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New measures of consistency and coverage for social research based on neutrosophic logic

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Abstract: The mathematical methods systematically used in social sciences often rely on statistical tools like correlation, which may not optimally model social phenomena, frequently described using words. Set theory, as proposed by C.C. Ragin, provides a more suitable tool, leveraging set asymmetry to measure set-theoretic consistency and coverage, which were later generalized to fuzzy sets. Subsequent extensions into the neutrosophic field are significant because the underlying binary logic in traditional models often fails to capture the complexities and nuances of social realities, such as those found in Afro-Latin American and Caribbean cosmovisions that encompass ambiguity and indeterminacy. This paper proposes new, logically grounded measures for consistency and coverage, derived naturally from logical operators known as neutrosophic R-implications (generalizations of fuzzy R-implications). By explicitly incorporating Indeterminacy alongside Truth and Falsity, these new measures gain a deeper theoretical connotation, emphasize the logical relationships central to Ragin's methods, and offer tools better aligned with complex, non-exclusive realities.

Keywords: Fuzzy set, single-valued neutrosophic set, set-theoretic consistency, set-theoretic coverage, social research, neutrosophic R-implication, Diverse Epistemologies, Afro-Latin American and Caribbean cosmovisions

1. Introduction

A critical analysis of the methods used for mathematical modeling in the social sciences appears in the works of the social scientist Charles C. Ragin [1]. The author questions the use of statistical correlation as a recurring method in research within this field. On the one hand, correlation is not synonymous with causality; on the other hand, social sciences are mostly explained by the use of words. Set theory is a more appropriate tool for modeling with words when it is compared to correlation; however, the former is not sufficiently used.

This author uses simple and well-known measures to calculate the relationships between sets, these are the set-theoretic consistency and set-theoretic coverage [2]. He considers interesting the contrast between the symmetry of correlation and the asymmetry of operations with sets. This asymmetry allows us to consider the multicausality that can occur in the same outcome.

Set-theoretic consistency is a measure of the subset of causal conditions concerning the outcome. It determines to what degree the same causal conditions cause the same outcome. The other measure is the set-theoretic coverage that determines the degree to which an outcome is obtained from a specific causal condition or combination of causal conditions. This is interpreted as meaning that the greater the number of causal paths to obtain the same result of an outcome, the lower the numerical value of the theoretical set coverage.

While consistency is a theoretical measure of the relationship between sets, coverage is an empirical relationship. Even when the causal relationship between sets is sufficiently consistent, a low coverage value makes knowledge about the outcome insignificant, since it occurs under restricted causal conditions compared to other outcomes.

Ragin tested the effectiveness of these measures. Ultimately, all of them are used to obtain IF-THEN rules to represent logical cause-effect relationships that are useful for classification or prediction depending on the context in which they are applied. IF-THEN rules are also developed in the information systems of rough sets [3]. These measures are part of the method called Quality Comparative Analysis (QCA) and its extension to the fuzzy field fsQCA, which consists of algorithms with steps defined by Ragin himself [4-7].

In another sense, in the first method measures are proposed based on examples expressed in variables with dichotomous values for crisp sets. After being criticized, Ragin generalized this idea to the fuzzy case, where the variables take values in the interval [0,1]. The importance of the fuzzy set theory to represent linguistic values that are characterized by vagueness is well known. Even, L. Zadeh established some concepts within fuzzy logic such as the linguistic variable that can take linguistic values rather than numerical values, because daily human beings efficiently perform complex calculations with words and generally without the need to use numbers [8].

More recently these measures were generalized to neutrosophic sets, specifically single-valued neutrosophic sets, where each element is assigned a triple of truth values, such that one of them represents truthfulness, another indeterminacy, and the third falsehood [9]. The only restriction is that each one must be a numerical value in the interval [0,1].

This methodological gap concerning formal intervention analysis within existing neutrosophic approaches is significant because the underlying Aristotelian binary logic (true/false, either/or) inherent in many traditional quantitative and even standard models often fails to adequately capture the nuances of complex social realities. Decolonial critiques highlight the need for methodologies engaging with diverse epistemologies that operate beyond such binaries [10,11]. Afro-Latin American and Caribbean cosmovisions (with notable examples in places like Cuba), for example, forged through the African diaspora and complex syncretism, frequently embody perspectives where reality is understood differently. They often navigate simultaneous, seemingly contradictory identities (e.g., syncretic deities combining Orisha and Saint figures), acknowledge causal influences from an active and often ambiguous spiritual realm, and utilize practices like divination that inherently engage with uncertainty[12]. Such worldviews, readily accommodating ambiguity, paradox, and multi-valence, resonate strongly not merely with fuzzy logic (representing partial truth) but arguably more profoundly with Neutrosophy [13]. As a generalization of fuzzy logic, Neutrosophy's framework explicitly incorporates Indeterminacy (I) alongside Truth (T) and Falsity (F) [13], offering conceptual tools better aligned with these complex, non-exclusive realities. Methodologically, approaches like Neutrosophic Qualitative Comparative Analysis (NQCA) [9] attempt to operationalize this by using neutrosophic sets (T, I, F) to represent complex social conditions and configurations.

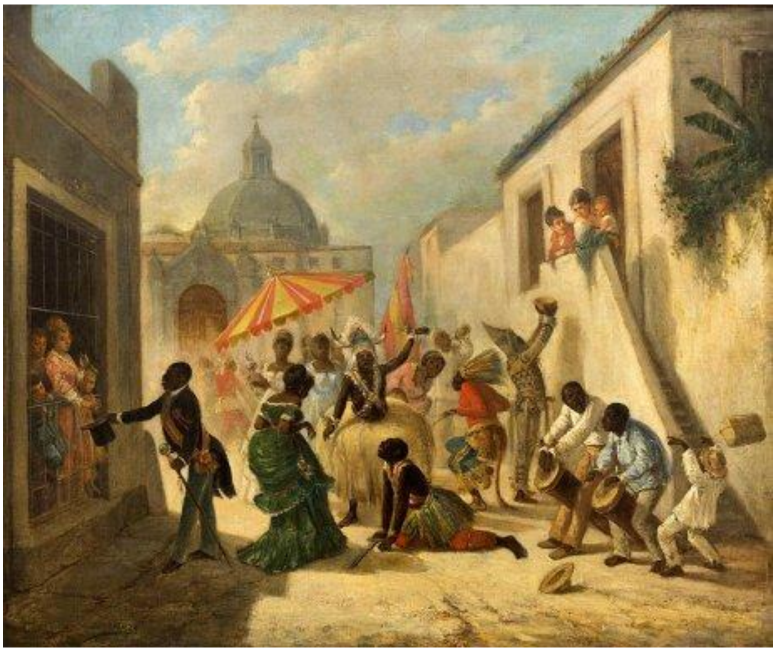

Figure 1. Día de Reyes en La Habana, by Víctor Patricio Landaluze (oil on canvas, 51 × 61 cm). *This 19th-century scene captures Afro-Cuban ritual, dance, and syncretism during Epiphany, highlighting colonial social hierarchies and cultural hybridization. The painting visually illustrates how Afro-Caribbean worldviews embody ambiguity, contradiction, and indeterminacy—core elements aligned with neutrosophic logic and decolonial epistemologies.*

The measures proposed by Ragin and later in the framework of neutrosophy are set-theist measures. In this paper, we introduce other measures that allow us to obtain similar numerical results, but that have a logical foundation that comes from classical logic and its extension to fuzzy and neutrosophic logic. Here we take into account the close relationship that exists between the concepts of logic and set. For example, if A and B are two fuzzy or neutrosophic sets, we can transfer the relationships between them to logic, when analyzing the relationship between the propositions $p_A(x)$: ="x is A" and $p_B(x)$: ="x is B" which is an equivalent way to the set-theist relationships of x ∈A and x ∈B. On the other hand, the methods proposed by Ragin have important logical components in their representation, especially causal ones, since they speak of necessary and sufficient relationships.

For this, we will base ourselves on single-valued neutrosophic sets. From here we can derive valid results for fuzzy sets when the triple (T(x),I(x),F(x))∈[0,1]^3 becomes (T(x),0,1-T(x)), or even if it becomes (T(x),1-T(x)-F(x),F(x))∈[0,1]^3, T(x)+F(x)≤1 we obtain results in the field of intuitionistic fuzzy sets.

The idea we follow is to obtain measures based on the R-implications defined by continuous t-norms [14]. Exactly, we use the neutrosophic R-implications or n-R-implications which are extensions of the R-implications to the Neutrosophy framework [15-17]. This idea is in line with what Ragin wanted to obtain intuitively as consistency and coverage measures. This is because the n-R-implications are logical operators where the concepts of necessity and sufficiency that Ragin talks about are legitimately handled. After all, the field of logic is where this makes more sense. Even more, the R-implications were the ones chosen by P. Hájek to develop his fuzzy logic theory in the narrow sense, where fuzzy logic is understood as an heir to classical mathematical logic and its concepts [14]. The n-R-implications, like the R-implications in fuzzy logic, satisfy the residuation condition which is a way of extending the Deduction theorem of bivalent classical logic.

In summary, we can affirm that the consistency and coverage measures that we propose in this article to measure the relationships between sets proposed by Ragin for the social sciences, can be replaced by others in the field of neutrosophic logic such that the valuations are similar. However, in our case, we will have a logical foundation that will allow us to link Ragin's theory with the theories of the neutrosophic logic or single-valued neutrosophic sets.

To this end, the article is divided according to the following structure; next it is a section of Preliminaries where the basic notions of the n-R-implications theory are recalled as well as neutrosophy and set-theoretic consistency and set-theoretic coverage according to Ragin. The section called New Measures of Consistency and Coverage, allows us to introduce the new proposed measures, their properties are demonstrated and an application example is presented. Next, a Discussion section follows. The last section gives the conclusions of the article.

1. Preliminaries

1.1. Neutrosophic R-Implications

Given a proposition p *in the propositional calculus, a Neutrosophic valuation is the triple ([18]):*

$v_N(p) = (t, i, f)$ (1)

Where, $(t, i, f) \in [0,1]^3$ *such that* t *is the degree of truthfulness,* i *is the degree of indeterminacy, and* f *is the degree of falseness.*

Given $v_1 = (t_1, i_1, f_1)$ *and* $v_2 = (t_2, i_2, f_2)$ *we have that* $v_1 \preccurlyeq_N v_2$ *if and only if:*

$t_1 \leq t_2$, $i_1 \geq i_2$, *and* $f_1 \geq f_2$ (2)

So, the maximum value of the neutrosophic valuation is $(1,0,0)$ *which is denoted by* $\bar{1}$, *and the minimum is* $(0,1,1)$ *which is denoted by* $\bar{0}$.

Definition 1 *([11]). Let* $T_N: [0,1]^3 \times [0,1]^3 \to [0,1]^3$ *be a mapping that satisfies the following conditions* $\forall x, y, z \in [0,1]^3$:

1. $T_N(x, y) = T_N(y, x)$ *(Commutativity),*
2. $T_N(T_N(x, y), z) = T_N(x, T_N(y, z))$ *(Associativity),*
3. $T_N(x, z) \preccurlyeq_N T_N(y, z)$ *for* $x \preccurlyeq_N y$ *(Monotonicity),*
4. $T_N(x, \bar{1}) = x$ *(Boundary conditions).*

Then we say that $T_N(\cdot,\cdot)$ *is a neutrosophic norm or n-norm.*

Definition 2 *([11]).. Let* $S_N: [0,1]^3 \times [0,1]^3 \to [0,1]^3$ *be a mapping that satisfies the following conditions* $\forall x, y, z \in [0,1]^3$:

1. $S_N(x, y) = S_N(y, x)$ *(Commutativity),*
2. $S_N(S_N(x, y), z) = S_N(x, S_N(y, z))$ *(Associativity),*
3. $S_N(x, z) \preccurlyeq_N S_N(y, z)$ *for* $x \preccurlyeq_N y$ *(Monotonicity),*
4. $S_N(x, \bar{0}) = x$ *(Boundary conditions).*

Thus, we say that $S_N(\cdot,\cdot)$ *is a neutrosophic conorm or n- conorm.*

Definition 3 *([11, 15]). A neutrosophic residual implication or n-R-implication is based on an n-norm* $T_N(x, y)$ *defined with the following equation:*

$RI_N(x, y) = sup\{u \in [0,1]^3 : T_N(x, u) \preccurlyeq_N y\}$ (3)

$\forall x, y \in [0,1]^3$.

Even after reviewing the literature dedicated to defining neutrosophic implicators, there are a few definitions of the axiomatic that a neutrosophic implicator must comply with, as indicated in the definition below:

Definition 4 ([15, 19]). A *single-valued neutrosophic implicator* (SVN-implicator for short) is an operator $I_N: [0,1]^3 \times [0,1]^3 \to [0,1]^3$, which satisfies the conditions shown below.

$\forall x, x', y, y' \in [0,1]^3$ it is fulfilled:

1. If $x' \preccurlyeq_N x$, then $I_N(x, y) \preccurlyeq_N I_N(x', y)$,

2. If $y \preccurlyeq_N y'$, then $I_N(x,y) \preccurlyeq_N I_N(x,y')$,
3. $I_N(\bar{0},\bar{0}) = I_N(\bar{0},\bar{1}) = I_N(\bar{1},\bar{1}) = \bar{1}$,
4. $I_N(\bar{1},\bar{0}) = \bar{0}$.

SVN-implicators can satisfy the following properties:

1. $I_N(\bar{1},x) = x$ (Neutrality principle),
2. $I_N(x,x) = \bar{1}$ $\forall x \in [0,1]^3$ (Identity principle),
3. $I_N(x,y) = I_N(n_N(y), n_N(x))$, where $n_N(x) = I_N(x,\bar{0})$ is an n-negator (Contrapositivity),
4. $I_N(x, I_N(y,z)) = I_N(y, I_N(x,z))$ (Interchangeability principle),
5. $x \preccurlyeq_N y$ if and only if $I_N(x,y) = \bar{1}$ (Confinement principle),
6. I_N is a continuous mapping (Continuity).

2.2. Set-theoretic consistency and set-theoretic coverage

Given a variable X representing a causal condition and a variable Y denoting the outcome, let us further denote by $X_i \in [0,1]$ the truth value of membership of the i-th case to the fuzzy set X. Similarly, $Y_i \in [0,1]$ is the fuzzy truth value of membership of the i-th case to the outcome Y_i. Then the consistency of the subset relation is expressed by the following equation ([2]):

$$consistency(X_i \le Y_i) = \frac{\sum min(X_i, Y_i)}{\sum X_i} \quad (4)$$

This measure satisfies that if all the fuzzy truth values of each causal condition are less than or equal to the truth values of their corresponding outcomes, then the consistency is maximum equal to 1. If for any case its causal truth value slightly exceeds the value of the outcome, then the consistency is slightly less than 1. Otherwise, the consistency is less than 1. This is a theoretical measure, Ragin sets a threshold value of 0.75 to determine that there is an acceptable degree of consistency, below this threshold the consistency should not be considered.

On the other hand, the coverage measure is defined by Equation 5 ([2]).

$$coverage(X_i \le Y_i) = \frac{\sum min(X_i, Y_i)}{\sum Y_i} \quad (5)$$

The formula above is used to calculate the degree to which the same outcome is obtained from different causes or causal combinations. Ragin recommends using this measure after realizing that there is a sufficient set-theoretic consistency. This is an empirical measure according to Ragin, where a low degree of coverage indicates that the probability of predicting the outcome is low since the same result is obtained from different causes or combinations of causes.

This method was generalized to the neutrosophic case in [9], using Neutrosophic Likert Scales. The first important definition in the proposed method is that of single-valued neutrosophic set.

Definition 5 ([9]). Let U be a universe of discourse. A *single-valued neutrosophic set* (SVNS) is defined as $N = \{(x, T(x), I(x), F(x)) : x \in U\}$, where $T, I, F : U \to [0,1]$ denote the membership functions of truthfulness, indeterminacy, and falseness, respectively, such that they satisfy the condition $0 \le T(x) + I(x) + F(x) \le 3$.

In the method the SVNS is converted into a fuzzy set as follows:

Let $A_N = \{(x, T_A(x), I_A(x), F_A(x)) : x \in U\}$ be a single-valued neutrosophic set, this becomes an equivalent fuzzy set by $A_F = \{(x, \mu_A(x)) : x \in U\}$, such that:

$\mu_A(x) = 1 - 0.5(1 - T_A(x) + max\ \{I_A(x), F_A(x)\})$ (6)

According to Vázquez et al., $\mu_A(x)$ is obtained from the measure of similarity between A_N and $(1,0,0)$ ([9]).

After converting the SVNS into a fuzzy set, the SVNN obtained in each case are converted into fuzzy truth values by using Equation 6.

Finally, Ragin's method for fuzzy sets is applied based on Equations 4 and 5.

2. New Measures of Consistency and Coverage

A fundamental property that n-R-implicators fulfill is the confinement principle, inherited from fuzzy R-implicators. It is known that $x \preccurlyeq_N y$ is a necessary and sufficient condition of $I_N(x,y) = \bar{1}$ when $I_N(x,y)$ is an n-R-implication. Also, the n-R-implication operators are defined from inequality as occurs in the measures proposed by Ragin, and therefore they are asymmetric relations as well.

If x_i is the valuation of the i-th causal condition for the proposition of being X and y_i is the valuation corresponding to the proposition of being Y, then the measures that we propose of neutrosophic consistency and coverage are the following:

$consistency_{NAve}(x_i \preccurlyeq_N y_i) = mean(\{x_i \Longrightarrow_N y_i\})$ (7)

$consistency_{NMin}(x_i \preccurlyeq_N y_i) = \cap_N\ (\{x_i \Longrightarrow_N y_i\})$ (8)

$coverage_{NAve}(x_i \preccurlyeq_N y_i) = mean(\{y_i \Longrightarrow_N x_i\})$ (9)

$coverage_{NMin}(x_i \preccurlyeq_N y_i) = \cap_N\ (\{y_i \Longrightarrow_N x_i\})$ (10)

In formulas 7 and 9 the arithmetic mean of the neutrosophic R-implications between a causal condition and its outcome is used. In formulas 8 and 10 the following operator is used:

$(T_1, I_1, F_1)\ \cap_N\ (T_2, I_2, F_2) = \ (min\{T_1, T_2\}, max\{I_1, I_2\}, max\{F_1, F_2\})$ (11)

Formulas that use the average combine logical properties with the use of the arithmetic mean as a measure of statistical central tendency. Meanwhile, measures that use the intersection between neutrosophic valuations are based on a joint measure defined in lattices.

$\cap_N$ is defined by $x \cap_N y = x *_N (x \Longrightarrow_N y)$, where $*_N$ is the n-norm and $\Longrightarrow_N$ is the neutrosophic R-implication defined from the n-norm. It is shown that with these components $x \cap_N y$ satisfies Equation 11 since it is the "meet" operator of the neutrosophic residuated lattice.

Properties

1. Every pair (x_i, y_i) satisfies $x_i \preccurlyeq_N y_i$ if and only if $consistency_{NAve}(x_i \preccurlyeq_N y_i) = consistency_{NMin}(x_i \preccurlyeq_N y_i) = \bar{1}$.
2. Every pair (x_i, y_i) satisfies $y_i \preccurlyeq_N x_i$ if and only if $coverage_{NAve}(y_i \preccurlyeq_N x_i) = coverage_{NMin}(y_i \preccurlyeq_N x_i) = \bar{1}$.
3. If $x_i, y_i \in \{\bar{0}, \bar{1}\}$ we have that $consistency_{NMin}(x_i \preccurlyeq_N y_i) = \bar{0}$, if there exists a pair (x_i, y_i) such that $x_i \not\preccurlyeq y_i$. Similarly $coverage_{NMin}(y_i \preccurlyeq_N x_i) = \bar{0}$, if there exists a pair (x_i, y_i) such that $y_i \not\preccurlyeq x_i$.
4. If $x_i, y_i \in \{\bar{0}, \bar{1}\}$ we have that $consistency_{NAve}(x_i \preccurlyeq_N y_i) = \left(\frac{m}{n}, 1 - \frac{m}{n}, 1 - \frac{m}{n}\right)$, where m is the number of pairs that satisfy $x_i \preccurlyeq_N y_i$ and n is the total number of pairs (x_i, y_i).

5. Similarly $coverage_{NAve}(y_i \preccurlyeq_N x_i) = \left(\frac{m}{n}, 1-\frac{m}{n}, 1-\frac{m}{n}\right)$, where m is the number of pairs fulfilling $y_i \preccurlyeq_N x_i$ and n is the total number of pairs (x_i, y_i).

Proof

1. Applying the confinement principle in Equations 7 and 8, we obtain the proof.
2. Equivalently, this proof is obtained from the confinement principle and Equations 9 and 10.
3. When $x_i, y_i \in \{\bar{0}, \bar{1}\}$ emulates the crisp case, we have that if there exists any pair where $x_i = \bar{1}$ and $y_i = \bar{0}$, then $(x_i \Longrightarrow_N y_i) = \bar{0}$ and it is enough that there exists such a case for the minimum to be $\bar{0}$.
   The proof to $coverage_{NMin}(y_i \preccurlyeq_N x_i)$ is equivalent.
4. $consistency_{NAve}(x_i \preccurlyeq_N y_i)$ is the arithmetic mean of $\bar{1}$ (when $x_i \preccurlyeq_N y_i$) and $\bar{0}$ (when $x_i \not\preccurlyeq_N y_i$) which is the proposed formula.
5. Adapting the same previous steps to $coverage_{NAve}(y_i \preccurlyeq_N x_i)$ we reach the proposed result.□

Note also that from the non-increasing property of $I_N(x, y)$ concerning x, then the larger is x for y, the smaller will be the value of the n-R-implication. This coincides with what is satisfied by the set-theoretic measures defined by Ragin.

The neutrosophic R-implications defined in the literature for valuations (t_x, i_x, f_x) and (t_y, i_y, f_y) are obtained as the triples defined below ([11-13]):

$$t_{\Rightarrow N\Pi} = \begin{cases} 1, if\ t_x \leq t_y \\ \frac{t_y}{t_x}, otherwise \end{cases}, \quad i_{\Rightarrow N\Pi} = \begin{cases} 0, if\ i_y \leq i_x \\ \frac{i_y - i_x}{1-i_x}, otherwise \end{cases}, \text{ and } f_{\Rightarrow N\Pi} = \begin{cases} 0, if\ f_y \leq f_x \\ \frac{f_y - f_x}{1-f_x}, otherwise \end{cases} \quad (12)$$

Called *Product*.

$$t_{\Rightarrow NG} = \begin{cases} 1, if\ t_x \leq t_y \\ t_y, otherwise \end{cases}, \quad i_{\Rightarrow NG} = \begin{cases} 0, if\ i_y \leq i_x \\ i_y, otherwise \end{cases}, \text{ and } f_{\Rightarrow NG} = \begin{cases} 0, if\ f_y \leq f_x \\ f_y, otherwise \end{cases} \quad (13)$$

Called *Gödel*'s.

$$t_{\Rightarrow NL} = \begin{cases} 1, if\ t_x \leq t_y \\ 1 - t_x + t_y, otherwise \end{cases}, \quad i_{\Rightarrow NL} = \begin{cases} 0, if\ i_y \leq i_x \\ i_y - i_x, otherwise \end{cases}, \text{ and } f_{\Rightarrow NL} = \begin{cases} 0, if\ f_y \leq f_x \\ f_y - f_x, otherwise \end{cases} \quad (14)$$

Called *Lukasiewicz*'s.

Below we illustrate with an example the use of the proposed measures in solving a real-life problem:

Example 1 ([9]):

The defined outcome is the perception of Academic Success (SUCCESS). A Likert scale is developed, represented as single-valued neutrosophic sets. The study also considers other variables: Academic Resources (RES), Motivation (MOT), and Quality of Teaching (QUAL). A survey was conducted with a group of 12 Software Engineering students at the University of Guayaquil (see Table 1).

Table 1. Survey Data. Taken from [9].

Case	RES	MOT	QUAL	SUCCESS
1	(0.9,0.9, 0.2)	(0.6, 1, 0.5)	(0.3, 0.7, 0.3)	(0.8, 0.6, 0.7)
2	(0.5, 0.5, 0.5)	(1,1,1)	(0.5,0.2,0.5)	(0.6, 0.6, 0.7)
3	(0.8, 0.7, 0.4)	(0.7, 0.9, 0.5)	(0.8, 0.5, 0.5)	(0.8, 0.5, 0.5)
4	(1,1,0)	(0.8,0.8,0)	(1,0.9,0.3)	(0.7, 1, 0.9)

Case	RES	MOT	QUAL	SUCCESS
5	(1,0.5,0)	(1,0.5,1)	(1,0.5,1)	(0.9, 0.6, 0.1)
6	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)	(0.9, 0.9, 0.9)
7	(0.2, 0.5, 0.8)	(1,0,0)	(0.5, 0.5, 0.5)	(0.8, 0.5, 0.2)
8	(1, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)	(0.9, 0.9, 0.1)
9	(1,1,0)	(0.8, 0.8, 0)	(1,0,0)	(0.9, 0, 0)
10	(0.7, 1, 0.2)	(0.9, 0.4, 0)	(0.6, 0.9, 0.1)	(1,0,0)
11	(0.4, 0.7, 0.2)	(0.3, 0.9, 0.4)	(0.8, 0.4, 0.6)	(0.4, 0.8, 0.3)
12	(0.6, 1, 0.6)	(0.6, 0.5, 0.2)	(0.2, 0.5, 0.7)	(1,0,1)

Consistency and coverage calculations for each n-norm and each of the cases are summarized in Tables 2-4. Note that we retain the notation * to indicate what Ragin calls set intersection, but which in the logical context is the conjunction of the causal conditions using $\cap_N$.

Table 2. The consistency and coverage measures of Equations 7-10 corresponding to the n-R-implication Product are calculated for four combinations of causal conditions.

Conditions tested	$\text{Consistency}_{\text{NMin}}$	$\text{Consistency}_{\text{NAve}}$	$\text{Coverage}_{\text{NMin}}$	$\text{Coverage}_{\text{NAve}}$
RES	(0.7, 0.333333, 1.0)	(0.940741, 0.0611111, 0.276389)	(0.25, 1.0, 0.75)	(0.865278, 0.345833, 0.0791667)
MOT	(0.6, 1.0, 1.0)	(0.93125, 0.141667, 0.208333)	(0.6, 1.0, 1.0)	(0.896991, 0.416667, 0.178571)
QUAL	(0.5, 1.0, 1.0)	(0.916667, 0.197222, 0.235714)	(0.2, 0.9, 1.0)	(0.802778, 0.1375, 0.158631)
RES*MOT*QUAL	(0.875, 0.2, 1.0)	(0.98125, 0.0166667, 0.188095)	(0.2, 1.0, 1.0)	(0.731019, 0.525, 0.281548)

Table 3. The consistency and coverage measures of Equations 7-10 corresponding to Gödel's n-R-implication are calculated for four combinations of causal conditions.

Conditions tested	$\text{Consistency}_{\text{NMin}}$	$\text{Consistency}_{\text{NAve}}$	$\text{Coverage}_{\text{NMin}}$	$\text{Coverage}_{\text{NAve}}$
RES	(0.7, 0.8, 1.0)	(0.933333, 0.166667, 0.35)	(0.2, 1.0, 0.8)	(0.833333, 0.383333, 0.0833333)
MOT	(0.6, 1.0, 1.0)	(0.916667, 0.175, 0.233333)	(0.3, 1.0, 1.0)	(0.825, 0.458333, 0.2)
QUAL	(0.4, 1.0, 1.0)	(0.908333, 0.25, 0.275)	(0.2, 0.9, 1.0)	(0.758333, 0.175, 0.183333)
RES*MOT*QUAL	(0.7, 0.6, 1.0)	(0.966667, 0.05, 0.216667)	(0.2, 1.0, 1.0)	(0.633333, 0.566667, 0.3)

Table 4. The consistency and coverage measures of Equations 7-10 corresponding to Lukasiewicz's n-R- implication are calculated for four combinations of causal conditions.

Conditions tested	$Consistency_{NMin}$	$Consistency_{NAve}$	$Coverage_{NMin}$	$Coverage_{NAve}$
RES	(0.7, 0.1, 0.9)	(0.941667, 0.025, 0.191667)	(0.4, 1.0, 0.6)	(0.883333, 0.291667, 0.0666667)
MOT	(0.6, 0.5, 0.9)	(0.933333, 0.0666667, 0.175)	(0.6, 0.8, 0.9)	(0.916667, 0.25, 0.108333)
QUAL	(0.6, 0.4, 0.6)	(0.925, 0.0833333, 0.125)	(0.2, 0.9, 0.9)	(0.825, 0.125, 0.133333)
RES*MOT*QUAL	(0.9, 0.1, 0.6)	(0.983333, 0.00833333, 0.0916667)	(0.2, 1.0, 0.9)	(0.775, 0.358333, 0.191667)

To convert the elements of Tables 2-4 into fuzzy values we can use Equation 6, resulting in the values shown in Tables 5-7. This allows us to compare the results obtained with those appearing in [9].

Table 5. The consistency and coverage measures of Table 2 are converted into fuzzy for four combinations of causal conditions.

Conditions tested	$Consistency_{NMin}$	$Consistency_{NAve}$	$Coverage_{NMin}$	$Coverage_{NAve}$
RES	0.35	0.8322	0.125	0.75972
MOT	0.30	0.8614583	0.300	0.74016
QUAL	0.25	0.8405	0.099	0.82207
RES*MOT*QUAL	0.4375	0.8966	0.099	0.60301

Table 6. The consistency and coverage measures of Table 3 are converted into fuzzy for four combinations of causal conditions.

Conditions tested	$Consistency_{NMin}$	$Consistency_{NAve}$	$Coverage_{NMin}$	$Coverage_{NAve}$
RES	0.35	0.79166	0.099	0.725
MOT	0.30	0.84166	0.150	0.683
QUAL	0.199	0.8166	0.099	0.787499
RES*MOT*QUAL	0.35	0.875	0.099	0.533

Table 7. The consistency and coverage measures of Table 4 are converted into fuzzy for four combinations of causal conditions.

Conditions tested	$Consistency_{NMin}$	$Consistency_{NAve}$	$Coverage_{NMin}$	$Coverage_{NAve}$
RES	0.399	0.875	0.199	0.795833
MOT	0.35	0.879166	0.35	0.833
QUAL	0.5	0.899	0.1499	0.845833
RES*MOT*QUAL	0.6499	0.945833	0.099	0.70833

As can be seen, the measurements based on Equations 8 and 10 give lower results than the measurements based on Equations 7 and 9. This is due to the use of an n-norm in the former instead of an average in the latter.

In all cases, the consistency of the combination of all causal variables (RES*MOT*QUAL) is greater for all operators when compared to the causal variables of a single operator (either RES, MOT, or QUAL). While the coverage is lower.

In Tables 2-4, it can be seen that the proposed method results in neutrosophic values, thus preserving accuracy, unlike the method in [9]. These values are feasible to become fuzzy values as seen in Tables 5-7. It is also possible to include values that cannot be defined in fuzzy logic or another like intuitionistic fuzzy logic, for example (1,1,1), which expresses maximum indeterminacy with maximum contradiction.

We recommend using the measures based on Equations 8 and 10 in problems where rules with very high truth value results are needed. On the other hand, the measures based on Equations 7 and 9 mean that the results are acceptable on average, so they are more advisable in this context.

In summary, measures based on the minimum are recommended for generating rules with high qualitative results since their values are high when the worst value of all is high, while rules generated by measures based on the arithmetic mean are better when quantitatively significant results are desired since high values are obtained even when the values of some individual results are not high.

Now let us compare the results with those obtained in [9], as seen in Table 8.

Table 8. Results of the example according to the method used in [9].

Conditions tested	**Consistency**	**Raw Coverage**	**Combined**
RES	0.927928	0.730496	0.841773
MOT	0.903226	0.794326	0.873243
QUAL	0.903226	0.794326	0.873243
RES*MOT*QUAL	0.957447	0.638298	0.794931

Table 8 shows that the variables are ordered according to consistency in $RES*MOT*QUAL \succ RES \succ MOT = QUAL$.

Guided by the Consistency$_{Nave}$ of Tables 5-7 we have the order relation $RES*MOT*QUAL \succ MOT \succ QUAL \succ RES$ for the product and Gödel n-R-implications. In the case of Lukasiewicz n-R-implication, this is $RES*MOT*QUAL \succ QUAL \succ MOT \succ RES$.

Regarding the comparison for Coverage$_{NAve}$ in Table 8 it can be seen that: $RES \succ MOT = QUAL \succ RES*MOT*QUAL$ in the method of Leyva et al., while in the proposed method we have: $QUAL \succ RES \succ MOT \succ RES*MOT*QUAL$ for the product and Gödel n-R-implications. For Lukasiewicz n-R-implication, we have $QUAL \succ MOT \succ RES \succ RES*MOT*QUAL$. In any case, the truth values obtained show a consistency greater than 0.79 and coverage of more than 0.5 as a truth value.

Let us now calculate the results for the * between two variables. For simplicity, we directly state the fuzzy values for the measures based on the arithmetic mean, see Tables 9-11.

Table 9. Consistency$_{NAve}$ and Coverage$_{NAve}$ fuzzified based on product n-R-implication.

Conditions tested	Consistency$_{NAve}$	Coverage$_{NAve}$
RES*MOT	0.89479166	0.63946759
RES*QUAL	0.86130952	0.71284722
MOT*QUAL	0.89657738	0.65196759
RES*MOT*QUAL	0.8966	0.60301

Table 10. Consistency$_{NAve}$ and Coverage$_{NAve}$ fuzzified based on Gödel n-R-implication.

Conditions tested	Consistency$_{NAve}$	Coverage$_{NAve}$
RES*MOT	0.875	0.566666
RES*QUAL	0.84166	0.675
MOT*QUAL	0.875	0.579166
RES*MOT*QUAL	0.875	0.533

Table 11. Consistency$_{NAve}$ and Coverage$_{NAve}$ fuzzified based on Lukasiewicz n-R-implication.

Conditions tested	Consistency$_{NAve}$	Coverage$_{NAve}$
RES*MOT	0.929166	0.741666
RES*QUAL	0.916666	0.754166
MOT*QUAL	0.945833	0.754166
RES*MOT*QUAL	0.945833	0.70833

Recall that in [9] the result was as shown in Table 12.

Table 12. Consistency and Coverage obtained in [9], for pairwise variable combinations.

Conditions	consistency	coverage	combined
RES*MOT	0.957895	0.645390	0.799335
RES*QUAL	0.950000	0.673759	0.812578
MOT*QUAL	0.960000	0.680851	0.821001
RES*MOT*QUAL	0.957447	0.638298	0.794931

As can be seen from Table 12, the consistency of RES*MOT*QUAL is slightly lower than the consistency of MOT*QUAL and RES*MOT. This is not expected because RES*MOT*QUAL is an intersection of more sets, so its value should be lower and the consistency is assumed to be higher.

If compared with the calculations in Tables 9-11, this is satisfied.□

In the software also called fsQCA, Ragin introduces other measures, for example, the one he calls *coincidence*, which he defines by the equation:

$coincidence(\mathrm{X}_i \leq \mathrm{Y}_i) = \frac{\sum min(\mathrm{X}_i,\mathrm{Y}_i)}{\sum max(\mathrm{X}_i,\mathrm{Y}_i)}$ (15)

In this paper, we introduce two new coincidence measures based on the ideas developed here, see Equations 16 and 17:

$coincidence_{NAve}(X_1, X_2, \cdots, X_n) = mean(\{(x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n) \Leftrightarrow_N (x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n)\})$(16)

$coincidence_{NMin}(X_1, X_2, \cdots, X_n) = min(\{(x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n) \Leftrightarrow_N (x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n)\})$ (17)

Where $x \Leftrightarrow_N y \coloneqq (x \Longrightarrow_N y) \cap_N (y \Longrightarrow_N x)$, it is about the neutrosophic bi-implication.

Some properties of these measures are as follows:

Properties

1. $coincidence_{NAve}(X_1, X_2, \cdots, X_n) =$ $mean(\{(x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n) \Longrightarrow_N (x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n)\})$ and $coincidence_{NMin}(X_1, X_2, \cdots, X_n) = min(\{(x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n) \Longrightarrow_N (x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n)\})$.
2. If $p(j)$ is a permutation of the indices $\{1, 2, \cdots, n\}$ then: $coincidence_{NAve}(X_1, X_2, \cdots, X_n) = coincidence_{NAve}(X_{p(1)}, X_{p(2)}, \cdots, X_{p(n)})$ and $coincidence_{NMin}(X_1, X_2, \cdots, X_n) = coincidence_{NMin}(X_{p(1)}, X_{p(2)}, \cdots, X_{p(n)})$. (Symmetry)
3. $coincidence_{NAve}(X_1, X_2, \cdots, X_n) \preccurlyeq_N coincidence_{NAve}(X_1, X_2, \cdots, X_{n-1})$ and $coincidence_{NMin}(X_1, X_2, \cdots, X_n) \preccurlyeq_N coincidence_{NMin}(X_1, X_2, \cdots, X_{n-1})$. (Non-increasing monotonicity)
4. If all $x_1, x_2, \cdots, x_n \in \{\bar{0}, \bar{1}\}$ then:
   $coincidence_{NMin}(X_1, X_2, \cdots, X_n) = \begin{cases} \bar{1}, if\ every\ x_i\ is\ equal\ to\ each\ other \\ \bar{0}, \quad otherwise \end{cases}$ and
   $coincidence_{NAve}(X_1, X_2, \cdots, X_n) = \left(\frac{m}{n}, 1 - \frac{m}{n}, 1 - \frac{m}{n}\right)$, where m is the number of elements with equal evaluations for each case for all variables.

Proof

1. Taking into account that $x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n \preccurlyeq x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n$ then $x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n \Longrightarrow_N x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n := \bar{1}$. Considering that the neutrosophic bi-implication is formed by the neutrosophic conjunction of the n-R-implication in both directions, in addition to the neutrosophic conjunction complies with the neutrality principle, then the property is proven.
2. Obvious.
3. From property 1 and the fact that $x_1 \cup_N x_2 \cup_N \cdots \cup_N x_{n-1} \preccurlyeq x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n$ and $x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n \preccurlyeq x_1 \cap_N x_2 \cap_N \cdots \cap_N x_{n-1}$, and the fact that the n-R-implications are non- increasing for the first argument and non- decreasing for the second argument, then comparing $x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n \Longrightarrow_N x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n$ with $x_1 \cup_N x_2 \cup_N \cdots \cup_N x_{n-1} \Longrightarrow_N x_1 \cap_N x_2 \cap_N \cdots \cap_N x_{n-1}$ we have that the former one is less than the last one. Then, applying the min and mean operators for all cases maintains this property.
4. Applying the confinement principle and taking into account Property 1 the only way it is fulfilled $coincidence_{NMin}(X_1, X_2, \cdots, X_n) = \bar{1}$ is when $x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n = x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n$, it is equivalent to saying that all the elements are equal for all cases. When these equalities are fulfilled, coincidence is $\bar{1}$ even when the elements are not $\bar{1}$ or $\bar{0}$.

When these are only $\bar{1}$ or $\bar{0}$ and one is different from the others then $x_1 \cap_N x_2 \cap_N \cdots \cap_N x_n \prec x_1 \cup_N x_2 \cup_N \cdots \cup_N x_n$, which implies that $coincidence_{NMin}(X_1, X_2, \cdots, X_n) = \bar{0}$.

On the other hand, if n is the total number of cases analyzed, while m is the number of cases in which all values $x_1, x_2, \cdots, x_n$ are equal to each other, then by calculating the average of Equation 17, it is satisfied that we have the average of the set containing m times $\bar{1}$s and $n-m$ times $\bar{0}$s. This is the formula that must be demonstrated in point 4 of the proposition.□

Example 2. In this example, we calculate the coincidence measure according to Equations 16 and 17 for the values in the table in Example 1, concerning the variables RES, MOT, and QUAL.

Table 13. Calculation of coincidence for the three variables RES, MOT, and QUAL. Conjoint neutrosophic results using Equations 16 and 17 for the three implications, and fuzzyfied results.

Coincidence	Equation	Neutrosophic	Fuzzyfied result
Set-theist	**From Ragin**	-	0.594937
Based on Π	**Coincidence$_{NMin}$**	[0.2, 1.0, 1.0]	0.099999
	Coincidence$_{NAve}$	[0.6486, 0.6778, 0.413889]	0.485417
Based on Gödel	**Coincidence$_{NMin}$**	[0.2, 1.0, 1.0]	0.099999
	Coincidence$_{NAve}$	[0.6083, 0.69167, 0.46667]	0.458333
Based on Lukasiewicz	**Coincidence$_{NMin}$**	[0.2, 1.0, 1.0]	0.099999
	Coincidence$_{NAve}$	[0.69167, 0.4000, 0.34167]	0.645833

As can be seen from the Table above, the results obtained with min are much stricter than those obtained with average.

3. Discussion

In the measures proposed so far in this article and taking into account the example studied, the effectiveness of calculating consistency, coverage, and coincidence using neutrosophic R-implications is demonstrated. From a practical point of view, the measures based on min show results that are too restrictive because this operator is too strict concerning the results. While the measures that use the average are more in line with what can be expected.

There are also differences between these measures concerning the type of implication used. Of the three of them, Lukasiewicz is more sensitive to changes in values. That is why we recommend it if we want to better differentiate different cases. On the other hand, the product and Gödel measures show greater robustness and less accuracy.

The proposed measures are a logical approximation to Ragin's method, so the results should be interpreted as truth values. In this article, we work with single-valued neutrosophic numbers and that is why more general input values are allowed than with the Ragin fuzzy method. Note in the example that the value (1,1,1) was included, which cannot be represented in any extension of fuzzy logic except for neutrosophic logic.

On the other hand, unlike the method proposed in ([9]), in this method, it is possible to perform calculations directly with single-valued neutrosophic numbers and the results can be fuzzified for representing them with a single numerical value. Furthermore, in the proposed method, unlike the set method, the coincidence is never undefined. This can be seen if the coincidence is calculated where the values of the sets are all 0, then it is undefined in Ragin's fuzzy method and not in the one proposed here.

4. Conclusions

Ragin's creation of the QCA and fuzzy QCA methods, with the ease of using software to perform the calculations, established consistency, coverage, and coincidence measures based on set theory. When data is in the form of single-valued neutrosophic numbers, a common approach has been to fuzzify them and apply the fuzzy QCA method as proposed in [9]. However, this can obscure the inherent indeterminacy. This paper addresses a methodological gap by proposing logical, non-set-based measures founded on neutrosophic R-implications. This approach is significant because the underlying Aristotelian binary logic inherent in many traditional models often fails to capture the nuances of complex social realities, such as those found in diverse epistemologies like Afro-Latin American and Caribbean cosmovisions which readily accommodate ambiguity, paradox, and multi-valence. Our logical method links several theories, including the Deduction theorem implicit in neutrosophic R-implications. We have proven theoretically and demonstrated with examples that our proposed measures offer good results, comparable to Ragin's set-based method. Crucially, the advantage we offer is that operating directly on single-valued neutrosophic numbers (incorporating Truth, Indeterminacy, and Falsity) allows calculation with valuations not representable in fuzzy logic and, more importantly, provides conceptual tools better aligned with complex, non-exclusive realities. Unlike the method in [9], our proposed method is applied directly to single-valued neutrosophic numbers, yielding valuations that preserve indeterminacy, thus offering a more adequate approach for phenomena where binary or fuzzy representations fall short.

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