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Multineutrosophic Analysis of the Relationship Between Survival and Business Growth in the Manufacturing Sector of Azuay Province, 2020–2023, Using Plithogenic n-Superhypergraphs

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Abstract. This study analyzed the relationship between survival and business growth in the manufacturing sector of Azuay Province. To achieve this, Plithogenic n-SuperHyperGraphs and Multineutrosophic logic were employed to examine financial data defined by a set of n-indicators, n-records, and n-entities between 2020 and 2023. Additionally, the Springate Z-Score model was used, along with the logarithmic variation of assets, multiple regression, and factor analysis. The results showed that business survival was determined by variables related to vertices of financial stability and operational efficiency, while growth was primarily associated with asset expansion. Factor analysis identified three key components: firm size, financial solvency, and profitability, grouped within the dominant vertex. These findings demonstrate an integrated financial structure underlying business survival. In conclusion, the results provide an empirical basis for designing sustainability strategies in Azuay's manufacturing sector, with potential applications in other regions and sectors of Ecuador.

Keywords: business survival, business growth, manufacturing, Multineutrosophy, Plithogenic n-SuperHyperGraphs.

1. Introduction

Business continuity and growth are interdependent processes conditioned by economic, operational, and structural factors [1]. Models such as the Z-Score, especially the Springate model, allow for the evaluation of financial stability and anticipation of insolvency risks, being key in uncertain environments [2]. Additionally, growth is explained by internal factors such as innovation, human capital, and management, as well as by external conditions such as market structure and economic cycles [3].

In fact, in the manufacturing sector of Azuay, these processes face particular challenges stemming from economic slowdown and the structural effects of the pandemic. Despite favorable indicators in 2020, weaknesses in profitability and efficiency persist, especially in microenterprises [4].

Given this scenario, there emerges a need to understand how business survival and growth are related in the manufacturing sector of Azuay during the 2020-2023 period. This research analyzes this relationship with the objective of establishing conditions that facilitate the design of economic systems oriented toward strengthening the sector. Specifically, three objectives are proposed: identifying the factors that influence both processes, quantitatively estimating the relationship between survival and growth through econometric methods and determining the structural elements that support their interdependence.

In this sense, the study seeks to provide useful empirical evidence for the development of strategies that promote business sustainability [5]. To this end, this study implements different tools such as Z-Score models, Plithogenic n-SuperHyperGraphs, and multiple regression in the evaluation and interaction between the financial and operational dimensions of companies in the sector. In fact, this interaction between elements (referred to as vertices) and their groups (defined as super-vertices) helps identify factors affecting the manufacturing sector in Azuay. Complementarily, multineutrosophic logic is used to examine *n-indicators* (financial indicators) based on *n-resources* (files or financial information records) over a time period (2020 to 2023). Therefore, the integration of these instruments allows for understanding these factors, strengthening Azuay's industrial fabric, promoting employment stability, and contributing to regional economic development [6].

2. Materials and Methods

For the study, *Plithogenic n-SuperHyperGraphs* were used to model the interrelationships between financial, operational, and structural variables associated with business survival and growth in the manufacturing sector of Azuay (defined as vertices and their interconnections). This methodology, defined by Smarandache, allowed for the incorporation of uncertainty and revealed complex patterns not identifiable through conventional methods (according to the consulted methodology [7]).

Complementarily, a descriptive, explanatory, and correlational approach was adopted to analyze factors influencing business continuity and expansion, based on historical data obtained from secondary sources (databases from the Superintendence of Companies, Securities, and Insurance). The use of econometric models facilitated the evaluation of interactions between key variables, providing a comprehensive view of financial stability in the manufacturing sector. For data processing and analysis, the business survival model and the Springate Z-score model [8] were employed, defining the following equation:

$Business\ survival\ (Z) = 1.03A + 3.07B + 0.66C + 0.40D$(2)

Where:

- A = Working Capital / Total Assets
- B = Net income before interest and taxes / Total Assets
- C = Net income before taxes / Current liabilities
- D = Sales / Total Assets

The Springate model, created by Gordon Springate, is based on four financial ratios to calculate a Zeta-Score that evaluates the probability of bankruptcy. For a company to be considered healthy, it must have a minimum score of 0.862. On the other hand, business growth was measured using the logarithmic difference of total assets between two consecutive periods, following Evans' (1987) methodology:

$$Growth\ (Gt) = Ln\frac{S_t}{S_t - 1} = Ln\ S_t - Ln\ S_{t-1}$$

Where:

- St represented the total assets of the company in the current period, reflecting its financial structure at the present time.
- $St - 1$ corresponded to the total assets of the company in the previous period, allowing the estimation of the relative rate of variation in the company's size.

Subsequently, a Pearson correlation analysis was applied to determine the relationship between business survival and firm growth, verifying the normality of the data and evaluating the direction and intensity of this relationship. Then, a principal component factor analysis with statistical validity criteria (Bartlett's test and KMO) was used to identify underlying common factors. In fact, it allowed for grouping variables or vertices associated with survival and growth without altering the model structure. Thus, it facilitated the interpretation of their connections, determinants, and the distinction between integrated or independent dimensions within the structural analysis of the *Plithogenic n-SuperHyperGraphs*.

2.1 Multineutrosophic set

Definition 1 [9]. The *Neutrosophic set* N is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^-0$ and 1^+ belong to the set of hyperreal numbers.

Definition 2. The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$ and $F_A: U \rightarrow [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *Single-Valued Neutrosophic Number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3 ([10])**.** The MultiNeutrosophic Set (or Subset MultiNeutrosophic Set SMNS).

Let $\mathcal{U}$ be a universe of discourse and M a subset of it. Then, a MultiNeutrosophic Set is: $M = \{x, x(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)\}, x \in U$,

where p, r, s are integers $\geq 0, p + r + s = n \geq 2$ and at least one of p, r, s $is \geq 2$, in order to ensure the existence of multiplicity of at least one neutrosophic component: truth/membership, indeterminacy, or falsehood/nonmembership; all subsets $T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s \subseteq [0,1]$;

$$0 \leq \sum_{j=1}^{p} inf\ T_j + \sum_{k=1}^{r} inf\ I_k + \sum_{l=1}^{s} inf\ F_l \leq \sum_{j=1}^{p} sup\ T_j + \sum_{k=1}^{r} sup\ I_k + \sum_{l=1}^{s} sup\ F_l \leq n.$$

No other restrictions apply on these neutrosophic multicomponents.

$T_1, T_2, \ldots, T_p$ are multiplicities of the truth, each one provided by a different source of information (expert).

Similarly, $I_1, I_2, \ldots, I_r$ are multiplicities of the indeterminacy, each one provided by a different source.

And $F_1, F_2, \ldots, F_s$ are multiplicities of the falsehood, each one provided by a different source.

3 The Study

3.1 Key variables in the Azuay manufacturing sector: Structure of the Plithogenic n-SuperHyperGraph.

The analysis of results allowed for identifying the factors that influenced business survival and growth in the manufacturing sector of Azuay during the study period. From the characterization of key variables, it became evident that financial stability, operational efficiency, and adaptive capacity played a determining role in business continuity. Through the application of statistical techniques, the relationship between business survival and growth was established. For the analysis and development of the study, the Plithogenic n-SuperHyperGraph structure was proposed, identifying 3 dimensions or sets of vertices to measure and evaluate business survival and growth in the manufacturing sector of Azuay (see Table 1).

Table 1: Structure of the Plithogenic n-SuperHyperGraph of the manufacturing sector of Azuay. Source: Own elaboration.

Dimension / Vertex	Vertex attribute	Sublevel vertex	Multineutrosophic sublevel vertex
Financial stability (V_1)	Indicator (V_{11})	Period (V_{111})	V_{111n}: $\{2020, 2021, 2022, 2023\}$, $\{entity, record, \$, \%\}$
	Business survival(V_{12})	Springate Z-score model (V_{121})	V_{1211n}: $\{2020, 2021, 2022, 2023\}$, $\{High, medium, low, critical\}$
	Growth rate(V_{13})	Evans Methodology(V_{131})	V_{131n}: $\{2020, 2021, 2022, 2023\}$, $\{High, medium, low, contraction\}$
	Entities(V_{14})	Business size (V_{141})	$(V_{141})\{big, medium, small\}$
Macro environment structure (V_2)	Economic situation (V_{21})	(V_{21n}), includes multiple Vertex sub attributes n that make up the vertexV_{21}{inflation, interest rates, structural barriers, epidemics, etc., n-elements}. These elements are presented according to the time period analyzed.	(V_{21nn})*{period; impact of the current element on the entity, …, n-elements (Multineutrosophic criteria: High, medium, low,)}*These elements are presented according to the time period analyzed.
	Market dynamics (V_{22})	(V_{22n}), includes multiple Vertex sub attributes n that make up the vertexV_{22}*{Competition, market demand, …, n-elements}*These elements are presented according to the time period analyzed.	(V_{22nn})*{period; impact of the current event on the entity, …, n-items (Multineutrosophic criterion: High, medium, low)}* These elements are presented according to the time period analyzed.
Working capital management ()V_3	Operational efficiency(V_{31})	-	(V_{3111}){Operating efficiency ratio}
	Adaptability(V_{32})	Organizational flexibility(V_{321})	$(V_{3211})\ \{High, medium, low\}$
		Strategic innovation(V_{322})	$(V_{3221})\ \{High, medium, low, null\}$

In Table 1, it was observed that vertex V_1 is composed of *n-indicators* to determine the health of the entity according to the evaluation determined in a *time period* framed between 2020 to 2023. Each of these evaluations were performed on *n-entities* in the manufacturing sector of Azuay through *n-records* (multiple resources that record a variety of information within the multineutrosophic set of financial indicators). Therefore, to structure the study according to multineutrosophic logic, the following pair is proposed:

$$\mathrm{V}_1: \{\, n - indicators\} \rightarrow \{n - records, time\ period, n - entities\}$$

Therefore, to determine business survival (V_{12}) and growth rate (V_{13}), a subset of V_1 is evaluated according to the Springate Z-score model (V_{121}) and Evans' methodology (V_{131}) (See sections 3.2, 3.3 and 3.4). However, the analysis of V_2 and V_3 would be presented in section 3.

3.2 Characterization of the factors affecting business survival and growth.

The following table presents the financial ratios affecting business survival, including the evolution of the *Working Capital/Total Assets* indicator in the manufacturing sector of Azuay during the 2020-2023 period (see Table 2). The data show that in 2020 this indicator stood at 23.36%, followed by an increase to 26.54% in 2021. Subsequently, 2022 saw a decrease to 21.60%, a trend that continued in 2023 when the value dropped to 12.29%.

Table 2: Subset of financial ratios from vertex V_{111n} (2020-2023). Source: Prepared by the authors based on (Superintendency of Companies, Securities and Insurance, 2024).

General Description	Year 2020	Year 2021	Year 2022	Year 2023
A: Working Capital/Total Assets	23.36%	26.54%	21.60%	12.29%
B: Earnings Before Interest and Taxes / Total Assets	11.64%	0.76%	2.22%	2.52%
C: Earnings Before Taxes / Current Liabilities	29.84%	13.83%	29.83%	38.62%
D: Sales / Total Assets	106.00%	104.36%	94.31%	92.02%

These results show variations in the working capital to the total assets ratio throughout the period analyzed. The increase in 2021 indicates that manufacturing companies increased their working capital relative to total assets that year, while subsequent years showed a progressive decline in this indicator. The decreases recorded in 2022 and 2023 suggest that working capital's share of total assets was significantly reduced by the end of the period. This trend can be evaluated alongside other financial indicators to understand the financing structure and liquidity of manufacturing companies in Azuay province.

The evolution of the EBIT/Total Assets ratio in Azuay's manufacturing sector during 2020-2023 is presented. In 2020, the ratio reached 11.64%, followed by a sharp decline to 0.76% in 2021. From 2022 onward, a gradual recovery was recorded, increasing to 2.22%, with a further rise to 2.52% in 2023. These values reflect changes in operating profits relative to total assets during the study period. The decline between 2020 and 2021 indicates a significant reduction in operating profitability relative to assets, while the subsequent recovery suggests improved operating income generation capacity relative to asset size. Analyzing this trend alongside other financial indicators enables a more precise assessment of the manufacturing sector profitability evolution in Azuay province.

Similarly, the evolution of the Pretax Income/Current Liabilities ratio in Azuay's manufacturing sector during 2020-2023 is presented. In 2020, the ratio stood at 29.84%, followed by a decrease to 13.83% in 2021. From 2022 onward, a recovery was observed, rising to 29.83%, and reaching its peak in 2023 at 38.62%. This pattern shows an initial reduction in pretax income generation relative to current liabilities, followed by progressive recovery in the last two years analyzed. The 2021 decline indicates lower pretax profits relative to short-term obligations, while the 2022-2023 recovery reflects increased operating profitability relative to current liabilities. Evaluating this trend alongside other financial indicators provides insights into the evolution of payment capacity and financial stability of manufacturing companies in Azuay province.

However, the Sales/Total Assets ratio in Azuay's manufacturing sector reached 106.00% in 2020, followed by a slight decrease to 104.36% in 2021. Subsequent years showed more pronounced declines, with 94.31% in 2022 and a further drop to 92.02% in 2023. These results demonstrate a progressive reduction in sales relative to total assets, indicating diminished revenue generation compared to asset structure throughout the study period. This trend suggests decreasing efficiency in asset utilization for sales generation, potentially associated with various economic and operational factors within the manufacturing sector. Analyzing this variation alongside other financial indicators provides better understanding of the evolution in productive and commercial performance of companies in Azuay province.

In contrast, Figure 1 shows the evolution of the Natural Logarithm of Assets in Azuay's manufacturing sector, comparing the current period with the previous period during 2020-2023. The data show that in 2020, the average natural logarithm of assets was 12.63, while the previous period's value was 12.61. In 2021, both indicators increased, reaching 12.69 and 12.63, respectively. In 2022, the average natural logarithm of assets remained stable at 12.71, while the previous period's value reached 12.69. Finally, in 2023, both values converged at 12.71 and 12.70, respectively.

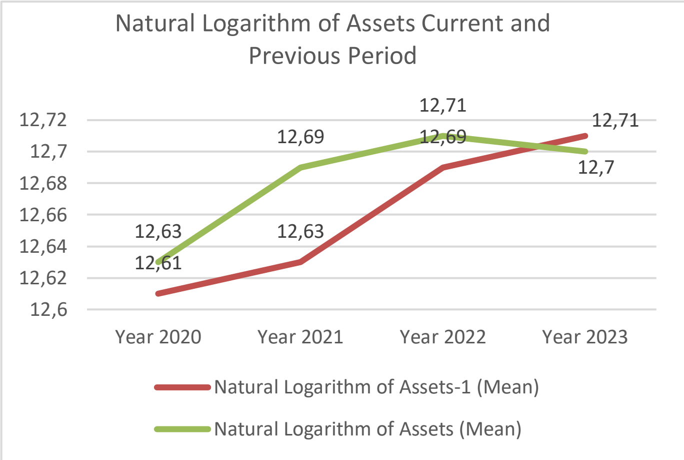


Figure 1: Natural logarithm of assets - current and previous periods. Source: Own elaboration based on (Superintendency of Companies, Securities and Insurance, 2024).

These results reflect a gradual growth trend in total assets of manufacturing companies in Azuay province throughout the analyzed period. The convergence observed in 2023 values of current and previous period natural logarithm of assets suggests stabilization in company asset evolution. Analysis of this trend alongside other financial indicators enables better understanding of growth dynamics and capital accumulation in the manufacturing sector.

Additionally, Figure 2 shows the evolution of average growth in Azuay's manufacturing sector during the 2020-2023 period. In 2020, average growth was 2.12%, followed by an increase to 3.01% in 2021. In 2022, growth maintained a positive trend, reaching 3.17%. However, 2023 saw a significant decline to -0.26%, indicating a contraction in manufacturing company performance that year. These results reflect moderate growth during the first three years of analysis, with acceleration in 2022 followed by a reversal in 2023. The observed decrease in the final year suggests sector companies experienced a reduction in total assets, potentially associated with various economic and financial conditions. Analyzing this pattern alongside other indicators helps evaluate factors influencing business growth dynamics in Azuay province.

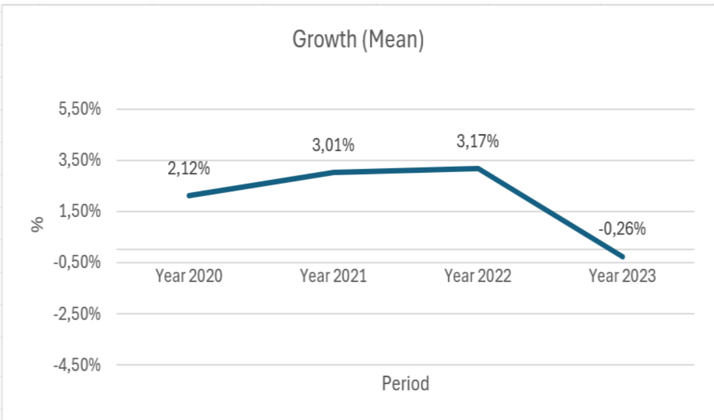


Figure 2: Business growth. Source: Own elaboration based on (Superintendency of Companies, Securities and Insurance, 2024).

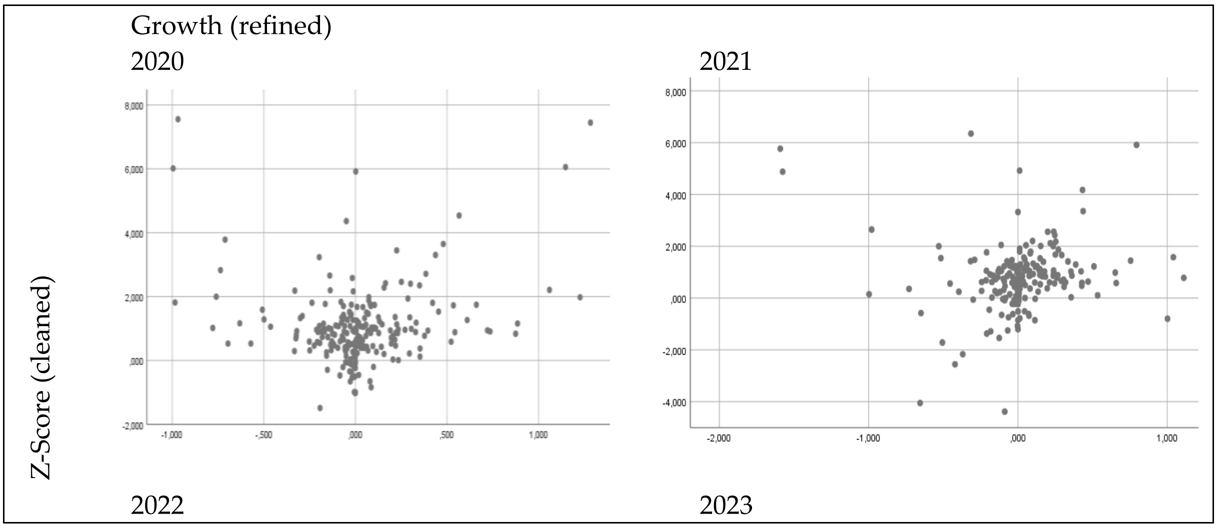
3.3 Statistical relationship between business survival and growth (internal connections of vertex V_1).

Table 3 (Z-Score 2020) presents the results of the estimation of the relationship between business growth and the Z-Score index from 2020 to 2023. In fact, the coefficient of the constant was 1.054 with a statistical significance of 0.000, indicating that it is a relevant parameter within the model. On the other hand, the coefficient of the variable "Growth 2020" was 0.350 with a standard error of 0.257 and a t-value of 1.364. The significance associated with this variable was 0.174, indicating that business growth in 2020 did not exhibit a statistically significant relationship with the Z-Score index in that same year.

Table 3: Regression model coefficients between growth and Z-Score (2020–2023). Source: Own elaboration based on data from the Superintendency of Companies, Securities and Insurance (2024).

Coefficients*							
Model		Non-standardized coefficients		Standardized coefficients	t	Next.	a. Dependent variable
		B	Dev. Error	Beta			
1	(Constant)	1,054	0.081		12.963	0.000	Z - Score 2020
	Growth 2020	0.350	0.257	0.091	1,364	0.174	
2	(Constant)	0.789	0.090		8.773	0.000	Z - Score 2021
	Growth 2021	0.096	0.284	0.024	0.340	0.734	
3	(Constant)	0.791	0.100		7.902	0.000	Z - Score 2022
	Growth 2022	1.273	0.270	0.316	4.721	0.000	
4	(Constant)	0.751	0.091		8.283	0.000	Z - Score 2023
	Growth 2023	1.323	0.257	0.338	5.148	0.000	

Figure 3 (Z-Score 2020) illustrates the relationship between both variables through a scatter plot. It can be observed that the distribution of the points does not show a clear trend, suggesting that business growth in 2020 did not have a relevant impact on the Z-Score index. The data dispersion indicates significant variability without an apparent correlation between the variables analyzed. This result is consistent with the coefficient analysis, where the low value of the standardized beta (0.091) and the lack of statistical significance reinforce the absence of a strong relationship between business growth and financial stability in the year 2020.



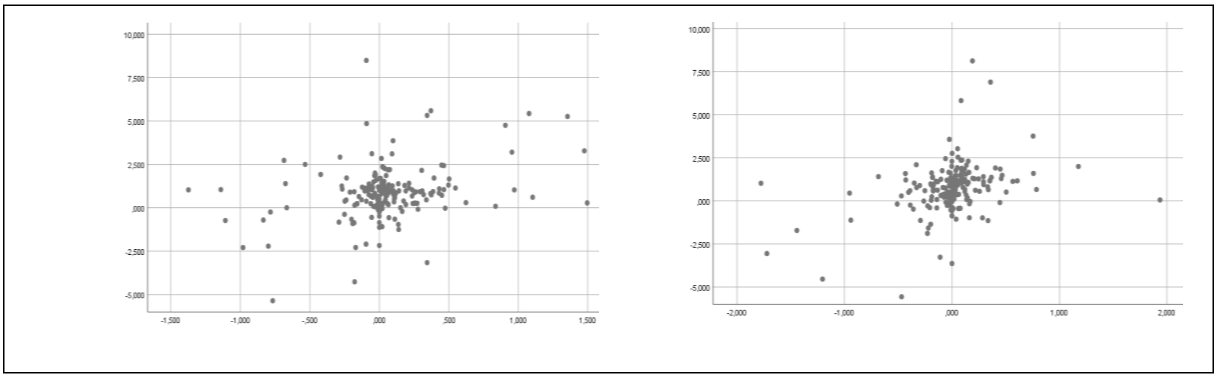


Figure 3: Regression model between growth and Z-Score (2020–2023). Source: Own elaboration based on data from the Superintendency of Companies, Securities and Insurance (2024).

On the other hand, the results of the estimation of the relationship between business growth and the Z-Score index in 2021 are presented (see Table 3, Z-Score 2021). The coefficient of the constant was 0.789, with a statistical significance of 0.000, indicating that its effect within the model is relevant. However, the coefficient of the variable "Growth 2021" was 0.096, with a standard error of 0.284 and a t-value of 0.340. The significance associated with this variable was 0.734, indicating that business growth in 2021 did not show a statistically significant relationship with the Z-Score index for that same year.

Meanwhile, Figure 3 (Z-Score 2021) represents the relationship between both variables through a scatter plot. It can be observed that the data does not show a clear trend suggesting a strong correlation between business growth and the financial survival index. The dispersion of the points indicates that the variability of the Z-Score in 2021 was not directly related to business growth in that year. These results are consistent with those obtained in the coefficient table, where the low magnitude of the beta coefficient (0.024) and the lack of statistical significance reinforce the absence of a relevant relationship between these variables in the period analyzed.

In contrast, Table 3 (Z-Score 2022) presents the results of the regression analysis evaluating the relationship between business growth and the Z-Score index in 2022. The coefficient of the constant was 0.791 with a significance level of 0.000, indicating that its effect within the model is statistically relevant. The coefficient of the variable "Growth 2022" was 1.273 with a standard error of 0.270 and a t-value of 4.721. The significance associated with this variable was 0.000, indicating a statistically significant relationship between business growth and the Z-Score index in that year.

Moreover, Figure 3 (Z-Score 2022) represents the relationship between both variables through a scatter plot. Unlike previous years, in this case, a clearer trend is observed in the distribution of the points, suggesting a stronger correlation between business growth and financial stability as measured by the Z-Score. These results are consistent with those obtained in the coefficient table, where the standardized beta coefficient of 0.316 and its statistical significance confirm the existence of a positive relationship between business growth and financial stability in the manufacturing sector of Azuay in 2022.

Likewise, Table 3 (Z-Score 2023) presents the results of the estimation of the relationship between business growth and the Z-Score index in 2023. The coefficient of the constant was 0.751, with a statistical significance of 0.000, indicating that it is a relevant parameter within the model. The variable "Growth 2023" obtained a coefficient of 1.323, with a standard error of 0.257 and a t-value of 5.148. The associated significance was 0.000, confirming the existence of a statistically significant relationship between business growth and the Z-Score index in the year analyzed.

Similarly, Figure 3 (Z-Score 2023) shows the distribution of the data in a scatter plot, revealing a more defined relationship compared to previous years. The greater concentration of points suggests that as business growth increases, so does the Z-Score index, indicating a possible positive relationship between both variables. These results are consistent with those obtained in the coefficient table, where the standardized beta coefficient of 0.338 and its high statistical significance reinforce the existence of a correlation between business growth and financial stability in the manufacturing sector of Azuay in 2023.

3.4 Factors determining business growth and survival.

The analysis of determining factors for business growth and survival allowed for the identification of dimensions that explained the stability and expansion of companies in the manufacturing sector. Factor analysis was applied to reduce variables into latent components that captured the underlying data structure. Business survival was represented by two factors associated with financial stability and operational efficiency. Meanwhile, business growth was explained by a single component based on the logarithmic variation of assets. The results confirmed that financial and operational structure influenced company continuity, while growth responded to a one-dimensional dynamic.

3.4.1 Business survival.

The following table shows the proportion of variance explained by each component extracted using the principal component analysis method (see Table 4). Indeed, the first two components extracted through factor analysis present eigenvalues greater than 1, justifying their retention according to Kaiser's criterion. Together, they explain 60.91% of the total variance of the set of variables analyzed. The first component contributes 33.88% and the second contributes 27.03%, indicating that a significant proportion of the variability in the data can be synthesized in these two dimensions. This result suggests the existence of common latent structures that simultaneously integrate characteristics associated with business survival and growth. The factorial reduction obtained allows grouping financial variables without substantial loss of statistical information, which improves the multivariate interpretation of the behavior of companies in the manufacturing sector. In contrast, components from the third onward present eigenvalues less than 1 and make marginal contributions to the total explanation, which is why they were not considered in the definitive factorial solution. To identify the principal components, factor loadings greater than 0.55 were considered.

Table 4: Total variance explained by the components. Source: Own elaboration based on data from the Superintendency of Companies, Securities and Insurance (2024).

Total variance explained									
	Initial eigenvalues			Sums of squared charges of extraction			Sums of charges squared by rotation		
Component	Total	% variance	% accumulated	Total	% variance	% accumulated	Total	% variance	% accumulated
1	1,898	23,722	23,722	1,898	23,722	23,722	1,808	22,601	22,601
2	1,749	21,862	45,583	1,749	21,862	45,583	1,522	19,025	41,626
3	1,195	14,936	60,519	1,195	14,936	60,519	1,511	18,893	60,519
4	,995	12,436	72,955						
5	,864	10,801	83,756						
6	,671	8,384	92,140						
7	,432	5,399	97,539						
8	,197	2,461	100,000						

Extraction method: principal component analysis.

Similarly, the rotated component matrix allows for the observation of latent groupings between growth and business survival variables. Varimax rotation with Kaiser normalization was used to improve the interpretation of the factor loadings (see Table 5). The first component shows high loadings on variables associated with business size, specifically the natural logarithm of assets in the current (0.944) and previous (0.940) periods, indicating that this factor explains business size as measured by total assets. In contrast, the second component groups variables related to solvency and operational liquidity, including working capital (0.705), solvency (0.602), and the survival index (0.705), reflecting short-term financial balance and business stability.

Table 5: Component matrix. Source: Own elaboration based on data from the Superintendency of Companies, Securities and Insurance (2024).

Rotated component matrix[a]			
	Component		
	1	2	3
Growth	,146	,358	,019
LN Active -1	,940	,008	-,006
LN Active	,944	,086	,032
Working Capital	-,073	,705	-,405
Operating Profitability	-,039	,172	,756
Solvency	-,036	,602	,226
Sales efficiency	,055	,007	,713
Survival Rate	-,014	,705	,464

- Extraction method: principal component analysis.
- Rotation method: Varimax with Kaiser normalization.
- a. The rotation has converged after 4 iterations.

The analysis shows that variables describing business dynamics do not group according to their theoretical nature (growth or survival), but rather according to integrated latent structures, which confirms the relevance of joint factor analysis. Finally, in a third component, the variables operational profitability and sales efficiency were grouped, so this factor was called profitability. The existence of three well-defined components allows synthesizing financial information into interpretable and statistically relevant dimensions to characterize the behavior of the manufacturing sector. Therefore, three underlying factors are determinants in business survival: company size, financial solvency, and profitability, belonging to V_1, defined as the dominant vertex within the structure of the Plithogenic n-SuperHyperGraph.

On the other hand, Figure 4 shows the scree plot corresponding to the joint factor analysis of the variables (vertices) related to growth (V_{13}) and business survival (V_{12}). It shows a sharp drop in the eigenvalues between components 1 and 2, which empirically supports the selection of two main factors according to the Kaiser criterion. The decreasing slope from the third component onward indicates a progressively marginal contribution of the remaining factors to the explanation of the total variance. This graphical configuration confirms that the factor model can be adequately represented by two latent dimensions without significant loss of statistical information. The selection of these factors is consistent with the eigenvalues presented in Table 4 and validates the methodological decision to limit the interpretation to those components with the greatest explanatory capacity.

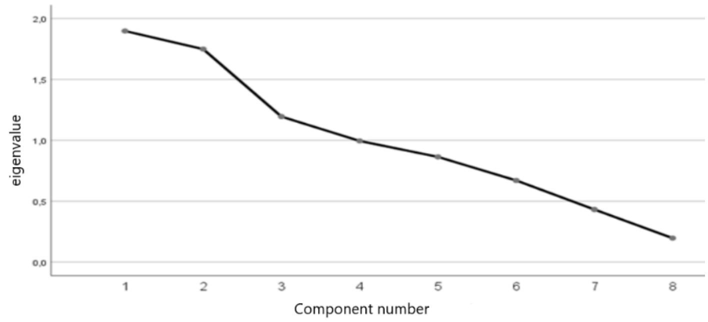


Figure 4: Scree plot of business survival. Source: Own elaboration based on data from the Superintendency of Companies, Securities and Insurance (2024).

4 Discussion

The empirical evidence obtained in this research shows similarities with various theoretical approaches and previous studies on business survival and growth. The business life cycle theory by Gort and Klepper (1982) establishes that firms go through different stages from their creation to maturity and possible decline. In fact, the findings confirm that financial stability plays a decisive role in business continuity [11].

Similarly, Altman's Z-Score model has been widely used to assess financial stability based on indicators such as profitability, liquidity, and leverage [12]. The results obtained through the application of this model reflect its predictive capacity within the manufacturing sector, aligning with findings from studies that highlight the usefulness of bankruptcy prediction models in the analysis of business viability [13]. According to the theory of market "natural selection," firms with fewer capabilities tend to exit the market, while those with greater adaptability manage to consolidate [14]. Therefore, the identified connection between the increase in the number of firms and financial soundness supports this view, especially in the years when growth was positive and statistically significant.

Regarding the increase in firms, Evans (1987) proposes that smaller firms tend to grow more rapidly due to their greater flexibility in adjusting to environmental changes. The results obtained in this study are consistent with this assumption, as more pronounced growth was identified in certain periods. Thus, endogenous growth highlights the accumulation of knowledge and human capital as key elements in firm expansion, which is supported by the observed connection between growth and financial soundness in the analyzed manufacturing firms [15].

However, discrepancies were identified with certain previous theoretical propositions. Gibrat's Law states that firm growth is a random process, and that the growth rate does not depend on the firm's initial size [16]. Nevertheless, the results obtained in this study suggest that growth is not entirely random, as a notable connection was observed in some periods between financial soundness and the expansion of manufacturing firms in Azuay. Similarly, Barney's resource-based view holds that the accumulation of tangible and intangible assets strengthens firms' sustainability [17].

In contrast, the results obtained indicate that access to economic resources does not always translate into long-term stability, suggesting the presence of other factors influencing firms' longevity. Pantoja et al. (2021) argue that effective working capital management (*defined as vertex* V_3) contributes to firms' survival [18]. However, the results did not show a notable link between these components in certain periods, indicating that other variables, such as the structure of the environment and industry dynamics (*defined as vertices* V_2 *and* V_{22}), may play a more significant role. Jovanovic's model (1982) states that firm growth is constrained by internal management capacity and efficiency in resource allocation [19]. Nevertheless, the analysis did not find strong support for this assumption, as the connection between growth and financial soundness was not consistent throughout all the years analyzed.

Similarly, Sutton's model (1997) highlights the priority of sectoral competitiveness (V_{22}) and economies of scale in determining growth rates [20]. However, the results obtained suggest that contextual factors, such as the economic situation and structural barriers (*defined as vertex* V_{21}), may differentially influence the connection between growth and survival.

It should be noted that the investigation was based on economic data provided by the Superintendence of Companies, Securities, and Insurance, which limited the analysis to formally registered firms, excluding those operating in informality, which represent a significant portion of the manufacturing sector in Azuay (partial or unknown information). Moreover, the cross-sectional nature of the data did not allow for an accurate capture of the temporal dynamics and strategic decisions adopted by the firms over time. In fact, these interrelations remain undefined in indeterminate vertices $\{V_{22}, V_{31}, V_{321}, V_{322}\}$ for a multineutrosophic set undefined by:

V_1: $\{\, n - indeterminate\ indicators\} \rightarrow \{n - indeterminate\ records, n - indeterminate\ time\ periods, n - indeterminate\ entities\}$).

The heterogeneity of the manufacturing sector constitutes another limitation, as it includes firms of different sizes and levels of specialization (*defined as vertex* V_{141}, V_{142}), which may have affected the generalizability of the results. Furthermore, the study did not incorporate qualitative variables related to firm management, investment in innovation, and environmental strategies (V_3), which could contribute to a more comprehensive understanding of the components that influence the survival and growth of firms. Therefore, from the structure of the plithogenic n-SuperHyperGraph and the multi-neutrosophic analysis, the following pairs of vertex interrelations are derived:

$$\left\{\{V^{*}_{111n}\ \cap \{V_{12}, V_{13}, V_{14}\}\} \cap \{V_{21n} \cup V_{22n}\}\right\} \cap \left\{V_{31} \cup \{V_{321} \cup V_{322}\}\right\}$$

Where $\left\{V^{*}_{111n} \cap \{V_{12}, V_{13}, V_{14}\}\right\}$ is defined as P, which is a subset of determined indicators in intersection with a subset of equally classified entities (V_{14}) from the manufacturing sector in Azuay, and with a defined outcome in the vertices of survival (V_{12}) and business growth (V_{13}) over an n-period of time H.

The other pair, $\{V_{21n} \cup V_{22n}\}$, defined as R, represents the union of the economic context (V_{21}) and market dynamics (V_{22}) over an n-period of time H. In this way, all macro-external factors that affect the evaluated entity are visualized.

Finally, the last pair composed of $\left\{V_{31} \cup \{V_{321} \cup V_{322}\}\right\}$ is defined by the letter S, which represents the union of operational efficiency (V_{31}) and organizational flexibility (V_{321}) combined with strategic innovation (V_{322}). This configuration aims to reveal the strategies adopted by management during an n-period of time H.

Therefore, the following function of the multineutrosophic set is proposed, defined as $f(H): \rightarrow P \cap R \cap S$.. This function allows the evaluation of each vertex within the plithogenic n-SuperHyperGraph to be included in the development of future research. For example, for an *n-period of time (H)* defined in the year 2020, the following result is obtained:

$f(2023): \rightarrow P \cap R \cap S$, where results are obtained for those groups of heterogeneous entities evaluated and considered in the year 2020 that, under certain macro external factors, exhibited a multineutrosophic degree of business survival and growth, defined by n-results from a subset of n-indicators determined by management. Likewise, it reflects the actions planned by management in response to the n-market dynamics and the strategic decisions adopted by firms during a given *n-period of time*.

5. Conclusion

The permanence of companies in the market depended on elements that provided information about their economic situation and their operational methods. These elements were calculated through the relationship between funds available for immediate use and the total value of company assets, between earnings (without considering loan and tax obligations) and the total value of company assets. Between earnings (without considering tax obligations) and short-term debt, and between sales income and the total value of company assets. To determine company growth, the change in the value of all assets from one period to another was calculated. Although in 2020 and 2021 no clear statistical relationship between growth and permanence was found, data from 2022 and 2023 showed a significant positive relationship between both events. This shows behavior that is not consistent at all times but depends on the current situation in the manufacturing sector.

The statistical calculation of the relationship between permanence and growth of companies, through models that look for patterns in the data, revealed that the increase in company size, calculated by the change in asset value, can help at times to improve the Z-score, an element that indicates economic status. However, this relationship was not maintained consistently throughout all the years analyzed, suggesting that growth does not guarantee business permanence per se, nor vice versa. The partial dependence between these variables reflected the structural complexity of the sector and the influence of external factors not directly observed in the applied models.

Factor analysis and the implementation of the Plithogenic n-SuperHyperGraph together allowed grouping the variables of business growth and survival into two main components, without distinguishing between pre-established theoretical categories. The first component captured the dimension associated with company size through the natural logarithm of assets, while the second grouped variables related to working capital, solvency, and the survival index. This integrated factorial structure suggests that growth and stability processes should not be treated in isolation, but as part of a system of latent interrelationships that shape the financial behavior of manufacturing companies in Azuay. The absence or indeterminacy of a factorial segmentation between growth and survival reinforces the need for methodological approaches that allow interpreting these phenomena in their multidimensional complexity. For this purpose, a solution is proposed based on the inclusion of multineutrosophic functions that encompass the spectrum of the evaluated set from n-information resources and their impact on the entity's financial stability.

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