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Application of Neutrosophic SuperHyperSoft Sets in MADM

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Abstract: In this work, we introduce the concept of Neutrosophic SuperHyperSoft Sets (NSHSS) by considering a universe of discourse and disjoint attribute structures, and develop fundamental operations including addition, multiplication, scalar multiplication, and scalar exponentiation. A novel Neutrosophic SuperHyperSoft Weighted Aggregation (NSHSWA) operator is proposed, and a new entropy measure is defined to quantify uncertainty, with its theoretical properties validated. Building on this, we propose a Multi-Attribute Decision-Making (MADM) framework for selecting optimal mobile phones by combining NSHSS with entropy-based weighting. Five primary criteria- operational efficiency, power endurance, imaging capability, cost, and robustness are evaluated using power sets and neutrosophic modelling to capture uncertainty in user preferences. Subjective linguistic evaluations are mapped to neutrosophic values, and aggregated using the NSHSWA operator. Among 162 propositions constructed, the top alternatives are identified and ranked using a neutrosophic score function. The proposed methodology offers a comprehensive and reliable tool for guiding consumers toward selecting high quality mobile phones based on personalized needs, while providing a strong foundation for future uncertain information modelling and decision support.

Keywords: Neutrosophic SuperHyperSoft Sets, SuperHyperSoft Sets, Neutrosophic Soft Sets, Neutrosophic Sets.

1. Introduction and Preliminaries

Decision-making is in general a complex task and requires the frameworks capable of dealing with ambiguity, uncertainty and a lack of knowledge. After being introduced by Smarandache in 1999, Neutrosophic Sets (NSs) offer a mathematically sound method to express indeterminacy with three independent parameters: truth, indeterminacy and falsehood. NSs are especially very useful in modelling complex situations involving overlapping and contradiction information due to this extension of IFSs. The concept of Soft Sets (SSs) was introduced by Molodtsov in 1999. They offer a flexible mathematical framework for managing uncertainty without the need for additional elements like membership grades or probabilities. SSs have been found very useful in several decision-making problems due to their possibility of effective expression and interpretation of qualitative and quantitative data. By coupling neutrosophic logic with soft sets, Maji et al. (2003) introduced Neutrosophic Soft Sets (NSSs), which extend the decision-making capabilities by apprehend the varying degrees of truth, indeterminacy and falsity. Abbas with his colleagues (2020) instigated the concept of HyperSoft Sets (HSSs) to deal with multi-attribute problems having a hierarchical data structure. The HSSs generalize SSs by introducing sub-attributes and enable more extensive analysis of complex systems.

Later, the extension of neutrosophic concepts into Hyper Soft Sets (HSSs) led to the creation of Neutrosophic HyperSoft Sets (NHSSs), which combined the advantages of Hyper Soft Sets and Neutrosophic Sets to simultaneously address uncertainty and granularity (Saeed et al., 2020). The latest development in soft set theory is the 2023 foundation by Smarandache of SuperHyperSoft Sets (SHSSs). Compared to HSSs, SHSSs have greater representational power due to the incorporation of multi-dimensional analysis and higher levels of granularity. The Neutrosophic extension, NSHSSs, further strengthens the framework's ability to represent ambiguity, uncertainty and interdependencies among criteria (Smarandache, 2024). The present paper is an attempt to apply NSHSSs to Multi-Attribute Decision Making, with a view to showing how their strong representational capabilities can be used in solving real-life decision-making problems.

Definition 1.1 [7]: Consider X is universe of discourse, $\aleph(X)$ the power set of X *and* $\mathscr{b}_1, \mathscr{b}, \dots \mathscr{b}_n$ $(n \geq 1)$ distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots\ \wp_n$. Consider $\aleph(\wp_i)$ denote the powerset of $\wp_i$ for $i = 1, 2, \dots n$. A SuperHyperSoft Set (SHSS) over X is defined as the pair $(\gamma, \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n))$ where $\gamma : \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n) \rightarrow \aleph(X)$.

Definition 1.2 [7,8]: Consider X is universe of discourse, $\aleph(X)$ the powerset of X and $\mathscr{b}_1, \mathscr{b}_2, \dots \mathscr{b}_n$ $(n \geq 1)$ distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots\ \wp_n$. Let $\aleph(\wp_i)$ denote the powerset of $\wp_i$, for $i = 1, 2, \dots n$. A Neutrosophic SuperHyperSoft Set (NSHSS) over X is defined as the pair $\left(\gamma,\ \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n)\right)$ where $\gamma : \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n) \rightarrow \aleph(X)$ and
$\gamma = \{ b, < a, \mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(\mathrm{a}), \mathcal{F}_{\gamma(b)}(\mathrm{a}) > :\ a \in X, b \in \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n)\}$.
Here $\mathcal{T}_{\gamma(b)}(\mathrm{a}), \mathcal{I}_{\gamma(b)}(\mathrm{a}), \mathcal{F}_{\gamma(b)}(\mathrm{a}) : X \rightarrow [0,1]$ designate the membership, indeterminacy and non-membership degrees of $x \in X$ for every y, satisfying: $0 \leq \mathcal{T}_{\gamma(b)}(\mathrm{a}) + \mathcal{I}_{\gamma(b)}(\mathrm{a}) + \mathcal{F}_{\gamma(b)}(\mathrm{a}) \leq 3$.

Definition 1.3 [8]: Consider X is universe of discourse, $\aleph(X)$ the powerset of X *and* $\mathscr{b}_1, \mathscr{b}_2, \dots \mathscr{b}_n$ $(n \geq 1)$ distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \dots\ \wp_n$. Let $\aleph(\wp_i)$ denote the powerset of $\wp_i$, for $i = 1, 2, \dots n$. Let γ, δ be two NSHSS over X is defined as the pair $(\gamma, \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n))$ and $(\delta, \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n))$ where $\gamma, \delta : \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n) \rightarrow \aleph(X)$ and
$\gamma = \{ b, < x, \mathcal{T}_{\gamma(b)}(\mathrm{a}), \mathcal{I}_{\gamma(y)}(\mathrm{a}), \mathcal{F}_{\gamma(y)}(\mathrm{a}) > :\ a \in X, b \in \aleph(\wp_1) \times \aleph(\wp_2) \times . . \times \aleph(\wp_n)\}$
$\delta = \{ b, < c, \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\delta(b)}(c) > :\ c \in X,\ b \in \aleph(\wp_1) \times \aleph(\wp_2) \times \dots \times \aleph(\wp_n)\}$.
Then the basic operators are defined as

1. $\gamma \leq \delta = < \mathcal{T}_{\gamma(b)}(a) \leq \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(y)}(a) \leq \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) \geq \mathcal{F}_{\delta(b)}(c) >$.
2. $\gamma = < \mathcal{T}_{\gamma(b)}(a) = 0,\ \mathcal{I}_{\gamma(b)}(a) = 0, F_{\gamma(b)}(a) = 0 >$.
3. $\gamma^{\,c} = < \mathcal{F}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{T}_{\gamma(b)}(a) >$.
4. $\gamma \cup \delta = < max\{\mathcal{T}_{\gamma(b)}(a), \mathcal{T}_{\delta(b)}(c)\}, \frac{\mathcal{I}_{\gamma(b)}(a) + \mathcal{I}_{\delta(b)}(c)}{2}, min\{\mathcal{F}_{\gamma(b)}(a), \mathcal{F}_{\delta(b)}(c)\} >$.
5. $\gamma \cap \delta = < min\{\mathcal{T}_{\gamma(b)}(a), \mathcal{T}_{\delta(b)}(c)\}, \frac{\mathcal{I}_{\gamma(b)}(a) + \mathcal{I}_{\delta(b)}(c)}{2}, max\{\mathcal{F}_{\gamma(b)}(a), \mathcal{F}_{\delta(b)}(c)\} >$.

2. Operators in Neutrosophic SuperHypersoft Sets

Definition 2.1: Consider X is universe of discourse, $\aleph(X)$ the powerset of X *and* $\mathscr{b}_1, \mathscr{b}_2, \ldots \mathscr{b}_n$ $(n > 1)$ distinct attributes with disjoint corresponding sets $\wp_1, \wp_2, \ldots \wp_n$. Let $\aleph(\wp_i)$ denote the powerset of $\wp_i$ for $i = 1, 2, \ldots n$.
Let γ, δ be two NSHSS over X is defined as the pair $(\gamma, \aleph(\wp_1)\times \aleph(\wp_2) \times \ldots \times \aleph(\wp_n))$ and $(\delta, \aleph(\wp_1)\times \aleph(\wp_2) \times \ldots \times \aleph(\wp_n))$ where $\gamma, \delta : \aleph(\wp_1)\times \aleph(\wp_2) \times \ldots \times \aleph(\wp_n) \to \aleph(X)$ and
$\gamma = \{ b, < a, \mathcal{T}_{\gamma(b)}(a), \mathcal{I}_{\gamma(b)}(a), \mathcal{F}_{\gamma(b)}(a) > : a \in X, b \in \aleph(\wp_1)\times \aleph(\wp_2) \times \ldots \times \aleph(\wp_n)\}$
$\delta = \{ b, < c, \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\delta(b)}(c) > : c \in X, b \in \aleph(\wp_1)\times \aleph(\wp_2) \times \ldots \times \aleph(\wp_n)\}$. Then the basic operators are defined as,

1. $\gamma \oplus \delta = <\mathcal{T}_{\gamma(b)}(a) + \mathcal{T}_{\delta(b)}(c) - \mathcal{T}_{\gamma(b)}(a) . \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(b)}(a) . \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) . \mathcal{F}_{\delta(b)}(c)>$.
2. $\gamma \otimes \delta = < \mathcal{T}_{\gamma(b)}(a). \mathcal{T}_{\delta(b)}(c), \mathcal{I}_{\gamma(b)}(x) + \mathcal{I}_{\delta(b)}(c) - \mathcal{I}_{\gamma(b)}(a) . \mathcal{I}_{\delta(b)}(c), \mathcal{F}_{\gamma(b)}(a) + \mathcal{F}_{\delta(b)}(c) - \mathcal{F}_{\gamma(b)}(a) . \mathcal{F}_{\delta(b)}(c)>$.
3. $\lambda . \gamma = < 1 - \left(1 - \mathcal{T}_{\gamma(b)}(a)\right)^{\lambda}, \left(\mathcal{I}_{\gamma(b)}(a)\right)^{\lambda}, \left(\mathcal{F}_{\gamma(b)}(a)\right)^{\lambda} >$, where $\lambda > 0$.
4. $\gamma^{\lambda} = < (\mathcal{T}_{\gamma(b)}(a))^{\lambda}, 1 - (1 - \mathcal{I}_{\gamma(b)}(a))^{\lambda}, 1 - \left(1 - \mathcal{F}_{\gamma(b)}(a)\right)^{\lambda} >$, where $\lambda > 0$.

Theorem 2.2: Let $\gamma_i = (\mathcal{T}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)), i = 1,2,3.\ldots n$ be a collection of NSHSS, then Neutrosophic SuperHyperSoft Weighted Aggregation (NSHSWA) operator value is also a NSHSS and
$NSHS\text{WA}(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_n) =$

$$\left(1 - \prod_{i,j=1}^{n}\left(1 - \mathcal{T}_{\gamma(b)}(a_i)\right)^{w_j}, \prod_{i,j=1}^{n}\left(\mathcal{I}_{\gamma(b)}(a_i)\right)^{w_j}, \prod_{i,j=1}^{n}\left(\mathcal{F}_{\gamma(b)}(a_i)\right)^{w_j}\right) \quad \text{……….. } \mathbf{(1)}$$

where $w = (w_1, w_2, w_3, \ldots w_n)^T$ be the weight vector of $\gamma_j (j = 1,2,3, \ldots . n)$ and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Proof. Prove this theorem by applying mathematical induction on n.
If n = 2, we have $NSHS\text{WA}(\gamma_1, \gamma_2) = w_1\gamma_1 \oplus w_2\gamma_2$
By Definition 2.1, we can see that both $w_1\gamma_1$ *and* $w_2\gamma_2$ are NSHSS and the value of $w_1\gamma_1 \oplus w_2\gamma_2$ is also a NSHSS. From the operational laws of NSHSS, we have

$$w_1\gamma_1 = \left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}, \left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}, \left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\right);$$

$$w_2\gamma_2 = \left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}, \left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}, \left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}\right)$$

Then, $NSHS\text{WA}(\gamma_1, \gamma_2) = w_1\gamma_1 \oplus w_2\gamma_2$

$$=< 2 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1} - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2} - \left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\right)\left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}\right),$$
$$\left(\mathcal{I}_{\gamma(b)}(a_1)\right)^{w_1}\left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}, \left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\left(\mathcal{F}_{\gamma(b)})(\mathrm{a}_2)\right)^{w_2} >$$

$$=< 1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}, \left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2}, \left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_1)\right)^{w_1}\left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_2)\right)^{w_2} >.$$

If n = k, then Equation (1) holds, i.e.,
$NSHS\text{WA}(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_{\mathrm{n}}) = w_1\gamma_1 \oplus w_2\gamma_2 \ldots \ldots \ldots \oplus w_k\gamma_k$

$$=< 1 - \prod_{j=1}^{k}\left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}, \prod_{j=1}^{k}\left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}, \prod_{j=1}^{k}\left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj} >$$

and the aggregated value is a NSHSS, then when n = k + 1, by the operational laws of NSHSS, we have
$NSHS\text{WA}(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_{\mathrm{k+1}}) = w_1\gamma_1 \oplus w_2\gamma_2 \ldots \ldots \ldots \oplus w_k\gamma_k \oplus w_{k+1}\gamma_{k+1}$

$$= \langle 1 - \prod_{j=1}^{k}\left(1 - \mathcal{T}_{\gamma(b)}(a_i)\right)^{wj} + \left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_{k+1})\right)^{w_{k+1}}\right) - \left(1 - \prod_{j=1}^{k}\left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}\right)\left(1 - \left(1 - \mathcal{T}_{\gamma(b)}(a_{k+1})\right)^{w_{k+1}}\right), \prod_{j=1}^{k+1}\left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}, \prod_{j=1}^{k+1}\left(\mathcal{F}_{\gamma(b)})(\mathrm{a}_i)\right)^{wj}\rangle$$

$$=< 1 - \prod_{j=1}^{k+1}\left(1 - \mathcal{T}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}, \prod_{j=1}^{k+1}\left(\mathcal{I}_{\gamma(b)}(\mathrm{a}_i)\right)^{wj}, \prod_{j=1}^{k+1}\left(\mathcal{F}_{\gamma(b)}(a_i)\right)^{wj} >.$$

By which aggregated value is also a NSHSS, Therefore, when n = k + 1, Equation (1) holds. Thus, by steps 1 and 2, we recognize that Equation (1) sustain for all n.

Definition 2.3. Let $\gamma_{\mathrm{i}} = \{ (\mathcal{T}_{\gamma(b)}(a_i),\ \mathcal{I}_{\gamma(b)}(a_i), \mathcal{F}_{\gamma(b)}(a_i)) | a_i \in X\}$ be a NSHSS set on X. Then the entropy of γ_{i} is defined as,

$$E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}_{\gamma(b)}(a_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{F}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{I}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) - 1)\right|) \quad \ldots\ldots \textbf{(2)}$$

Theorem 2.4. The proposed entropy on NSHSS(X) fulfilling the conditions:

1. $E(\gamma_{\mathrm{i}}) = 0$, if γ is a crisp set ie., $\gamma_{\mathrm{i}} = \left(\mathcal{T}_{\gamma(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right) = (1,0,0)$ or $(0,0,1)$ for all $\mathrm{a}_i \in X$.
2. $E(\gamma_{\mathrm{i}}) = 1$, if $\gamma_{\mathrm{i}} = \{\left(\mathrm{a}_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) | \mathrm{a}_i \in X\}$
3. $E(\gamma_i) = E(\gamma_i{}^C)$, for all $\gamma_i \in NSHSS(X)$;
4. $E(\gamma_i) \leq E(\delta_i)$ if either $\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) \leq \mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(x_i) \leq \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i) \leq \mathcal{F}_{\delta(b)}(\mathrm{a}_i)$ when $max\left(\mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\delta(b)})(\mathrm{a}_i)\right) \leq \frac{1}{2}$ *or*
   $\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) \geq \mathcal{T}_{\delta(b)}(\mathrm{a}_i),\ \mathcal{I}_{\gamma(b)}(\mathrm{a}_i) \geq \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i) \geq \mathcal{F}_{\delta(b)}(\mathrm{a}_i)$ when
   $min\left(\mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\delta(b)}(\mathrm{a}_i)\right) \geq \frac{1}{2}$.

Proof:

1. Let $\gamma_{\mathrm{i}} = \left(\mathcal{T}_{\gamma(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right) = (1,0,0)$ for all $\mathrm{a}_i \in X$. Then,

$$E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((1+0)|(2\times 0 - 1)| + (1+0)|(2\times 0 - 1)| + (0+0)|(2\times 1 - 1)|)$$

$$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}(1+1) = 1 - \frac{1}{2m}\sum_{i=1}^{m} 2 = 0$$

Similarly, If $\gamma_{\mathrm{i}} = \left(\mathcal{T}_{\gamma(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right) = (0,1,0)$ and $\left(\mathcal{T}_{\gamma(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i),\ \mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right) = (0,0,1)$ $\forall \mathrm{a}_i \in X$, then $E(\gamma_{\mathrm{i}}) = 0$.

2. Consider $\gamma_{\mathrm{i}} = \left\{\left(\mathrm{a}_i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \middle| \mathrm{a}_i \in X\right\}$. Then

$$E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}\left(\left(\frac{1}{2} + \frac{1}{2}\right)\right)\left|\left(2\left(\frac{1}{2}\right) - 1\right)\right| + \left(\frac{1}{2} + \frac{1}{2}\right)\left|\left(2\left(\frac{1}{2}\right) - 1\right)\right| + \left(\frac{1}{2} + \frac{1}{2}\right)\left|\left(2\left(\frac{1}{2}\right) - 1\right)\right|$$
$$= 1 - 0 = 1.$$

3. Let $E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{F}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{I}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) - 1)\right|)$

$\Rightarrow E(\gamma_{\mathrm{i}}) = 1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{T}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{I}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) - 1)\right|) = E(\gamma_{\mathrm{i}}{}^C)$.

4. Let $\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) \leq \mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i) \leq \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i) \leq \mathcal{F}_{\delta(b)}(\mathrm{a}_i)$ and $max\left(\mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\delta(b)}(\mathrm{a}_i)\right) \leq \frac{1}{2}$.

Then, $1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{F}_{\gamma(b)}(\mathrm{a}_i)) - 1)\right| + (\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{F}_{\gamma(b)}(\mathrm{a}_i))\left|(2\mathcal{I}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| + \left(\mathcal{F}_{\gamma(b)}(\mathrm{a}_i) + \mathcal{I}_{\gamma(b)}(\mathrm{a}_i)\right)\left|(2\mathcal{T}_{\gamma(b)}(\mathrm{a}_i) - 1)\right| \leq$

$1 - \frac{1}{2m}\sum_{i=1}^{m}((\mathcal{T}_{\delta(b)}(\mathrm{a}_i) + \mathcal{I}_{\delta(b)}(\mathrm{a}_i))\left|(2\mathcal{F}_{\delta(b)}(\mathrm{a}_i) - 1)\right| + (\mathcal{T}_{\delta(b)}(\mathrm{a}_i) + \mathcal{F}_{\delta(b)}(\mathrm{a}_i))\left|(2\mathcal{I}_{\delta(b)}(\mathrm{a}_i) - 1)\right| +$

$(\mathcal{F}_{\delta(b)}(\mathrm{a}_i) + \mathcal{I}_{\delta(b)}(\mathrm{a}_i))\left|(2\mathcal{T}_{\delta(b)}(\mathrm{a}_i) - 1)\right|)$
When $max\left(\mathcal{T}_{\delta(b)}(\mathrm{a}_i), \mathcal{I}_{\delta(b)}(\mathrm{a}_i), \mathcal{F}_{\delta(b)}(\mathrm{a}_i)\right) \le \frac{1}{2}$.

3. NSHSS in MADM

Choosing the highest quality from such a large number of alternatives is quite tricky, as it requires balancing several parameters that vary depending on personal preferences. This research study will compare mobile phones in terms of operational efficiency, power endurance, imaging capability, cost, and robustness to identify the top four models that perform well. This approach unifies power sets and neutrosophic methods in handling uncertainty and imprecision through the NSHSS model. The proposed methodology benefits customers by enabling informed choices, making the review process both comprehensive and reliable.

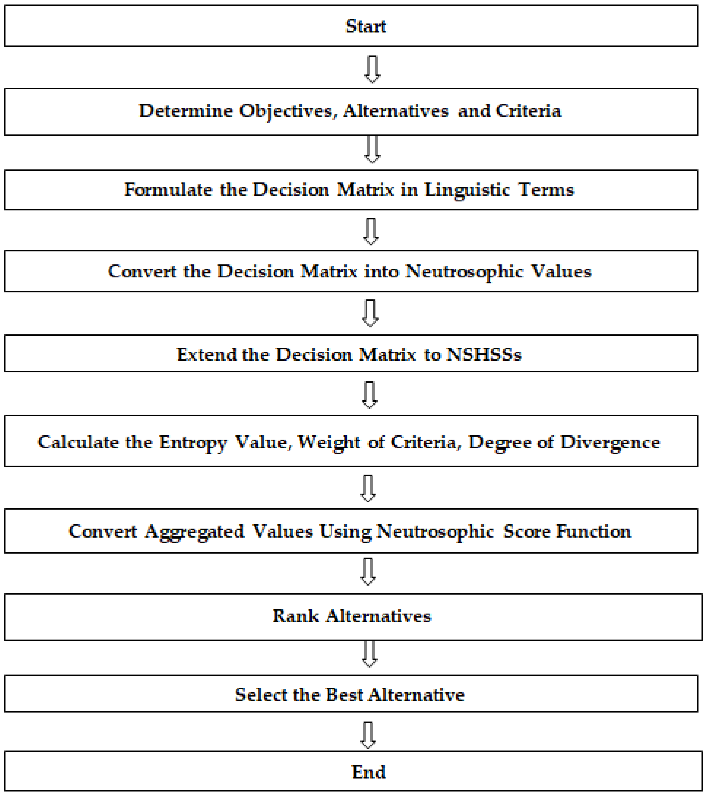
3.1 Algorithm: MADM Using NSHSS and Entropy

Step 1: To evaluate each criterion, it is necessary to first identify the decision-makers
$\kappa = \{\kappa_1, \kappa_2, \ldots \kappa_n\}$ criteria $\nabla=\{\nabla_1, \nabla_2, \ldots, \nabla_n\}$, and alternatives $\Omega = \{\Omega_1, \Omega_2 \ldots, \Omega n\}$, then gather input data from the decision-makers.
Step 2: Select the decision-makers $\kappa = \{\kappa_1, \kappa_2, \ldots, \kappa_n\}$ who will assess the criteria
$\nabla=\{\nabla_1, \nabla_2, \ldots, \nabla_n\}$ and alternatives after it has been established which criteria bear on the decision-making process.
Step 3: Utilise attributes and sub-attributes to create power sets of criteria and an NSHSS architecture. The evaluation of each alternative is then represented as follows: $\gamma_{\mathrm{i}} = \left(\mathcal{T}_{\gamma(b)}(a_i), \mathcal{I}_{\gamma(b)}(\mathrm{a}_i), \mathcal{F}_{\gamma(b)}(\mathrm{a}_i)\right)$.
Step 4: Decision-makers' linguistic evaluations for each of the NSHSS's criteria $\nabla=\{\nabla_1, \nabla_2, \ldots, \nabla_n\}$. To denote membership ($\mathcal{T}$), indeterminacy ($\mathcal{I}$) and non-membership ($\mathcal{F}$), propositions are converted into neutrosophic values.
Step 5: Apply the proposed entropy to evaluate the weight wi for each criterion Ci such that $\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w_i} = 1$.
Step 6: Use the NSHSWA operator to combine the evaluations of all criteria for each alternative

$$NSHS\mathrm{WA}(\gamma_1, \gamma_2, \gamma_3 \ldots \gamma_n) = < 1 - \prod_{i,j=1}^{n}\left(1 - \mathcal{T}_{\gamma(b)}(a_i)\right)^{w_j}, \prod_{i,j=1}^{n}\left(\mathcal{I}_{\gamma(b)}(a_i)\right)^{w_j}, \prod_{i,j=1}^{n}\left(\mathcal{F}_{\gamma(b)}(a_i)\right)^{w_j} >$$

Step 7: Evaluate the alternatives by using a Score Function $S = \frac{2+\mathcal{T}_a - \mathcal{I}_a - \mathcal{F}_a}{3}$ .... **(3)**
Step 8: Based on their scores rank the alternatives $\Omega = \{\Omega_1, \Omega_2 \ldots, \Omega n\}$.
Step 9: Designate the foremost Alternative(s).
Step 10: Termination the procedure.

3.2 Flow Chart for MADM Using NSHSS



Evaluation Criteria:

1. **Operational Efficiency (ψ1):** The operational efficiency of a smartphone is vital for user convenience, as it reflects how smoothly the device executes various tasks. This factor is divided into three categories: Superior (ψ11), Moderate (ψ12), and Inferior (ψ13).
2. **Power Endurance (ψ2):** The battery endurance of a smartphone is important for individuals who need their device to function throughout the day. This aspect is classified into three types: Extended (ψ21), Average (ψ22), and Limited (ψ23).
3. **Imaging Capability (ψ3):** As smartphones have become the primary devices for capturing photos and recording videos, the quality of the camera has gained significant importance. This criterion is separated into three levels: Outstanding (ψ31), Satisfactory (ψ32), and Subpar (ψ33).
4. **Cost (ψ4):** Financial considerations are a key element in selecting a smartphone. This parameter is categorized into two groups: Economical (ψ41) and High-costd (ψ42).
5. **Robustness (ψ5):** The robustness of a smartphone influences its longevity and resistance to wear and tear. It is evaluated at three levels: Strong (ψ51), Medium (ψ52), and Weak (ψ53).

Attributes and Sub-Attributes Representation:

Let ψ1, ψ2, ψ3, ψ4 and ψ5 are the utilized attributes and ψ1 = {ψ11, ψ12, ψ13}, ψ2 = {ψ21, ψ22, ψ23}, ψ3 = {ψ31, ψ32, ψ33}, ψ4 = {ψ41, ψ42} and ψ5 = {ψ51, ψ52, ψ53} are sub-attributes that have been utilize to assess the mobile phones X= {ϑ1, ϑ2, ϑ3, ϑ4, ϑ5}.

Power Sets of Criteria:

The power sets of X, ψ1, ψ2, ψ3, ψ4 and ψ5 are denoted by ℵ(X), ℵ(ψ1), ℵ(ψ2), ℵ(ψ3), ℵ(ψ4) and ℵ(ψ5), respectively. ℵ (X) = {∅, {ϑ1}, {ϑ2}, {ϑ3}, {ϑ4}, {ϑ5}, {ϑ1, ϑ2}, {ϑ1, ϑ3}, {ϑ1, ϑ4}, {ϑ1, ϑ5}, {ϑ2, ϑ3}, {ϑ2, ϑ4}, {ϑ2, ϑ5}, {ϑ3, ϑ4}, {ϑ3, ϑ5}, {ϑ4, ϑ5}, {ϑ1, ϑ2, ϑ3}, {ϑ1, ϑ2, ϑ4}, {ϑ1, ϑ2, ϑ5}, {ϑ1, ϑ3, ϑ4}, {ϑ1, ϑ3, ϑ5}, {ϑ1, ϑ4, ϑ5}, {ϑ2, ϑ3, ϑ4}, {ϑ2, ϑ3, ϑ5}, {ϑ2, ϑ4, ϑ5}, {ϑ3, ϑ4, ϑ5}, {ϑ1, ϑ2, ϑ3, ϑ4}, {ϑ1, ϑ2, ϑ3, ϑ5}, {ϑ1, ϑ2, ϑ4, ϑ5}, {ϑ1, ϑ3, ϑ4, ϑ5}, {ϑ2, ϑ3, ϑ4, ϑ5}, {ϑ1, ϑ2, ϑ3, ϑ4, ϑ5}}

P(ψ1) = {∅, {ψ11}, {ψ12}, {ψ13}, {ψ11, ψ12}, {ψ11, ψ13}, {ψ12, ψ13}, {ψ11, ψ12, ψ13}},

P(ψ2) = {∅, {ψ21}, {ψ22}, {ψ23}, {ψ21, ψ22}, {ψ21, ψ23}, {ψ22, ψ23}, {ψ21, ψ22, ψ23}},

P(ψ3) = {∅, {ψ 31}, {ψ32}, {ψ33}, {ψ31, ψ32}, {ψ31, ψ33}, {ψ32, ψ33}, {ψ31, ψ32, ψ33}},

P(ψ4) = {∅, {ψ41}, {ψ42}, {ψ41, ψ42}},

P(ψ5) = {∅, {ψ51}, {ψ52}, {ψ53}, {ψ51, ψ52}, {ψ51, ψ53}, {ψ52, ψ53}, {ψ51, ψ52, ψ53}}

Neutrosophic SuperHyperSoft Set:

Let F: ℵ(ψ1) × ℵ(ψ2) × ℵ(ψ3) × ℵ(ψ4) × ℵ(ψ5) → ℵ(X), where '×' indicate the Cartesian product for this equation. As a result, this is investigated as NSHSSs over X. The Cartesian product of ℵ(ψ1), ℵ(ψ2), ℵ(ψ3), ℵ(ψ4) and ℵ(ψ5) has 2048 elements.

P(ψ1) × P(ψ2) × P(ψ3) × P(ψ4) × P(ψ5) = {(∅, ∅, ∅, ∅, ∅), ({ψ11}, ∅, ∅, ∅, ∅), (∅, {ψ21}, ∅, ∅, ∅), ({ψ12}, {ψ22}, ∅, {ψ41}, ∅), ({ψ11, ψ12}, {ψ21, ψ22}, {ψ31, ψ32},{ ψ41}, {ψ51}), ({ψ13}, {ψ23}, {ψ33}, {ψ42}, {ψ51, ψ52}), (∅, {ψ21, ψ22, ψ23}, {ψ31, ψ32, ψ33}, {ψ41, ψ42}, {ψ51, ψ52, ψ53}), ({ψ11, ψ12, ψ13}, {ψ21, ψ22}, {ψ31, ψ33}, ∅, {ψ52, ψ53}), ({ψ12, ψ13}, {ψ21, ψ23}, {ψ32, ψ33}, {ψ41}, {ψ51, ψ53}), ({ψ11}, { ψ22}, {ψ31}, {ψ41, ψ42}, {ψ51, ψ52, ψ53}), ({ψ11}, ∅, {ψ31, ψ32}, {ψ42}, {ψ51}), ({ψ11, ψ13}, {ψ21}, {ψ33}, ∅, {ψ51, ψ53}), ({ψ12}, {ψ22, ψ23}, {ψ31, ψ33}, {ψ41}, {ψ51, ψ52}), ({ψ11, ψ12}, ∅, {ψ31, ψ32}, {ψ41}, {ψ51, ψ52, ψ53}), ({ψ12, ψ13}, {ψ21, ψ22}, {ψ32, ψ33}, {ψ41}, {ψ52, ψ53}), ({ψ11, ψ12}, {ψ21, ψ23}, {ψ32}, {ψ42}, {ψ51, ψ52}), ({ψ11}, {ψ22}, {ψ32}, {ψ41}, {ψ52}), ({ψ12}, {ψ23}, {ψ31, ψ33}, {ψ41, ψ42}, {ψ51, ψ53}), ({ψ11, ψ12}, {ψ21, ψ22}, { ψ33}, {ψ42}, {ψ51}), (∅, {ψ23}, {ψ31, ψ32}, {ψ41}, {ψ51, ψ53}), etc.}

Evaluation Process:

Table 1: The way that the DMs evaluated the alternatives through experts' opinions, in which linguistic variables are justifiable for assessment of the alternatives under prefixed standards. It also shows a way of converting linguistic inputs from experts to NSs.

Linguistic Term	Notation	$(\mathcal{T}, \mathcal{I}, \mathcal{F})\times 10^{-2}$
Feeble	$\Pi 1$	(0, 100, 100)
Frail	Π 2	(20, 95, 85)
Fragile	$\Pi 3$	(30, 80, 75)
Delicate	$\Pi 4$	(40, 75, 70)
Below Average	$\Pi 5$	(50, 65, 60)
Average	$\Pi 6$	(60, 60, 50)
Above Average	$\Pi 7$	(70, 45, 40)
Slightly Strong	$\Pi 8$	(80, 30, 35)
Strong	$\Pi 9$	(85, 25, 30)
Very Strong	$\Pi 10$	(90, 20, 20)
Extremely Strong	$\Pi 11$	(100, 10, 15)

Table 1: Mapping of Linguistic Strength Terms to Neutrosophic Set (NS) Values

The decision matrix is presented in Table 2, where evaluations of the decision-makers on each alternative (ϑ1, ϑ2, ϑ3 and ϑ4) are carried out with respect to identified criteria and sub-criteria. Evaluations will be made using pre-defined linguistic phrases in order to express the different levels of performance as depicted below.

Criteria	Sub-criteria	ϑ1	ϑ2	ϑ3	ϑ4
ψ1	ψ11	П1	П2	П4	П3
	ψ12	П5	П6	П7	П8
	ψ13	П9	П10	П11	П10
ψ2	ψ21	П2	П4	П3	П3
	ψ22	П6	П6	П7	П8
	ψ23	П9	П10	П10	П11
ψ3	ψ31	П1	П2	П3	П4
	ψ32	П6	П7	П9	П8
	ψ33	П10	П11	П11	П10
ψ4	ψ41	П10	П9	П7	П6
	ψ42	П3	П2	П4	П5
ψ5	ψ51	П2	П4	П3	П1
	ψ52	П6	П7	П8	П9
	ψ53	П9	П10	П11	П10

Table 2: Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Table 3: Decision matrix in neutrosophic values. Here each alternative ϑ1, ϑ2, ϑ3, ϑ4 is evaluated w.r.t. criteria and sub-criteria. Each evaluation will be made by using three components: Truth ($\mathcal{T}$), Indeterminacy ($\mathcal{I}$) and Falsity ($\mathcal{F}$). This will provide a complete description of the extent to which a criterion is satisfied, uncertain, or unsatisfied for each alternative.

Criteria	Sub-criteria	ϑ1 ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$)×10^{-2}	ϑ2 ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$) ×10^{-2}	ϑ3 ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$) ×10^{-2}	ϑ4 ($\mathcal{T}$, $\mathcal{I}$, $\mathcal{F}$) ×10^{-2}
ψ1	ψ11	(0,100,100)	(20,95,85)	(40,75,70)	(30,80,75)
	ψ12	(50,65,60)	(60,60,50)	(70,45,40)	(80,30,35)
	ψ13	(85,25,30)	(90,20,20)	(10,1,1.5)	(90,20,20)
ψ2	ψ21	(20,95,85)	(40,75,70)	(3,8,7.5)	(30,80,75)
	ψ22	(60,60,50)	(60,60,50)	(7,4.5,4)	(80,30,35)
	ψ23	(85,25,30)	(90,20,20)	(9,2,2)	(100,10,15)
ψ3	ψ31	(0,100,100)	(20,95,85)	(3,8,7.5)	(40,75,70)
	ψ32	(60,60,50)	(70,45,40)	(8.5,2.5,3)	(80,30,35)
	ψ33	(90,20,20)	(100,10,15)	(10,1,1.5)	(90,20,20)
ψ4	ψ41	(90, 20, 20)	(85, 25, 30)	(70, 45, 40)	(60, 60, 50)
	ψ42	(30,80,75)	(20,95,85)	(4,7.5,7)	(50,65,60)
ψ5	ψ51	(20,95,85)	(40,75,70)	(3,8,7.5)	(0,100,100)
	ψ52	(60,60,50)	(70,45,40)	(8,3,3.5)	(85,25,30)
	ψ53	(85,25,30)	(90,20,20)	(100,10,15)	(90,20,20)

Table 3: Decision Matrix Expressed in Neutrosophic Set Values

We assess each proposition in light of the aforementioned standards in order to conduct the MADM analysis.

Proposition 1: (Superior, Extended, Outstanding, Economical, Strong)
Proposition 2: (Superior, Extended, Outstanding, Economical, Medium)
Proposition 3: (Superior, Extended, Outstanding, Economical, Weak)
Proposition 4: (Superior, Extended, Outstanding, High-cost, Strong)
Proposition 5: (Superior, Extended, Outstanding, High-cost, Medium)
Proposition 6: (Superior, Extended, Outstanding, High-cost, Weak)
Proposition 7: (Superior, Extended, Satisfactory, Economical, Strong)
Proposition 8: (Superior, Extended, Satisfactory, Economical, Medium)
Proposition 9: (Superior, Extended, Satisfactory, Economical, Weak)
Proposition 10: (Superior, Extended, Satisfactory, High-cost, Strong)
Proposition 11: (Superior, Extended, Satisfactory, High-cost, Medium)
Proposition 12: (Superior, Extended, Satisfactory, High-cost, Weak)
Proposition 13: (Superior, Extended, Subpar, Economical, Strong)
Proposition 14: (Superior, Extended, Subpar, Economical, Medium)
Proposition 15: (Superior, Extended, Subpar, Economical, Weak)
Proposition 16: (Superior, Extended, Subpar, High-cost, Strong)
Proposition 17: (Superior, Extended, Subpar, High-cost, Medium)
Proposition 18: (Superior, Extended, Subpar, High-cost, Weak)
Proposition 19: (Superior, Average, Outstanding, Economical, Strong)
Proposition 20: (Superior, Average, Outstanding, Economical, Medium)
………………………………………………………………………………..
………………………………………………………………………………..
………………………………………………………………………………..
Proposition 153: (Inferior, Limited, Outstanding, High-cost, Strong)
Proposition 154: (Inferior, Limited, Outstanding, High-cost, Medium)
Proposition 155: (Inferior, Limited, Outstanding, High-cost, Weak)
Proposition 156: (Inferior, Limited, Satisfactory, Economical, Strong)
Proposition 157: (Inferior, Limited, Satisfactory, Economical, Medium)
Proposition 158: (Inferior, Limited, Satisfactory, Economical, Weak)
Proposition 159: (Inferior, Limited, Satisfactory, High-cost, Strong)
Proposition 160: (Inferior, Limited, Satisfactory, High-cost, Medium)
Proposition 161: (Inferior, Limited, Satisfactory, High-cost, Weak)
Proposition 162: (Inferior, Limited, Subpar, Economical, Strong)

Each proposition represents a unique combination of these criteria and out of 162 propositions the top three propositions are selected based on optimal performance across these factors.

Top 3 Propositions

The top three propositions are sort-out based on their balanced qualities after the 162 propositions have been analysed:

1. **Proposition 1: ψ13, ψ23, ψ33, ψ41, ψ52**
2. **Proposition 2: ψ13, ψ23, ψ33, ψ41, ψ53**.
3. **Proposition 3: ψ13, ψ23, ψ33, ψ41, ψ51**

The MADM strategy is to simplify decision-making for customers by focusing on the best-performing smartphones, balancing cost and performance to ensure value and quality across key aspects.

Proposition 1: This model scores inferior on operational efficiency (ψ13), limited on power endurance (ψ23), and subpar on imaging capability (ψ33). It is economical (ψ41) in cost and has medium robustness (ψ52), making it an affordable choice with moderate durability but with lower performance in battery life and camera quality.

Mobile / Criteria	ψ13	ψ23	ψ33	ψ41	ψ52
ϑ1	*Π*9	*Π*9	*Π*10	*Π*10	*Π*6
ϑ2	*Π*10	*Π*10	*Π*11	*Π*9	*Π*7
ϑ3	*Π*11	*Π*10	*Π*11	*Π*7	*Π*8
ϑ4	*Π*10	*Π*11	*Π*10	*Π*6	*Π*9
ϑ5	*Π*11	*Π*11	*Π*11	*Π*9	*Π*7

Table 4: Proposition-1-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile / Criteria	ψ13×10^{-2}	ψ23×10^{-2}	ψ33×10^{-2}	ψ41×10^{-2}	ψ52×10^{-2}
ϑ1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(60, 60, 50)
ϑ2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(70, 45, 40)
ϑ3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(80, 30, 35)
ϑ4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(85, 25, 30)
ϑ5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(70, 45, 40)

Table 5: Proposition-1-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 5, we will get the following Table 6.

Methods / Criteria	ψ13×10^{-3}	ψ23×10^{-3}	ψ33×10^{-3}	ψ41×10^{-3}	ψ52×10^{-3}
Entropy value	144	144	90	444	562
Degree of divergence D_j^p	856	856	910	556	438
weight of the criteria W_j^p	237	237	252	154	121

Table 6: Proposition 1-Criteria Weights

Aggregated values of proposition 1 using equation (1) : ϑ1= (860, 254, 271)×10^{-3},
ϑ2= (1000, 192, 215)×10^{-3}, ϑ3= (1000, 107, 207)×10^{-3}, ϑ4= (1000, 206, 226)×10^{-3}
ϑ5= (1000, 138, 188)×10^{-3}.

Score value of proposition 1 using equation (3): ϑ1= 777×10^{-3}, ϑ2= 864×10^{-3}, ϑ3= 875×10^{-3}, ϑ4=856×10^{-3}, ϑ5=891×10^{-3}.

Ranking: The ranking of the mobiles is ϑ5> ϑ3> ϑ2> ϑ4> ϑ1 by the score values of Proposition 1.

Proposition 2: This model scores inferior on operational efficiency (ψ13), limited on power endurance (ψ23), and subpar on imaging capability (ψ33). It is economical (ψ41) in cost and has weak robustness (ψ53), making it a budget-friendly option, but with lower durability and performance in key areas.

Mobile / Criteria	ψ13	ψ23	ψ33	ψ41	ψ53
ϑ1	*Π*9	*Π*9	*Π*10	*Π*10	*Π*9
ϑ2	*Π*10	*Π*10	*Π*11	*Π*9	*Π*10
ϑ3	*Π*11	*Π*10	*Π*11	*Π*7	*Π*11
ϑ4	*Π*10	*Π*11	*Π*10	*Π*6	*Π*10
ϑ5	*Π*11	*Π*11	*Π*11	*Π*9	*Π*11

Table 7: Proposition-2-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile/ Criteria	ψ13×10^{-2}	ψ23×10^{-2}	ψ33×10^{-2}	ψ41×10^{-2}	ψ53×10^{-2}
ϑ1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(85, 25, 30)
ϑ2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(90, 20, 20)
ϑ3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(100, 10, 15)
ϑ4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(90, 20, 20)
ϑ5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(100, 10, 15)

Table 8: Proposition-2-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 8, we will get the following Table 9.

Methods / Criteria	ψ13×10^{-3}	ψ23×10^{-3}	ψ33×10^{-3}	ψ41×10^{-3}	ψ53×10^{-3}
Entropy value	144	144	90	444	144
Degree of divergence D_j^p	856	856	910	556	856
weight of the criteria W_j^p	212	212	226	138	212

Table 9: Proposition 2-Criteria Weights

Aggregated values of proposition 2 using equation (1): ϑ1= (870, 231, 259) ×10^{-3}, ϑ2= (1000, 176, 198) ×10^{-3}, ϑ3=(1000, 143, 183) ×10^{-3}, ϑ4= (1000, 201, 214) ×10^{-3}, ϑ5= (1000, 114, 165) ×10^{-3}.

Score value of proposition 2 using equation (3): ϑ1= 794×10^{-3}, ϑ2= 875×10^{-3},ϑ3= 892×10^{-3}, ϑ4=862×10^{-3}, ϑ5=907×10^{-3}.

Ranking: The ranking of the mobiles is ϑ5> ϑ3> ϑ2> ϑ4> ϑ1 by the score values of Proposition 2.

Proposition 3: This model scores inferior on operational efficiency (ψ13), limited on power endurance (ψ23), and subpar on imaging capability (ψ33). It is economical (ψ41) in cost and has strong robustness (ψ51), making it a durable and affordable option, though with lower performance in power endurance and imaging quality.

Mobile / Criteria	ψ13	ψ23	ψ33	ψ41	ψ51
ϑ1	*Π*9	*Π*9	*Π*10	*Π*10	*Π*2
ϑ2	*Π*10	*Π*10	*Π*11	*Π*9	*Π*4
ϑ3	*Π*11	*Π*10	*Π*11	*Π*7	*Π*9

ϑ4	**Π**10	**Π**11	**Π**10	**Π**6	**Π**1
ϑ5	**Π**11	**Π**11	**Π**11	**Π**9	**Π**4

Table 10: Proposition-3-Decision Matrix Based on Strength-Oriented Linguistic Evaluations

Mobile / Criteria	**ψ13×10^{-2}**	**ψ23× 10^{-2}**	**ψ33× 10^{-2}**	**ψ41× 10^{-2}**	**ψ51× 10^{-2}**
ϑ1	(85, 25, 30)	(85, 25, 30)	(90, 20, 20)	(90, 20, 20)	(20, 95, 85)
ϑ2	(90, 20, 20)	(90, 20, 20)	(100, 10, 15)	(85, 25, 30)	(40, 75, 70)
ϑ3	(100, 10, 15)	(90, 20, 20)	(100, 10, 15)	(70, 45, 40)	(30, 80, 75)
ϑ4	(90, 20, 20)	(100, 10, 15)	(90, 20, 20)	(60, 60, 50)	(00, 100, 100)
ϑ5	(100, 10, 15)	(100, 10, 15)	(100, 10, 15)	(85, 25, 30)	(40, 75, 70)

Table 11: Proposition-3-Decision Matrix Expressed in Neutrosophic Set Values

Using equation (2) in Table 11, we will get the following Table 12.

Method / Criteria	**ψ13×10^{-3}**	**ψ23×10^{-3}**	**ψ33×10^{-3}**	**ψ41×10^{-3}**	**ψ51×10^{-3}**
Entropy value	144	144	90	444	-123
Degree of divergence D_j^p	856	856	910	556	1123
weight of the criteria W_j^p	199	199	212	129	261

Table 12: Proposition-3-Criteria weights

Aggregated values of proposition 3 using equation (1): ϑ1= (800, 328, 343) ×10^{-3}, ϑ2= (1000, 251, 275) ×10^{-3}, ϑ3= (1000, 240, 275) ×10^{-3}, ϑ4= (1000, 306, 324) ×10^{-3}, ϑ5= (1000, 191, 245) ×10^{-3}.

Score value of proposition 3 using equation (3): ϑ1= 709×10^{-3}, ϑ2= 825×10^{-3}, ϑ3= 829×10^{-3}, ϑ4=790×10^{-3}, ϑ5=855×10^{-3}.

Ranking: The ranking of the mobiles is ϑ5> ϑ3> ϑ2> ϑ4> ϑ1 by the score values of Proposition 3.

3.3 Graphical Representation of Entropy and Proposition Rankings

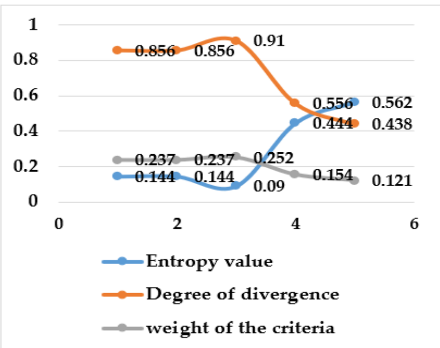


Figure 1: Comparison of Entropy Degree of divergence and criteria weight of Proposition 1

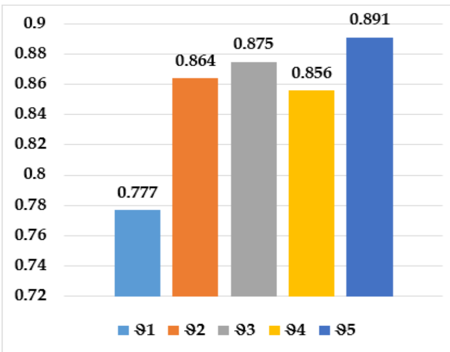


Figure 2: Comparison of Ranks of Proposition1

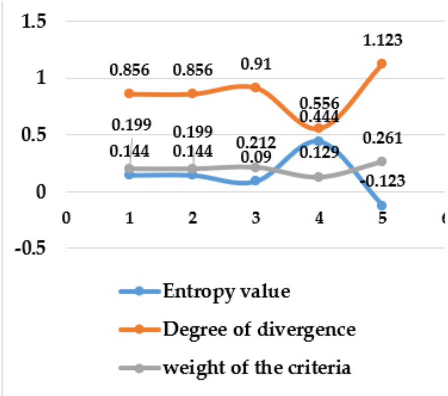


Figure 5: Comparison of Entropy Degree of divergence and criteria weight of Proposition 3

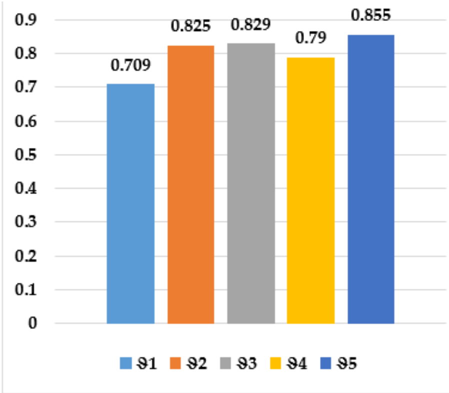


Figure 6: Comparison of Ranks of Proposition3

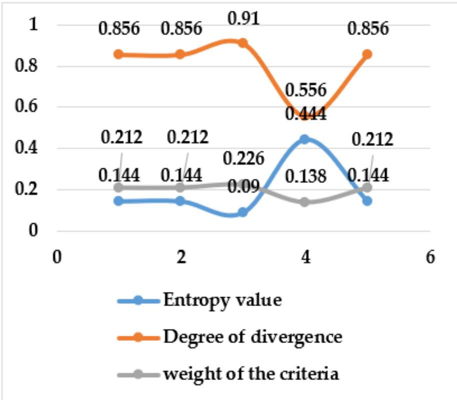


Figure 3: Comparison of Entropy Degree of divergence and criteria weight of Proposition 2

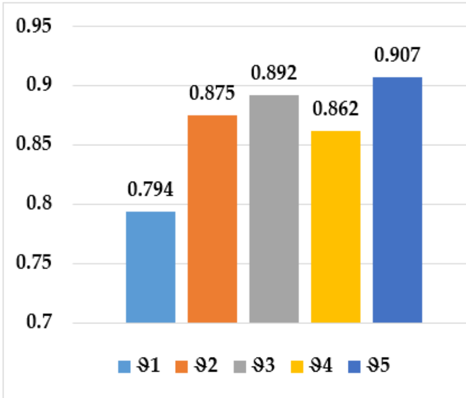


Figure 4: Comparison of Ranks of Proposition2

4. Conclusions

This study demonstrates the efficacy of NSHSSs in addressing complex decision-making scenarios involving multiple conflicting criteria. The proposed entropy-based MADM framework successfully incorporates uncertainty, ambiguity and granularity, offering a robust methodology for ranking alternatives. By applying this approach to mobile phone selection, the study illustrates the practical utility of NSHSSs in real-world applications. The results validate the capability of the NSHSS framework to evaluate alternatives comprehensively and rank them reliably, ensuring optimal decision-making outcomes. The innovative integration of power sets and neutrosophic principles allows for a detailed and hierarchical analysis of criteria, making NSHSSs particularly suitable for multi-attribute problems with intricate data structures. The methodology's adaptability to various decision-making contexts, from recruitment to sustainable logistics, underscores its potential for broad applicability.

Future research can explore expanding the application of Neutrosophic SuperHyperSoft Sets (NSHSSs) to diverse fields such as healthcare, environmental sustainability and artificial intelligence, validating their adaptability to complex, real-world problems. Integrating NSHSSs with machine learning techniques could automate the evaluation and ranking processes, enhancing scalability and precision.

Additionally, hybrid approaches combining NSHSSs with optimization algorithms like genetic algorithms and particle swarm optimization may address dynamic decision-making challenges. Comparative studies with other advanced decision-making frameworks will further highlight the strengths and limitations of NSHSSs, while the development of user-centric decision-support systems will make this robust methodology accessible to practitioners across various domains.

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