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Inverse Neutrosophic Planar Graphs and its application in optimizing the expenditure in inter-cropping.

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Abstract: Though Neutrosophic graphs have high impact on dealing with problems of indeterminacy, it may not directly address all aspects of edge membership degree constraints such as the truth membership degree of all edge is greater than or equal to the minimum of the membership degrees of its nodes is a specific structural constraint that may not be the case with the general framework of Neutrosophic graphs. This paper introduces a novel concept in the realm of Neutrosophic graphs defining a new graph variation called "Inverse Neutrosophic Graph". This idea has the opportunity to enhance the field of Neutrosophic graph and provide new perspective into handling ambiguity in complex systems. Additionally, the basic properties and algebraic operations are studied in Inverse Neutrosophic graph. Moreover, we define Inverse Neutrosophic multi-graph and planarity in this paper. The related theories that establish a Neutrosophic Multi-graph's strong (weak) planarity and score functions are then introduced. In our final section, we demonstrate how planarity in Inverse Neutrosophic Multi-graphs can be used to make decisions about how to minimize the expense of managing crops in inter cropping.

Keywords: Inverse Neutrosophic graphs, Inverse Neutrosophic (multi)graphs, planarity, decision making.

1.Introduction

Neutrosophic graphs are a new, emerging discipline area in graph theory and fuzzy logic, which falls under the larger theme of neutrosophy. Neutrosophy, initiated by Florentin Smarandach, is a generalization of classical logic that brings in the perception of indeterminacy and uncertainty, which provides models for complex and real-world systems. Neutrosophy introduced by Florentin Smarandache in the late 1990s, expands the classical ideas of true, false, and indeterminacy. It emphasizes the study of indeterminate and uncertain phenomena more than binary logic. In a neutrosophic graph, each edge or vertex is associated with a triplet of membership functions representing truth, indeterminacy, and falsity. Early research on neutrosophic graphs was primarily concerning on the extension of the classical graph theory concepts, such as connectivity and paths, to the neutrosophic framework. These studies were designed to define basic properties and operations within neutrosophic graphs, including the concept of neutrosophic distance and centrality. Also advanced studies were developed by focusing on algorithm for shortest path, minimum spanning tree and other graph related theories into neutrosophic theory. Furthermore, hybrid modelling was established by incorporating fuzzy or probabilistic graphs in neutrosophy structure to enhance the modelling of complex structure.

Arindam Dey and co-authors explores various types of operations on neutrosophic graph [2], Naz delves into the operations on Single valued neutrosophic graphs along with its application [4]. Zeng S; Shoaib M and co-authors, reviewed Certain Properties of Single-Valued Neutrosophic Graph with Application in Food and Agriculture Organization [11]. Akram analyses the planarity concept in single valued neutrosophic graph and presented their exciting properties [1]. The Limitations in neutrosophic planar graph were unveiled by Rupkumar Mahapatra, Sovan Samanta & Madhumangal Pal through the investigation of generalized neutrosophic planar graphs by determining the overall planarity score including truth, falsity and indeterminacy. Also, their properties were analysed and effective utilisation of GNPG is explored with a real time applications. The Forgotten Topological Index (ToI) and the Edge Forgotten Index (EFI) that provide deeper insight for quantifying uncertainty and the quality of connections in a neutrosophic graph along with its application were explore by Vetrivel, G., Mullai, M., & Buvaneshwari, R, moreover these authors present a comprehensive evaluation that showcases the transition from fuzzy logic to neutrosophic logic in graph theory. [16,17]

The perception of Inverse Neutrosophic mixed graph were brought forward by Thempaavai and Antony crispin sweety that deals with the scenario, where truth membership degrees of the edges are $\geq$ to the minimum truth membership degrees of the associated vertices, and where the false membership and indeterminacy values of the edges are $\leq$ to the maximum false membership and indeterminacy values of the corresponding vertices. incorporating directed and undirected edges together[15]

The article probes deeper into the concept of Inverse Neutrosophic graph and provider in-depth perspective on

- Commencing with an introductory section that likely explains the motivation behind Inverse NG that is debuted as the solution for the scenario possessing inverse relationships.
- Exploring various mathematical concepts related to Inverse NG, such as complement, degrees, subgraphs and various operations and associated theorems within this framework. These concepts are clarified and demonstrated using examples.
- Familiarizing the concept of multigraphs, planarity, strong/week planarity within the context of Inverse NG and its application that offers a significant insight.

2. Preliminary

Within this segment, the basic definitions are introduced.

DEFINITION 2.1

A fuzzy graph G (σ, μ) is an ordered pair, where σ signifies the fuzzy subset of vertices and μ signifies edge set that is a fuzzy relation on σ.

DEFINITION 2.2

A graph $G^* = [V, \sigma\ (T_1, I_1, F_1), \mu\ (T_2, I_2, F_2)]$ is triplet that is said to be Neutrosophic graph, where $\sigma\ (T_1, I_1, F_1)$ signifies the neutrosophic subset of vertices and $\mu\ (T_2, I_2, F_2)]$ signifies edge set that is a neutrosophic connection on $\sigma\ (T_1, I_1, F_1)$.

DEFINITION 2.3

The Inverse fuzzy graph containing a non-void set V, $E \subseteq V \times V$ and G = $\{V, E, \mu, \tau)$, where $\mu: V \to [0,1]$, $\tau: E \to [0,1]$ such that $\tau\,(ab) \geq \mu(a) \wedge \mu(b)$, $\forall$ ab $\in E$where μ signifies membership value of vertices and τ signifies membership value of the edges.

DEFINITION 2.4

A Neutrosophic multi graph, the triplet $G^* = [V, \sigma\ (T_1, I_1, F_1), \mu\ (T_{2n}, I_{2n}, F_{2n})]$ where

$V = \{v1, v2, v3, \ldots vn\}$ such that $T_1 : V \longrightarrow [0,1]$, $I_1 : V \longrightarrow [0,1]$ and $F_1 : V \rightarrow [0,1]$ and $T_{2n} : V \times V \longrightarrow [0,1]$, $I_{2n}: V \times V \longrightarrow [0,1]$ and $F_{2n}: V \times V \rightarrow [0,1]$, n = 1,2,…m $| (v_i, v_j) \in V \times V \}$ such that

$$T_{2n}(v_i, v_j) \leq \min \{T_1(v_i), T_1(v_j)\}$$

$$I_{2n}(v_i, v_j) \leq \min \{I_1(v_i), I_1(v_j)\}$$

$$F_2n(v_i, v_j) \leq \max \{F_1(v_i), F_1 v_j)\}$$

and $0 \leq T_i(v) + I_i\ (v) + F_i\ (v) \leq 3\ T_{2n}(v_i, v_j) + I_{2n}(v_i, v_j) + F_{2n}(v_i, v_j) \leq 3\ \forall\ vi \in V\ (i = 1,2,3, \ldots n)$ here, (T_1, I_1, F_1)signify the degree of truth membership function, indeterminacy membership function, and falsity membership function of the vertex $v1 \in V$ correspondingly and $T_{2n}\ (v_i, v_j)$, $I_{2n}(v_i v_j)$ and $F_{2n}\ (v_i, v_j)$ signify the degree of truth-membership function, indeterminacy–membership function, and falsity-membership function of the edge (v_i, v_j) correspondingly.

DEFINITION 2.5

A graph is termed to be planar if it can be embedded in the plane in a manner that no edges intersect, except at their vertices.

3. INVERSE NEUTROSOPHIC GRAPH

Within this segment the Inverse Neutrosophic Graph is presented. Initially the Inspiration behind this work is given, along with some preliminary concepts are analyzed.

3.1 Inspiration behind this work

Despite numerous real time application of Neutrosophic graphs, they are unable to address many problems where graphs are employed for its representation. For the case, according to mixed cropping in agricultural, when we cultivate two crops X and Y together in the same field minimizes the expenditure and increases profitability over cultivating the crops individually, consequently Neutrosophic graphs is inefficient in addressing this scenario. In general, according to probability theory for mutually exclusive events, the probability of their union is equal to the sum of the probabilities of each individual event. whereas for independent events the probability of union of two events is equal to the sum of probabilities of each individual events from which the probabilities of its intersection are subtracted. Additionally, the probabilities of intersection of two events are ≤ to minimum of individual events. Also, the probability of union of two events is ≥ to maximum of individual events that is greater than equal to minimum of two individual events in both the scenarios. Considering the event X to be the vertices of the graph, the true, indeterminacy and false membership values for each vertex can be assigned such that membership values can be equal to the probability of the associated events. more over the edge between two vertices X and Y represents the event happening X ∪ Y, the truth membership of edge will be ≥ to minimum of the membership degrees of its vertices, indeterminacy and false membership degree of edge will be ≤ to maximum of the membership degrees of its vertices. To deal with the issues aforementioned, we propose the Inverse Neutrosophic Graph to handle the problem where traditional graphs fall short. We evaluate the properties of Inverse NG and examine the planarity in Inverse NG to determine the effectiveness and developed the relates theorems. Finally explore the application for this model.

3.2. Frame work of Inverse NG

DEFINITION 3.1

Let $\mathbb{V}$ be a finite vertex set and $\mathbb{E}$ be a finite edge set contained in $\mathbb{V} \times \mathbb{V}$. Then $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is said to be an Inverse Neutrosophic Graph, were $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$: $\mathbb{V} \to [0, 1]$, (χ_T, χ_I, χ_F): $\mathbb{E} \to [0,1]$ such that,

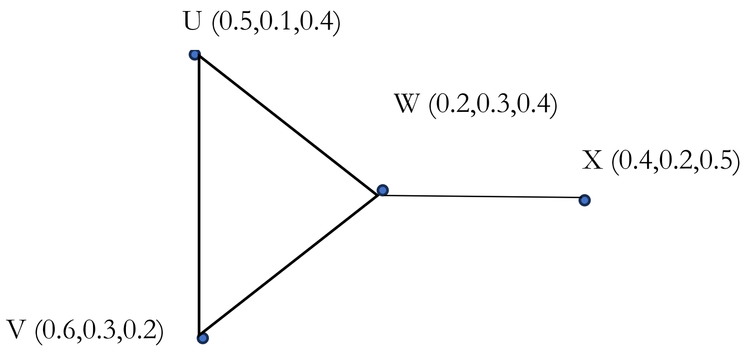
$$\chi_T(mn) \geq \varepsilon_T(m) \wedge \varepsilon_T(n), \forall\ mn \in \mathbb{E}$$

$$\chi_I(mn) \leq \varepsilon_I(m) \vee \varepsilon_I(n), \forall\ mn \in \mathbb{E}$$

$$\chi_F(mn) \leq \varepsilon_F(m) \vee \varepsilon_F(n), \forall\ mn \in \mathbb{E}$$

thereabouts $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$ denotes the true – membership degree of vertices, indeterminacy – membership degree of vertices and false membership degree of vertices consequently, (χ_T, χ_I, χ_F) denotes the true-membership degree of edges, indeterminacy - membership degree of edges and false membership degree of edges consequently.

Example 3.1



Edges

uv (0.7,0.2,0.3)

vw (0.4,0.1,0.3)

wx (0.2,0.1,0.4)

uw (0.4,0.2,0.3)

Figure 1.

DEFINITION 3.2

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)]$ be two Inverse NG. Then ζ'' is termed as the partial Inverse Neutrosophic subgraph, if $\mathbb{V}'' \subseteq \mathbb{V}'$, where $\chi''_{Ti} \geq \chi'_{Ti}$, $\chi''_{Ii} \leq \chi'_{Ii}$ and $\chi''_{Fi} \leq \chi'_{Fi}$, for all $m_i \in \mathbb{V}''$. ζ'' is termed as an Inverse Neutrosophic subgraph, if

$\mathbb{V}'' \subseteq \mathbb{V}'$, where $\chi''_{Tij} = \chi'_{Tij}$, $\chi''_{Iij} = \chi'_{Iij}$ and $\chi''_{Fij} = \chi'_{Fij}$ for all $(m_i n_j) \in \mathbb{E}''$.

Corollary: It's obvious that Inverse Neutrosophic subgraph is partial Inverse NG but reverse is not factual generally.

Example 3.2

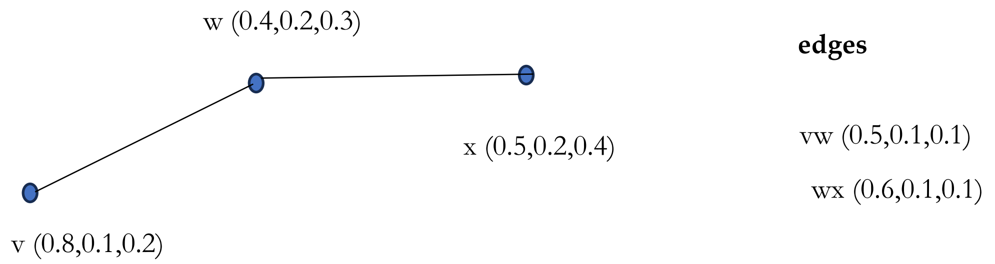


Figure 2.a

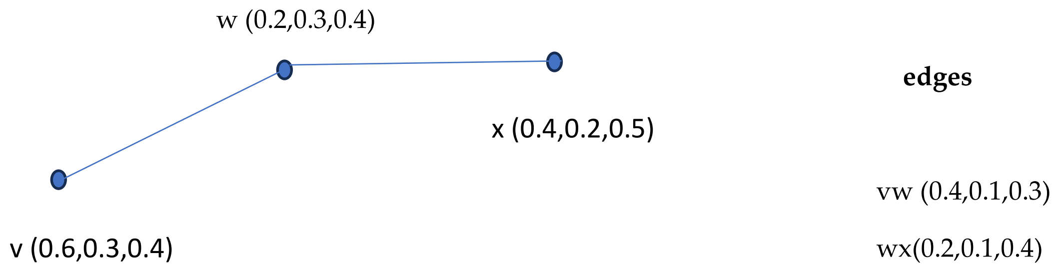


Figure 2.b.

DEFINITION 3.3

Assume $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently

(i) The vertex degree of any vertex m $\in \mathbb{V}$ is signified by deg (m) and described as

deg (m) = $(deg_T(m), deg_I(m), deg_F(m))$ where

$$deg_T(m) = \sum_{mn\in\mathbb{E}} \chi_T(mn),$$

$$deg_I(m) = \sum_{mn\in\mathbb{E}} \chi_I(mn),$$

$$deg_F(m) = \sum_{mn\in\mathbb{E}} \chi_F(mn).$$

(ii) The order of ζ is signified by O (ζ) and well-defined as

$$O(\zeta) = \sum_{m\in\mathbb{V}} (\varepsilon_T(m), \varepsilon_I(m), \varepsilon_F(m)).$$

(iii) The **size** of ζ is signified by S (ζ) and well-defined as $S = \sum_{mn\in\mathbb{E}}(\chi_T(mn), \chi_I(mn), \chi_F(mn))$

Theorem 3.1

Assume that ζ is an Inverse NG. subsequently $\sum_{m\in\mathbb{V}} \deg(m) = 2\sum_{mn\in\mathbb{E}} \chi_T(mn), \chi_I(mn), \chi_F(mn)$

Proof: straight forward

Example 3.3

Consider the Inverse NG ζ as in Figure 3.

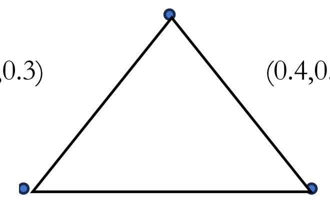


Figure.3

The degree of vertices of ζ, deg(u) = (0.9,0.4,0.6), deg(v) = (0.7,0.3,0.7), deg(w) = (0.6,0.3,0.7)

The order of ζ, O(ζ) = (1.1,0.6,1.3) and the size of ζ, S(ζ) = (1.1,0.5,1).

Also, $\sum_{m\in\mathbb{V}} \deg(m)$ = deg(u) = (0.9,0.4,0.6) + deg(v) = (0.7,0.3,0.7) + deg(w) = (0.6,0.3,0.7)

= (2.2,1,2) = 2 $\sum_{mn\in\mathbb{E}} \chi_T(mn), \chi_I(mn), \chi_F(mn)$ = 2(1.1,0.5,1).

DEFINITION 3.4

Assume $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently ζ is said to have stable vertex if for any $mn \in \mathbb{E}$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$ (m) = $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$ (n) and to have stable edge if for any $mn \in \mathbb{E}$,$(\chi_T, \chi_I, \chi_F)(mn) = (\chi_T, \chi_I, \chi_F)(uv)$ and said to be stable if ζ has both stable vertex and stable edge.

Example 3.4

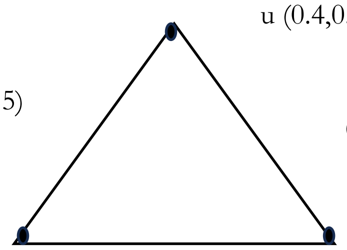


Figure 4.

In the above figure, vertex u and v are said to be a stable vertex and edges uv & uw are said to be stable edges.

DEFINITION 3.5

Assume $\zeta = [\mathbb{V}, \mathbb{E}, (\chi_T, \chi_I, \chi_F), (\varepsilon_T, \varepsilon_I, \varepsilon_F)]$ is an Inverse NG, subsequently **complete** Inverse NG is defined as

$$\chi_T(mn) = \varepsilon_T(m) \wedge \varepsilon_T(n), \forall\, mn \in \mathbb{E}$$
$$\chi_I(mn) = \varepsilon_I(m) \vee \varepsilon_I(n), \forall\, mn \in \mathbb{E}$$
$$\chi_F(mn) = \varepsilon_F(m) \vee \varepsilon_F(n), \forall\, mn \in \mathbb{E}$$

Example 3.5

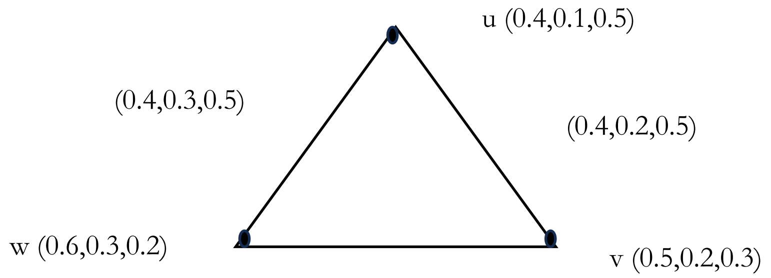


Figure 5.

4. Operations on Inverse NG.

We explore several operations involving Inverse NG, such as union, intersection and cartesian product.

DEFINITION 4.1

Assume $\zeta' = [\mathbb{V}', \mathbb{E}', (\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)]$ and $\zeta'' = [\mathbb{V}'', \mathbb{E}'', (\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)]$ be two ING. Subsequently, we denote and define the union of Inverse NG ζ' and ζ'' as

$$\zeta' \cup \zeta'' = [(\mathbb{V}' \cup \mathbb{V}''), (\mathbb{E}' \cup \mathbb{E}''), (\chi'_T \cup \chi''_T), (\chi'_I \cup \chi''_I), (\chi'_F \cup \chi''_F), (\varepsilon'_t \cup \varepsilon''_t), (\varepsilon'_I \cup \varepsilon''_I), (\varepsilon'_F \cup \varepsilon''_F)]$$

$$\varepsilon'_T \cup \varepsilon''_T(m) = \begin{cases} \varepsilon'_T(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \text{ and } \mathrm{m} \notin \mathbb{V}'' \\ \varepsilon''_T(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}'' \text{ and } \mathrm{m} \notin \mathbb{V}' \\ \varepsilon'_T(m) \vee \varepsilon''_T(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_I \cup \varepsilon''_I(m) = \begin{cases} \varepsilon'_I(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \text{ and } \mathrm{m} \notin \mathbb{V}'' \\ \varepsilon''_I(m) \ when \ \ \mathrm{m} \in \mathbb{V}'' \text{ and } \mathrm{m} \notin \mathbb{V}' \\ \varepsilon'_I(m) \wedge \varepsilon''_I(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_F \cup \varepsilon''_F(m) = \begin{cases} \varepsilon'_F(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \text{ and } \mathrm{m} \notin \mathbb{V}'' \\ \varepsilon''_F(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}'' \text{ and } \mathrm{m} \notin \mathbb{V}' \\ \varepsilon'_F(m) \wedge \varepsilon''_F(m) \ \ when \ \ \mathrm{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\chi'_T \cup \chi''_T(mn) = \begin{cases} \chi'_T(mn) \ when \ mn \in \mathbb{E}' \ and \ mn \notin \mathbb{E}'' \\ \chi''_T(mn) \ when \ mn \in \mathbb{E}'' \ and \ mn \notin \mathbb{E}' \\ \chi'_T(mn) \vee \chi''_T(mn) \ when \ mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_I \cup \chi''_I(mn) = \begin{cases} \chi'_I(mn) \ when \ mn \in \mathbb{E}' \ and \ mn \notin \mathbb{E}'' \\ \chi''_I(mn) \ when \ mn \in \mathbb{E}'' \ and \ mn \notin \mathbb{E}' \\ \chi'_I(mn) \wedge \chi''_T(mn) \ when \ mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_F \cup \chi''_F(mn) = \begin{cases} \chi'_F(mn) \ when \ mn \in \mathbb{E}' \ and \ mn \notin \mathbb{E}'' \\ \chi''_F(mn) \ when \ mn \in \mathbb{E}'' \ and \ mn \notin \mathbb{E}' \\ \chi'_F(mn) \wedge \chi''_T(mn) \ when \ mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

Example 4.1

Table 1: Two Inverse Neutrosophic graphs ζ_1 *and* ζ_2 as in **Figure 6.a**

Vertices of ζ_1	Edges of ζ_1	Vertices of ζ_2	Edges of ζ_2
v_2(0.1,0.2,0.1)	v_2v_3(0.4,0.1,0.1)	v_1(0.5,0.1,0.4)	v_1v_2(0.7,0.2,0.2)
v_3(0.5,0.2,0.4)	v_4v_3(0.3,0,0.2)	v_2(0.6,0.3,0.2)	v_2v_3(0.8,0.1,0.3)
v_4(0.1,0.2,0.2)	v_2v_4(0.2,0.1,0.1)	v_3(0.2,0.3,0.4)	v_4v_3(0.3,0.1,0.3)
		v_4(0.4,0.2,0.5)	v_2v_4(0.5,0.1,0.4)

Table 2: The union of two Inverse Neutrosophic graph is **Figure 6.b**

,

Vertices of $\zeta_1 \cup \zeta_2$	Edges of $\zeta_1 \cup \zeta_2$
v_1(0.5,0.1,0.4)	v_1v_2(0.7,0.2,0.2)
v_2(0.6,0.2,0.1)	v_2v_3(0.8,0.1,0.1)
v_3(0.5,0.2,0.4)	v_4v_3(0.3,0,0.2)
v_4(0.4,0.2,0.2)	v_2v_4(0.5,0.1,0.1)

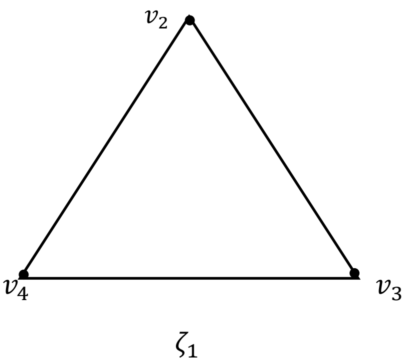


Figure 6.a

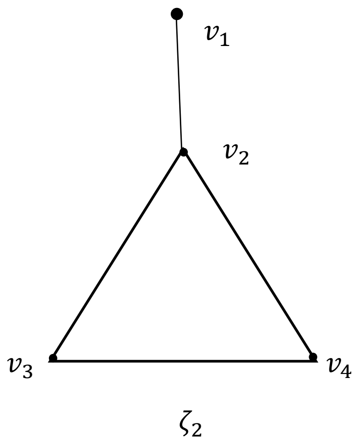
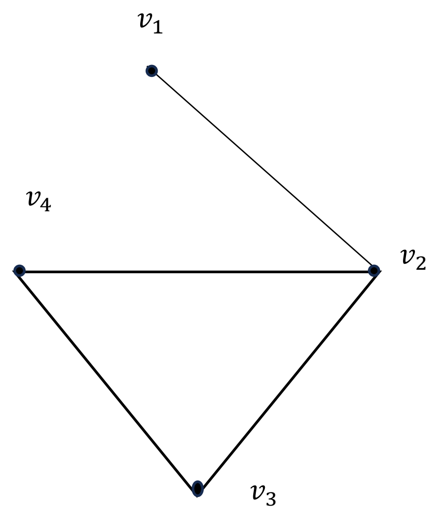


Figure 6.b



$\zeta_1 \cup \zeta_2$

Figure 6.c

From the above example it is evident that union of two inverse Neutrosophic graph is not an inverse Neutrosophic graph. Therefore, a new operation called quasi – Union is introduced which ensures that resulting graph maintains the characteristics of Inverse Neutrosophic graph.

DEFINITION 4.1

Assume ζ'= [$\mathbb{V}'$, $\mathbb{E}'$,$(\chi'_T, \chi'_I, \chi'_F), (\varepsilon'_t, \varepsilon'_I, \varepsilon'_F)$] and ζ''= [$\mathbb{V}''$, $\mathbb{E}''$,$(\chi''_T, \chi''_I, \chi''_F), (\varepsilon''_t, \varepsilon''_I, \varepsilon''_F)$] be two ING. Subsequently, we denote and define the quasi-union of Inverse NG ζ' and ζ'' as

$\zeta' \cup \zeta''$= [$(\mathbb{V}' \cup \mathbb{V}''), (\mathbb{E}' \cup \mathbb{E}''), (\boldsymbol{\chi'_T \cup \chi''_T}), (\boldsymbol{\chi'_I \cup \chi''_I}), (\boldsymbol{\chi'_F \cup \chi''_F}), (\boldsymbol{\varepsilon'_t \cup \varepsilon''_t}), (\boldsymbol{\varepsilon'_I \cup \varepsilon''_I}), (\boldsymbol{\varepsilon'_F \cup \varepsilon''_F})$]

$$\varepsilon'_T \cup \varepsilon''_T\,(m) = \begin{cases} \varepsilon'_T(m) \;\; when \;\; \text{m} \in \mathbb{V}' \text{ and m} \notin \mathbb{V}'' \\ \varepsilon''_T(m) \;\; when \;\; \text{m} \in \mathbb{V}'' \text{ and m} \notin \mathbb{V}' \\ \varepsilon'_T(m) \wedge \varepsilon''_T(m) \;\; when \;\; \text{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_I \cup \varepsilon''_I\,(m) = \begin{cases} \varepsilon'_I(m) \;\; when \;\; \text{m} \in \mathbb{V}' \text{ and m} \notin \mathbb{V}'' \\ \varepsilon''_I(m) \;\; when \;\; \text{m} \in \mathbb{V}'' \text{ and m} \notin \mathbb{V}' \\ \varepsilon'_I(m) \vee \varepsilon''_I(m) \;\; when \;\; \text{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\varepsilon'_F \cup \varepsilon''_F\,(m) = \begin{cases} \varepsilon'_F(m) \;\; when \;\; \text{m} \in \mathbb{V}' \text{ and m} \notin \mathbb{V}'' \\ \varepsilon''_F(m) \;\; when \;\; \text{m} \in \mathbb{V}'' \text{ and m} \notin \mathbb{V}' \\ \varepsilon'_F(m) \vee \varepsilon''_F(m) \;\; when \;\text{m} \in \mathbb{V}' \cap \mathbb{V}'' \end{cases}$$

$$\chi'_T \cup \chi''_T\,(mn) = \begin{cases} \chi'_T(mn) \;\; when \;\; mn \in \mathbb{E}' \; and \; mn \notin \mathbb{E}'' \\ \chi''_T(mn) \;\; when \; mn \in \mathbb{E}'' \; and \; mn \notin \mathbb{E}' \\ \chi'_T(mn) \vee \chi''_T(mn) \;\; when \; mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_I \cup \chi''_I\,(mn) = \begin{cases} \chi'_I(mn) \;\; when \;\; mn \in \mathbb{E}' \; and \; mn \notin \mathbb{E}'' \\ \chi''_I(mn) \;\; when \;\; mn \in \mathbb{E}'' \; and \; mn \notin \mathbb{E}' \\ \chi'_I(mn) \wedge \chi''_T(mn) \;\; when \;\; mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

$$\chi'_F \cup \chi''_F (mn) = \begin{cases} \chi'_F(mn) \text{ when } mn \in \mathbb{E}' \text{ and } mn \notin \mathbb{E}'' \\ \chi''_F(mn) \text{ when } mn \in \mathbb{E}'' \text{ and } mn \notin \mathbb{E}' \\ \chi'_F(mn) \wedge \chi''_T(mn) \text{ when } mn \in \mathbb{E}' \cap \mathbb{E}'' \end{cases}$$

Theorem 4.1

The Quasi - Union of every two ING is an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs.

For every mn $\in \mathbb{E}' \cup \mathbb{E}''$, one of these three instances occur.

Instance I:

Assume, mn $\in \mathbb{E}'$ and mn $\notin \mathbb{E}''$, Thus

(i) $\chi_T(mn) = \chi'_T(mn) \geq \chi'_T(m) \wedge \chi'_T(n)$

given that, mn $\in \mathbb{E}'$ and mn $\notin \mathbb{E}''$ therefore m $\in \mathbb{V}'$ and m $\notin \mathbb{V}''$ or m $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_T(m) = \varepsilon'_T(m)$ or $\varepsilon_T(m) = \varepsilon'_T(m) \wedge \varepsilon''_T(m)$, thus $\varepsilon'_T(m) \geq \varepsilon_T(m)$.

Furthermore $\varepsilon'_T(n) \geq \varepsilon_T(n)$.

Therefore $\chi_T(mn) = \chi'_T(mn) \geq \chi'_T(m) \wedge \chi'_T(n) \geq \chi_T(m) \wedge \chi_T(n)$.

$(\boldsymbol{ii})$ $\chi_I(mn) = \chi'_I(mn) \leq \chi'_I(m) \vee \chi'_I(n)$

given that, mn $\in \mathbb{E}'$ and mn $\notin \mathbb{E}''$ therefore m $\in \mathbb{V}'$ and m $\notin \mathbb{V}''$ or m $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_I(m) = \varepsilon'_I(m)$ or $\varepsilon_I(m) = \varepsilon'_I(m) \vee \varepsilon''_I(m)$, thus $\varepsilon'_I(m) \leq \varepsilon_I(m)$.

Furthermore $\varepsilon'_I(n) \leq \varepsilon_I(n)$.

Therefore $\chi_I(mn) = \chi'_I(mn) \leq \chi'_I(m) \vee \chi''_I(n) \leq \chi_I(m) \vee \chi_I(n)$

$(\boldsymbol{iii})$ $\chi_F(mn) = \chi'_F(mn) \leq \chi'_F(m) \vee \chi'_F(n)$

given that, mn $\in \mathbb{E}'$ and mn $\notin \mathbb{E}''$ therefore m $\in \mathbb{V}'$ and m $\notin \mathbb{V}''$ or m $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_F(m) = \varepsilon'_F(m)$ or $\varepsilon_F(m) = \varepsilon'_F(m) \vee \varepsilon''_F(m)$, thus $\varepsilon'_F(m) \leq \varepsilon_F(m)$.

Furthermore $\varepsilon'_F(n) \leq \varepsilon_F(n)$.

Therefore $\chi_F(mn) = \chi'_F(mn) \leq \chi'_F(m) \vee \chi''_F(n) \leq \chi_F(m) \vee \chi_F(n)$

Instance II:

Assume, mn $\in \mathbb{E}''$ and mn $\notin \mathbb{E}'$, Thus in similar way we obtain,

$\chi_T(mn) \geq \chi_T(m) \wedge \chi_T(n), \chi_I(mn) \leq \chi_I(m) \vee \chi_I(n), \chi_F(mn) \leq \chi_F(m) \vee \chi_F(n)$

Instance III:

Assume mn $\in \mathbb{E}' \cap \mathbb{E}''$, Thus

(i) $\chi_T(mn) = \chi'_T(mn) \vee \chi''_T(mn)$, where m, n $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_T(m) = \varepsilon'_T(m) \wedge \varepsilon''_T(m)$ and $\varepsilon_T(n) = \varepsilon'_T(n) \wedge \varepsilon''_T(n)$.

Therefore, $\chi_T(mn) = \chi'_T(mn) \vee \chi''_T(mn) \geq \{(\varepsilon'_T(m) \wedge \varepsilon'_T(n)) \vee (\varepsilon''_T(m) \wedge \varepsilon''_T(n))\}$

$$\geq \chi_T(m) \vee \chi_T(n).$$

$(ii)\ \chi_I(mn) = \chi_I'(mn) \wedge\ \chi_I''(mn)$, where m, n $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_I(m) = \varepsilon_I'(m) \vee\ \ \varepsilon''_I(m)$ and $\varepsilon_I(n) =\ \varepsilon_I'(n) \vee\ \varepsilon''_I(n)$.

Therefore, $\chi_I(mn) = \chi_I'(mn) \wedge\ \chi_I''(mn) \leq \{(\varepsilon_I'(m) \vee \varepsilon_I'(n)) \wedge (\varepsilon_I''(m) \vee\ \varepsilon''_I(n))\}$

$$\leq \chi_I(m) \wedge \chi_I(n).$$

$(iii)\ \chi_F(mn) = \chi_F'(mn) \wedge\ \chi_F''(mn)$, where m, n $\in \mathbb{V}' \cap \mathbb{V}''$

(i.e.) $\varepsilon_F(m) = \varepsilon_F'(m) \vee\ \ \varepsilon''_F(m)$ and $\varepsilon_F(n) =\ \varepsilon_F'(n) \vee\ \ \varepsilon''_F(n)$.

Therefore, $\chi_F(mn) = \chi_F'(mn) \wedge\ \chi_F''(mn) \leq \{(\varepsilon_F'(m) \vee \varepsilon_F'(n)) \wedge (\varepsilon_F''(m) \vee\ \varepsilon''_F(n))\}$

$$\leq \chi_F(m) \wedge \chi_F(n).$$

Hence, The Quasi - Union of $\zeta'\ and\ \zeta''$ is an Inverse Neutrosophic Graph.

Hence the proof

Example 4.2

Considering two Inverse Neutrosophic Graph in **Figure 6.a & 6.b,**

The quasi union of $\zeta_1 and\ \zeta_2$ is

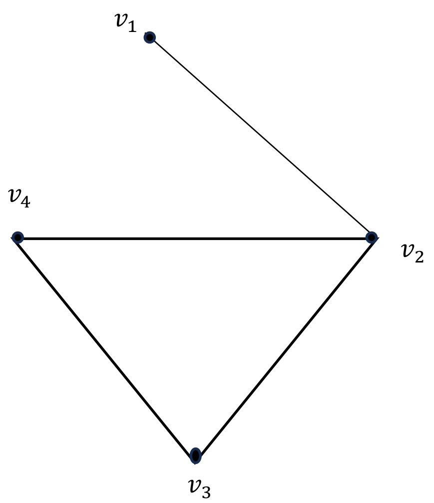


Figure 7

Table 3: Quasi – union of $\zeta_1 and\ \zeta_2$

Vertices of $\zeta_1 \cup \zeta_2$	Edges of $\zeta_1 \cup \zeta_2$
v_1(0.5,0.1,0.4)	v_1v_2(0.7,0.2,0.2)
v_2(0.1,0.3,0.2)	v_2v_3(0.8,0.1,0.1)
v_3(0.2,0.3,0.4)	v_4v_3(0.3,0,0.2)
v_4(0.1,0.2,0.5)	v_2v_4(0.5,0.1,0.1)

DEFINITION 4.3

Assume ζ'= [$\mathbb{V}'$, $\mathbb{E}'$,$(\chi_T', \chi_I', \chi_F')$, $(\varepsilon_t', \varepsilon_I', \varepsilon_F')$] and ζ''= [$\mathbb{V}''$, $\mathbb{E}''$,$(\chi''_T, \chi_I'', \chi_F'')$, $(\varepsilon''_t, \varepsilon_I'', \varepsilon_F'')$] be two ING. Subsequently, we denote and define the intersection of Inverse NG ζ' and ζ'' as

$$\zeta' \cap \zeta'' = [(\mathbb{V}' \cap \mathbb{V}''), (\mathbb{E}' \cap \mathbb{E}''), (\chi_T' \cap \chi''_T), (\chi_I' \cap \chi''_I), (\chi_F' \cap \chi''_F), (\varepsilon_t' \cap \varepsilon''_t), (\varepsilon_I' \cap \varepsilon_I''), (\varepsilon_F' \cap \varepsilon_F'')]$$

$$\varepsilon_T' \cap \varepsilon_T''\ (m) = \varepsilon_T'\ (m) \wedge \varepsilon_T''\ (m) \quad \forall\ m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\varepsilon_I' \cap \varepsilon_I''\ (m) = \varepsilon_I'\ (m) \vee \varepsilon_I''\ (m) \quad \forall\ m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\varepsilon_F' \cap \varepsilon_F''\ (m) = \varepsilon_F'\ (m) \vee \varepsilon_F''\ (m) \quad \forall\ m \in \mathbb{V}' \cap \mathbb{V}''$$

$$\chi'_T \cap \chi''_T\,(mn) = \chi'_T(mn)\ \wedge \chi''_T(mn)\ \ when\ \ mn\ \in \mathbb{E}' \cap\ \mathbb{E}''$$
$$\chi'_I \cap\ \chi''_I\,(mn) = \chi'_I(mn) \vee \chi''_I(mn)\ \ when\ \ mn\ \in \mathbb{E}' \cap\ \mathbb{E}''$$
$$\chi'_F \cap\ \chi''_F\,(mn) = \chi'_F(mn) \vee \chi''_F(mn)\ \ when\ \ mn\ \in \mathbb{E}' \cap\ \mathbb{E}''$$

Theorem 4.2

The Intersection of any two ING is an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs, whereby $\mathbb{V}' \cap \mathbb{V}''$ is a filled set.

Then for all mn $\in\ \mathbb{E}' \cap \mathbb{E}''$,

(i) χ'_T(mn) $\geq \varepsilon'_T(m) \wedge \varepsilon'_T(n)$ and χ'_T(mn) $\geq \varepsilon''_T(m) \wedge \varepsilon''_T(n)$

Therefore,

$$\begin{aligned}\chi_T(mn) &= \chi'_T(\text{mn}) \wedge \chi''_T(\text{mn})\\ &\geq \{\ \varepsilon'_T(m) \wedge \varepsilon'_T(n)\} \wedge \{\varepsilon''_T(m) \wedge \varepsilon''_T(n)\}\\ &\geq\ \chi_T(m) \wedge \chi_T(n).\end{aligned}$$

(ii) χ'_I(mn) $\leq \varepsilon'_I(m) \vee \varepsilon'_I(n)$ and χ'_I(mn) $\leq \varepsilon''_I(m) \vee \varepsilon''_I(n)$

Therefore,

$$\begin{aligned}\chi_I(mn) &= \chi'_I(\text{mn}) \vee \chi''_I(\text{mn})\\ &\leq \{\ \varepsilon'_I(m) \vee \varepsilon'_I(n)\} \vee \{\varepsilon''_I(m) \vee \varepsilon''_I(n)\}\\ &\leq\ \chi_I(m) \vee \chi_I(n).\end{aligned}$$

(iii) χ'_F(mn) $\leq \varepsilon'_F(m) \vee \varepsilon'_F(n)$ and χ'_F(mn) $\leq \varepsilon''_F(m) \vee \varepsilon''_F(n)$

Therefore,

$$\begin{aligned}\chi_F(mn) &= \chi'_F(\text{mn}) \vee \chi''_F(\text{mn})\\ &\leq \{\ \varepsilon'_F(m) \vee \varepsilon'_F(n)\} \vee \{\varepsilon''_F(m) \vee \varepsilon''_F(n)\}\\ &\leq\ \chi_F(m) \vee \chi_F(n).\end{aligned}$$

Consequently, $\zeta = \zeta' \cap \zeta''$ is an Inverse Neutrosophic Graph.

Hence the proof

DEFINITION 4.3

Assume ζ'= [$\mathbb{V}'$, $\mathbb{E}'$,($\chi'_T, \chi'_I, \chi'_F$), ($\varepsilon'_t, \varepsilon'_I, \varepsilon'_F$)] and ζ''= [$\mathbb{V}''$, $\mathbb{E}''$,($\chi''_T, \chi''_I, \chi''_F$), ($\varepsilon''_T, \varepsilon''_I, \varepsilon''_F$)] be two ING. Subsequently, we denote and define the cartesian product of Inverse NG ζ' and ζ'' as

$$\zeta' \times \zeta'' = [(\mathbb{V}' \times \mathbb{V}''), (\mathbb{E}' \times \mathbb{E}''), (\boldsymbol{\chi'_T} \times \boldsymbol{\chi''_T}), (\boldsymbol{\chi'_I} \times \boldsymbol{\chi''_I}), (\boldsymbol{\chi'_F} \times \boldsymbol{\chi''_F}), (\boldsymbol{\varepsilon'_T} \times\ \boldsymbol{\varepsilon''_T}) (\boldsymbol{\varepsilon'_I} \times\ \boldsymbol{\varepsilon''_I}), (\boldsymbol{\varepsilon'_F} \times \boldsymbol{\varepsilon''_F})]$$

$$\varepsilon_T(m\,n) = \epsilon'_T(\text{m}) \wedge\ \epsilon''_T(\text{n}),$$

$$\chi_T\{(\text{m}\,n')(\text{m}\,n'')\} = \epsilon'_T(\text{m}) \wedge\ \chi''_T(n'n''), \chi_T\{(\,m'n)(m''n)\} = \chi'_T(m'm'') \wedge \epsilon''_T(\text{n})$$

$$\varepsilon_I(m\,n) = \epsilon_I'(\mathrm{m}) \vee \epsilon_I''(\mathrm{n}),$$

$$\chi_I\{(\mathrm{m}\,n')(\mathrm{m}\,n'')\} = \epsilon_I'(\mathrm{m}) \vee \chi_I''(n'n''), \chi_I\{(m'n)(m''n)\} = \chi_I'(m'm'') \vee \epsilon_I''(\mathrm{n})$$

$$\varepsilon_F(m\,n) = \epsilon_F'(\mathrm{m}) \vee \epsilon_F''(\mathrm{n}),$$

$$\chi_F\{(\mathrm{m}\,n')(\mathrm{m}\,n'')\} = \epsilon_F'(\mathrm{m}) \vee \chi_F''(n'n''), \chi_F\{(m'n)(m''n)\} = \chi_F'(m'm'') \vee \epsilon_F''(\mathrm{n})$$

Theorem 4.3

The cartesian product of any two ING is also an ING.

Proof:

Consider ζ' and ζ'' be two Inverse Neutrosophic graphs, the cartesian product of two ING possess two categories of edges

(i) (m n') (m n'') that fulfills $\chi_T\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} = \varepsilon_T'(\mathrm{m}) \wedge \chi_T''(n'n'')$.

In this category, Given that, $\chi_T''(n'n'') \geq \varepsilon_T''(n') \wedge \varepsilon_T''(n'')$.

Thus, $\varepsilon_T'(\mathrm{m}) \wedge \chi_T''(n'n'') \geq \varepsilon_T'(\mathrm{m}) \wedge [\varepsilon_T''(n') \wedge \varepsilon_T''(n'')]$.

$= \varepsilon_T(\mathrm{m}\,n') \wedge \varepsilon_T(\mathrm{m}\,n'')$

Therefore, $\chi_T\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} \geq \varepsilon_T(\mathrm{m}\,n') \wedge \varepsilon_T(\mathrm{m}\,n'')$

(ii) (m n') (m n'') that fulfills $\chi_I\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} = \varepsilon_I'(\mathrm{m}) \vee \chi_I''(n'n'')$.

Given that, $\chi_I''(n'n'') \leq \varepsilon_I''(n') \vee \varepsilon_I''(n'')$.

Thus, $\varepsilon_I'(\mathrm{m}) \vee \chi_I''(n'n'') \leq \varepsilon_I'(\mathrm{m}) \vee [\varepsilon_I''(n') \vee \varepsilon_I''(n'')]$.

$= \varepsilon_I(\mathrm{m}\,n') \vee \varepsilon_I(\mathrm{m}\,n'')$

Therefore, $\chi_I\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} \leq \varepsilon_I(\mathrm{m}\,n') \vee \varepsilon_I(\mathrm{m}\,n'')$

(ii) (m n') (m n'') that fulfills $\chi_F\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} = \varepsilon_F'(\mathrm{m}) \vee \chi_F''(n'n'')$.

Given that, $\chi_F''(n'n'') \leq \varepsilon_F''(n') \vee \varepsilon_F''(n'')$.

Thus, $\varepsilon_F'(\mathrm{m}) \vee \chi_F''(n'n'') \leq \varepsilon_F'(\mathrm{m}) \vee [\varepsilon_F''(n') \vee \varepsilon_F''(n'')]$.

$= \varepsilon_F(\mathrm{m}\,n') \vee \varepsilon_F(\mathrm{m}\,n'')$

Therefore, $\chi_F\{(\mathrm{m}\,n')\,(\mathrm{m}\,n'')\} \leq \varepsilon_F(\mathrm{m}\,n') \vee \varepsilon_F(\mathrm{m}\,n'')$

b. $(m'n)\,(m''n)$ that fulfills $\chi_T[(m'n)\,(m''n)] = \chi'(m'm'') \wedge \varepsilon_T''(n)$

In this category, by using the same approach, we can demonstrate

$\chi_T\{(m'n)(m''n)\} \geq \epsilon_T(m'n) \wedge \epsilon_T(m''n)$

$\chi_I\{(m'n)(m''n)\} \leq \epsilon_I(m'n) \vee \epsilon_I(m''n)$

$\chi_F\{(m'n)(m''n)\} \leq \epsilon_F(m'n) \vee \epsilon_F(m''n)$

Consequently, $\zeta = \zeta' \times \zeta''$ is an Inverse Neutrosophic Graph.

Hence the proof

5. Inverse Neutrosophic Multigraphs

DEFINITION 5.1

Let $\mathbb{V}$ is a non-void set, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$: $\mathbb{V} \rightarrow [0,1]$ is Neutrosophic Set and $\mathbb{E}$ = {(x,y), $[(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$ (x,y)],i= 1,2,…m | (x,y) ∈ $\mathbb{V} \times \mathbb{V}$ } be a Neutrosophic multiset of $\mathbb{V} \times \mathbb{V}$ such that χ_{Ti} (xy) ≥ ε_T (x) ∧ ε_T(y) , χ_{Ii} (xy) ≤ ε_I(x) ∨ ε_I(y) , χ_{Fi} (xy) ≤, ε_F(x) ∨ , ε_F(y) for i= 1,2,…m where m = maximum { i : χ_{Ti} (xy), χ_{Ii} (xy) χ_{Fi} (xy) ≠ 0 }. Then ζ = ({$\mathbb{V}$, $\mathbb{E}$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$, $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$} is called as Inverse Neutrosophic Multi graphs where $\varepsilon_T(x)$, $\varepsilon_I(x), \varepsilon_F(x)$ signify true membership, indeterminacy membership and false membership of vertices x and $\chi_{Ti}(xy)$, $\chi_{Ii}(xy), \chi_{Fi}(xy)$ signifies the true membership, indeterminacy membership and false membership of edges (xy) within ζ correspondingly.

6. Inverse Neutrosophic Multigraphs and its planarity.

Within this segment, the planarity in terms of some key related concepts, such as interconnecting value and inverse Neutrosophic planarity values are defined.

Let ζ = ({$\mathbb{V}$, $\mathbb{E}$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$, $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$} be an Inverse Neutrosophic Multi-graph and a specific geometric illustration it has two Inverse Neutrosophic edges ((x, y), $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$ (x, y)) and ((p, q), $(\chi_{Tj}, \chi_{Ij}, \chi_{Fj})$ (p, q)) which are interconnected by a single point P, where i, j are constant integers. In Neutrosophic sense, if at least one of ((x, y), $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$ (x, y)) and $(\chi_{Tj}, \chi_{Ij}, \chi_{Fj})$ (p, q)) is close to zero, then the overlapping won't be significant in the illustration. However if both of ((x, y), $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$ (x, y)) and $(\chi_{Tj}, \chi_{Ij}, \chi_{Fj})$ (p, q)) are close to one, then the overlapping will be considered significant in the illustration.

Building on the concept, let us describe the interconnecting value and Inverse Neutrosophic planarity values.

DEFINITION 6.1

Consider ζ = ({$\mathbb{V}$, $\mathbb{E}$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$, $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$} to be an Inverse Neutrosophic multi-graph possessing a certain geometric illustration which has the interconnecting Point S among inverse Neutrosophic edges ((x, y), $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$ (x, y)) and ((p, q), $(\chi_{Tj}, \chi_{Ij}, \chi_{Fj})$ (p, q)),Then the value of intersecting edges at the point S is defined by

$$S = (S_T, S_I, S_F) = \left(\frac{\chi_{Ti}(x,y)+\chi_{Tj}(p,q)}{2}, \frac{\chi_{Ii}(x,y)+\chi_{Ij}(p,q)}{2}, \frac{\chi_{Fi}(x,y)+\chi_{Fj}(p,q)}{2} \right)$$

If the number of interconnecting points in an Inverse Neutrosophic multi-graph rises, then planarity value declines. Also, if the value of interconnecting points declines, then planarity value rises. Therefore, IP is inversely related to the planarity value. Based on this concept, the idea of planarity value of an Inverse Neutrosophic multi-graph is defined.

DEFINITION 6.2

Let ζ = ({$\mathbb{V}$, $\mathbb{E}$, $(\varepsilon_T, \varepsilon_I, \varepsilon_F)$, $(\chi_{Ti}, \chi_{Ii}, \chi_{Fi})$} be an Inverse Neutrosophic multi-graph with a specific geometric illustration, which has n intersection Points $S_1, S_2, S_3 \ldots S_n$. Then the Inverse Neutrosophic planarity value of S' of this geometric illustration of χ is well-defined by

$$S' = \frac{(2+S_T'-S_I'-S_F')}{3}$$

Where $S'_T = \frac{1}{1+\{S_{T1}+S_{T2}+\cdots+S_{Tn}\}}$, $S'_I = \frac{S_{I1}+S_{I2}+\cdots+S_{In}}{n}$, $S'_F = \frac{S_{F1}+S_{F2}+\cdots+S_{Fn}}{n}$

It is evident that 0 ≤ S' ≤ 1 and we assert that each geometric illustration of Inverse Neutrosophic multi-graph is planar by specific planarity value S'.

Examples 6.1

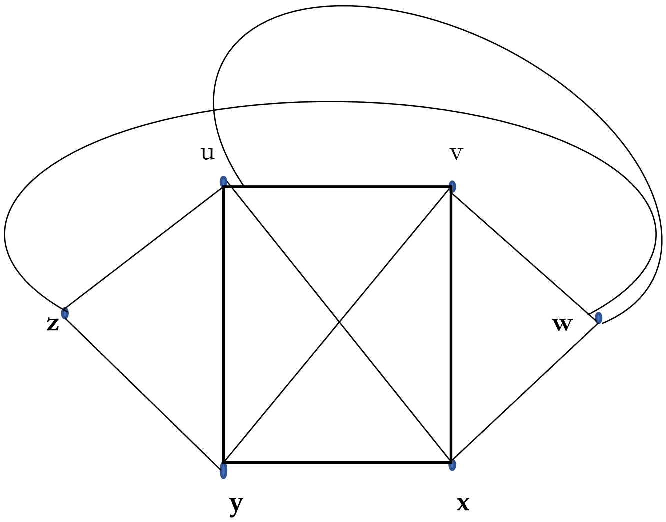


Figure. 7

Table 4: Vertices & Edges of Figure. 7

Vertices	Edges
u (0.6,0.6,0.4)	uz(0.8,0.4,0.2)
v (0.8,0.7,0.2)	uw(0.7,0.5,0.3)
w (0.3,0.4,0.6)	zw(0.5,0.1,0.2)
x (0.6,0.1,0.7)	xy (0.5,0.6,0.3)
y (0.5,0.8,0.4)	yz(0.5,0.4,0.1)
z (0.7,0.5,0.3)	ux(0.6,0.3,0.2)
	vy(0.7,0.5,0.1)

The geometric illustration of ζ has 2 intersection points, S_1 and S_2.

The intersection value of edges at the point S_1 and S_2 is

$S_{T_1}=\frac{0.6+0.7}{2}$ = 0.65, $S_{I_1}=\frac{0.3+0.5}{2}$= 0.4, $S_{F_1} = \frac{0.2+0.1}{2}$= 0.15,

$S_{T_2}=\frac{0.8+0.7}{2}$= 0.75, $S_{I_2}=\frac{0.5+0.4}{2}$ =0.45, $S_{F_2} = \frac{0.3+0.2}{2}$= 0.25.

S'_T = 0.42, S'_I = 0.41, S'_F = 0.2,

Therefore, the planarity value $S'$of this geometric illustration of ζ is S' = 0.6.

Lemma 6.1

Provided that the degree of planarity of an ING be (S'_T, S'_I, S'_F), consequently $S'_T + S'_I + S'_F \leq 3$.

Proof.

Let ζ be an ING. Therefore $0 \leq \chi_T$ (xy)≤ 1, $0 \leq \chi_I$ (xy)≤ 1, $0 \leq \chi_F$ (xy)≤ 1.

Hence, the score of connection amid two edges are $0 \leq S_T \leq 1, 0 \leq S_I \leq 1, 0 \leq S_F \leq 1$.

Consequently, the degree of planarity value $0 \leq S'_T \leq 1, 0 \leq S'_I \leq 1, 0 \leq S'_F \leq 1$.

Therefore $S'_T + S'_I + S'_F \leq 3$.

2. Provided that the score of planarity value of an ING is S′ consequently 0 ≤ S′ ≤ 1.

Proof.

Let S' be the score of planarity value of an ING, therefore $S' = \frac{(2+S'_T-S'_I-S'_F)}{3}$ and $0 \leq S'_T \leq 1, 0 \leq S'_I \leq 1, 0 \leq S'_F \leq 1$. Furthermore, it is noted that the value of S' is max if the value of S'_T is max and S'_I , S'_F are minimal.

So, the max value of S' is $S' = \frac{(2+1-0-0)}{3} = 1$.

Again, the value of S' is minimal. if S'_T is minimal. and S'_I , S'_F are max. So, the minimal. value of S' is $S' = \frac{(2+0-1-1)}{3} = 0$. Then $0 \leq S' \leq 1$.

Theorem 6.1

Each Inverse neutrosophic multi-graph ζ with the geometric illustration having n intersection points

$$\frac{1}{1+n} \leq S' \leq \frac{1}{1+\alpha n}$$

Where $\alpha = \min \{(\varepsilon_T, \varepsilon_I, \varepsilon_F)(x) : x \in \mathbb{V}\}$.

Proof:

In accordance to Definition 6.2:

$$S' = \frac{(2+S'_T - S'_I - S'_F)}{3}$$

Wherein, $S'_{Ti} = \frac{1}{1+\{S_{T1}+S_{T2}+\cdots+S_{Tn}\}}$, $S'_{Ii} = \frac{S_{I1}+S_{I2}+\cdots+S_{In}}{n}$, $S'_{Fi} = \frac{S_{F1}+S_{F2}+\cdots+S_{Fn}}{n}$

and in accordance to the definition 6.1 and 6.2,

$(S_T, S_I, S_F) = \left(\frac{\chi_{Ti}(x,y)+\chi_{Tj}(p,q)}{2}, \frac{\chi_{Ii}(x,y)+\chi_{Ij}(p,q)}{2}, \frac{\chi_{Fi}(x,y)+\chi_{Fj}(p,q)}{2} \right)$

and lemmas, we attain $(S'_{Ti}, S'_{Ii}, S'_{Fi}) \leq 1$

for i = {1, 2, ..., n}, we attain $\{S_{T1} + S_{T2} + \cdots + S_{Tn} \leq n$ and via lemma, $S' = \frac{(2+S'_T-S'_I-S'_F)}{3} \geq \frac{1}{1+n}$

conversely,

χ_T (xy) $\geq \varepsilon_T$ (x) $\wedge$ ε_T(y), χ_I (xy) $\leq \varepsilon_I$(x) $\vee$ ε_I(y), χ_F (xy) $\leq \varepsilon_F$(x) $\vee$ ε_F(y) and for each inverse Neutrosophic edge.

As $\alpha = \min \{(\varepsilon_T, \varepsilon_I, \varepsilon_F)(x) : x \in \mathbb{V}\}$. and we attain $\chi_{Ti}(x,y) \geq \alpha$ so, $(S'_{Ti}, S'_{Ii}, S'_{Fi}) \geq 1$

for i = {1, 2, ..., n}, we attain $\{S_{T1} + S_{T2} + \cdots + S_{Tn} \geq n\, \alpha$ and via lemma so

$S' = \frac{(2+S'_T-S'_I-S'_F)}{3} \leq \frac{1}{1+\alpha n}$.

DEFINITION 6.3:

Consider $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\}$ to be an Inverse Neutrosophic Multi-graphs with a specific geometric illustration having n intersecting points $S_1, S_1, \ldots, S_n$ so that m of these interconnecting points fulfills $I_{S_i} \geq 0.5$, for i = 1, 2,..,m. Therefore, this geometric illustration of ζ is termed as inverse **Neutrosophic strong planar multi-graph** S > 0.5 and m < $\frac{n}{2}$.

This indicates that a geometric illustration of ζ is an inverse Neutrosophic strong planar multi-graph, provided the inverse Neutrosophic planarity value is superior than 0.5 and the interconnecting points that fulfil $I_S \geq 0.5$ are fewer than half of the total interconnecting points n. Alternatively, this geometric illustration of ζ is termed as inverse **Neutrosophic weak planar multi-graph**.

Example 6.2

Consider $\zeta = (\{\mathbb{V}, \mathbb{E}, (\varepsilon_T, \varepsilon_I, \varepsilon_F), (\chi_{Ti}, \chi_{Ii}, \chi_{Fi})\}$ to be an Inverse Neutrosophic Multi-graph with a specific geometric illustration as depicted in the figure as follows,

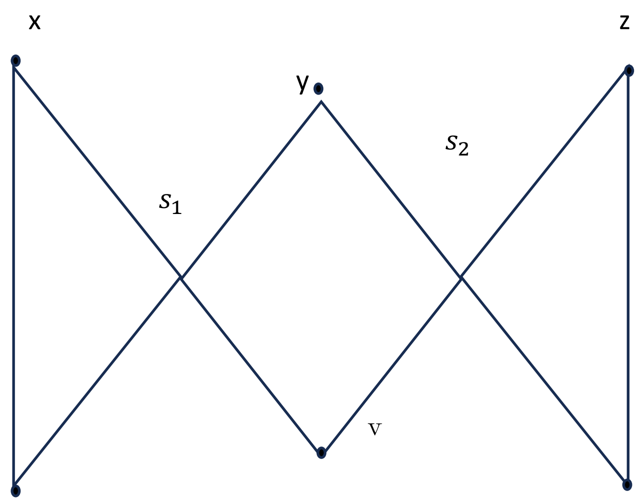


Figure. 8

Table 5: Vertices & edges of Figure 8

Vertices	edges
x(0.8, 0.4, 0.4)	xv (0.4,0.4,0.3)
y(0.2,0.4,0.3)	yu (0.3, 0.3,0.5
z(0.9,0.2,0.1)	xw(0.8,0.5,0.2)
u(0.5,0.6,0.6)	zu(0.6,0.4,0.5)
v(0.1,0.5, 0.4)	yw (0.2, 0.2,0.1)
w(0.7,0.6,0.1)	vz (0.5,0.4,0.3)

Consequently, this geometric representation of ζ has two intersecting points, S_1 andS_2. The interconnecting value at the location S_1 is S_{T_1}= 0.3, S_{I_1}= 0.3, S_{F_1}= 0.2, S_{T_2}= 0.4, S_{I_2}= 0.35, S_{F_2}= 0.4. S'_T = 0.588, S'_I = 0.325, S'_F = 0.3, therefore $\boldsymbol{S'}$ **= 0.654**. Hence, this geometric illustration from ζ is **strong planar**.

By drawing the Inverse Neutrosophic multi-graph ζ in alternative geometric illustration presented within Figure beneath,

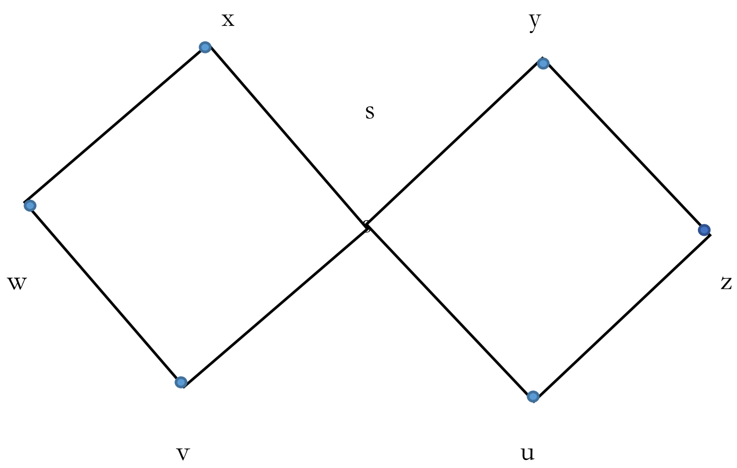


Figure. 9

Table 6: vertices & Edges of Figure. 9

Vertices	Edges
u (0.9,0.6,0.2)	xu (0.8,0.8,0.2)
v (0.7,0.4,0.6)	yv (1,0.6,0.2)
w (0.7,0.5,0.3)	xw (0.6,0.5,0.1)
x (0.5,0.9,0.3)	wv (0.8,0.3,0.5)
y (0.8,0.7,0.5)	yz(0.7,0.6,0.3)
z (0.5,0.8,0.4)	zu (0.8,0.3,0.4)

Consequently, the above geometric illustration of ζ has one intersecting point S (0.8, 0.7, 0.2), thus the planarity value of the intersecting edge S is S'_T = 0.56, S'_I = 1, S'_F = 0.7, therefore S' = **0.5**.

Hence aforementioned geometric representation from ζ is **weak planar.**

Remark: In the Inverse Neutrosophic graph the circumstances is distinct, since as per the application, a particular number of intersection point is sometime needed to resolve our issue. In the inverse Neutrosophic graph, our focus is not on finding a geometric illustration devoid of intersecting points. Instead, based on the problem requirements, we identify the required number of intersecting points and aim to determine a geometric illustration that meets our condition. Additionally, we attempt to minimize the danger associated with these intersecting points (lowest $\{I_{S_1}\}$) and maximize the inverse planarity value (S'), as will be demonstrated in the application.

Example 6.3

The geometric illustration of ζ that is depicted in Figure 8, having 2 intersecting points such that S_1 and S_2, S'_T = 0.42, S'_I = 0.41, S'_F = 0.2 and $\boldsymbol{S' = 0.6}$. Despite P' > 0.5, the condition $\mathbf{m < \frac{n}{2}}$ fails to be met.

Therefore, the aforementioned geometric illustration of ζ is **weak planar**.

7. Application: Decision making in mixed cropping using Inverse Neutrosophic graph:

Mixed cropping or inter cropping is an agricultural practice that involves two or more crops are grown together in the same field. This approach integrates different farming activities to maximize resource use, improve productivity, and enhance sustainability. Mixed cropping offers numerous advantages in terms of resource efficiency, economic stability, and environmental benefits. However, it also presents challenges related to management complexity and economic risks. Successful mixed cropping relies on effective planning, resource management, and integration of sustainable practices to optimize the benefits of combining different crop production.

For instance, combining of crops can be done with minimizing expenditure that becomes more significant. Each crop is considered as a vertex of a graph as expenditure is considered as significant in our study, we assign a membership degree representing its resource management expenditure. This membership degree can be regularized by dividing every expected expenditure for each crops with average total expenditure, the expenditure is varying on certain circumstances, the expected expenditure for resource management of the particular crops is considered as truth membership, there is a chance of high level of uncertainty about the exact expenditure, possibly due to complex management, pest and disease management & economic and market risk which is considered as the indeterminacy membership and There may be chance that the stated expenditure of is inaccurate or misleading which is considered as false membership.

Accordingly, provided that the expenditure (that incorporates labor expenditure, crop maintenance charge) for different crops w, x, y, z is {3200, 2000, 2500, 4000} also the satisfactory factor of the expected expenditure is {90%,80%,70%,85%}

The Membership values are denoted as

Truth membership value (TMV) = Ratio of estimated expenditure to average total expenditure (provided that the estimated expenditure is greater than average total expenditure, then TMV is considered as 1.

Indeterminacy Membership Value (IMV) = 20 % of the estimated expenditure.

FMV = considering the satisfactory factor False membership of a vertex is 1- satisfactory.

$(\varepsilon_T, \varepsilon_I, \varepsilon_F)$(w) = (1, 0.2, 0.1)

$(\varepsilon_T, \varepsilon_I, \varepsilon_F)$(x) = (0.7,0.14,0.2)
$(\varepsilon_T, \varepsilon_I, \varepsilon_F)$(y) = (0.8,0.16,0.3)
$(\varepsilon_T, \varepsilon_I, \varepsilon_F)$(z) = (1,0.2,0.15)

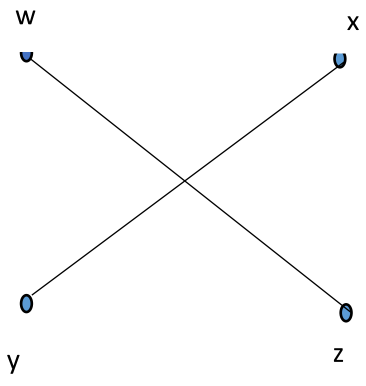
Our objective is combining of crops so that we can allocate crop in one field. To evaluate our options, we construct an inverse Neutrosophic graph for each situation. In this graph, we connect vertices representing combining of crops with edges that intersect at a point P, representing the same farm.

For n farms (particular area), each graph will feature n intersecting points, each representing a farm the particular area. The vertices connected by intersecting edges denote the two crops in same area field of the farm.

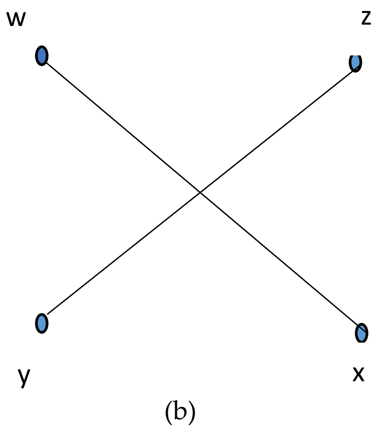
Each graph consists of four vertices (each crop), two edges (illustrates combining of crops), and one intersecting point (illustrates the same field of the farm). This configuration forms an inverse Neutrosophic graph where each edge is allocated a membership degree, indeterminacy degree and false degree representing the required expenditure of maintaining crops in the same field of the farm. To ensure accuracy, these membership degrees are normalized by considering average of the total maintenance expenditure. It's crucial to note that each edge's membership degree (illustrates combining of crops) will inevitably exceed the expenditure of maintaining crops separately. Optimal planning involves selecting pairs of crops that can be combined in same field of the form that leads to minimization of maintenance and maximizing the production and profit. So, to estimate the best combination, the study of planarity in every possible inverse neutrosophic graph is sufficient and to select the one which is strong planar that has the greatest inverse neutrosophic planarity value P'. If all the selections are weak planar, then it is sufficient to choose the one with the greatest inverse Neutrosophic planarity value P'.

In our instance we have 3 possible options. First option, to combine crop w and crop x in the same field of the fam, crop y and crop z in the other field of the farm.

We signify this option by the inverse Neutrosophic graph ζ_1 revealed in the Figure 10 (a)



(a)



(b)

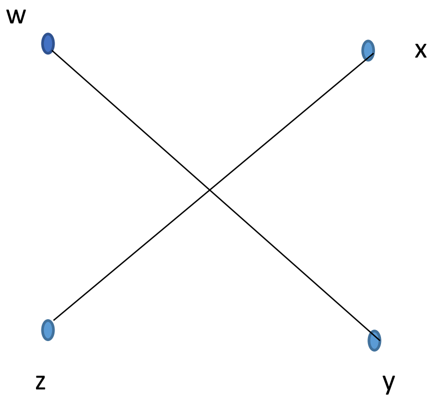
Figure 10.

Second option, to combine crop w and z in the particular field of the farm and crops x and y in another field of the farm.

We signify this option by the Neutrosophic fuzzy graph ζ_2 revealed in Figure 10 (b)

Third option, to combine crop w and x in particular field of the farm and crops z and y in another field of the farm.

We signify this option by the Neutrosophic fuzzy graph ζ_3 revealed in Figure 10 (c).



(c)

To determine the optimal option, the calculation of the Inverse Neutrosophic planarity value for every option are required.

Within ζ_1, we obtain S'_T = 0.54, S'_I = 0.05, S'_F = 0.075 and S' = 0.8 indicating that ζ_1 is strong planar.
Within ζ_2, we obtain S'_T = 0.5, S'_I = 0.075, S'_F = 0.12 and S' = 0.79 indicating that ζ_2 is strong planar.
Within ζ_3 , we obtain S'_T = 0.57, S'_I = 0.1, S'_F = 0.1 and S' = 0.7 indicating that ζ_3 is strong planar.

As a result, the Inverse Neutrosophic strong planar graph exhibiting highest Inverse Neutrosophic planarity value serves as the optimal option, considering that the expenditure will be minimized. It means that the First choice is the optimal one in our instance.

8. Conclusion:

The core intension of developing Inverse Neutrosophic graph was to provide an enhanced and flexible framework interpreting multifaced system where uncertainty and the directionality of interactions are significant. Explores both the similarities and differences between Inverse Neutrosophic graph and Neutrosophic graphs. Then, perception of Inverse Neutrosophic Multi-graphs and defined the notation for the planarity of such structures using intersecting values and inverse Neutrosophic planarity values. Several theorems were presented to establish extremes for the inverse neutrosophic planarity value. Furthermore, we defined strong and weak planarity and applications related to these concepts. The significant crop combinations for intercropping are identified using the notion of planarity in Inverse Neutrosophic Graphs that minimizes the expenditure in maintenance of the crops. Looking ahead, future research will focus on further exploring planarity and investigating the concepts of faces and duality in inverse Neutrosophic multi-graphs.

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The authors affirms that they possess no competing interests.

Authors Contribution

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References:

[1] Akram, M., (2016). Single-Valued Neutrosophic Planar Graphs, *International Journal of Algebra and Statistics*, 5(2) pp. 157-167.https://doi.org/10.20454/ijas.2016.1207.

[2] Akram, M., & Shahzadi, S., (2017). Neutrosophic soft graphs with application, *Journal of Intelligent & Fuzzy Systems*,32(1), pp. 841–858.https://doi.org/10.3233/JIFS-16090.

[3] Akram, M.,Shahzadi ,S., & Saeid, A., (2018).Single-valued neutrosophic hypergraphs, *TWMS Journal of Applied and Engineering Mathematics*, 8(1), pp. 122–135.

[4] Alshehri, N., & Akram, M., (2014). Intuitionistic fuzzy planar graphs, *Discrete Dynamics in Nature and Society*, 4(1), pp. 1-9.http://dx.doi.org/10.1155/2014/397823.

[5] Arindam dev, Ranjan Kumar, Broumi Said & Pritam Bhowmik., (2023). Different Types of Operations on Neutrosophic Graphs, *International Journal of Neutrosophic science*, 19(2), pp. 87-94. http://doi.org/10.54216/IJNS.19028.

[6] Broumi, S., Talea, M., Bakali, A., & Smarandache, F., (2016). Single-valued neutrosophic graphs, *Journal of New Theory*, 10(1), pp. 86–101.http://dx.doi.org/10.5281/zenodo.50940.

[7] Borzooei ,R.A., Almallah,R., Jun,Y.B., & Ghaznavi,H., (2020). Inverse Fuzzy Graphs with Applications, *New Mathematics and Natural Computation (NMNC)*, *16*(2), pp. 397-418. http://doi.org/ 10.1142/S1793005720500246.

[8] Borzooei ,R.A., & Almallah,R.,(2022). Inverse Fuzzy Multigraphs and Planarity with Application in Decision-making, Soft Computing, 26(6), pp :1-9. http://doi.org/10.1007/s00500-021-06681-6.

[9] Dhavaseelan, R., Vikramaprasad, R., & Krishnaraj, V., (2015). Certain types of neutrosophic graphs, *International Journal of Mathematical Sciences and Applications*, 5(2), pp. 333–339.

[10] Naz Sumera, Rashmanlou, Hossein, & Malik, M. Aslam, (2017), Operations on single valued neutrosophic graphs with application, *Journal of Intelligent & Fuzzy Systems*, 32(3), pp. 2137-2151. http://doi.org/10.3233/JIFS-161944.

[11] Pal, A., Samanta, S., & Pal, M., (2014), New concept of fuzzy planar graphs, *International Journal of Advanced Research in Artificial Intelligence*, 3(1), pp. 52–59. http://doi.org/10.14569/IJARAI.2014.030108.

[12] Rupkumar Mahapatra, Sovan Samanta, & Madhumangal Pal,(2020), Generalized neutrosophic planar graphs and its application, *Journal of Applied Mathematics and Computing*, 6, pp. 693–712. https://www.doi.org/10.1007/S12190-020-01411-X

[13] Soumitra Poilik, & Ganesh Ghoral, (2022). New concepts of inverse fuzzy mixed graphs and its application, *Granular Computing*, *7*(3**)**, pp.549–559.https://doi.org/10.1007/s41066-021-00284-0

[14] Zeng S, Shoaib M,Ali S,Smarandache , Rashmanlou H, & Mofidnakhaei F,(2021), Certain Properties of Single-Valued Neutrosophic Graph with Application in Food and Agriculture Organization, *International Journal of Computational Intelligence Systems*, 14(1), pp 1875-6891. https://doi.org/10.2991/ijcis.d.210413.001

[15] Thempaavai Jayaprakash & Antony Cripsin Sweety Charles Selvaraj (2024), Inverse Neutrosophic Mixed Graphs, *Journal of Fuzzy Extension and Applications*, *5*(3), pp. *395–415*. https://doi.org/10.22105/jfea.2024.429140.1356

[16] Vetrivel, G., Mullai, M., & Buvaneshwari, R. (2024). Forgotten Topological Index and its Properties on Neutrosophic Graphs. *Neutrosophic Systems with Applications*, **21**, PP.36- 45. https://doi.org/10.61356/j.nswa.2024.21356

[17] Vetrivel, G., Mullai, M., & Rajchakit, G. (2024). Product Perspective from Fuzzy to Neutrosophic Graph Extension- A Review of Literature. *Neutrosophic Systems with Applications*, **23**, pp. 13-22. https://doi.org/10.61356/j.nswa.2024.23377.

Books:

[18] Akram, M., (2018). Single–valued Neutrosophic Graphs, *Infosys Science Foundation Series in Mathematical Sciences*, *Springer*. Doi: 10.1007/978-981-13-3522-8.

[19] Florentin Smarandache., W.B vasantha kandasamy., & Illendendral kandasamy., (2015) Neutrosophic Graphs: A new dimension to graph theory, *Brussels: Europa Nova, 2015*. pp. 1-128. https://digitalrepository.unm.edu/math_fsp.

[20] Florentin Smarandache., (1999). A unifying field in logics in Neutrosophy: Neutrosophic Probability, Set and Logic, *Rehoboth: American Research Press*.

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