



An Integrated of Neutrosophic-ANP Technique for Supplier Selection

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Abstract. This study provides a novel integrated multi-criteria decision-making approach to supplier selection problems in neutrosophic environment. The main objective is to study the Analytic network process (ANP) technique in environment of neutrosophic and present a new method for formulation problem of Multi-Criteria Decision-Making (MCDM) in network structure out of neutrosophic and present a way of checking and calculating consistency consensus degree of decision makers. We have used neutrosophic set theory in ANP to overcome the problem that the decision makers might be have restricted knowledge or differences opinions of individuals participating in group decision making to specify deterministic valuation values to comparison judgments. We have formulated that each pairwise comparison judgment as a trapezoidal neutrosophic number. The decision makers specify the weight criteria of each criteria in the problem and compare between each criteria and effect of each criteria on other criteria Whenever number of alternatives increasing it's difficult to make a consistent judgments because the workload of giving judgments by each expert. We have introduced a real life example in the research of how to select personnel mobile according to opinion of decision makers. Through solution of a numerical example we present steps of how formulate problem in ANP by Neutrosophic.

Keywords: Triangular neutrosophic number; ANP method; supplier selection; Consistency; MCDM

1 Introduction

This The Analytic Network Process (ANP) is a new theory that extends the Analytic Hierarchy Process (AHP) to cases of dependence and feedback and generalizes on the supermatrix approach introduced in Saaty (1980) for the AHP [1]. This research focuses on ANP method, which is a generalization of AHP. Analytical Hierarchy Process (AHP) [2] is a multi-criteria decision making method that given the criteria and alternative solutions of a specific model, a graph structure is created and the decision maker is asked to pairwise compare the components, in order to determine their priorities. On the other hand, ANP supports feedback and interaction by having inner and outer dependencies among the models components [2]. We deal with the problem and analyze it and specify alternatives and the critical factors that change the decision. ANP consider one of the most technique that used for dealing with multi criteria decision making using network hierarchy.

The ANP is an expansion of AHP and it's a multi-criteria decision making technique. It's advanced by Saaty in 1996 for considering dependency and feedback between elements of decision making problem. The analytic network process models the decision making problems as a network not as hierarchies as with the analytic hierarchy process. In the analytic hierarchy process it's assumed that the alternatives depend on criteria and criteria depend on goal. So, in AHP the criteria don't depend on alternatives, criteria don't affect depend on each other and also alternatives don't depend on each other. But in the analytic network process the dependencies between decision making elements are allowed. The differences between ANP and AHP presented with the structural graph as in Fig.1. The upper side of Fig.1 shows the hierarchy of AHP in which elements from the lower level have influence on the higher level or in other words the upper level depends on the lower level. But in the lower side of Fig.1

which shows the network model of ANP, we have a cluster network and there exist some dependencies between them. The dependencies may be inner-dependencies when the cluster influence itself or may be outer-dependencies when cluster depend on another one. The complex decision making problem in real life may be contain dependencies between problem elements but AHP doesn't consider this, so it may lead to less optimal decisions and ANP is more appropriate.

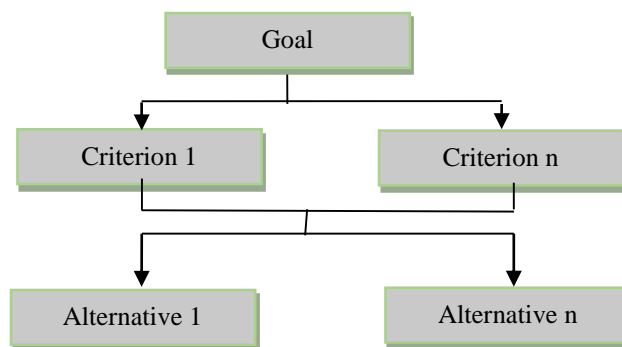
Neutrosophic is a generalization of the intuitionistic fuzzy set whilst fuzzy using true and false for express relationship, Neutrosophic using true membership, false membership and indeterminacy membership [3, 12]. ANP using network structure, dependence and feedback [4, 11]. (MCDM) is a formal and structured decision making methodology for dealing with complex problems. ANP fuzzy integrated with many researches as SWOT method. An overview of integrated ANP with intuitionistic fuzzy. Then, this research of proposed model ANP with neutrosophic represents ANP in neutrosophic environments.

The main achievements of this research are:

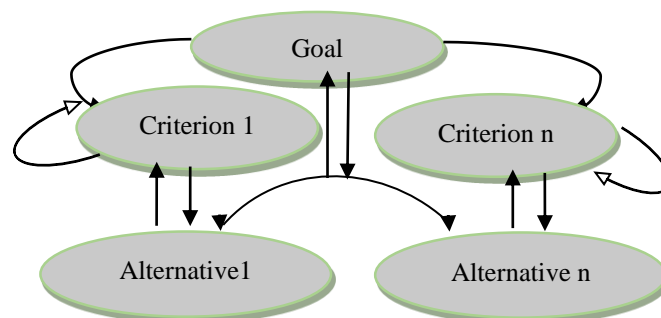
- Considering the significance of integrating of ANP method and VIKOR method under the environment of neutrosophic.
- Recognizing a comprehensive the most effective criteria for supplier's selection.

The research is organized as it is assumed up:

Section 2 gives an insight into some basic preliminaries about neutrosophic. Section 3 explains the proposed methodology of neutrosophic ANP group decision making model. Section 4 introduces numerical example. Lastly, presents conclusion.



(a) The AHP hierarchy.



(b) The ANP network.

Figure 1: The structural difference between hierarchy and network model.

2 Preliminaries

In this section, the essential definitions involving neutrosophic set, single valued neutrosophic sets, trapezoidal neutrosophic numbers and operations on trapezoidal neutrosophic numbers are defined.

Definition 1. [5, 6, 10] Let X be a space of points and $x \in X$. A neutrosophic set A in X is definite by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $] -0, 1+[$. That is $T_A(x):X \rightarrow] -0, 1+[$, $I_A(x):X \rightarrow] -0, 1+[$ and $F_A(x):X \rightarrow] -0, 1+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0- \leq \sup(x) + \sup x + \sup x \leq 3+$.

Definition 2. [5, 6, 7] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object taking the form $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$, where $T_A(x):X \rightarrow [0, 1]$, $I_A(x):X \rightarrow [0, 1]$ and $F_A(x):X \rightarrow [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A , respectively. For convenience, a SVN number is represented by $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

Definition 3. [8, 9] Suppose that $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0, 1]$ and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ where $a_1 \leq a_2 \leq a_3 \leq a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special neutrosophic set on the real line set \mathbb{R} whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as:

$$T_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\beta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & \text{otherwise} \end{cases}, \tag{1}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\theta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & \text{otherwise} \end{cases}, \tag{2}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \leq x \leq a_2) \\ \alpha_{\tilde{a}} & (a_2 \leq x \leq a_3) \\ \frac{(x-a_3+\beta_{\tilde{a}}(a_4-x))}{(a_4-a_3)} & (a_3 \leq x \leq a_4) \\ 1 & \text{otherwise} \end{cases}, \tag{3}$$

Where $\alpha_{\tilde{a}}, \theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ and represent the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued trapezoidal neutrosophic number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ may express an ill-defined quantity of the range, which is approximately equal to the interval $[a_2, a_3]$.

Definition 4. [6, 8] Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \theta_{\tilde{b}}, \beta_{\tilde{b}} \rangle$ be two single valued trapezoidal neutrosophic numbers and $\Upsilon \neq 0$ be any real number. Then,

1. Addition of two trapezoidal neutrosophic numbers
 $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$
2. Subtraction of two trapezoidal neutrosophic numbers
 $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$

3. Inverse of trapezoidal neutrosophic number

$$\tilde{a}^{-1} = \left\langle \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle \quad \text{where } (\tilde{a} \neq 0)$$

4. Multiplication of trapezoidal neutrosophic number by constant value

$$Y\tilde{a} = \begin{cases} \langle (Ya_1, Ya_2, Ya_3, Ya_4); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (Y > 0) \\ \langle (Ya_4, Ya_3, Ya_2, Ya_1); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle & \text{if } (Y < 0) \end{cases}$$

5. Division of two trapezoidal neutrosophic numbers

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \left\langle \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \left\langle \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

6. Multiplication of trapezoidal neutrosophic numbers

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3, a_4b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 > 0, b_4 > 0) \\ \langle (a_1b_4, a_2b_3, a_3b_2, a_4b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 > 0) \\ \langle (a_4b_4, a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle & \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

3 Methodology

In this study, we present the steps of proposed model we identify criteria, evaluating them and decision makers also evaluate their judgments using neutrosophic trapezoidal numbers. Since most previous researches using AHP to solve problems but AHP using hierarchy structure so not use in problems with feedback and interdependence so we presenting ANP with neutrosophic to deal with the complex problems. We present a new scale from 0 to 1 to avoid this drawbacks. We use (n-1) judgments to obtain consistent trapezoidal neutrosophic preference relations instead of $\frac{n \times (n-1)}{2}$ to decrease the workload and not tired decision makers. ANP is used for ranking and selecting the alternatives. The model of ANP with neutrosophic quantifies four criteria to combine them for decision making into one global variable. To do this, we first present the concept of ANP and determine the weight of each criteria based on opinion of decision makers. Then each alternative is evaluated with other criteria and considering the effects of relationship among criteria. The ANP technique composed of four steps.

The steps of our model ANP-Neutrosophic can be introduced as:

Step1 Constructing model and problem structuring

1. Selection of decision makers (DMs).
2. Form the problem in a network
3. Preparing the consensus degree

Step2 Pairwise comparison matrices and determine weighting

1. Identify the alternatives of a problem $A = \{A1, A2, A3... Am\}$.
2. Identify the criteria and sub criteria and the interdependence between it $C = \{C1, C2, C3...Cm\}$.
3. Determine the weighting matrix of criteria that is defined by DMs for each criteria $W1$.
4. Determine the relationship interdependence among the criteria and weight of effect of each criteria on another in range from 0 to 1.
5. Determine the interdependence matrix from multiplication of weighting matrix in step 3 and interdependence matrix in step 4.

- Decision makers make pairwise comparisons matrix between Alternatives compared to each criterion.

$$\tilde{R} = \begin{bmatrix} (l_{11}, m_{11l}, m_{11u}, u_{11}) & (l_{11}, m_{11l}, m_{11u}, u_{11}) & \dots & (l_{1n}, m_{1nl}, m_{1nu}, u_{1n}) \\ (l_{21}, m_{21l}, m_{21u}, u_{21}) & (l_{22}, m_{22l}, m_{22u}, u_{22}) & \dots & (l_{2n}, m_{2nl}, m_{2nu}, u_{2n}) \\ \dots & \dots & \dots & \dots \\ (l_{n1}, m_{n1l}, m_{n1u}, u_{n1}) & (l_{n2}, m_{n2l}, m_{n2u}, u_{n2}) & \dots & (l_{nn}, m_{nnl}, m_{nnu}, u_{nn}) \end{bmatrix} \quad (4)$$

- After making the matrix is consistent we transform neutrosophic matrix to pairwise comparisons deterministic matrix by adding (α, θ, β) and using the following equation to calculate the accuracy and score

$$S(\tilde{a}_{ij}) = \frac{1}{16} [a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}) \quad (5)$$

Step3: formulation of supermatrix

- Determine Scale and weighting data for the n alternatives against n criteria $W_{21}, W_{22}, W_{23}, W_{2n}$
- Determine the interdependence weighting matrix of criteria comparing it to another criteria in range from 0 to 1 is defined as
- We obtaining the weighting criteria $W_c = W_3 \times W_1$
- Determine the interdependence matrix $\tilde{A}_{criteria}$ among the alternatives with respect to each criterion.

Step4 selection of the best alternatives

- Determine the priorities matrix of the alternatives with respect to each of the n criteria W_{An} where n number of criteria.

$$\begin{aligned} \text{Then } W_{A1} &= W_{\tilde{A}_{C1}} \times W_{21} \\ W_{A2} &= W_{\tilde{A}_{C1}} \times W_{22} \\ W_{A3} &= W_{\tilde{A}_{C1}} \times W_{23} \\ W_{An} &= W_{\tilde{A}_{Cn}} \times W_{2n} \end{aligned}$$

$$\text{Then } W_A = [W_{A1}, W_{A2}, W_{A3}, \dots, W_{An}]$$

- In the last we ranking the priorities of criteria and obtaining the best alternatives by multiplication of the W_A matrix by the Weighting criteria matrix W_c .

$$= W_A \times W_c$$

4 Practical example

In this section, to illustrate the ANP Neutrosophic we present an example. This example is that the selecting the best personnel mobile from four alternative Samsung that is alternative A1, Huawei that is alternative A2, iPhone that is alternative A3, Infinix is alternative A4. With four criteria, the four criteria are as follows: C_1 for price, C_2 for processor, C_3 for color, C_4 for model. The criteria to be considered is the supplier selections are determined by the experts from a decision group.

Step 1: In order to compare the criteria, the decision makers assuming that there is no interdependence among criteria. The weighting matrix of criteria that is defined by decision makers is as $W_1 = (P, P, C \text{ and } M) = (0.33, 0.40, 0.22 \text{ and } 0.05)$

Step 2: Assuming that there is no interdependence among the four alternatives, (A_1, A_2, A_3, A_4) , they are compared against each criterion yielding. Decision makers determine the relationships between each criterion and Alternatives Determine the neutrosophic Decision matrix between four Alternatives (A_1, A_2, A_3, A_4) and four criteria (C_1, C_2, C_3, C_4)

$$R = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{bmatrix} (0.3,0.5,0.2,0.5; 0.3,0.4,0.6) & (0.6,0.7,0.9,0.1; 0.4,0.3,0.5) & (0.7,0.2,0.4,0.6; 0.8,0.4,0.2) & (0.3,0.6,0.4,0.7; 0.4,0.5,0.6) \\ (0.6,0.3,0.4,0.7; 0.2,0.5,0.8) & (0.2,0.3,0.6,0.9; 0.6,0.2,0.5) & (0.6,0.7,0.8,0.9; 0.2,0.5,0.7) & (0.3,0.5,0.2,0.5; 0.5,0.7,0.8) \\ (0.3,0.5,0.2,0.5; 0.4,0.5,0.7) & (0.3,0.7,0.4,0.3; 0.2,0.5,0.9) & (0.8,0.2,0.4,0.6; 0.4,0.6,0.5) & (0.2,0.5,0.6,0.8; 0.4,0.3,0.8) \\ (0.4,0.3,0.1,0.6; 0.2,0.3,0.5) & (0.1,0.4,0.2,0.8; 0.7,0.3,0.6) & (0.5,0.3,0.2,0.4; 0.3,0.4,0.7) & (0.6,0.2,0.3,0.4; 0.6,0.3,0.4) \end{bmatrix} \end{matrix}$$

$$S(\tilde{a}_{ij}) = \frac{1}{16} [a_1 + b_1 + c_1 + d_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}})$$

The deterministic matrix can obtain by S (\tilde{a}_{ij}) equation in the following step:

$$R = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_3 \end{matrix} & \begin{bmatrix} 0.122 & 0.23 & 0.261 & 0.163 \\ 0.113 & 0.238 & 0.188 & 0.10 \\ 0.113 & 0.085 & 0.163 & 0.17 \\ 0.123 & 0.169 & 0.105 & 0.178 \end{bmatrix} \end{matrix}$$

Scale and weighting data for four alternatives against four criteria is derived by dividing each element by sum of each column. The comparison matrix of four alternatives and four criteria is the following: Scale and weighting data for four alternatives against four criteria:

$$\begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_3 \end{matrix} & \begin{bmatrix} 0.259 & 0.319 & 0.364 & 0.268 \\ 0.240 & 0.329 & 0.262 & 0.164 \\ 0.240 & 0.118 & 0.227 & 0.278 \\ 0.261 & 0.234 & 0.146 & 0.291 \end{bmatrix} \\ & w_{21} & w_{22} & w_{23} & w_{24} \end{matrix}$$

Step 3: The interdependence among the criteria is next considered by decision makers. The interdependence weighting matrix of criteria is defined as:

$$w_3 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} & \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \end{matrix}$$

$$w_c = w_3 \times w_1 = \begin{bmatrix} 1 & 0.8 & 0.4 & 0 \\ 0 & 0.2 & 0.5 & 0.6 \\ 0 & 0 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \times \begin{bmatrix} 0.33 \\ 0.40 \\ 0.22 \\ 0.05 \end{bmatrix} = \begin{bmatrix} 0.738 \\ 0.220 \\ 0.037 \\ 0.005 \end{bmatrix}$$

Thus, it is derived that $w_c = (C_1, C_2, C_3 \text{ and } C_4) = (0.738, 0.220, 0.037, 0.005)$.

Step 4: Interdependence among the alternatives with respect to each criterion

a. First Criteria

$$\tilde{A}_{C1} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{bmatrix} (0.5, 0.5, 0.5, 0.5) & (0.3, 0.2, 0.4, 0.5; 0.7, 0.2, 0.5) & (0.1, 0.1, 0.3, 0.8; 0.5, 0.2, 0.1) & (0.1, 0.3, 0.2, 1.0; 0.5, 0.2, 0.1) \\ (0.5, 0.6, 0.8, 0.7; 0.7, 0.2, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.1, 0.2, 0.4, 0.8; 0.4, 0.5, 0.6) & (0.1, 0.2, 0.5, 1.0; 0.5, 0.1, 0.2) \\ (0.2, 0.7, 1.0, 1.0; 0.8, 0.2, 0.1) & (0.0, 0.4, 1.0, 1.0; 0.3, 0.1, 0.5) & (0.5, 0.5, 0.5, 0.5) & (0.2, 0.3, 0.4, 0.7; 0.7, 0.2, 0.5) \\ (1.0, 0.8, 1.0, 1.0; 0.6, 0.2, 0.3) & (0.2, 0.5, 1.0, 1.0; 0.6, 0.2, 0.3) & (0.3, 0.6, 0.7, 0.8; 0.9, 0.4, 0.6) & (0.5, 0.5, 0.5, 0.5) \end{bmatrix} \end{matrix}$$

Then, Sure that the matrix be deterministic or transform the previous matrix to be deterministic pairwise comparisons matrix and calculate the weight of each criteria using Eq.5. The deterministic matrix can obtain by S (\tilde{a}_{ij}) equation in the following step:

$$\tilde{A}_{C1} = \begin{bmatrix} 0.5 & 0.175 & 0.179 & 0.22 \\ 0.325 & 0.5 & 0.122 & 0.25 \\ 0.453 & 0.265 & 0.5 & 0.2 \\ 0.38 & 0.354 & 0.285 & 0.5 \end{bmatrix}$$

We present the weight of each alternatives according to each criteria from the deterministic matrix easily by dividing each entry by the sum of the column, we obtain the following matrix as:

$$\tilde{A}_{C1} = \begin{bmatrix} 0.30 & 0.135 & 0.165 & 0.188 \\ 0.196 & 0.386 & 0.112 & 0.214 \\ 0.273 & 0.198 & 0.460 & 0.171 \\ 0.229 & 0.274 & 0.262 & 0.427 \end{bmatrix}$$

b. Second Criteria

We present the weight of each alternatives according to each criteria from the deterministic matrix easily by dividing each entry by the sum of the column, we obtain the following matrix as:

$$\tilde{A}_{c2} = \begin{bmatrix} 0.50 & 0.215 & 0.244 & 0.192 \\ 0.216 & 0.503 & 0.161 & 0.175 \\ 0.273 & 0.182 & 0.495 & 0.197 \\ 0.229 & 0.356 & 0.259 & 0.436 \end{bmatrix}$$

c. Third Criteria

$$\tilde{A}_{c3} = \begin{bmatrix} 0.43 & 0.27 & 0.30 & 0.22 \\ 0.08 & 0.35 & 0.26 & 0.20 \\ 0.15 & 0.16 & 0.31 & 0.30 \\ 0.33 & 0.21 & 0.12 & 0.27 \end{bmatrix}$$

d. Four Criteria

$$\tilde{A}_{c4} = \begin{bmatrix} 0.40 & 0.16 & 0.16 & 0.15 \\ 0.19 & 0.43 & 0.14 & 0.19 \\ 0.23 & 0.23 & 0.5 & 0.23 \\ 0.18 & 0.18 & 0.18 & 0.42 \end{bmatrix}$$

Step 4: The overall priorities for the candidate alternatives are finally calculated by multiplying W_A and W_c and given by and presented in Fig.2.

$$= W_A \times W_c = \begin{bmatrix} 0.199 & 0.303 & 0.327 & 0.222 \\ 0.172 & 0.294 & 0.209 & 0.216 \\ 0.273 & 0.251 & 0.210 & 0.305 \\ 0.299 & 0.347 & 0.241 & 0.250 \end{bmatrix} \times \begin{bmatrix} 0.738 \\ 0.220 \\ 0.005 \\ 0.037 \end{bmatrix} = \begin{bmatrix} 0.426 \\ 0.400 \\ 0.507 \\ 0.365 \end{bmatrix}$$

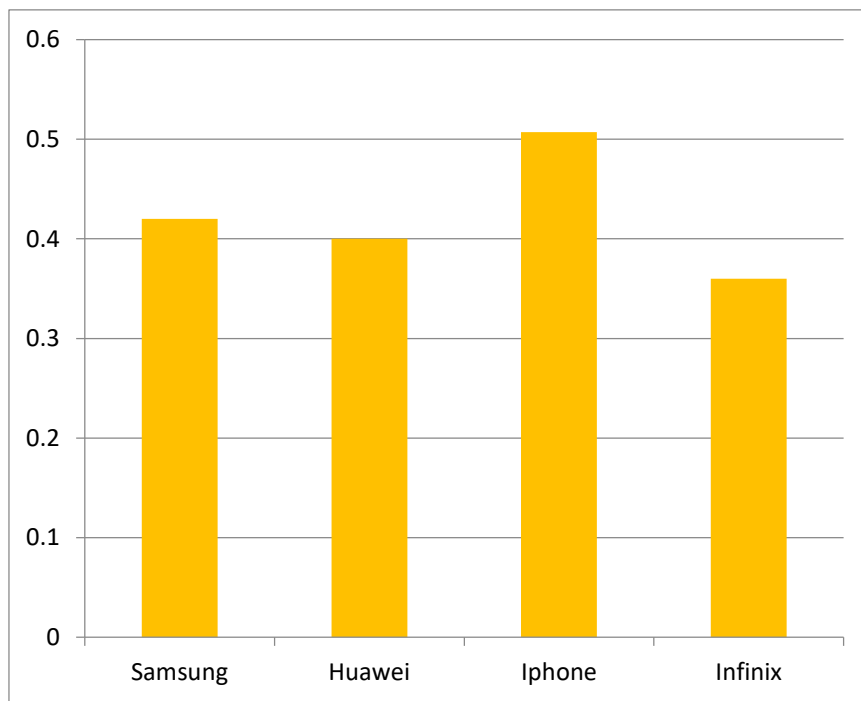


Figure 2: Ranking the alternatives using ANP under Neutrosophic.

5 Conclusion

This research presented the technique of ANP in the neutrosophic environments for solving complex problem with network structure not hierarchy and show the interdependence among criteria and feedback and relative weight of DMs. Firstly, we have presented ANP and how determine the weight for each criteria. Next, we show the interdependence among criteria and calculating effecting of each criteria on another and calculating the

weighting of each criteria to each alternatives. We have using a new scale from 0 to 1 instead of 1-9. In the future, we will apply ANP in environments of neutrosophic by integrating it by other technique such as TOPSIS and other technique. The case study we have presented is a real life example about selecting the best personnel mobile for using that the DMs specify the criteria and how select the best alternatives.

References

- [1] Saaty, T. L. (2001). Analytic network process. *Encyclopedia of Operations Research and Management Science*, Springer: 28-35.
- [2] Thomas L. Saaty. *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation* (Decision Making Series). Mcgraw- Hill, 1980.
- [3] Smarandache, F. (2010). "Neutrosophic set-a generalization of the intuitionistic fuzzy set." *Journal of Defense Resources Management* **1**(1): 107.
- [4] L Saaty, T. (2008). "The analytic network process." *Iranian Journal of Operations Research* **1**(1): 1-27.
- [5] Saaty, T. L., & Vargas, L. G. (2006). *Decision making with the analytic network process*. Springer Science+ Business Media, LLC.
- [6] Hezam, I. M., Abdel-Baset, M., & Smarandache, F. (2015). Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem. *Neutrosophic Sets and Systems*, 10, 39-46.
- [7] El-Hefenawy, N., Metwally, M. A., Ahmed, Z. M., & El-Henawy, I. M. (2016). A review on the applications of neutrosophic sets. *Journal of Computational and Theoretical Nanoscience*, 13(1), 936-944.
- [8] Abdel-Baset, M., Hezam, I. M., & Smarandache, F. (2016). Neutrosophic Goal Programming. *Neutrosophic Sets and Systems*, 112.
- [9] Mahdi, I. M., Riley, M. J., Fereig, S. M., & Alex, A. P. (2002). A multi-criteria approach to contractor selection. *Engineering Construction and Architectural Management*, 9(1), 29-37.
- [10] Abdel-Baset, M., et al. (2017). Neutrosophic Integer Programming Problems, *Infinite Study*.
- [11] Mohamed, M., et al. (2017). Using neutrosophic sets to obtain PERT three-times estimates in project management, *Infinite Study*.
- [12] Abdel-Basset, M., et al. (2018). "A novel group decision-making model based on triangular neutrosophic numbers." *Soft Computing* **22**(20): 6629-6643.
- [13] Hussian, A. N., Mohamed, M., Abdel-Baset, M., & Smarandache, F. (2017). Neutrosophic Linear Programming Problems. *Infinite Study*.

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