{C}_\mathcal{H} + \mathcal{U}_\mathcal{H} - I_\mathcal{H} - \mathcal{K}_\mathcal{H} - \mathcal{F}_\mathcal{H} \le 3$$

$$-1 \le \frac{\mathcal{T}_\mathcal{H} + \mathcal{M}_\mathcal{H} + \mathcal{C}_\mathcal{H} + \mathcal{U}_\mathcal{H} - I_\mathcal{H} - \mathcal{K}_\mathcal{H} - \mathcal{F}_\mathcal{H}}{3} \le \frac{3}{4}$$

$$-1 \le \mathcal{A}_\mathcal{H} \le \frac{3}{4}$$

Hence the proof.

4.3 Heptapartitioned Neutrosophic Weighted Arithmetic Averaging Operator (HPNWAAO)

Definition 4.3.1: Let $\mathcal{H}_j = \left\{\delta; \mathcal{T}_{\mathcal{H}_j}(\delta), \mathcal{M}_{\mathcal{H}_j}(\delta), \mathcal{C}_{\mathcal{H}_j}(\delta), \mathcal{U}_{\mathcal{H}_j}(\delta), I_{\mathcal{H}_j}(\delta), \mathcal{K}_{\mathcal{H}_j}(\delta), \mathcal{F}_{\mathcal{H}_j}(\delta)\right\}$ where $j = 1,2,3, \dots, n$ be the collection of HPNNs is the set of real numbers given by $HPNWAA: (Re)^n \to Re$. Let $HPNWAA$ operator is represented by $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n)$ is defined as $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) = w_1\mathcal{H}_1 + w_2\mathcal{H}_2 + \dots + w_n\mathcal{H}_n = \sum_{j=1}^{n} w_j\mathcal{H}_j$, $w_j(j = 1,2, \dots, n)$ denotes the weightage of HPNNs $\mathcal{H}_j(j = 1,2, \dots, n)$, and also $\sum_{j=1}^{n} w_j = 1$, where $w_j \in [0,1]$.

We now propose the following theorem by making use of the basic operations of HPNNs.

Theorem 1. Let

$\mathcal{H}_j = \left\{\delta; \mathcal{T}_{\mathcal{H}_j}(\delta), \mathcal{M}_{\mathcal{H}_j}(\delta), \mathcal{C}_{\mathcal{H}_j}(\delta), \mathcal{U}_{\mathcal{H}_j}(\delta), I_{\mathcal{H}_j}(\delta), \mathcal{K}_{\mathcal{H}_j}(\delta), \mathcal{F}_{\mathcal{H}_j}(\delta)\right\} (j = 1,2, \dots, n))$ be a collection of heptapartitioned neutrosophic value (HPNVs) in the set of real numbers. The aggregated value of HPNWAA is also an HPNV, and

$$HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) = w_1\mathcal{H}_1 + w_2\mathcal{H}_2 + \cdots + w_n\mathcal{H}_n = \sum_{j=1}^{n} w_j\mathcal{H}_j$$

$$= \begin{Bmatrix} 1 - \prod_{j=1}^{n}\left(1 - \mathcal{T}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \mathcal{M}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \mathcal{C}_{\mathcal{H}_j}\right)^{w_j}, \\ \prod_{j=1}^{n} \mathcal{U}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{n} I_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{n} \mathcal{K}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{n} \mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{Bmatrix} \quad (4.1)$$

whereas the weight of HPNV $\mathcal{H}_j (j = 1,2, \dots, n)$ is $w_j \in [0,1]$ with the condition $\sum_{j=1}^{n} w_j = 1$.

Proof: By mathematical induction, we prove this theorem.

For $n = 1$, it is trivial.

For $n = 2$, $\sum_{j=1}^{2} w_j\mathcal{H}_j = w_1\mathcal{H}_1 + w_2\mathcal{H}_2$.

$$= \left\{ \begin{matrix} \begin{bmatrix} 1 - \left(1 - \mathcal{T}_{\mathcal{H}_1}\right)^{w_1}, 1 - \left(1 - \mathcal{M}_{\mathcal{H}_1}\right)^{w_1}, 1 - \left(1 - \mathcal{C}_{\mathcal{H}_1}\right)^{w_1}, \\ \mathcal{U}_{\mathcal{H}_1}{}^{w_1}, I_{\mathcal{H}_1}{}^{w_1}, \mathcal{K}_{\mathcal{H}_1}{}^{w_1}, \mathcal{F}_{\mathcal{H}_1}{}^{w_1} \end{bmatrix} \\ + \\ \begin{bmatrix} 1 - \left(1 - \mathcal{T}_{\mathcal{H}_2}\right)^{w_2}, 1 - \left(1 - \mathcal{M}_{\mathcal{H}_2}\right)^{w_2}, 1 - \left(1 - \mathcal{C}_{\mathcal{H}_2}\right)^{w_2}, \\ \mathcal{U}_{\mathcal{H}_2}{}^{w_2}, I_{\mathcal{H}_2}{}^{w_2}, \mathcal{K}_{\mathcal{H}_2}{}^{w_2}, \mathcal{F}_{\mathcal{H}_2}{}^{w_2} \end{bmatrix} \end{matrix} \right\}$$

$$= \begin{Bmatrix} 1 - \left(1 - \mathcal{T}_{\mathcal{H}_1}\right)^{w_1} + 1 - \left(1 - \mathcal{T}_{\mathcal{H}_2}\right)^{w_2} - \left(1 - \left(1 - \mathcal{T}_{\mathcal{H}_1}\right)^{w_1}\right)\left(1 - \left(1 - \mathcal{T}_{\mathcal{H}_2}\right)^{w_2}\right), \\ 1 - \left(1 - \mathcal{M}_{\mathcal{H}_1}\right)^{w_1} + 1 - \left(1 - \mathcal{M}_{\mathcal{H}_2}\right)^{w_2} - \left(1 - \left(1 - \mathcal{M}_{\mathcal{H}_1}\right)^{w_1}\right)\left(1 - \left(1 - \mathcal{M}_{\mathcal{H}_2}\right)^{w_2}\right), \\ 1 - \left(1 - \mathcal{C}_{\mathcal{H}_1}\right)^{w_1} + 1 - \left(1 - \mathcal{C}_{\mathcal{H}_2}\right)^{w_2} - \left(1 - \left(1 - \mathcal{C}_{\mathcal{H}_1}\right)^{w_1}\right)\left(1 - \left(1 - \mathcal{C}_{\mathcal{H}_2}\right)^{w_2}\right), \\ \mathcal{U}_{\mathcal{H}_1}{}^{w_1}\mathcal{U}_{\mathcal{H}_2}{}^{w_2}, I_{\mathcal{H}_1}{}^{w_1}I_{\mathcal{H}_2}{}^{w_2}, \mathcal{K}_{\mathcal{H}_1}{}^{w_1}\mathcal{K}_{\mathcal{H}_2}{}^{w_2}, \mathcal{F}_{\mathcal{H}_1}{}^{w_1}\mathcal{F}_{\mathcal{H}_2}{}^{w_2} \end{Bmatrix}$$

$$= \begin{Bmatrix} 1 - \prod_{j=1}^{2}\left(1 - \mathcal{T}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{2}\left(1 - \mathcal{M}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{2}\left(1 - \mathcal{C}_{\mathcal{H}_j}\right)^{w_j}, \\ \prod_{j=1}^{2} \mathcal{U}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{2} I_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{2} \mathcal{K}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{2} \mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{Bmatrix}$$

Hence it satisfies for $n = 2$.

For $n = k$, we assume that the theorem holds good.

Therefore, $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) = w_1\mathcal{H}_1 + w_2\mathcal{H}_2 + \cdots + w_n\mathcal{H}_n = \sum_{j=1}^{n} w_j\mathcal{H}_j$

$$= \begin{Bmatrix} 1 - \prod_{j=1}^{k}\left(1 - \mathcal{T}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{k}\left(1 - \mathcal{M}_{\mathcal{H}_j}\right)^{w_j}, 1 - \prod_{j=1}^{k}\left(1 - \mathcal{C}_{\mathcal{H}_j}\right)^{w_j}, \\ \prod_{j=1}^{k} \mathcal{U}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k} I_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k} \mathcal{K}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k} \mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{Bmatrix}$$

For $n = k + 1$,

$$HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) = \sum_{j=1}^{k} w_j\mathcal{H}_j + w_{k+1}\mathcal{H}_{k+1}$$

$$= \left\{ \begin{array}{c} 1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j}+1-\left(1-\mathcal{T}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}-(1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j})(1-\left(1-\mathcal{T}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}), \\ 1-\prod_{j=1}^{k}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j}+1-\left(1-\mathcal{M}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}-(1-\prod_{j=1}^{k}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j})(1-\left(1-\mathcal{M}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}), \\ 1-\prod_{j=1}^{k}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j}+1-\left(1-\mathcal{C}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}-(1-\prod_{j=1}^{k}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j})(1-\left(1-\mathcal{C}_{\mathcal{H}_{k+1}}\right)^{w_{k+1}}), \\ \prod_{j=1}^{k}\mathcal{U}_{\mathcal{H}_j}{}^{w_j}.\mathcal{U}_{\mathcal{H}_{k+1}}{}^{w_{k+1}},\prod_{j=1}^{k}I_{\mathcal{H}_j}{}^{w_j}.I_{\mathcal{H}_{k+1}}{}^{w_{k+1}}, \\ \prod_{j=1}^{k}\mathcal{K}_{\mathcal{H}_j}{}^{w_j}.\mathcal{K}_{\mathcal{H}_{k+1}}{}^{w_{k+1}},\prod_{j=1}^{k}\mathcal{F}_{\mathcal{H}_j}{}^{w_j}.\mathcal{F}_{\mathcal{H}_{k+1}}{}^{w_{k+1}} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} 1-\prod_{j=1}^{k+1}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j},1-\prod_{j=1}^{k+1}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j},1-\prod_{j=1}^{k+1}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j}, \\ \prod_{j=1}^{k+1}\mathcal{U}_{\mathcal{H}_j}{}^{w_j},\prod_{j=1}^{k+1}I_{\mathcal{H}_j}{}^{w_j},\prod_{j=1}^{k+1}\mathcal{K}_{\mathcal{H}_j}{}^{w_j},\prod_{j=1}^{k+1}\mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{array} \right\}$$

Hence, we observe that the theorem holds good for $n = k + 1$.

By mathematical induction, the theorem is verified for all values of n.

The three membership functions of $\mathcal{H}_j$ lies between $[0,1]$ which satisfies the conditions:

$0 \le 1-\prod_{j=1}^{n}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j} \le 1$, $0 \le 1-\prod_{j=1}^{n}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j} \le 1$, $0 \le 1-\prod_{j=1}^{n}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j} \le 1$, $0 \le \prod_{j=1}^{n}\mathcal{U}_{\mathcal{H}_j}{}^{w_j} \le 1$, $0 \le \prod_{j=1}^{n}I_{\mathcal{H}_j}{}^{w_j} \le 1$, $0 \le \prod_{j=1}^{n}\mathcal{K}_{\mathcal{H}_j}{}^{w_j} \le 1$, $0 \le \prod_{j=1}^{n}\mathcal{F}_{\mathcal{H}_j} \le 1$, further it holds the relation,

$$0 \le \left\{ \begin{array}{c} 1-\prod_{j=1}^{n}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j}+1-\prod_{j=1}^{n}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j}+1-\prod_{j=1}^{n}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j} \\ +\prod_{j=1}^{n}\mathcal{U}_{\mathcal{H}_j}{}^{w_j}+\prod_{j=1}^{n}I_{\mathcal{H}_j}{}^{w_j}+\prod_{j=1}^{n}\mathcal{K}_{\mathcal{H}_j}{}^{w_j}+\prod_{j=1}^{n}\mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{array} \right\} \le 7$$

Hence the theorem is proved.

Property 1. Idempotency

If all $\mathcal{H}_j (j = 1,2,\ldots,n)$ are equal, then

$$\mathcal{H}_j = \mathcal{H} = \{\langle x; \mathcal{T}_{\mathcal{H}}(x), \mathcal{M}_{\mathcal{H}}(x), \mathcal{C}_{\mathcal{H}}(x), \mathcal{U}_{\mathcal{H}}(x), I_{\mathcal{H}}(x), \mathcal{K}_{\mathcal{H}}(x), \mathcal{F}_{\mathcal{H}}(x) : x \in \mathcal{H}\rangle\}$$

then $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n) = \mathcal{H}$.

Proof:

For proving this, we use eqn. (4.1).

$$HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n) = HPNWAA(\mathcal{H}, \mathcal{H}, \ldots, \mathcal{H}) = \sum_{j=1}^{n} w_j \mathcal{H}$$

$$= \left\{ \begin{array}{c} 1-\prod_{j=1}^{k}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j}, \\ \prod_{j=1}^{k}\mathcal{U}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k}I_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k}\mathcal{K}_{\mathcal{H}_j}{}^{w_j}, \prod_{j=1}^{k}\mathcal{F}_{\mathcal{H}_j}{}^{w_j} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} 1-(1-\mathcal{T}_{\mathcal{H}})^{\sum_{j=1}^{n} w_j}, 1-(1-\mathcal{M}_{\mathcal{H}})^{\sum_{j=1}^{n} w_j}, 1-(1-\mathcal{C}_{\mathcal{H}})^{\sum_{j=1}^{n} w_j}, \\ \mathcal{U}_{\mathcal{H}}{}^{\sum_{j=1}^{n} w_j}, I_{\mathcal{H}}{}^{\sum_{j=1}^{n} w_j}, \mathcal{K}_{\mathcal{H}}{}^{\sum_{j=1}^{n} w_j}, \mathcal{F}_{\mathcal{H}}{}^{\sum_{j=1}^{n} w_j} \end{array} \right\}$$

$$= \{\langle x, \mathcal{T}_{\mathcal{H}}(x), \mathcal{M}_{\mathcal{H}}(x), \mathcal{C}_{\mathcal{H}}(x), \mathcal{U}_{\mathcal{H}}(x), I_{\mathcal{H}}(x), \mathcal{K}_{\mathcal{H}}(x), \mathcal{F}_{\mathcal{H}}(x)\rangle\} = \mathcal{H}$$

Hence the proof.

Property 2. Boundedness

Let $\mathcal{H}_j = \mathcal{H} = \{\langle x, \mathcal{T}_{\mathcal{H}}(x), \mathcal{M}_{\mathcal{H}}(x), \mathcal{C}_{\mathcal{H}}(x), \mathcal{U}_{\mathcal{H}}(x), I_{\mathcal{H}}(x), \mathcal{K}_{\mathcal{H}}(x), \mathcal{F}_{\mathcal{H}}(x) : x \in \mathcal{H}\rangle\} (j = 1,2, \dots, n)$ be a collection of HPNVs in the set of real numbers.

$$\text{Consider } \mathcal{H}^+ = \left\{ \begin{array}{c} \max_j\left(\mathcal{T}_{\mathcal{H}_j}\right), \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right), \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right) \\ \min_j\left(\mathcal{U}_{\mathcal{H}_j}\right), \min_j\left(I_{\mathcal{H}_j}\right), \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right), \min_j\left(\mathcal{F}_{\mathcal{H}_j}\right) \end{array} \right\}$$

$$\mathcal{H}^- = \left\{ \begin{array}{c} \min_j\left(\mathcal{T}_{\mathcal{H}_j}\right), \min_j\left(\mathcal{M}_{\mathcal{H}_j}\right), \min_j\left(\mathcal{C}_{\mathcal{H}_j}\right) \\ \max_j\left(\mathcal{U}_{\mathcal{H}_j}\right), \max_j\left(I_{\mathcal{H}_j}\right), \max_j\left(\mathcal{K}_{\mathcal{H}_j}\right), \max_j\left(\mathcal{F}_{\mathcal{H}_j}\right) \end{array} \right\}$$

where $j = 1,2, \dots, n$. Then $\mathcal{H}^- \le HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) \le \mathcal{H}^+$.

Proof: We infer that

$$\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right) \le \mathcal{T}_{\mathcal{H}_j} \le \max_j\left(\mathcal{T}_{\mathcal{H}_j}\right), \quad \min_j\left(\mathcal{M}_{\mathcal{H}_j}\right) \le \mathcal{M}_{\mathcal{H}_j} \le \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right), \quad \min_j\left(\mathcal{C}_{\mathcal{H}_j}\right) \le \mathcal{C}_{\mathcal{H}_j} \le \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right),$$

$$\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right) \le \mathcal{U}_{\mathcal{H}_j} \le \max_j\left(\mathcal{U}_{\mathcal{H}_j}\right), \quad \min_j\left(I_{\mathcal{H}_j}\right) \le I_{\mathcal{H}_j} \le \max_j\left(I_{\mathcal{H}_j}\right), \quad \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right) \le \mathcal{K}_{\mathcal{H}_j} \le \max_j\left(\mathcal{K}_{\mathcal{H}_j}\right),$$

$$\min_j\left(\mathcal{F}_{\mathcal{H}_j}\right) \le \mathcal{F}_{\mathcal{H}_j} \le \max_j\left(\mathcal{F}_{\mathcal{H}_j}\right) \text{ for } j = 1,2, \dots, n. \tag{4.2}$$

Then, $1-\prod_{j=1}^{n}\left(1-\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right)\right)^{w_j} \le 1-\prod_{j=1}^{n}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j} \le 1-\prod_{j=1}^{n}\left(1-\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right)\right)^{w_j}$

$$1-\left(1-\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right)\right)^{\sum_{j=1}^{n} w_j} \le 1-\prod_{j=1}^{n}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j} \le 1-\left(1-\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right)\right)^{\sum_{j=1}^{n} w_j}$$

$$\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right) \le 1-\prod_{j=1}^{n}\left(1-\mathcal{T}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{T}_{\mathcal{H}_j}\right)$$

By eq. (4.2), for $j = 1,2, \dots, n$.

Similarly,

$$\min_j\left(\mathcal{M}_{\mathcal{H}_j}\right) \le 1-\prod_{j=1}^{n}\left(1-\mathcal{M}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right),$$

$$\min_j\left(\mathcal{C}_{\mathcal{H}_j}\right) \le 1-\prod_{j=1}^{n}\left(1-\mathcal{C}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right),$$

$$\prod_{j=1}^{n}\left(\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right)\right)^{w_j} \le \prod_{j=1}^{n}\left(\mathcal{U}_{\mathcal{H}_j}\right)^{w_j} \le \prod_{j=1}^{n}\left(\max_j\left(\mathcal{U}_{\mathcal{H}_j}\right)\right)^{w_j},$$

$$\left(\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right)\right)^{\sum_{j=1}^{n} w_j} \le \prod_{j=1}^{n}\left(\mathcal{U}_{\mathcal{H}_j}\right)^{w_j} \le \left(\max_j\left(\mathcal{U}_{\mathcal{H}_j}\right)\right)^{\sum_{j=1}^{n} w_j},$$

$$\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right) \le \prod_{j=1}^{n}\left(\mathcal{U}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{U}_{\mathcal{H}_j}\right).$$

Similarly,

$$\min_j\left(I_{\mathcal{H}_j}\right) \le \prod_{j=1}^{n}\left(I_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(I_{\mathcal{H}_j}\right), \quad \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right) \le \prod_{j=1}^{n}\left(\mathcal{K}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{K}_{\mathcal{H}_j}\right),$$

$$\min_j\left(\mathcal{F}_{\mathcal{H}_j}\right) \le \prod_{j=1}^{n}\left(\mathcal{F}_{\mathcal{H}_j}\right)^{w_j} \le \max_j\left(\mathcal{F}_{\mathcal{H}_j}\right).$$

Let $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n) \le \mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{M}_{\mathcal{H}}, \mathcal{C}_{\mathcal{H}}, \mathcal{U}_{\mathcal{H}}, I_{\mathcal{H}}, \mathcal{K}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}})$

The score function of $\mathcal{H}$ is,

$$\mathcal{S}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}}}{3} + \frac{\mathcal{U}_{\mathcal{H}} + I_{\mathcal{H}} + \mathcal{K}_{\mathcal{H}} + \mathcal{F}_{\mathcal{H}}}{4}$$

$$\le \left[\frac{\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right)}{3} + \frac{\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right) + \min_j\left(I_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{4}\right] = \c{S}(\mathcal{H}^+)$$

In the same way,

$$\mathcal{S}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}}}{3} + \frac{\mathcal{U}_{\mathcal{H}} + I_{\mathcal{H}} + \mathcal{K}_{\mathcal{H}} + \mathcal{F}_{\mathcal{H}}}{4}$$

$$\ge \left[\frac{\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{M}_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{C}_{\mathcal{H}_j}\right)}{3} + \frac{\max_j\left(\mathcal{U}_{\mathcal{H}_j}\right) + \max_j\left(I_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{K}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{4}\right] = \c{S}(\mathcal{H}^-)$$

Here we discuss the different cases:

Case (i) If $\mathcal{S}(\mathcal{H}) < \mathcal{S}(\mathcal{H}^+)$ and $\mathcal{S}(\mathcal{H}) > \mathcal{S}(\mathcal{H}^-)$ then, $\mathcal{H}^- < HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n) < \mathcal{H}^+$.

Case (ii) If $\mathcal{S}(\mathcal{H}) = \mathcal{S}(\mathcal{H}^+)$, we consider

$$\mathcal{S}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}}}{3} + \frac{\mathcal{U}_{\mathcal{H}} + I_{\mathcal{H}} + \mathcal{K}_{\mathcal{H}} + \mathcal{F}_{\mathcal{H}}}{4}$$

$$= \left[\frac{\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right)}{3} + \frac{\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right) + \min_j\left(I_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{4}\right]$$

Then it follows,

$$\frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}}}{3} = \frac{\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{M}_{\mathcal{H}_j}\right) + \max_j\left(\mathcal{C}_{\mathcal{H}_j}\right)}{3}$$

$$\frac{\mathcal{U}_{\mathcal{H}} + I_{\mathcal{H}} + \mathcal{K}_{\mathcal{H}} + \mathcal{F}_{\mathcal{H}}}{4} = \frac{\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right) + \min_j\left(I_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{K}_{\mathcal{H}_j}\right) + \min_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{4}$$

The accuracy function,

$$\mathcal{A}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}} + \mathcal{U}_{\mathcal{H}} - I_{\mathcal{H}} - \mathcal{K}_{\mathcal{H}} - \mathcal{F}_{\mathcal{H}}}{3}$$

$$= \frac{\max_j\left(\mathcal{T}_{\mathcal{H}_j}\right)+\max_j\left(\mathcal{M}_{\mathcal{H}_j}\right)+\max_j\left(\mathcal{C}_{\mathcal{H}_j}\right)+\min_j\left(\mathcal{U}_{\mathcal{H}_j}\right)-\min_j\left(I_{\mathcal{H}_j}\right)-\min_j\left(\mathcal{K}_{\mathcal{H}_j}\right)-\min_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{3} = A(\mathcal{H}^+) \quad (4.3)$$

which implies $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) \le \mathcal{H}^+$.

In the same way,

$$\mathcal{A}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}} + \mathcal{U}_{\mathcal{H}} - I_{\mathcal{H}} - \mathcal{K}_{\mathcal{H}} - \mathcal{F}_{\mathcal{H}}}{3}$$

$$= \frac{\min_j\left(\mathcal{T}_{\mathcal{H}_j}\right)+\min_j\left(\mathcal{M}_{\mathcal{H}_j}\right)+\min_j\left(\mathcal{C}_{\mathcal{H}_j}\right)+\max_j\left(\mathcal{U}_{\mathcal{H}_j}\right)-\max_j\left(I_{\mathcal{H}_j}\right)-\max_j\left(\mathcal{K}_{\mathcal{H}_j}\right)-\max_j\left(\mathcal{F}_{\mathcal{H}_j}\right)}{3} = A(\mathcal{H}^-) \quad (4.4)$$

which implies $HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) \ge \mathcal{H}^-$.

From eq. (4.3) and (4.4), we infer that $\mathcal{H}^- \le HPNWAA(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) \le \mathcal{H}^+$.

Hence the proof is verified.

5. Multi Criterion Decision Making using HPNWAA Operator.

To resolve MCDM technique with heptapartitioned neutrosophic numbers which is represented in the form of HPNVs with m alternatives $L = \{L_1, L_2, \dots, L_m\}$ and attributes are given by $C = \{C_1, C_2, \dots, C_n\}$ and their weights be $\mathcal{W} = \{w_1, w_2, \dots w_n\}^T$ with $w_j \ge 0$ and $\sum_{j=1}^{n} w_j = 1$ for $j = 1,2, \dots, n$. The decision matrix is given by,

$$\mathfrak{D}_{\mathcal{H}} = \left(\dot{d}_{ij}\right)_{m\times n} = [\mathcal{T}_{\mathcal{H}}, \mathcal{M}_{\mathcal{H}}, \mathcal{C}_{\mathcal{H}}, \mathcal{U}_{\mathcal{H}}, I_{\mathcal{H}}, \mathcal{K}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}}]_{m\times n}$$

where $(\mathcal{T}_{\mathcal{H}}, \mathcal{M}_{\mathcal{H}}, \mathcal{C}_{\mathcal{H}}, \mathcal{U}_{\mathcal{H}}, I_{\mathcal{H}}, \mathcal{K}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}}) \subset [0,1]$

with the condition $0 \le \mathcal{T}_{\mathcal{H}} + \mathcal{M}_{\mathcal{H}} + \mathcal{C}_{\mathcal{H}} + \mathcal{U}_{\mathcal{H}}, I_{\mathcal{H}}, \mathcal{K}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}} \le 7$.

Step 1: To find aggregate value of the attributes in terms of heptapartitioned neutrosophic values.

Step 2: By using $HPNWAA$ operator, find the aggregate value corresponding to each alternative by using eqn. (4.1).

Step 3: For the aggregated values, obtain the score value for each alternative using

$\mathcal{S}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}}+\mathcal{M}_{\mathcal{H}}+\mathcal{C}_{\mathcal{H}}}{3} + \frac{\mathcal{U}_{\mathcal{H}}+ I_{\mathcal{H}}+ \mathcal{K}_{\mathcal{H}}+ \mathcal{F}_{\mathcal{H}}}{4}$.

Step 4: Valuate the ranking order, by using MCDM technique for the different attributes and alternates under consideration.

Step 5: Pertain the best choice in accordance to the ranking order.

Illustrative Example 3.

Consider five different packaging services M_1, M_2, M_3, M_4, M_5 to deliver the goods on time with less transportation cost. Three attributes were taken into consideration such as, A - Delivery Charges, B-Time of Delivery, C – Safety of goods with different weightages as (0.3, 0.5, 0.2). The best packaging company should be selected among the five different alternates based on the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) ranking order.

5.1 Algorithm for TOPSIS method:

1. Construct the heptapartitioned decision matrix based on the response of attributes and alternatives.
2. To find the score value for HPNVs:

$$\mathcal{S}_{\mathcal{H}} = \frac{\mathcal{T}_{\mathcal{H}}+\mathcal{M}_{\mathcal{H}}+\mathcal{C}_{\mathcal{H}}}{3} + \frac{\mathcal{U}_{\mathcal{H}}+ I_{\mathcal{H}}+ \mathcal{K}_{\mathcal{H}}+ \mathcal{F}_{\mathcal{H}}}{4} \quad (4.5)$$

3. Normalize the matrix $\overline{\mathfrak{N}}_{ij} = \frac{\mathfrak{N}_{ij}}{\sqrt{\sum_{j=1}^{n} \mathfrak{N}_{ij}^2}}$ (4.6)
4. Calculate the weighted normalized matrix $\mathfrak{U}_{ij} = \mathfrak{N}_{ij} * w_j$ (4.7)
5. Calculate the ideal best $(\mathfrak{U}_j^+)$ and ideal worst $(\mathfrak{U}_j^-)$ values for each of the attributes.
6. For Beneficial Criteria (B): Ideal best will be the maximum of the values.
   Ideal worst will be the minimum of the values.
7. For Non-Beneficial Criteria (N.B): Ideal best will be the minimum of the values.
   Ideal worst will be the maximum of the values.
8. Calculate the Euclidean distance from the ideal best $\mathfrak{x}_i^+ = \sqrt{\sum_{j=1}^{n} (\mathfrak{U}_{ij} - \mathfrak{U}_j^+)^2}$ (4.8)
9. Calculate the Euclidean distance from the ideal worst $\mathfrak{x}_i^- = \sqrt{\sum_{j=1}^{n} (\mathfrak{U}_{ij} - \mathfrak{U}_j^-)^2}$ (4.9)
10. Calculate the performance score $\mathfrak{P}_i = \frac{\mathfrak{x}_i^-}{\mathfrak{x}_i^- + \mathfrak{x}_i^+}$ (4.10)
11. Based on the performance score, rank the alternatives.

Table 1: Heptapartitioned neutrosophic values for different alternates and attributes.

	A	***B***	***C***
M_1	(0.72,0.32,0.45,0.81,0.28, 0.66,0.16)	(0.80,0.47,0.92,0.77,0.61, 0.54,0.38)	(0.64,0.82,0.62,0.58,0.71, 0.47,0.46)
M_2	(0.35,0.56,0.47,0.34,0.87, 0.54,0.78)	(0.75,0.62,0.45,0.56,0.68, 0.89,0.32)	(0.45,0.87,0.56,0.42,0.66, 0.72,0.85)
M_3	(0.51,0.83,0.58,0.47,0.34, 0.64,0.26)	(0.60,0.77,0.53,0.36,0.82, 0.28,0.87)	(0.74,0.82,0.34,0.52,0.74, 0.67,0.32)
M_4	(0.92,0.56,0.74,0.61,0.37, 0.49,0.82)	(0.85,0.71,0.64,0.80,0.22, 0.16,0.45)	(0.56,0.87,0.95,0.52,0.32, 0.14,0.78)
M_5	(0.80,0.53,0.64,0.74,0.23, 0.18,0.43)	(0.56,0.89,0.64,0.32,0.44, 0.66,0.43)	(0.90,0.25,0.56,0.47,0.88, 0.32,0.16)

Using the above data, the score values are calculated by using eqn. (4.5).

Table 2: Score Values for HPNVs

	A	***B***	***C***
M_1	0.974167	1.305	1.248333
M_2	1.0925	1.219167	1.289167

M_3	1.0675	1.215833	1.195833
M_4	1.3125	1.140833	1.233333
M_5	1.051667	1.159167	1.0275

Based on the score values, normalized values for HPNVs are obtained by using eqn. (4.6)

Table 3: Normalized values for HPNVs

	A	B	C
M_1	0.394081	0.482579	0.464351
M_2	0.44195	0.450839	0.47954
M_3	0.431837	0.449606	0.444822
M_4	0.530947	0.421871	0.458771
M_5	0.425432	0.428651	0.382206

Weighted normalized matrix along with the ideal best and ideal worst are calculated using the eqns. (4.7), (4.8), (4.9). Finally based upon the ideal best - ideal worst values, performance score has been calculated using the eqn. (4.10). Ranking is done based upon the performance scores.

Table 4: Weighted Normalized Matrix and Ideal Best - Ideal Worst for HPNVs

	A	B	C	Si+	Si-	Pi	Rank
M_1	0.118224	0.241289	0.09287	0.003038	0.053639	0.946401	1
M_2	0.132585	0.225419	0.095908	0.021403	0.036077	0.627645	2
M_3	0.129551	0.224803	0.088964	0.021174	0.035117	0.62385	3
M_4	0.159284	0.210936	0.091754	0.05123	0.015313	0.230122	5
M_5	0.12763	0.214325	0.076441	0.034561	0.031836	0.479475	4
V+	0.118224	0.241289	0.095908	-	-	-	-
V-	0.159284	0.210936	0.076441	-	-	-	-

6. Comparative study of MCDM by HPNWAA Operator TOPSIS method with MOORA Method.

This section gives the comparative study of the MCDM solved by TOPSIS method in the previous section, is compared with MOORA method. MOORA is designed to handle situations where multiple factors or criteria are important in decision-making, like cost, quality, performance, and environmental impact, among others.

6.1 Algorithm for MOORA method:

Step 1: Start with a decision matrix where rows represent alternatives and columns represent criteria.

Step 2: Normalize the decision matrix to handle different units of measurement. This can be done using methods like vector normalization.

$$\bar{\mathfrak{N}}_{ij} = \frac{\mathfrak{N}_{ij}}{\sqrt{\sum_{j=1}^{n} {\mathfrak{N}_{ij}}^2}} \ (i = 1,2, \ldots m)$$

Step 3: Calculate the ratio of the normalized values for each criterion for each alternative.

Step 4: Multiply the ratio values by the weight of the respective criterion if weights are assigned to the criteria.

Step 5: Estimation of Assessment Values by using the formula mentioned below:

$$y_i = \sum_{j=1}^{g} w_j \bar{\mathfrak{N}}_{ij} - \sum_{j=g+1}^{n} w_j \bar{\mathfrak{N}}_{ij}, (j = 1,2, \dots n)$$

Step 5: Rank the alternatives based on their overall performance.

Table 5: Normalization, Aggregation and Rank for HPNVs by MOORA method

	A	B	C	Normalized	B.C	N.B	y_i	Rank
M1	0.9742	1.305	1.2483	0.2938	0.1877	0.0721	0.1717	1
M2	1.0925	1.2192	1.2892	0.3295	0.1754	0.0745	0.1510	2
M3	1.0675	1.2158	1.1958	0.3219	0.1749	0.0691	0.1474	3
M4	1.3125	1.1408	1.2333	0.3958	0.1641	0.0712	0.1166	5
M5	1.0517	1.1592	1.0275	0.3171	0.1668	0.0594	0.1310	4

Table 6: Rank Comparison by TOPSIS with MOORA method

	$\mathfrak{P}_i$	y_i	**Rank by TOPSIS**	**Rank by MOORA**
M1	0.946401	0.171713	1	1
M2	0.627645	0.151019	2	2
M3	0.62385	0.14741	3	3
M4	0.230122	0.116622	5	5
M5	0.479475	0.130967	4	4

7. Conclusion

In this paper, we introduce the heptapartitioned neutrosophic weighted averaging operator, designed to address multi-criteria decision-making (MCDM) problems in uncertain environments. Building on neutrosophic logic, which represents uncertainty through three components—truth, indeterminacy, and falsity—the heptapartitioned neutrosophic system refines these components into more detailed partitions, offering a deeper representation of uncertainty.

The operator aggregates uncertain information, providing a single value that reflects both the criteria being evaluated and the confidence in the data. This approach is crucial in real-world scenarios where data is imprecise or incomplete. We also prove algebraic properties of this operator, solidifying its reliability in mathematical contexts. To further enhance decision-making, we incorporate score and accuracy functions. Using the proposed operator, we applied the TOPSIS method to solve an MCDM problem under uncertainty, comparing it with the MOORA method. While both methods produce rankings, the heptapartitioned neutrosophic operator within TOPSIS provides a more detailed ranking by better handling uncertainty.

This approach is beneficial in real-world decision-making, where traditional deterministic models fall short due to incomplete or imprecise data. By accommodating various degrees of uncertainty, the heptapartitioned neutrosophic logic offers more flexible and effective decision-making.

Future work will focus on proving additional properties of the heptapartitioned neutrosophic sets and expanding its application to other domains, such as healthcare, finance, and logistics, where uncertainty plays a significant role in decision-making.

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