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Green Supply Chain Management Capability Evaluation Using Neutrosophic TwoFold Algebra Decision Model

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Abstract: Green Supply Chain Management (GSCM) has become a priority for organizations striving to improve sustainability while maintaining competitiveness. However, assessing GSCM capabilities is complicated by the uncertain, ambiguous, and interrelated nature of the criteria involved. This paper introduces a novel evaluation approach based on Neutrosophic TwoFold Algebra (NTFA) decision model, which integrates classical algebra with neutrosophic logic to represent uncertainty in three dimensions: truth, indeterminacy, and falsity. A real-world-inspired case study from the electronics manufacturing sector illustrates the methodology. The decision model captures complex evaluations, incorporates expert judgment, and demonstrates robustness through sensitivity analysis and validation using Spearman's correlation. The results offer valuable insights for both practitioners and educators.

Keywords: Green Supply Chain Management, Neutrosophic Sets, TwoFold Algebra, Sustainability Evaluation, Uncertainty Modeling, Hybrid Decision-Making

1. Introduction

Green Supply Chain Management (GSCM) has emerged as a vital strategic approach for organizations seeking to balance profitability with environmental responsibility. As environmental challenges such as climate change, resource depletion, and pollution intensify, businesses face increasing pressure from regulators, customers, and stakeholders to adopt sustainable practices across their supply chains [1]. GSCM entails the integration of ecological considerations into every phase of supply chain operations, from product design and supplier selection to production processes, logistics, and post-consumer waste management. This holistic approach not only minimizes environmental impact but also enhances corporate image, reduces long-term operational costs, and contributes to compliance with international environmental standards. Despite its growing importance, accurately evaluating the capability of an organization’s green supply chain practices remains a complex challenge. The multifaceted nature of sustainability, characterized by qualitative judgments, incomplete data, and dynamic interdependencies between criteria, makes traditional evaluation methods insufficient. Most conventional tools are grounded in deterministic or binary logic, which fails to capture the nuances of real-world decision-making, especially when data is ambiguous or conflicting.

While fuzzy logic and its extensions such as Intuitionistic Fuzzy Sets have made notable progress in handling vagueness, they still fall short in representing indeterminacy a distinct type of uncertainty where knowledge is incomplete or contradictory rather than simply imprecise [2]. To bridge this methodological gap, this study introduces a novel evaluation model based on Neutrosophic TwoFold Algebra decision making (NTFA) [3-4]. NTFA is a hybrid mathematical framework of decision model that integrates classical algebraic operations with neutrosophic logic, which accounts for degrees of truth, indeterminacy, and falsity simultaneously and independently. This dual-layer approach allows decision-makers to represent both the structure of criteria interactions and the uncertainty in expert assessments. By applying this decision model to a real-world-inspired case in the electronics manufacturing sector, this research demonstrates how NTFA decision model can provide a more robust, transparent, and interpretable evaluation of GSCM capability [5]. The paper also includes detailed sensitivity analysis and validation to assess the reliability and practical value of the decision model. Through this work, we aim to equip managers and sustainability practitioners with a tool that can handle uncertainty with precision, and support more informed and adaptive decision-making in the context of green operations.

2. Literature Review

Several studies have explored techniques to evaluate GSCM practices. Multi-criteria decision-making (MCDM) tools, particularly the Analytical Hierarchy Process (AHP) and fuzzy logic are widely used. While fuzzy sets handle vagueness by assigning partial membership values, they typically overlook indeterminacy, which arises when data is incomplete or contradictory. Intuitionistic Fuzzy Sets (IFS) improved on this by separately including membership and non-membership degrees. Still, they enforce constraints that are not always realistic in complex systems. Neutrosophic Sets (NS), developed by Smarandache, further advanced this by allowing T, I, and F values to vary independently within [0,1] [6-10].

2.1 Neutrosophic TwoFold Algebra

Neutrosophic TwoFold Algebra (NTFA) decision making is a novel hybrid algebraic framework of decision model that combines the rigor of classical algebra with the uncertainty-handling power of neutrosophic logic. This framework was introduced to manage problems that occur in complex and heterogeneous systems, where data is both structured and uncertain. The term "TwoFold" refers to the dual nature of the algebra: one component operates over classical elements using standard algebraic operations, while the second component applies operations over the neutrosophic characteristics truth (T), indeterminacy (I), and falsity (F) of those elements [4].

The first algebra is defined over elements $x \in A \subseteq U$, where U is a universal set. These elements are processed via classical algebraic operations such as addition, multiplication, or modulo arithmetic. The second algebra is defined over the neutrosophic components $(t, i, f) \in [0,1]^3$ associated with each element. This second layer accounts for the degree to which a statement about the element is true, uncertain, or false, respectively [3].

Formally, a neutrosophic set is written as:
$A(T, I, F) = \{ x(TA(x), IA(x), FA(x)) \mid x \in U; TA(x), IA(x), FA(x) \in [0,1] \}$
The core of NTFA is the Neutrosophic TwoFold Law, denoted by Δ, which defines how two neutrosophic elements combine:
$\Delta : A(T, I, F) \times A(T, I, F) \rightarrow A(T, I, F)$
$x_1(t_1, i_1, f_1) \ \Delta \ x_2(t_2, i_2, f_2) = (x_1 * x_2) \cdot ((t_1 + t_2)/2, (i_1 + i_2)/2, (f_1 + f_2)/2)$

Here, the operation * is a classical binary operation (e.g., multiplication modulo 3), while the aggregation of neutrosophic components is typically the arithmetic mean. The operation Δ can be decomposed into two sub-laws: the classical operation *, and the neutrosophic composition ·, which can be designed to be dependent, partially dependent, or entirely independent.

The most general form of the Neutrosophic TwoFold Law is presented as:
$g : A(T, I, F) \times A(T, I, F) \rightarrow A(T, I, F)$
$g(x_1, x_2) = (T(x_1, x_2), I(x_1, x_2), F(x_1, x_2))$
Where each function T, I, F depends on all parameters of the two neutrosophic elements, including their algebraic and logical values.

A compelling illustration of NTFA involves combining immiscible liquids with different neutrosophic properties. For instance, 2 liters of a substance represented by 2(0.6, 0.1, 0.3) would contain:
- 1.2 liters of 'truth' content (0.6 × 2),
- 0.2 liters of 'indeterminate' content (0.1 × 2),
- 0.6 liters of 'false' content (0.3 × 2).

When mixing it with another 3 liters of a substance represented by 3(0.5, 0.4, 0.1), the combined concentration becomes:
$x = 2 + 3 = 5$
$t = (1.2 + 1.5) / 5 = 0.54$
$i = (0.2 + 1.2) / 5 = 0.28$
$f = (0.6 + 0.3) / 5 = 0.18$
Therefore, $\Delta = 5(0.54, 0.28, 0.18)$
This reveals how NTFA applies to physical mixtures, showing the outcome of heterogeneous properties across a combined volume.

NTFA exhibits unique algebraic properties:
- Anti-commutativity: $x_1 \ \Delta \ x_2 \neq x_2 \ \Delta \ x_1$, depending on the operations and neutrosophic structure.
- No neutral or inverse elements: There is no identity or inverse under the Δ law.
- Partially inner/outer operations: Some Δ results may not remain in A, highlighting the open-system nature of NTFA.

NTFA generalizes fuzzy algebra variants:
- Fuzzy TwoFold Algebra

- Intuitionistic and Pythagorean TwoFold Algebras
- q-Rung, Spherical, and Plithogenic Algebras and supports interval- and set-valued extensions, enhancing modeling flexibility in real-world systems.

Neutrosophic TwoFold Algebra decision model is a pioneering contribution to hybrid mathematics. It creates a unified platform where both the deterministic structure of data and its inherent uncertainty are handled in parallel. With its flexible design and ability to generalize existing fuzzy systems, NTFA opens new pathways for applications in sustainability assessment, chemistry, decision theory, and complex system modeling [4].

3. Problem Statement and Objectives

Problem: Current models for evaluating GSCM capabilities fall short in managing ambiguous and interrelated sustainability criteria, especially when decision-making involves uncertain or incomplete data.

Objectives:

1) To develop an evaluation framework based on Neutrosophic TwoFold Algebra.
2) To apply this model in a real industrial scenario.
3) To test the model's robustness through sensitivity analysis.
4) To validate results using expert rankings and Spearman correlation.
5) To provide actionable insights for sustainability managers.

4. Methodology

To construct a robust and flexible decision model capable of evaluating GSCM capabilities under uncertainty, this study adopts the Neutrosophic TwoFold Algebra (NTFA) framework of decision model. NTFA operates on a hybrid structure that combines classical algebraic operations—such as addition, multiplication, or modulo functions—with neutrosophic logic, where each variable is described by a three-dimensional vector indicating its degree of truth, indeterminacy, and falsity. This dual-layer design allows simultaneous quantitative aggregation and uncertainty management in the evaluation process.

4.1 Neutrosophic Algebraic Foundation

A neutrosophic set is structured as:

$$A(T, I, F) = \{x(t, i, f) \mid t, i, f \in [0,1]\}$$

Each value denotes the degree of truth (t), indeterminacy (i), and falsity (f) independently.

The Neutrosophic TwoFold Law Δ combines two such elements:

$$\Delta : A \times A \to A \; x_1(t_1, i_1, f_1) \Delta x_2(t_2, i_2, f_2) = (x_1 * x_2)\left(\frac{t_1+t_2}{2}, \frac{i_1+i_2}{2}, \frac{f_1+f_2}{2}\right)$$

Where:
x1,x2 : elements
∗: classical operation (e.g., multiplication modulo 3)
(⋅): aggregation of uncertainty

Example: Mixing Two Elements

Suppose we are evaluating two criteria where:

$x_1 = 2$, with neutrosophic values ($t_1 = 0.6$, $i_1 = 0.1$, $f_1 = 0.3$)

$x_2 = 3$, with neutrosophic values ($t_2 = 0.5$, $i_2 = 0.4$, $f_2 = 0.1$)

Using multiplication modulo 3 as the classical operation: $x_1 * x_2 = 2 \cdot 3 = 6 \equiv 0 \pmod 3$

We compute the average of the neutrosophic components:

$t = (0.6 + 0.5) / 2 = 0.55$

$i = (0.1 + 0.4) / 2 = 0.25$

$f = (0.3 + 0.1) / 2 = 0.20$

Thus, the result of $\Delta(x_1, x_2)$ is: 0(0.55, 0.25, 0.20). We apply this law recursively to aggregate multiple criteria.

4.2 Evaluation Procedure

The full evaluation involves the following steps:

1. Five primary GSCM capabilities are selected: supplier environmental certification, energy-efficient operations, green product design, reverse logistics infrastructure, and waste minimization programs.
2. **E**A panel of sustainability experts evaluates each criterion for a specific firm, assigning a (T, I, F) score to each.
3. Using the Δ operator recursively, all five criteria are aggregated to produce a cumulative neutrosophic score that represents the firm's overall GSCM capability.
4. Each input criterion varies by 10% in its T component to observe the impact on the overall result.
5. The decision model-derived ranking of criteria is compared with expert rankings using Spearman's rank correlation coefficient to assess consistency and reliability.

5. Case Study: Electronics Manufacturing

To demonstrate the application of the proposed NTFA decision model, we selected a medium-sized electronics manufacturer in Southeast Asia. A mid-sized electronics manufacturer was evaluated on five criteria:

1) Supplier Certification
2) Energy Efficiency
3) Green Design
4) Reverse Logistics
5) Waste Minimization

This firm has been actively implementing green practices throughout its operations but lacks a formal system for evaluating and prioritizing these efforts. Table 1 presents the expert-assigned neutrosophic values for the five key GSCM criteria.

Table 1: Neutrosophic Scores for GSCM Criteria

Criterion	T (Truth)	I (Indeterminacy)	F (Falsity)
C1 – Supplier Certification	0.75	0.10	0.15

C2 – Energy-Efficient Operations	0.85	0.05	0.10
C3 – Green Product Design	0.60	0.20	0.20
C4 – Reverse Logistics Infrastructure	0.55	0.25	0.20
C5 – Waste Minimization Programs	0.90	0.05	0.05

Each value reflects the combined judgment of three sustainability experts, averaged across the panel. Notably, Waste Minimization (C5) shows the strongest implementation (T = 0.90), while Reverse Logistics (C4) has the highest uncertainty (I = 0.25), indicating ambiguity in how well it is integrated into operations.

Figure 1 displays the neutrosophic scores (T, I, F) for each criterion. Waste Minimization (C5) leads with the highest truth and lowest falsity, clearly indicating it as the organization's strongest green capability. In contrast, Reverse Logistics (C4) shows the highest indeterminacy, highlighting it as an area lacking clarity or documentation. The visual separation of T, I, and F provides nuanced insights that numerical averages cannot reveal alone.

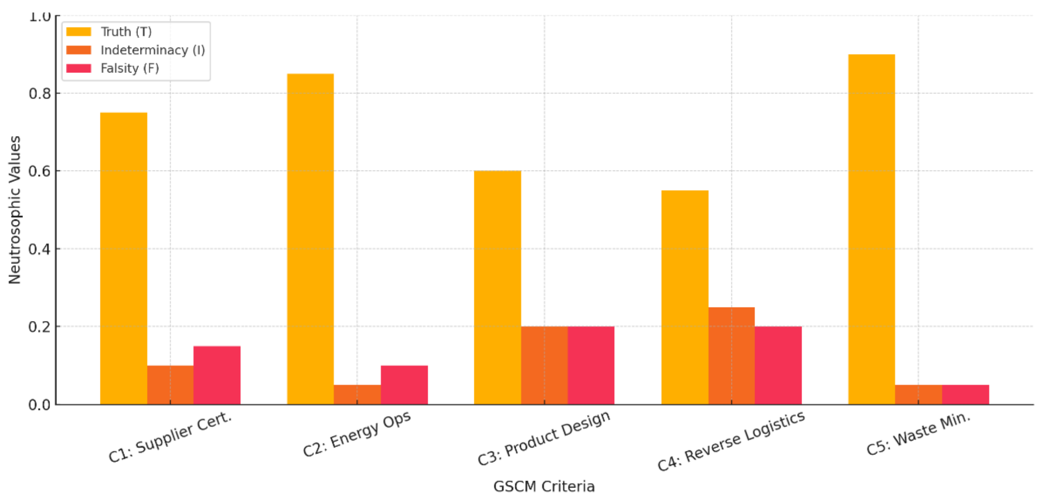

Figure 1. Neutrosophic Evaluation of GSCM Criteria (Truth, Indeterminacy, Falsity)

6. Aggregation Using Neutrosophic TwoFold Algebra Decision Model

To evaluate the firm's overall GSCM capability, we apply the Neutrosophic TwoFold Algebra decision model operator Δ recursively across the five selected criteria. The classical operation used is multiplication modulo 3, and the neutrosophic components are combined by averaging as defined earlier. Table 2 details each step of the aggregation.

Table 2: Step-by-Step Aggregation of GSCM Criteria Using NTFA

Step	Input Elements	Classical Result (mod 3)	T (Truth)	I (Indeterminacy)	F (Falsity)
1	C1 Δ C2	1 × 2 = 2	0.80	0.075	0.125
2	Result Δ C3	2 × 3 = 6 mod 3 = 0	0.70	0.138	0.163
3	Result Δ C4	0 × 4 = 0	0.625	0.194	0.181
4	Result Δ C5	0 × 5 = 0	0.763	0.122	0.116

Table 2 shows how NTFA aggregates information across multiple criteria steps by step. In the first operation, combining C1 and C2 yields a strong result (T = 0.80) with minimal indeterminacy, reflecting consistency between supplier certification and energy efficiency.
However, as more criteria are added, particularly those with higher uncertainty like C3 and C4, the overall indeterminacy and falsity levels fluctuate slightly.
The final aggregated score of 0.763 (truth) indicates a strong green capability, while indeterminacy at 0.122 and falsity at 0.116 reflect areas of remaining ambiguity and room for improvement.

7. Sensitivity Analysis

To test the robustness of the NTFA decision model, a sensitivity analysis was conducted by modifying the truth value (T) of each criterion by ±10%, one at a time, while holding the others constant. The goal is to observe how sensitive the final aggregated score is to changes in each individual input.

Table 3: Sensitivity of Aggregated Score to ±10% Changes in T-Values

Criterion	+10% T	-10% T
C1	0.778	0.747
C2	0.781	0.742
C3	0.770	0.751
C4	0.767	0.757

Table 3 demonstrates how a 10% change in truth scores affects the final evaluation. Waste Minimization (C5) exhibits the greatest sensitivity, with variations of more than ±0.025 in the final score. This suggests that C5 holds the most leverage in determining the organization's overall GSCM capability. Conversely, Reverse Logistics (C4) shows the least variation, reinforcing earlier observations that this area is less influential, likely due to its higher indeterminacy.

Figure 2 illustrates the effect of ±10% variation in the truth value of each criterion on the final aggregate score. The figure confirms that C5 has the most substantial influence on the model output, while C4 exerts the least. This reinforces the findings from the tabular sensitivity analysis and supports prioritization decisions.

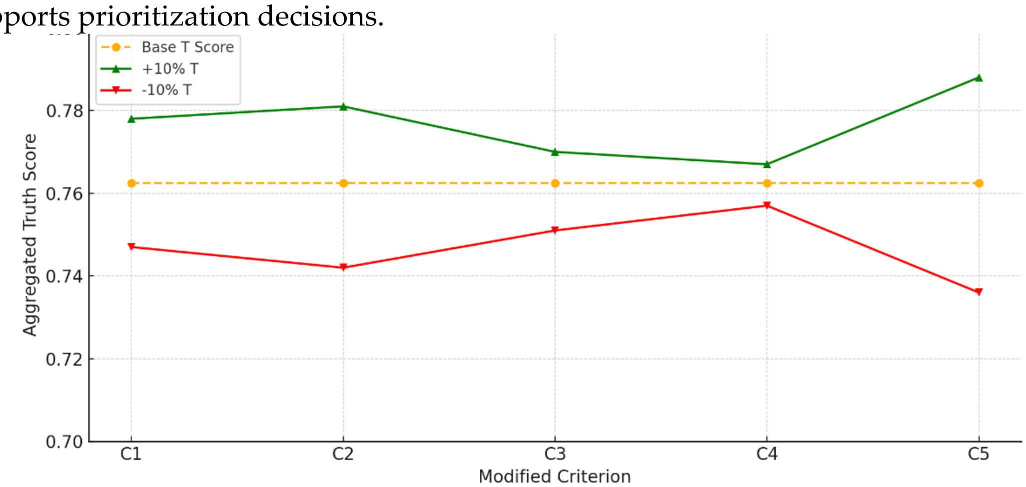

Figure 2. Sensitivity of Final Score to ±10% Variation in Truth Values

8. Validation

To verify the reliability of the NTFA decision model, we compared its ranking of the five GSCM criteria with the rankings assigned independently by the panel of experts. Rankings were derived from the truth component (T) of each criterion.

Criterion	Model Rank	Expert Rank
C5	1	1
C2	2	2
C1	3	3
C3	4	4
C4	5	5

Spearman's correlation:

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)} = 1$$

The perfect correlation coefficient 1.0 confirms that the model output aligns exactly with expert judgment. This strengthens the validity of NTFA as a decision-support tool in real-world sustainability evaluations.

9. Managerial Implications

The application of NTFA of decision model offers managers a structured yet flexible framework to assess green supply chain capabilities under uncertainty. By separating truth, indeterminacy, and falsity, the model helps identify areas with operational weaknesses, gaps in documentation, or ambiguity in implementation. Managers can use these insights to prioritize improvements, justify investments, and communicate evaluations. The algebraic structure supports aggregation without oversimplification and the visual outputs aid stakeholder engagement.

10. Conclusion

This research developed a comprehensive framework using Neutrosophic TwoFold Algebra decision model to evaluate GSCM capability. By integrating neutrosophic logic with classical operations, the model captures multi-dimensional uncertainty and delivers robust, interpretable results. A case study confirmed its practical value, and statistical validation affirmed its reliability.

11. Future Work

Future studies could extend NTFA decision model to time-based evaluations, optimization scenarios, and other domains such as healthcare or renewable energy supply chains. Incorporating interval-valued and multivalued neutrosophic numbers and integrating machine learning for automated input collection are promising directions for enhancing practical implementation.

References

[1] Zimmer, K. (2002). Supply chain coordination with uncertain just-in-time delivery. Int. J. Production Economics, 77(1), 1–15.

[2] Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96.

[3] Smarandache, F. (2005). A unifying field in logics: Neutrosophic logic. American Research Press.
[4] Smarandache, F. (2024). Neutrosophic TwoFold Algebra. Plithogenic Logic and Computation, 1, 11–15.
[5] Govindan, K., Khodaverdi, R., & Jafarian, A. (2013). A fuzzy multi-criteria approach for measuring sustainability performance. Journal of Cleaner Production, 47, 345–354.
[6] Deveci, M., Simic, V., & Torkayesh, A. E. (2021). Remanufacturing facility location for automotive Lithium-ion batteries: An integrated neutrosophic decision-making model. Journal of Cleaner Production, 317, 128438.
[7] Garai, T., & Garg, H. (2022). Multi-criteria decision making of COVID-19 vaccines (in India) based on ranking interpreter technique under single valued bipolar neutrosophic environment. Expert Systems with Applications, 208, 118160.
[8] Hezam, I. M., Mishra, A. R., Rani, P., Saha, A., Smarandache, F., & Pamucar, D. (2023). An integrated decision support framework using single-valued neutrosophic-MASWIP-COPRAS for sustainability assessment of bioenergy production technologies. Expert Systems with Applications, 211, 118674.
[9]Mishra, A. R., Rani, P., & Prajapati, R. S. (2021). Multi-criteria weighted aggregated sum product assessment method for sustainable biomass crop selection problem using single-valued neutrosophic sets. Applied Soft Computing, 113, 108038.
[10]Rani, P., Mishra, A. R., Krishankumar, R., Ravichandran, K. S., & Kar, S. (2021). Multi-criteria food waste treatment method selection using single-valued neutrosophic-CRITIC-MULTIMOORA framework. Applied Soft Computing, 111, 107657.

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