Neutrosophic Sets and Systems, Vol. 85, 2025
University of New Mexico

Basic Statistical Concepts On Neutrosophic Soft Sets

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Abstract: This study explores the basic statistical measures—mean, standard deviation, covariance, and correlation—within the framework of neutrosophic soft sets. Neutrosophic soft sets provide a powerful mathematical model for handling uncertainty, indeterminacy, and inconsistency in real-world data analysis. The study defines and formulates these statistical concepts under neutrosophic soft environments, incorporating true, indeterminate, and false memberships. A systematic approach to computing the neutrosophic soft mean and standard deviation is presented to measure central tendency and data dispersion. Furthermore, neutrosophic soft covariance and correlation are examined to analyze relationships between uncertain datasets. Practical applications in decision-making, medical diagnostics, and financial analysis demonstrate the effectiveness of these statistical tools. This research enhances statistical methodologies for uncertain data, offering a robust analytical foundation for fields requiring flexible and reliable data interpretation.

Keywords: Neutrosophic soft set(NSS); Neutrosophic soft mean; Neutrosophic soft covariance; Neutrosophic soft correlation coefficient.

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1. Introduction

Making decisions and analyzing data frequently require dealing with ambiguity, uncertainty, and insufficient information. Fuzzy sets, intuitionistic fuzzy sets and soft sets are some of the mathematical models previously created to deal with these kinds of problems. Nevertheless, these models are limited in their ability to handle unclear, inconsistent, and partial data at the same time. Neutrosophic Soft Set(NSS) was pioneered by amalgamating the concepts of Neutrosophic Sets and Soft sets to overcome the limitations. This model effectively represents authenticity, ambiguity and non-authenticity components, making a more flexible and generalized tool for uncertainty modelling. Neutrosophic soft sets are extensively accustomed in decision-making, medical diagnosis, risk assessment, machine learning, and engineering applications where incomplete or conflicting data is involved.

Zadeh [15] initiated the study of uncertainties with the launch of fuzzy sets in 1965. Later, Atanassov pioneered the concept of intuitionistic fuzzy sets. Molodtsov [12] presented the idea of soft sets in 1999 and looked into its applications in a number of fields, including probability theory, theory of measurement, smoothness of functions, game theory, Perron integration, operation research and more. Later, basic soft set fundamentals were specified by Maji et al. [9].

In addition to introducing some new concepts and features Chen [5] identified mistakes in several of the findings of Maji et al. [9].Many scholars from a variety of fields are currently interested in studying the numerous characteristics and applications of soft set hypothesis. Since then, soft set theory used extensively in numerous scientific and social aspects. Maji et al. [8] evolved fuzzy soft sets which is a hybrid structure with the combination of fuzzy and soft sets. Maji et al later introduced intuitionistic fuzzy soft sets [10]. Neutrosophic set proposal , a computational tool which involves imprecise, ambiguity, and conflicting data, was first presented by Smarandache [8]. Neutrosophic soft sets were later introduced by Maji [18].

Applications of uncertainty-based statistical concepts pertaining to sociological problems, such as illegal immigration and human trafficking, can be found in Acharjee et al. [4], Mordeson et al. [16], and Acharjee and Mordeson [3]. Furthermore, Acharjee [1], Acharjee and Tripathy [2], and numerous others have hybrid structures associated with soft sets.

We lay groundwork for statistics of neutrosophic soft sets in this study. Here, we develop statistical ideas based on neutrosophic soft set theory. We begin by defining the neutrosophic soft mean, which extends traditional averaging techniques to accommodate neutrosophic values, thereby enabling the calculation of central tendencies in uncertain datasets. Building on this foundation, we developed neutrosophic soft covariance, which measures the degree of variability and co-dependency between neutrosophic soft attributes. This measure is essential for understanding relationships in data characterized by ambiguous membership degrees.Furthermore, we develop neutrosophic soft attribute correlation coefficients to quantify the strength and direction of associations between attributes in neutrosophic soft sets. These coefficients provide a valuable statistical tool for analyzing the interplay between various attributes when dealing with uncertain information and given examples.

Key contributions made by this study include the following to the field of statistical analysis in neutrosophic soft set theory:

Extension of Statistical Measures: It formalizes the computation of fundamental statistical evaluations within the neutrosophic soft set framework, incorporating true, indeterminate, and false memberships.

Handling Uncertainty and Indeterminacy: By integrating statistical tools with neutrosophic soft theory, the study provides a robust approach for analyzing data that involves uncertainty, vagueness, and partial truth, making it applicable in real-world scenarios where classical statistical methods fall short.

Improved Decision-Making Framework: The developed methods enhance decision-making processes in areas such as medical diagnostics, financial risk assessment, and engineering systems, where uncertainty plays a significant role.

Practical Applications and Examples: The study demonstrates the real-world applicability of neutrosophic soft statistical measures through examples and case studies, validating their effectiveness in uncertain environments.

2. Preliminaries

Definition 2.1[7]: A pair (S, A) over the discourse universe η, where S is a mapping provided by $S : A \to \wp(\eta)$, is called a soft set. A parameterised family of universe subsets is a soft set over η. Here, the approximation of the soft set (S,A) elements is denoted by S(e).

Example 2.2: A university is evaluating candidates for a scholarship based on three parameters:

1. **Good in Mathematics**
2. **Good in Sports**
3. **Good in Music**

Consider the group of applicants be $\eta = \{\chi, \lambda, \gamma, \varpi, \rho\}$

Consider the group of parameters be E = {Math, Sports, Music}

Here, soft set F is defined as follows:

- F(Math)=$\{\chi, \lambda, \gamma\}$ (Candidates χ, λ, and γ are good at Math)
- F(Sports)=$\{\lambda, \gamma, \varpi\}$ (Candidates λ, γ, and ϖ are good at Sports)
- F(Music)=$\{\chi, \varpi, \rho\}$ (Candidates χ, ϖ, and ρ are good at Music)

So, the **soft set representation** is: F={(Math,$\{\chi, \lambda, \gamma\}$),(Sports,$\{\lambda, \gamma, \varpi\}$),(Music,$\{\chi, \varpi, \rho\}$)}

Definition 2.3 [8]: When X contains a generic element X represented by λ, let X be the space of points (objects). Real standard elements of [0,1] are represented by the authentic degree function ξ_A, ambiguity membership function ζ_A, and non-authentic function η_A, which characterise a neutrosophic set A in X.

It is depicted as $A = \left\{ \left\langle \lambda, (\xi_A(\lambda), \zeta_A(\lambda), \eta_A(\lambda) / \lambda \in E, \xi_A, \zeta_A, \eta_A \in]0^-, 1^+[\right\rangle \right\}$. There are no limitations on the sum of $\xi_A(\lambda)$, $\zeta_A(\lambda)$ and $\eta_A(\lambda)$ and so $0^- \le \xi_A(\lambda) + \zeta_A(\lambda) + \eta_A(\lambda) \le 3^+$.

Definition 2.4[18]: Suppose X represents objects in the universe and E represents a collection of parameters. A neutrosophic soft set over X with respect to E is a couple (F,P), where F:E $\rightarrow$P(X) is a function that associates each parameter $e \in E$ with a soft set $F(e) \subseteq X$, which represents a subset of X relevant to the parameter e.

Example 2.5: Consider a group of candidates for a job U= $\{\chi, \lambda, \gamma, \varpi\}$.

The parameters are E = $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$= {Experience, Skills, Attitude}

Suppose that, F(Experience) = {< χ, 0.5, 0.6, 0.3 >, < λ, 0.4, 0.7, 0.6 >, < γ,0.6, 0.2, 0.3 >, < ϖ, 0.7, 0.3, 0.2 >}, F(skills) = {< χ, 0.6, 0.3, 0.5 >, < λ, 0.7, 0.4, 0.3 >, < γ, 0.8, 0.1, 0.2 >, < ϖ, 0.7, 0.1, 0.3 > }, F(Attitude) = {< χ, 0.7, 0.4, 0.3 >, < λ, 0.6, 0.7, 0.2 >, < γ, 0.7, 0.2, 0.5 >, < ϖ, 0.5, 0.2, 0.6 > }.

The NSS (F, E) can be depicted tabularly as follows:

Table 1. representation of the aforementioned data in a NSS format

U	Experience	Skills	Attitude
χ	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
λ	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
γ	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)
ϖ	(0.7,0.3,0.2)	(0.7,0.1,0.3)	(0.5,0.2,0.6)

3. Neutrosophic soft mean(NSM) and neutrosophic soft standard deviation(NSSD) of NSS

Definition 3.1: Suppose that U represents objects in the universe and (η, A) represents a NSS over U where $\eta(\varepsilon_i)$ is a NS regarding the characteristic ε_i ∈A, then NSM of (η,A) is represented by $\overline{\eta_A} = \{(A, \eta(A))\}$ and η(A) is defined as

$$\eta(A) = \{\frac{x_k}{(\min\{a_k^i\}, \min\{b_k^i\}, \max\{c_k^i\})} : k \in \Delta, i \in I\}$$

Here a_k^i, b_k^i and c_k^i are authentic value, ambiguity value and non-authentic value of x_k respectively, for the attribute ε_i where $\varepsilon_i \in A$.

If $0 \le \sigma \le 1, 0 \le \tau \le 1$ and $0 \le \varsigma \le 1$ then

$$(\sigma, \tau, \varsigma)\overline{\eta}_A = ((\sigma_1, \tau_1, \varsigma_1), (\sigma_2, \tau_2, \varsigma_2), (\sigma_3, \tau_3, \varsigma_3)........(\sigma_n, \tau_n, \varsigma_n)......)$$

Where $\sigma_k = 1$ if $\min\{\sigma_k^i\} \ge \sigma$, otherwise 0,

$\tau_k = 1$ if $\min\{\tau_k^i\} \ge \tau$, otherwise 0 and ς_k =0 if $\max\{\varsigma_k^i\} \le \varsigma$, otherwise 1 for i∈ I .

Example 3.2: Consider an NSS (η,A) over a universe U, where (η,A)={(ε1, {[x1, 0.5, 0.4, 0.2],[x2, 0.6, 0.5, 0.4],[x3, 0.9,0.8,0.6]}, ε2, {[x1, 0.4, 0.6, 0.3],[x2, 0.7, 0.8, 0.4],[x3, 0.2,0.4,0.6]},ε3, {[x1, 0.5, 0.4, 0.3],[x2, 0.6, 0.4, 0.8],[x3, 0.4,0.6,0.7]]})}

Then (0.5,0.6,0.4) $\overline{\eta_A}$ =((1,0,0),(0,0,1),(0,0,0));

Definition 3.3: If (η,A) be an NSS and $\varepsilon_i \in A, i \in I$ then $(\sigma,\tau,\varsigma)\overline{\eta}(\varepsilon_1) = (v_1, v_2,....v_n,....)$ where $v_j = (\sigma_j, \tau_j, \varsigma_j)$. In this case $\sigma_j = 1$ if $\eta_j^1(\varepsilon_i)(x_j) \geq \sigma$ and 0; $\tau_j = 1$ if $\eta_J^2(\varepsilon_i)(x_j) \geq \tau$ and 0; otherwise and $\varsigma_j = 0$ if $\eta_J^3(\varepsilon_i)(x_j) \leq \varsigma$ and 1; otherwise. Here $\eta_j^1(\varepsilon_i)(x_j)$ indicates the truth value of x_j in j[th] position of $\eta(\varepsilon_i)\forall j \in \Delta$. Similarly, $\eta_j^2(\varepsilon_i)(x_j)$ indicates the indeterminacy value of x_j in j[th] position of $\eta(\varepsilon_i)\forall j \in \Delta$. $\eta_j^3(\varepsilon_i)(x_j)$ Indicates the non-authentic value of x_j in j[th] position of $\eta(\varepsilon_i)\forall j \in \Delta$.

Example 3.4: Consider Example 3.2,here (0.6,0.2,0.4) $\overline{\eta}(\varepsilon_1)$ =((0,1,0),(1,1,0),(1,1,1)),

Definition 3.5: Let (η, A) be a NSS then scale of (η, A) is denoted by h and it is defined as

$$h = \max\{\eta_j^1(\varepsilon_i)(x_j), \eta_j^2(\varepsilon_i)(x_j), \eta_j^3(\varepsilon_i)(x_j) \setminus \varepsilon_i \in A, x_j \in U, i \in I, j \in \Delta\}$$

Example 3.6: Let's look at an example 3.2, here h=0.9

Definition3.7: Let U be a universe of objects and a collection of characteristics E, where $A \subseteq E$ and $|A| = n$. If (η,A) be a NSS on U, then (σ,τ,ς)-cut NSSD of (η,A) is represented by $((\sigma,\tau,\varsigma)\overline{(\eta,A)})$ and it is interpret as

$$\left(\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(\sigma,\tau,\varsigma)\overline{\eta_k^1(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^1}\right\|^2}, \sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(\sigma,\tau,\varsigma)\overline{\eta_k^2(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^2}\right\|^2},\right.$$

$$\left.\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(\sigma,\tau,\varsigma)\overline{\eta_k^3(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^3}\right\|^2}\right)$$

Where

$$\left\|(\sigma,\tau,\varsigma)\overline{\eta_k^1(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^1}\right\|^2 = \left\langle(\sigma,\tau,\varsigma)\overline{\eta_k^1(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^1}, (\sigma,\tau,\varsigma)\overline{\eta_k^1(\varepsilon_i)} - (\sigma,\tau,\varsigma)\overline{\eta_{k,A}^1}\right\rangle$$

and so on for other part.

Here $(\sigma,\tau,\varsigma)\overline{\eta_k^1(\varepsilon_i)}$ and $(\sigma,\tau,\varsigma)\overline{\eta_{k,A}^1}$ indicate the first coordinate of $(\sigma,\tau,\varsigma)\overline{\eta_k(\varepsilon_i)}$ and $(\sigma,\tau,\varsigma)\overline{\eta_{k,A}}$ respectively, where k indicates the k-th ordered pair representation of the authentic, ambiguity and the non-authentic values for $(\sigma,\tau,\varsigma)\overline{\eta_k(\varepsilon_i)}$ and $(\sigma,\tau,\varsigma)\overline{\eta_{k,A}}$ respectively.

Example 3.8: Let's look at an example 3.2, where

$$(N,A) = \{(\varepsilon_1, \{[x_1, 0.5, 0.4, 0.2], [x_2, 0.6, 0.5, 0.4], [x_3, 0.9, 0.8, 0.6]\},$$

Let σ=0.3, τ=0.4, ς =0.2 then

(0.3,0.4,0.2) $\overline{\eta(\varepsilon_1)}$ =((1,1,1),(0,1,1),(1,1,1));

(0.3,0.4,0.2) $\overline{\eta(\varepsilon_2)}$=((1,1,1),(1,1,1),(0,1,1));

(0.3,0.4,0.2) $\overline{\eta(\varepsilon_3)}$=((1,1,1),(1,1,1),(1,1,1))

Now (0.3,0.4,0.2) $\overline{\eta_A}$ =((1,1,1),(0,1,1),(1,1,1))

$$\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(0.3,0.4,0.2)\overline{\eta_k^1(\varepsilon_i)}-(0.3,0.4,0.2)\overline{\eta_{k,A}^1}\right\|^2}$$

$$=\sqrt{\frac{1}{3}(0+0+0+1+1+1+0+0+0)}=\sqrt{\frac{3}{3}}=1$$

$$\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(0.3,0.4,0.2)\overline{\eta_k^2(\varepsilon_i)}-(0.3,0.4,0.2)\overline{\eta_{k,A}^2}\right\|^2}$$

$$=\sqrt{\frac{1}{3}(0+0+0+0+0+0+0+0+0)}=0$$

$$\sqrt{\frac{1}{n}\sum_{i\in I,k\in\Delta}\left\|(0.3,0.4,0.2)\overline{\eta_k^3(\varepsilon_i)}-(0.3,0.4,0.2)\overline{\eta_{k,A}^3}\right\|^2}$$

$$=\sqrt{\frac{1}{3}(1+0+0+0+0+0+0+0+0)}=\sqrt{\frac{1}{3}}$$

$$\rho((0.3,0.2,0.4)(\eta,A))=(1,0,\sqrt{\frac{1}{3}})$$

The above result indicates that standard deviation of $(0.3,0.2,0.4)(\eta,A)$ is 1 from truth membership, 0 from indeterminacy and $\sqrt{\frac{1}{3}}$ false membership values.

4. Neutrosophic soft covariance and Neutrosophic soft attribute correlation coefficient with (σ,τ,ς)-cut:

Definition 4.1: Consider the set of attributes E in a universe U. Let (η,ψ) and $(\wp,\varphi)$ be NSS where $\psi,\varphi\subseteq E$, $|\psi|=n>|\varphi|=m$. We extend φ to ϕ = $\varphi\cup\{\hbar_{\ell+1},\hbar_{\ell+2},....,\hbar_n\}$ } such that $\wp_i^1(\hbar_k)(x_i)=1,\wp_i^2(\hbar_k)(x_i)=0$ and $\wp_i^3(\hbar_k)(x_i)=0$ for all $k\in\{\ell+1,\ell+2.....n\}$. Then the (σ,τ,ς)-cut level neutrosophic soft covariance of (η,ψ) and $(\wp,\varphi)$ is denoted by (σ,τ,ς) NSSCov $((\eta,\psi),(\wp,\varphi))$ and is defined as

$$(\sigma,\tau,\varsigma)NSSCov((\eta,\psi),(\wp,\varphi))=(\frac{1}{n}\{\|\Delta_1\|^2+\|\Delta_2\|^2+....+\|\Delta_n\|^2\},\frac{1}{n}\{\|\Delta_1^{'}\|^2+\|\Delta_2^{'}\|^2+...\|\Delta_n^{'}\|^2\},$$
$$\frac{1}{n}\{\|\Delta_1^{''}\|^2+\|\Delta_2^{''}\|^2+...\|\Delta_n^{''}\|^2\})$$

Where

$$\Delta_j=(\min\{\sigma\eta_1^1(\varepsilon_j),\sigma\wp_1^1(\hbar_j)\},\min\{\sigma\eta_2^1(\varepsilon_j),\sigma\wp_2^1(\hbar_j)\},.......,\min\{\sigma\eta_i^1(\varepsilon_j),\sigma\wp_i^1(\hbar_j)\},.....),$$

$$\Delta_j^{'}=(\min\{\sigma\eta_1^2(\varepsilon_j),\sigma\wp_1^2(\hbar_j)\},\min\{\sigma\eta_2^2(\varepsilon_j),\sigma\wp_2^2(\hbar_j)\},.......,\min\{\sigma\eta_i^2(\varepsilon_j),\sigma\wp_i^2(\hbar_j)\},.....),$$

$$\Delta_j^{''}=(\min\{\sigma\eta_1^3(\varepsilon_j),\sigma\wp_1^3(\hbar_j)\},\min\{\sigma\eta_2^3(\varepsilon_j),\sigma\wp_2^3(\hbar_j)\},........,\min\{\sigma\eta_i^3(\varepsilon_j),\sigma\wp_i^3(\hbar_j)\},.....),$$

The attributes $\hbar_{\ell+1},\hbar_{\ell+2},....,\hbar_n$ with $\wp_i^1(\hbar_k)(x_i)=0,\wp_i^2(\hbar_k)(x_i)=0$ and $\wp_i^3(\hbar_k)(x_i)=0$ for all $k \in \{\ell+1, \ell+2,...., n\}, i \in \Delta$ are called neutrosophic soft statistical fictitious characteristics

for φ relative to ψ .

(ii)Let U be a universe of objects and E be a set of attributes E. Let (η,ψ) and $(\wp,\varphi)$ be NSS where $\psi,\varphi \subseteq E, |\psi| = n = |\varphi|$. Then the (σ,τ,ς)-cut level neutrosophic soft covariance of (η,ψ) and $(\wp,\varphi)$ is denoted by (σ,τ,ς) NSSCov $((\eta,\psi),(\wp,\varphi))$ and is defined as

$$(\sigma,\tau,\varsigma)NSSCov((\eta,\psi),(\wp,\varphi)) = (\frac{1}{n}\{\|\Delta_1\|^2 + \|\Delta_2\|^2 + ... + \|\Delta_n\|^2\}, \frac{1}{n}\{\|\Delta_1^{'}\|^2 + \|\Delta_2^{'}\|^2 + ...\|\Delta_n^{'}\|^2\},$$
$$\frac{1}{n}\{\|\Delta_1^{''}\|^2 + \|\Delta_2^{''}\|^2 + ...\|\Delta_n^{''}\|^2\})$$

Where

$$\Delta_j = (\min\{\sigma\eta_1^1(\varepsilon_j),\sigma\wp_1^1(\hbar_j)\}, \min\{\sigma\eta_2^1(\varepsilon_j),\sigma\wp_2^1(\hbar_j)\},......., \min\{\sigma\eta_i^1(\varepsilon_j),\sigma\wp_i^1(\hbar_j)\},.....),$$
$$\Delta_j^{'} = (\min\{\sigma\eta_1^2(\varepsilon_j),\sigma\wp_1^2(\hbar_j)\}, \min\{\sigma\eta_2^2(\varepsilon_j),\sigma\wp_2^2(\hbar_j)\},......., \min\{\sigma\mu_i^2(\varepsilon_j),\sigma\wp_i^2(\hbar_j)\},.....),$$
$$\Delta_j^{''} = (\min\{\sigma\eta_1^3(\varepsilon_j),\sigma\wp_1^3(\hbar_j)\}, \min\{\sigma\eta_2^3(\varepsilon_j),\sigma\wp_2^3(\hbar_j)\},......., \min\{\sigma\eta_i^3(\varepsilon_j),\sigma\wp_i^3(\hbar_j)\},.....),$$

and $\varepsilon_j \in A, \hbar_j \in A, i \in \Delta, j \in I$.

Definition 4.2: Let (η, A) be an NSS with at least two attributes $\varepsilon_1,\varepsilon_2 \in A$, then the neutrosophic soft attribute correlation coefficient of $(\sigma,\tau,\varsigma)\eta(\varepsilon_1)$ and $(\sigma,\tau,\varsigma)\eta(\varepsilon_2)$ is denoted by NSACC($(\sigma,\tau,\varsigma)\eta(\varepsilon_1),(\sigma,\tau,\varsigma)\eta(\varepsilon_2)$) and is defined as

$$\text{NSACC}(\ (\sigma,\tau,\varsigma)\eta(\varepsilon_1),(\sigma,\tau,\varsigma)\eta(\varepsilon_2)) = (\frac{\sum_{i\in\Delta}\Delta_i}{\|\overline{\sigma\eta(\varepsilon_1)}\|.\|\overline{\sigma\eta(\varepsilon_2)}\|}, \frac{\sum_{i\in\Delta}\Delta_i^{'}}{\|\overline{\tau\eta(\varepsilon_1)}\|.\|\overline{\tau\eta(\varepsilon_2)}\|}, \frac{\sum_{i\in\Delta}\Delta_i^{''}}{\|\overline{\varsigma\eta(\varepsilon_1)}\|.\|\overline{\varsigma\eta(\varepsilon_2)}\|})$$

where $\Delta_i = \min\{\sigma\eta_i(\varepsilon_1),\sigma\eta_i(\varepsilon_2)\}$,

$\Delta_i^{'} = \min\{\tau\eta_i(\varepsilon_1),\tau\eta_i(\varepsilon_2)\}$,

$\Delta_i^{''} = \min\{\varsigma\eta_i(\varepsilon_1),\varsigma\eta_i(\varepsilon_2)\}$

$$\|\overline{\sigma\eta(\varepsilon_j)}\| = \sqrt{\langle\overline{\sigma\eta(\varepsilon_j)},\overline{\sigma\eta(\varepsilon_j)}\rangle} = \sqrt{\sum_{i\in I}(\sigma\eta_i(\varepsilon_j))^2}, \|\overline{\sigma\eta(\varepsilon_1)}\| \neq 0, \|\overline{\sigma\eta(\varepsilon_2)}\| \neq 0$$

$$\|\overline{\tau\eta(\varepsilon_j)}\| = \sqrt{\langle\overline{\tau\eta(\varepsilon_j)},\overline{\tau\eta(\varepsilon_j)}\rangle} = \sqrt{\sum_{i\in I}(\tau\eta_i(\varepsilon_j))^2}, \|\overline{\tau\eta(\varepsilon_1)}\| \neq 0, \|\overline{\tau\eta(\varepsilon_2)}\| \neq 0$$

$$\|\overline{\varsigma\eta(\varepsilon_j)}\| = \sqrt{\langle\overline{\varsigma\eta(\varepsilon_j)},\overline{\varsigma\eta(\varepsilon_j)}\rangle} = \sqrt{\sum_{i\in I}(\varsigma\eta_i(\varepsilon_j))^2}, \|\overline{\varsigma\eta(\varepsilon_1)}\| \neq 0, \|\overline{\varsigma\eta(\varepsilon_2)}\| \neq 0$$

If any one of $\|\overline{\sigma\eta(\varepsilon_1)}\|, \|\overline{\sigma\eta(\varepsilon_1)}\|, \|\overline{\tau\eta(\varepsilon_1)}\|, \|\overline{\tau\eta(\varepsilon_2)}\|, \|\overline{\varsigma\eta(\varepsilon_1)}\|, \|\overline{\varsigma\eta(\varepsilon_2)}\|$ *is 0,* then NSACC($(\sigma,\tau,\varsigma)\eta(\varepsilon_1),(\sigma,\tau,\varsigma)\eta(\varepsilon_2)$) is not feasible. Here we employ the notation (∞,∞,∞), regardless what is $\Delta_i, \Delta_i^{'}, or \Delta_i^{''}$.

Theorem 4.3: If (η, A) be any NSS with at least two attributes $e_1, e_2 \in A$ over a universe U, then $(0,0,0) \leq$ NSACC($(\sigma,\tau,\varsigma)\eta(\varepsilon_1),(\sigma,\tau,\varsigma)\eta(\varepsilon_2)) \leq (1,1,1)$.

Proof:Since

$$\sum_{i\in\Delta}\Delta_i=\sum_{i\in\Delta}\min\{\sigma\eta_i(\varepsilon_1),\sigma\eta_i(\varepsilon_2)\}\le\sum_{i\in\Delta}\{\sigma\eta_i(\varepsilon_1),\sigma\eta_i(\varepsilon_2)\}\le\sqrt{\sum_{i\in\Delta}(\sigma\eta_i(\varepsilon_1))^2}\sqrt{\sum_{i\in\Delta}(\sigma\eta_i(\varepsilon_2))^2}$$

$$=\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\eta(\varepsilon_2)}\right\|$$

So $\dfrac{\sum_{i\in\Delta}\Delta_i}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\eta(\varepsilon_2)}\right\|}\le 1.$

Similarly we can show that $\dfrac{\sum_{i\in\Delta}\Delta_i^{'}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\eta(\varepsilon_2)}\right\|}\le 1$, $\dfrac{\sum_{i\in\Delta}\Delta_i^{''}}{\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\varsigma\overline{\eta(\varepsilon_2)}\right\|}\le 1$

NSACC($(\sigma,\tau,\varsigma)\eta(\varepsilon_1),(\sigma,\tau,\varsigma)\eta(\varepsilon_2))\le(1,1,1)$

Again $\sum_{i\in\Delta}\Delta_i\ge 0,\ \left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\eta(\varepsilon_2)}\right\|>0$ So $\dfrac{\sum_{i\in\Delta}\Delta_i}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\eta(\varepsilon_2)}\right\|}\ge 0.$

Similarly $\dfrac{\sum_{i\in\Delta}\Delta_i^{'}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\eta(\varepsilon_2)}\right\|}\ge 0$, $\dfrac{\sum_{i\in\Delta}\Delta_i^{''}}{\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\varsigma\overline{\eta(\varepsilon_2)}\right\|}\ge 0$

Hence proved.

Theorem 4.4: If $\eta(\varepsilon_1),\wp(\varepsilon_1)$ and $\Im(\varepsilon_1)$ are three NSSs of three NSSs (η, A),($\wp$, A) and ($\Im$, A) over U such that $\eta(e_1)\subseteq\wp(e_1)\subseteq\Im(e_1)$ and $e_1\in A$.Then

NSACC($(\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\Im(\varepsilon_1)})\le$ NSACC($(\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\wp(\varepsilon_1)})$

NSACC($(\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\Im(\varepsilon_1)})\le$ NSACC($(\sigma,\tau,\varsigma)\overline{\wp(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\Im(\varepsilon_1)})$

NSACC($(\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\wp(\varepsilon_1)})\le$ NSACC($(\sigma,\tau,\varsigma)\overline{\Im(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\wp(\varepsilon_1)})$

if $\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|-\left\|\sigma\overline{\wp(\varepsilon_1)}\right\|^2\le 0,,$

$\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\Im(\varepsilon_1)}\right\|-\left\|\tau\overline{\wp(\varepsilon_1)}\right\|^2\le 0,\ and\ \left\|\varsigma\overline{\wp(\varepsilon_1)}\right\|^2-\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\varsigma\overline{\Im(\varepsilon_1)}\right\|\le 0$

Proof: Let

$$\text{NSACC}(\ (\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\Im(\varepsilon_1)})=(\frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|},\frac{\sum_{i\in\Delta}\Delta_i^{'1}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\Im(\varepsilon_1)}\right\|},\frac{\sum_{i\in\Delta}\Delta_i^{''1}}{\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\varsigma\overline{\Im(\varepsilon_1)}\right\|})$$

$\Delta_i^1=\min\{\sigma\eta_i(\varepsilon_1),\sigma\Im_i(\varepsilon_1)\}=\sigma\eta_i(\varepsilon_1),$

$\Delta_i^{'1}=\min\{\tau\eta_i(\varepsilon_1),\tau\Im_i(\varepsilon_1)\}=\tau\Im_i(\varepsilon_1),$

$\Delta_i^{''1}=\min\{\varsigma\eta_i(\varepsilon_1),\varsigma\Im_i(\varepsilon_1)\}=\varsigma\Im_i(\varepsilon_1)$

$\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|\ne 0,\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|\ne 0,\left\|\tau\overline{\eta(\varepsilon_1)}\right\|\ne 0,\left\|\tau\overline{\Im(\varepsilon_1)}\right\|\ne 0,\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|\ne 0,\left\|\varsigma\overline{\Im(\varepsilon_1)}\right\|\ne 0$

Consider

$$\text{NSACC}(\ (\sigma,\tau,\varsigma)\overline{\eta(\varepsilon_1)},(\sigma,\tau,\varsigma)\overline{\wp(\varepsilon_1)})=(\frac{\sum_{i\in\Delta}\Delta_i^2}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|},\frac{\sum_{i\in\Delta}\Delta_i^{'2}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\Im(\varepsilon_1)}\right\|},\frac{\sum_{i\in\Delta}\Delta_i^{''2}}{\left\|\varsigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\varsigma\overline{\Im(\varepsilon_1)}\right\|})$$

$\Delta_i^1=\min\{\sigma\eta_i(\varepsilon_1),\sigma\wp_i(\varepsilon_1)\}=\sigma\eta_i(\varepsilon_1),$

$\Delta_i^{'1} = \min\{\tau\eta_i(\varepsilon_1), \tau\wp_i(\varepsilon_1)\} = \tau\Im_i(\varepsilon_1),$

$\Delta_i^{''1} = \min\{\varsigma\eta_i(\varepsilon_1), \varsigma\Im_i(\varepsilon_1)\} = \varsigma\Im_i(\varepsilon_1)$

$\left\|\sigma\overline{\eta(\varepsilon_1)}\right\| \neq 0, \left\|\sigma\overline{\Im(\varepsilon_1)}\right\| \neq 0, \left\|\tau\overline{\eta(\varepsilon_1)}\right\| \neq 0, \left\|\tau\overline{\Im(\varepsilon_1)}\right\| \neq 0, \left\|\varsigma\overline{\eta(\varepsilon_1)}\right\| \neq 0, \left\|\varsigma\overline{\Im(\varepsilon_1)}\right\| \neq 0$

Now

$$\frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|} - \frac{\sum_{i\in\Delta}\Delta_i^2}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\wp(\varepsilon_1)}\right\|} = \frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|} - \frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\wp(\varepsilon_1)}\right\|}$$

$$= \frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|}\left\{\frac{1}{\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|} - \frac{1}{\left\|\sigma\overline{\wp(\varepsilon_1)}\right\|}\right\} \leq 0$$

Since $\wp(\varepsilon_1) \subseteq \Im(\varepsilon_1)$, then $\left\|\sigma\overline{\wp(\varepsilon_1)}\right\| \leq \left\|\sigma\overline{\Im(\varepsilon_1)}\right\|$

Thus $\frac{\sum_{i\in\Delta}\Delta_i^1}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\Im(\varepsilon_1)}\right\|} \leq \frac{\sum_{i\in\Delta}\Delta_i^2}{\left\|\sigma\overline{\eta(\varepsilon_1)}\right\|.\left\|\sigma\overline{\wp(\varepsilon_1)}\right\|}$

Now $\Delta_i^1 = 0$ or 1 for all $i \in \Delta$. Thus squaring of Δ_i^1 does not alter the sum and so on for other cases

Again

$$\frac{\sum_{i\in\Delta}\Delta_i^{'1}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\Im(\varepsilon_1)}\right\|} - \frac{\sum_{i\in\Delta}\Delta_i^{'2}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\wp(\varepsilon_1)}\right\|} = \frac{1}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|}\left\{\frac{\sum_{i\in\Delta}\Delta_i^{'1}}{\left\|\tau\overline{\Im(\varepsilon_1)}\right\|} - \frac{\sum_{i\in\Delta}\Delta_i^{'2}}{\left\|\tau\overline{\wp(\varepsilon_1)}\right\|}\right\} -$$

$$= \frac{1}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|}\left\{\frac{\left\|\tau\overline{\Im(\varepsilon_1)}\right\|^2}{\left\|\tau\overline{\Im(\varepsilon_1)}\right\|} - \frac{\left\|\tau\overline{\wp(\varepsilon_1)}\right\|^2}{\left\|\tau\overline{\wp(\varepsilon_1)}\right\|}\right\} = \frac{1}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|}\left\{\left\|\tau\overline{\Im(\varepsilon_1)}\right\| - \left\|\tau\overline{\wp(\varepsilon_1)}\right\|\right\} \leq 0$$

So $\frac{\sum_{i\in\Delta}\Delta_i^{'1}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\Im(\varepsilon_1)}\right\|} \leq \frac{\sum_{i\in\Delta}\Delta_i^{'2}}{\left\|\tau\overline{\eta(\varepsilon_1)}\right\|.\left\|\tau\overline{\wp(\varepsilon_1)}\right\|}$

Hence proved.

Similarly we can prove for remaining parts.

5. Real data application:

Correlation between Customer Satisfaction & Service Quality in a Bank Scenario:

A bank wants to analyze whether **customer satisfaction** is correlated with **service quality** while considering uncertainty in responses. It collects data from five branches, where:

T (Truth Membership): Customers who are satisfied with the service.

I (Indeterminacy Membership): Customers who are neutral or uncertain.

F (Falsity Membership): Customers who are dissatisfied.

The following table shows customer satisfaction and service quality ratings for four branches.

Table 2. Representation of data in a NSS

Branch	Satisfaction (T, I, F)	Service Quality (T, I, F)

A	(0.8,0.1,0.3)	(0.9,0.05,0.25)
B	(0.6,0.2,0.4)	(0.7,0.15,0.35)
C	(0.7,0.1,0.52)	(0.8,0.1,0.6)
D	(0.9,0.45,0.05)	(0.95,0.52,0.03)

Here, **Satisfaction** and **Service Quality** each have three components: authenticity **(T), ambiguity(I), and non-authenticity (F).**

Here U = $\{\chi, \lambda, \gamma, \varpi\}$ and a set of attributes E= {satisfaction, service quality}

Let e_1 = satisfaction, e_2 =service quality

Then

$(\eta,E)=(\{\varepsilon_1,\{\chi,0.8,0.1,0.3\},\{\lambda,0.6,0.2,0.4\},\{\gamma,0.7,0.1,0.52\},\{\varpi,0.9,0.45,.05\}),(\{\varepsilon_2\{\chi,$

$0.9,0.05,0.25\},\{\lambda,0.7,0.15,0.35\},\{\gamma,0.8,0.1,0.6\},\{\varpi,0.95,0.52,0.03\})$

Let (σ,τ,ς) = (0.7, 0.2, 0.3)

$(0.7,0.2,0.3)\overline{\eta(\varepsilon_1)}$ ={(0,1,0),(1,1,0),(1,1,0),(0,0,1)}

$(0.7,0.2,0.3)\overline{\eta(\varepsilon_2)}$ ={(0,1,1),(1,1,0),(0,1,0),(0,0,1)}

Then $\Delta_1 = \min\{0,0\} = 0, \Delta_2 = \min\{1,1\} = 1, \Delta_3 = \min\{1,0\} = 0, \Delta_4 = \min\{0,0\} = 0$

$\Delta'_1 = \min\{1,1\} = 1, \Delta'_2 = \min\{1,1\} = 1, \Delta'_3 = \min\{1,1\} = 1, \Delta'_4 = \min\{0,0\} = 0$

$\Delta''_1 = \min\{0,1\} = 0, \Delta''_2 = \min\{0,0\} = 0, \Delta''_3 = \min\{0,0\} = 0, \Delta''_4 = \min\{1,1\} = 1$

Then NSACC($(0.7,0.2,0.3)\overline{\eta(\varepsilon_1)}$,$(0.7,0.2,0.3)\overline{\eta(\varepsilon_2)}$) $=((\frac{1}{\sqrt{2}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{2}})$ =(0.7,0.6,0.7)

6. Conclusion

In this study, we developed some fundamental aspects on neutrosophic soft sets using cut sets. The application of statistical concepts within the framework of neutrosophic soft sets delivers a powerful methodology for handling uncertainty, indeterminacy, and inconsistency in data analysis. Neutrosophic soft mean and standard deviation offer a structured approach to measuring central tendency and dispersion in uncertain datasets, while neutrosophic soft covariance and correlation facilitate the evaluation of relationships between variables in imprecise environments. These measures enhance decision-making processes in fields such as health care, finance, and engineering, where data often contain elements of vagueness and partial truth. By integrating statistical analysis with neutrosophic soft set theory, This study advances the creation of more efficient and adaptable tools for real-world data analysis. Future research can further optimize these methods and extend their applications to complex, high-dimensional datasets.

Funding: "This research received no external funding"

Acknowledgments: I thank the management of PVP siddhartha institute of Technology for their support in research.

Conflicts of Interest: The authors declare no conflict of interest

References

1. Acharjee, S. (2017). New results for Kharal's similarity measures of soft sets, New Math. Nat. Comput, 13 (01), 55–60.
2. Acharjee, S., & Tripathy, B. C. (2017). Some results on soft bitopology, Bol. Soc. Paran. Math, 35 (1), 269–279.

3. Acharjee, S., & Mordeson, J. N. (2017). Soft statistics with respect to utility and application in human trafficking, New Math. Nat. Comput., 13 (03), 289–310.
4. C¸ agman, N., Karatas¸, S., & Enginoglu, S. (2011). Soft topology, Comput. Math. Appl., 62, 351–358.
5. Chen, D. (2005). The parametrization reduction of soft sets and its application, Comput. Math. Appl., 49, 757–763.
6. Huang, H. L. (2017). An improved correlation coefficient of intuitionistic fuzzy sets, Jour. Intel. Syst., 13 pages
7. Li, H., & Shen, Y. (2012). Similarity measures of fuzzy soft sets based on different distances, Fifth Int. Symp. Comput. Intel. and Design., 28-29 Oct 2012, Hangzhou, China, 527–529.
8. Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets,Jour. Fuzzy Math., 9 (3), 589–602.
9. Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory, Comput. Math. Appl., 45, 555–562.
10. Maji, P. K., Roy, A. R., & Biswas, R. (2004). On Intuitionistic Fuzzy Soft Sets, Jour. FuzzyMath., 12 (3), 669–683.
11. Mitchell, H. B. (2004). A correlation coefficient for intuitionistic fuzzy sets, Int. J. Intel. Syst., 19 (5), 483–490.
12. Molodtsov, D. (1999). Soft set theory – first results, Comput. Math. Appl., 37, 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.
13. Pabitra Kumar Maji(2013),Neutrosophic soft sets,Annals of Fuzzy Mathematics and Informatics, 5(1),157–168.
14. Xiao, Z., & Zou, Y. (2014). A comparative study of soft sets with fuzzy sets and rough sets, Jour. Intel. Fuzzy Syst., 27, 425–434.
15. Zadeh, L. A. (1965). Fuzzy sets, Inform. Contrl.,8,338–353.
16. M., J., S., I., & R., P. (2023). Generalized Double Statistical Convergence Sequences on Ideals in Neutrosophic Normed Spaces. Neutrosophic Systems With Applications, 8, 50-60. https://doi.org/10.61356/j.nswa.2023.40
17. Kandemir, H. Şengül, Aral, N. D., Karakaş, M., & Et, M. (2024). Neutrosophic Statistical Analysis of Temperatures of Cities in the Southeastern Anatolia Region of Turkey. Neutrosophic Systems With Applications, 14, 50-59. https://doi.org/10.61356/j.nswa.2024.119
18. F. Smarandache(2005), Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Inter. J. Pure Appl. Math. 24 , 287–297.
19. Acharjee, S., Sarma, D. J., Hanneman, R. A., Mordeson, J. N., & Malik, D. S. (2018). Fuzzy soft attribute correlation coefficient and application to data of human trafficking, Proyecciones Jour. of Math., 37 (4), 637–681.
20. Atanassov, K. T. (1999). Intuitionistic Fuzzy Sets: Theory and Applications, Springer PhysicaVerlag, Heidelberg.
21. Mordeson, J. N., Mallenby, M., Mathew, S., & Acharjee, S. (2017). Human trafficking: Policy intervention, New Math. Nat. Comput., 13 (03), 341–358.
22. Pei, D., & Miao, D. (2005). From soft sets to information systems, Proceedings of the IEEE , international conference on granular computing, 25-27 July 2005, Beijing, China, 2, 617– 621.

Received: Nov. 19, 2024. Accepted: April 22, 2025