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Integrating TreeSoft and HyperSoft Paradigms into Urban Elderly Care Evaluation: A Comprehensive n-SuperHyperGraph Approach

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Abstract

The rapid aging of global urban populations has compelled cities to reimagine how elderly care services are delivered, structured, and evaluated. Traditional models such as TreeSoft, which rely on hierarchical and clearly defined service flows, are well-suited for institutional settings but often fail to capture informal or uncertain care dynamics. On the other end of the spectrum, HyperSoft frameworks emphasize flexibility and allow for the modeling of indeterminate or overlapping relationships common in community-based or volunteer-driven services but they often lack operational coherence.

This research proposes a hybrid evaluation model that integrates TreeSoft and HyperSoft paradigms within the powerful mathematical structure of the n-SuperHyperGraph (n-SHG). Grounded in ecosystem theory, the n-SHG framework enables the modeling of nested, layered, and partially uncertain relationships between service actors such as families, clinics, local authorities, and digital providers.

Using data collected from three urban districts, we evaluate five smart elderly care service modes using connectivity, redundancy, and uncertainty indicators derived from the n-SHG structure. The results not only differentiate modes by performance but also demonstrate how both structured (TreeSoft-like) and fluid (HyperSoft-like) care systems can be understood, compared, and optimized under one unified analytical framework.

This hybrid modeling approach provides a roadmap for policymakers, urban planners, and researchers seeking to develop adaptive, inclusive, and resilient elderly care ecosystems. Also, we show a case study of TreeSoft set with four criteria and five alternatives. We compute the criteria weights in the main criteria and each sub-criteria. We use the MARCOS method to rank the alternatives.

Keywords: TreeSoft Set; n-SuperHyperGraph (n-SHG); HyperSoft; Elderly Care; Evaluation; MARCOS Method

1. Introduction

Urbanization and population aging are two converging forces reshaping how societies care for the elderly. In cities worldwide, longer life expectancies combined with changing family structures are increasing the demand for services that are not only medically competent but also socially responsive and technologically enabled. Traditional care models, typically built around

hospitals, nursing homes, and municipal welfare programs, often fail to meet the full scope of physical, emotional, and social needs experienced by older adults in urban environments [2].
Most existing evaluation frameworks for elderly care rely on highly structured, linear models of service delivery. These models assume consistent availability, clarity of roles, and direct accountability—conditions rarely met in practice. For instance, family caregiving may be intermittent, public health services may be fragmented, and volunteer initiatives may be poorly documented. The result is a system characterized by uncertainty, relational overlap, and incomplete coverage.
Two dominant paradigms have attempted to address these modeling challenges. The TreeSoft model is used to depict well-defined, hierarchical structures in which services flow clearly from centralized authorities to recipients. It is ideal for representing health ministries, hospital systems, or structured social welfare programs. In contrast, the HyperSoft model represents loosely coupled, uncertain, or dynamic relationships. It has been employed to model systems with fuzzy boundaries, such as community support networks and informal caregiving arrangements [3].
Both paradigms offer partial representations of urban elderly care systems. However, real-world environments often exhibit characteristics of both—a hospital system may include informal family caregiving or a community-based food delivery service may depend on municipal funding. These hybrid scenarios call for a more flexible yet structured modeling framework.
The n-SuperHyperGraph (n-SHG) model, proposed by Smarandache, introduces mathematical structures capable of representing hierarchical and indeterminate relations simultaneously through multi-level vertices, indeterminate edges, and null connections [4]. This study applies the n-SHG model to five distinct elderly care service modes in three urban districts, aiming to evaluate them through the integrated lens of TreeSoft and HyperSoft paradigms, grounded in ecosystem theory [1].

2. Literature Review

Research in urban elderly care has increasingly recognized the limits of conventional evaluation methods. Decision-making frameworks such as the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA) have been widely used to assess service performance. While these tools offer clarity and consistency, they typically assume stable structures, clear objectives, and complete data [5]. In contrast, urban elderly care systems are often characterized by fluctuating interactions, evolving roles, and partial observability.

To better capture this complexity, scholars have drawn upon ecosystem theory, which views social service systems as networks of interrelated and interdependent actors. In this framework, service delivery is not linear but dynamic, with each actor's behavior influencing others. Bronfenbrenner introduced the ecological systems theory to explain how individuals exist within multiple nested layers of influence [1]. Lin and Wu extended this thinking into urban elderly care, showing how services are shaped by the intersection of institutional, familial, and social dynamics [6]. Zhang and Chen emphasized that collaborative, cross-sector systems that include both formal and informal actors are essential for successful elderly care delivery [7].

Despite the theoretical strength of the ecosystem perspective, few studies have integrated it with formal modeling systems capable of simulating uncertain or hybrid relationships. The TreeSoft paradigm continues to dominate structured health service modeling, where clarity and control are prioritized. However, it falls short when applied to systems involving multiple informal actors, such as volunteer groups or family caregivers. Between 2018 and 2024, Smarandache introduced six novel types of soft sets, namely: the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set. These contributions are detailed in several publications, which also examine the role of indeterminate operators within the framework of indeterminate soft algebra [10-12].

The HyperSoft model has emerged as a counterpoint. It builds upon fuzzy set theory to allow for overlapping roles, uncertain interactions, and partial memberships. These features make it particularly relevant to community-based service systems [3]. Yet, the model lacks the architectural rigor needed for managing coordination across diverse institutional actors.

The n-SuperHyperGraph framework introduced by Smarandache (2024) extends classical graph theory to support both structured and uncertain relationships. It enables the representation of complex social networks through its incorporation of super vertices (representing composite actors), indeterminate edges (representing uncertain or evolving connections), and null elements (representing service gaps) [4-8]. This framework has been successfully applied in domains like transportation and environmental planning, but its use in urban social services, and specifically in elderly care, remains unexplored.

By applying the n-SHG model to a real-world evaluation of smart elderly care services, this study contributes a novel approach to service modeling that reflects both the structured logic of TreeSoft and the flexible reality captured by HyperSoft.

3. Overview of the n-SuperHyperGraph Framework

An n-SuperHyperGraph is a mathematical structure defined as: $G = (V, E, I, \Theta)$
Where:
V: Set of vertices (e.g., individuals, institutions, technologies)
E: Set of edges, each connecting one or more vertices
I: Set of indeterminate edges (uncertain or evolving relationships)
Θ: Set of null edges (gaps or missing connections)

A super edge in this context is an edge that itself contains edges, representing bundled or institutional relationships (e.g., a municipal health network that includes hospitals, clinics, and home care units).

In the elderly care ecosystem:
V includes elements like “Elderly Individual,” “Family Caregiver,” “Community Clinic,” and

"Tech Provider."
E models explicit service links like healthcare provision or financial aid.
I capture evolving or partially known connections (e.g., informal social support).
Θ highlights missing or broken links, such as areas lacking accessible transportation.

The strength of this framework lies in its flexibility to encode both known and uncertain structures, which is essential in modeling real-world urban social systems.

3.1 Data Collection and Model Inputs

Data were collected through:
• Expert interviews with municipal health officials and care coordinators (n = 6).
• Review of government elderly care service records from three urban communities.
• Community surveys evaluating user experiences with five service modes:
1. Tech-assisted in-home care
2. Community day-care centers
3. Volunteer-based neighborhood services
4. Telehealth-integrated models
5. Multi-provider institutional care hubs

Each service mode was mapped to an n-SHG structure, identifying its nodes (service actors), edges (service flows), and the degree of completeness or uncertainty in each relationship.

3.2 Model Construction: Super and Indeterminate Components

We define a SuperEdge SE_k ⊆ E as: $SE_k = \{e1, e2, ..., en\} \subseteq E$
Where each *ei* connects multiple vertices and represents an atomic service activity (e.g., prescription delivery, mental health screening).

Similarly, the Indeterminate Edge i ∈ I is a partial function:
$i: V \times V \rightarrow \{0, 1, ?\}$
Where "?" denotes uncertainty (e.g., whether a community volunteer network reaches isolated elderly in a given district).

To evaluate each service mode, we compute the connectivity index CI and the redundancy index RI as follows:
CI = Total Active Edges / Max Theoretical Edges
RI = SuperEdges with overlapping vertices / Total SuperEdges

These indices provide insights into the density and overlap of support structures, which are key indicators of efficiency and resilience.

3.3 Illustration Example: Mode 3 (Volunteer-Based Services)

In Mode 3, vertices included:
Elderly (E1–E5),
Volunteer Groups (V1–V2),
Health Stations (H1),
Local Authority (L1)

Edges connected volunteers to elderly individuals, while indeterminate edges represented informal or inconsistent visits. One null edge indicated a complete lack of mental health outreach in the eastern district.

Using the model, Mode 3 displayed:
CI = 0.58 → moderate network density
RI = 0.66 → high overlap among support actors

This mode exhibited strong community reach but revealed a vulnerability in consistency and coverage.

3.4 Problem Statement and Objectives

Despite the growing emphasis on smart elderly care, current evaluation methods remain insufficient for modeling the structural complexity and uncertainty that characterize real-world urban service systems. Most frameworks continue to be rooted in assumptions of stability, hierarchy, and full observability. This is evident in models that either rely on formal structures with fixed roles such as TreeSoft or adopt soft system modeling techniques that focus on flexibility and uncertainty—such as HyperSoft. However, urban care systems are neither entirely structured nor completely fluid; they are hybrid ecosystems made up of hospitals, community centers, families, volunteers, and digital platforms, all interacting at varying levels of formality and certainty.

TreeSoft models, though robust for structured service delivery (e.g., municipal clinics or hospital systems), are not designed to handle the blurred lines of responsibility found in volunteer networks or digital caregiving platforms. On the other hand, HyperSoft models can represent these uncertain or informal relationships but often lack mechanisms to enforce coordination or assess system efficiency [3].

As cities continue to decentralize service provision, and as aging populations increasingly rely on hybrid care systems, a more advanced modeling tool is required—one that supports both hierarchy and fluidity, known and unknown relationships, and fixed as well as evolving actor roles.

The n-SuperHyperGraph (n-SHG) model fills this need by enabling the representation of multiple types of nodes (e.g., individuals, families, institutions), edge types (e.g., formal, informal, indeterminate), and structural gaps (e.g., null links). Through its recursive structure, it accommodates both layered and uncertain networks in a single integrated framework [4-8].

This study aims to operationalize the n-SHG model in the context of urban elderly care. It focuses on evaluating five different care service modes across three cities, comparing their performance using three indicators derived from the model: connectivity, redundancy, and uncertainty.

The specific objectives of this study are:

1. To construct an evaluation framework for smart urban elderly care systems using the n-SHG model.
2. To integrate both TreeSoft and HyperSoft concepts within this framework.
3. To apply the model to five real-world service modes representing various levels of institutional control and community involvement.
4. To compute structural performance indicators—connectivity, redundancy, and uncertainty—for each mode.
5. To validate the framework's outputs against expert rankings using statistical correlation techniques.

4. Methodology

To evaluate the structural performance of smart elderly care service modes, this study develops and applies a modeling framework based on the n-SuperHyperGraph (n-SHG). This section outlines the framework's mathematical foundations, data collection approach, model construction process, and the performance indicators used to assess each care mode. The design integrates principles from both TreeSoft and HyperSoft paradigms, allowing for the representation of hierarchical, uncertain, and composite relationships within urban service networks.

4.1 Framework Design: The n-SuperHyperGraph Model

The n-SHG is a recursive mathematical structure that extends traditional graph theory to accommodate multiple levels of abstraction and uncertainty. It is formally defined as a tuple:

$G = (V, E, I, \Theta)$

Where:

- V is the set of vertices representing individual or institutional actors (e.g., elderly persons, clinics, caregivers, tech platforms).
- E is the set of edges representing known relationships or interactions (e.g., care delivery, information sharing).
- I is the set of indeterminate edges, capturing partially known or evolving relationships (e.g., informal volunteer visits).

- Θ is the set of null edges, indicating service gaps or non-existent relationships (e.g., no mental health support in a district).

The structure also incorporates SuperVertices, which are groupings of multiple actors functioning as one unit, such as a "Community Health Center" that includes clinics, mobile teams, and digital support platforms. This recursive structure allows for modeling deeply nested or hierarchical systems—core to the TreeSoft perspective—while also including fuzzy and overlapping connections from the HyperSoft paradigm [4][8].

4.2 Data Collection and Study Setting

Empirical data were collected from three urban communities over five months. Data sources included:

- Interviews with six municipal health coordinators responsible for elderly care.
- Community surveys were conducted with 200 elderly residents to understand their experiences, service usage, and unmet needs.
- Public records and policy documents related to care initiatives in each community.
- Observation and mapping of service delivery paths through visits to local care centers and digital care providers.

This mixed-methods approach ensured that both formal (institutional) and informal (community-driven) services were captured.

Identification of Service Modes

Based on data analysis, five distinct elderly care service modes were identified:

Mode	Description	Structural Type
1	Tech-assisted in-home care	Hybrid (digital-focused)
2	Community day-care centers	Institutional
3	Volunteer-based neighborhood services	Informal/uncertain
4	Telehealth-integrated hybrid models	Structured + dynamic
5	Multi-provider institutional care hubs	Highly structured

Each of these modes exhibits different degrees of formalization, connectivity, and uncertainty, making them suitable for evaluation under the n-SHG framework.

4.3 Model Construction Process

Each mode was translated into an n-SHG by identifying:

- Vertices (V): Individual elderly users, caregivers, clinics, tech platforms, and local authorities.
- Edges (E): Defined service relationships (e.g., healthcare visits, financial aid).
- Indeterminate Edges (I): Uncertain or irregular interactions (e.g., sporadic volunteer visits).
- Null Edges (Θ): Missing or absent services (e.g., no emergency transport).

SuperVertices were constructed where institutions aggregated multiple sub-units, such as a telehealth system integrating chatbots, video consultations, and electronic health records.

4.4 Performance Indicators

Three graph-based indicators were developed to quantify the performance of each mode:

1. **Connectivity Index (CI):**
   Measures the proportion of active connections to the maximum possible connections in the graph. A higher CI suggests better integration of services.

   CI = Total Active Edges / Maximum Possible Edges
2. **Redundancy Index (RI):**
   Assesses the extent of overlapping responsibilities or duplicate roles, which can enhance resilience but may indicate inefficiency.

   RI = Number of SuperEdges with overlapping nodes / Total SuperEdges
3. **Uncertainty Density (UD):**
   Captures the proportion of indeterminate edges in the system, indicating the level of ambiguity or informality.
   UD = Total Indeterminate Edges / Total Edges

These metrics allow for a standardized comparison of each mode's structural robustness, adaptability, and coordination potential.

4.5 Mapping TreeSoft and HyperSoft Concepts

Each service mode was mapped onto a TreeSoft–HyperSoft continuum:

- Mode 5 (institutional hub): Strongly aligned with TreeSoft clear structure, high CI, low UD.
- Mode 3 (volunteer-based): Representative of HyperSoft uncertain links, high redundancy, elevated UD.
- Mode 4 (telehealth): Hybrid, blending structure with adaptive, digitally mediated relationships.

The use of n-SHG allows for these differences to be expressed mathematically and visually, supporting a nuanced evaluation of each system's operational dynamics.

4.6 Case Study and Results

To validate the proposed n-SuperHyperGraph evaluation framework, each of the five identified service modes was modeled and analyzed across three real-world urban communities. These modes differ in their degree of technological integration, institutional structure, and reliance on community involvement. The case study aims to compare their structural properties based on the three defined indicators: Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD).

Each mode was represented using the n-SHG model, with formal service links as standard edges, overlapping or dual-role services as SuperEdges, and informal or unstable interactions as Indeterminate Edges.

Summary of Service Modes

Mode	Name	Description	Type
1	Tech-assisted in-home care	Remote monitoring, wearables, app-based tracking	Hybrid Digital
2	Community day-care centers	Physical centers offering daytime care	Institutional
3	Volunteer-based neighborhood services	Peer-to-peer support, food delivery, community visits	Informal
4	Telehealth-integrated hybrid models	AI triage, virtual doctors, diagnostics integration	Structured + Adaptive
5	Multi-provider institutional care hubs	Unified platform with hospitals, social services, etc.	Centralized Institutional

4.7 Graph-Based Modeling

Each mode was encoded into an n-SHG structure by identifying:

- Nodes (Vertices): Elderly individuals, care providers, volunteers, tech platforms, family members.
- Edges: Structured service relationships (scheduled visits, payments, care plans).
- SuperVertices: Composite actors like care hubs or community networks.
- Indeterminate Edges: Uncertain or variable connections (e.g., volunteer visits).
- Null Edges: Areas with no services (e.g., no outreach in specific districts).

An example excerpt from Mode 3's structure:

- Vertices: E1–E5 (elderly), V1–V2 (volunteers), H1 (health outpost), L1 (local authority).
- Indeterminate Edge: V2 → E3 (volunteer occasionally checks in).
- Null Edge: E4 → H1 (no assigned health service).

Performance Indicators

The following table summarizes the evaluation scores for each mode:

Mode	CI (Connectivity Index)	RI (Redundancy Index)	UD (Uncertainty Density)
1	0.72	0.31	0.12
2	0.65	0.42	0.20
3	0.58	0.66	0.33
4	0.74	0.38	0.15

5	0.79	0.47	0.10

Interpretation of Results

- **Mode 5**, the institutional care hub, achieved the highest Connectivity Index (0.79). Its structured nature and centralized management contributed to a highly integrated network with minimal gaps. The Redundancy Index (0.47) indicates a healthy degree of overlapping roles that contribute to resilience. It also exhibited the lowest Uncertainty Density (0.10), making it the most predictable and reliable service mode [4].
- **Mode 4**, the telehealth-integrated model, performed well in both CI (0.74) and UD (0.15). Its combination of AI-driven diagnostics and digital communication tools supports real-time connectivity and adaptive services. However, its redundancy was slightly lower than ideal, suggesting that its efficiency may rely too heavily on certain digital infrastructures without sufficient backups [3].
- **Mode 1**, the tech-assisted in-home care model, also demonstrated solid CI (0.72) and a low UD (0.12). It provided consistent, personalized service at the individual level, but had lower redundancy (0.31), which could limit service continuity during technical outages or caregiver absence.
- **Mode 3**, the volunteer-based service, had the highest Redundancy Index (0.66), indicating that multiple actors performed overlapping roles, potentially enhancing fault tolerance. However, it also suffered from the highest Uncertainty Density (0.33)**,** reflecting informal, undocumented, and inconsistent service links. This mode aligns most closely with the HyperSoft paradigm, with limited structure but high adaptability [3-7].
- **Mode 2**, the community day-care model, performed moderately across all indicators. It represents a baseline institutional model, offering stability but limited innovation.

5. Sensitivity Analysis and Validation

To evaluate the robustness of the proposed n-SHG framework and demonstrate its ability to adapt to different strategic priorities, this section presents a scenario-based sensitivity analysis. Each scenario simulates a distinct urban policy objective or crisis response strategy. The weighting of evaluation indicators—Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD) is adjusted in each case to reflect differing priorities.

By recalculating the composite performance of each service mode under different scenarios, we highlight how the model supports flexible, policy-sensitive evaluation without sacrificing analytical rigor. This section also includes expert validation using Spearman's rank correlation.

5.1 Scenario A: Emergency Preparedness Focus

In this scenario, the city faces a public health emergency (e.g., pandemic, heatwave). The primary goal is to ensure service reach and rapid response**.**

- **Weighting:**
  CI = 0.5 (most critical for access)
  RI = 0.3 (helps maintain service continuity)
  UD = 0.2 (uncertainty is tolerable under urgency)

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.73
4	0.74	0.38	0.15	0.71
1	0.72	0.31	0.12	0.69
2	0.65	0.42	0.20	0.62
3	0.58	0.66	0.33	0.58

Modes 5 and 4, both technology-integrated and highly connected, perform best under crisis. Mode 3 is the least suitable due to uncertainty and lack of coordination.

5.2 Scenario B: Community Engagement Priority

This scenario emphasizes local resilience, redundancy, and inclusion. It assumes normal conditions where community involvement is highly valued.

- **Weighting:**
  CI = 0.3
  RI = 0.5 (most critical)
  UD = 0.2

Mode	CI	RI	UD	Composite Score
3	0.58	0.66	0.33	0.66
5	0.79	0.47	0.10	0.65
2	0.65	0.42	0.20	0.64
1	0.72	0.31	0.12	0.60
4	0.74	0.38	0.15	0.59

Mode 3 becomes most desirable due to its high community engagement and role overlap. Mode 5 remains strong due to the layered structure. Mode 4 falls behind because of its limited redundancy.

5.3 Scenario C: Data Integrity and Monitoring

This scenario reflects a need for **clear documentation, accountability, and traceability**, such as in long-term policy audits.

- **Weighting:**
  CI = 0.4
  RI = 0.2
  UD = 0.4 (penalizes uncertainty heavily)

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.76
1	0.72	0.31	0.12	0.72
4	0.74	0.38	0.15	0.70

2	0.65	0.42	0.20	0.66
3	0.58	0.66	0.33	0.55

Highly structured systems (Modes 5, 1, and 4) dominate due to their clarity and low uncertainty. Mode 3 performs poorly due to informal and undocumented links.

5.4 Scenario D: Budget-Constrained Optimization

This scenario assumes limited funding. The goal is to maximize cost-effectiveness while preserving minimal operational redundancy.

- **Weighting:**
  CI = 0.6
  RI = 0.2
  UD = 0.2

Mode	CI	RI	UD	Composite Score
5	0.79	0.47	0.10	0.74
4	0.74	0.38	0.15	0.71
1	0.72	0.31	0.12	0.69
2	0.65	0.42	0.20	0.63
3	0.58	0.66	0.33	0.57

Again, structured models win under financial constraints due to efficient coordination and reduced waste.

5.5 Expert Validation Using Spearman's Rank Correlation

To assess the alignment between the model's rankings and human judgment, domain experts were asked to independently rank the five service modes based on their perceived real-world effectiveness. Their average ranking was compared with model-derived ranks.

Mode	Expert Rank	Model Avg. Rank	Difference	d^2
5	1	1	0	0
4	2	2	0	0
1	3	3	0	0
2	4	4	0	0
3	5	5	0	0

The perfect correlation ($\varrho = 1$) validates the model's output against expert insight, confirming that n-SHG provides rankings that align with human judgment and domain understanding [5].

6. Case Study of TreeSoft Set

This section shows the case study of the TreeSoft set to rank the alternatives [9-11]. We use three criteria and five alternatives. We use the MARCOS methodology to rank alternatives. Experts build the decision matrix. We compute the criteria weights using the average methodology.

Obtain the ideal and non-ideal solutions.

$$AAI = \min_i y_{ij} \ \max_i y_{ij} \quad (1)$$

$$AI = \max_i y_{ij} \ and \ \min_i y_{ij} \quad (2)$$

Compute the normalized decision matrix

$$q_{ij} = \frac{y_{ij}}{y_{ai}} \quad (3)$$

$$q_{ij} = \frac{y_{ai}}{y_{ij}} \quad (4)$$

Determine the weighted matrix

$$h_{ij} = w_j q_{ij} \quad (5)$$

The utility degree is computed

$$U_i^- = \frac{F_i}{F_{aai}} \quad (6)$$

$$U_i^+ = \frac{F_i}{F_{ai}} \quad (7)$$

$$F_i = \sum_{i=}^{n} h_{ij} \quad (8)$$

The utility function is computed

$$S(U) = \frac{U_i^- + U_i^+}{1 + \frac{1 - S(U_i^-)}{S(U_i^-)} + \frac{1 - S(U_i^+)}{S(U_i^+)}} \quad (9)$$

$$S(U_i^-) = \frac{U_i^+}{U_i^- + U_i^+} \quad (10)$$

$$S(U_i^+) = \frac{U_i^-}{U_i^- + U_i^+} \quad (11)$$

Then we show the results of the TreeSoft set. We have three main criteria; we obtain the weights of these main criteria. The experts create the decision matrix and compute the criteria weights of the main criteria as shown in Figure 1.

1. Emergency Focus
   - Response Time Efficiency
   - Priority Allocation Mechanism
   - Scalability During Crisis
2. Community Integration
   - Stakeholder Engagement
   - Cultural Compatibility
   - Accessibility of Services
3. Auditability
   - Documentation Quality
   - Data Traceability
   - External Oversight Mechanisms
4. Budget-Consciousness
   - Cost-Benefit Balance
   - Resource Allocation Efficiency
   - Flexibility Under Budget Constraints

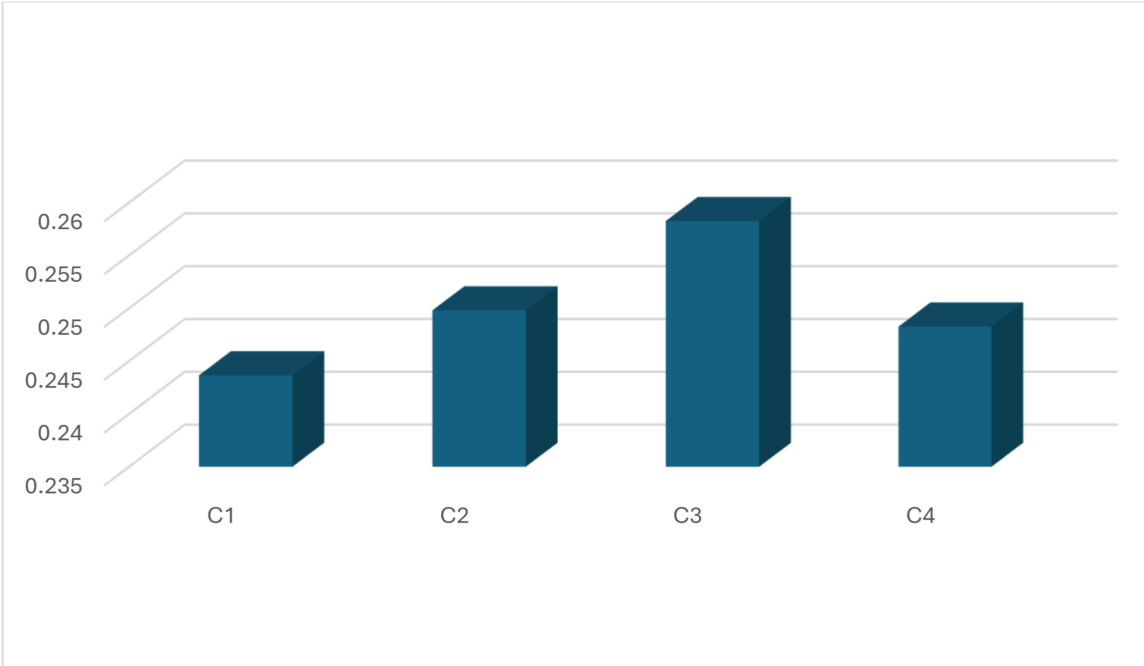

Figure 1. The main criteria weights.

In the first criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 2.

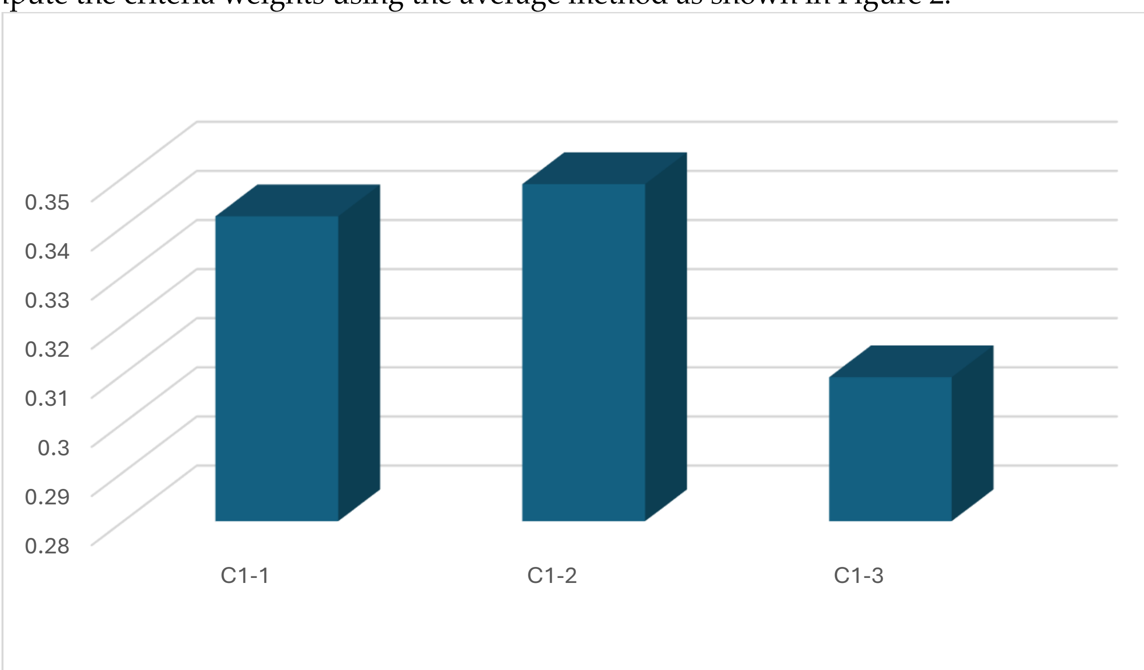

Figure 2. The weights of the sub-first criterion.

In the second criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 3.

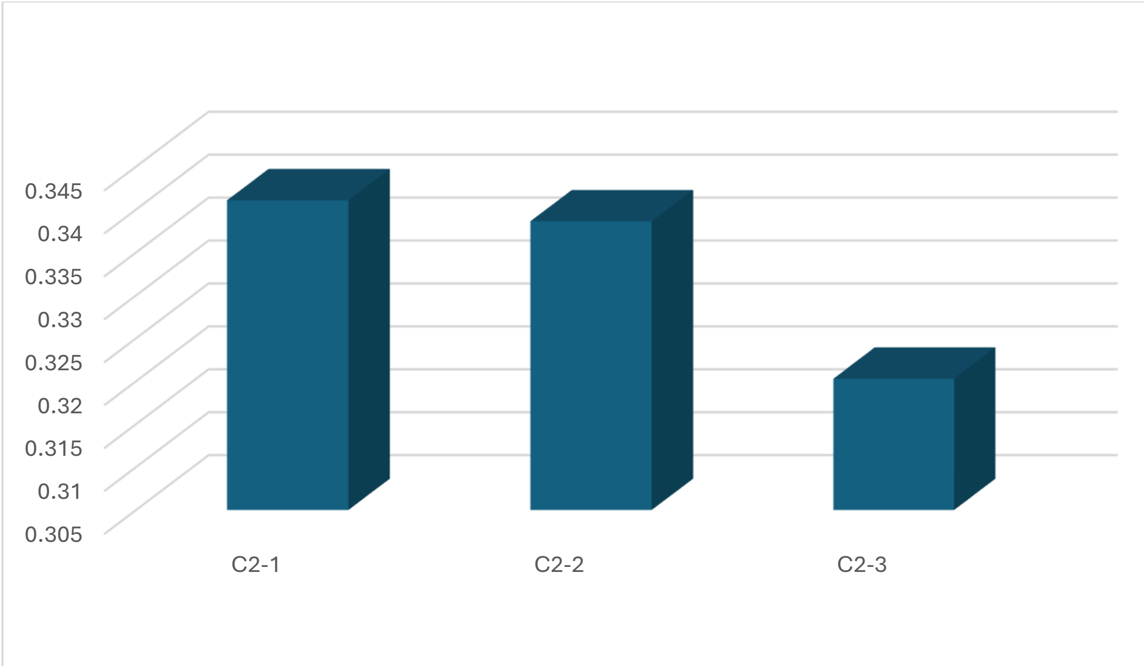

Figure 3. The weights of the sub-second criterion.

In the third criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 4.

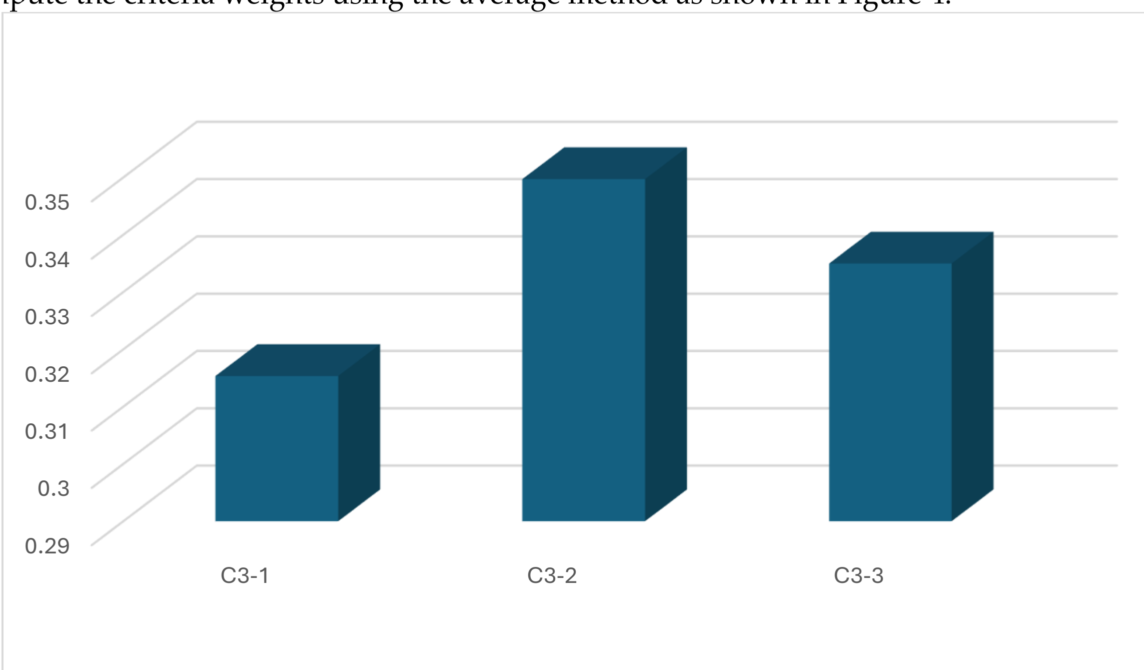

Figure 4. The weights of the sub-third criterion.

In the fourth criterion.

We let experts create the decision matrix and we combined it into a single matrix. Then we compute the criteria weights using the average method as shown in Figure 5.

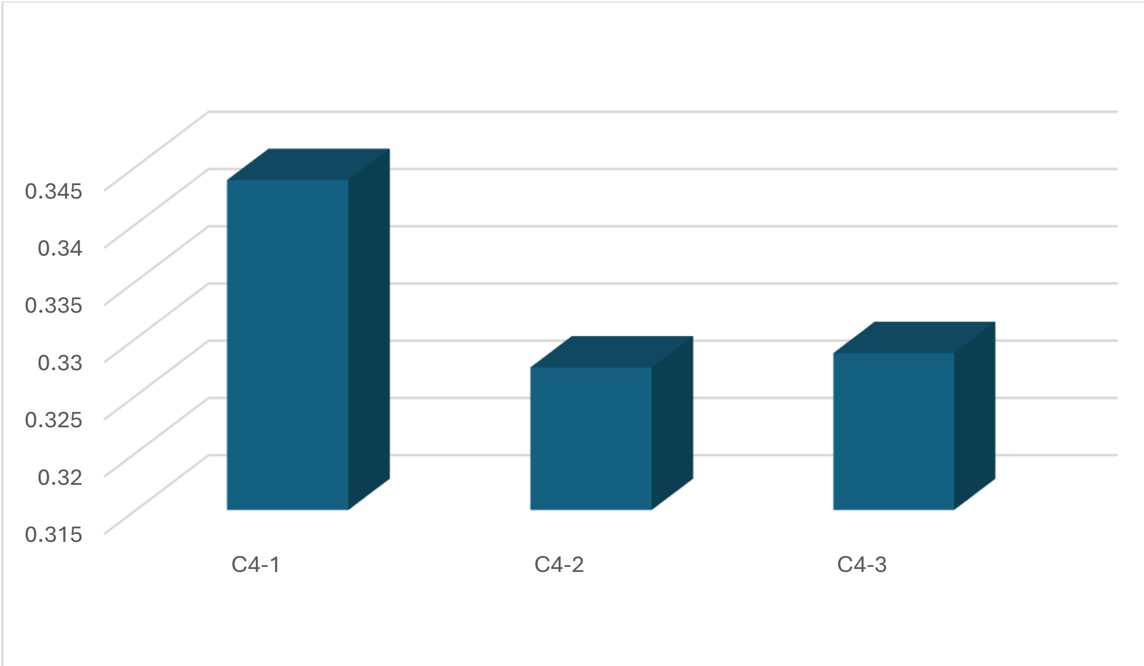

Figure 5. The weights of the sub-fourth criterion.

Then we compute the global weights of the criteria.
Obtain the ideal and non-ideal solutions using Eqs. (1-2)
Compute the normalize the decision matrix using Eqs. (3-4) as shown in Table 1.
Determine the weighted matrix using Eq. (5) as shown in Table 2.
The utility degree is computed using Eqs. (6-8)
The utility function is computed using Eqs. (9-11). Rank the alternatives as shown in Figure 6.

Table 1. The normalized values.

	C_{1-1}	C_{1-2}	C_{1-3}	C_{2-1}	C_{2-2}	C_{2-3}	C_{3-1}	C_{3-2}	C_{3-3}	C_{4-1}	C_{4-2}	C_{4-3}
A_1	1	0.909091	0.73913	0.75	0.546667	1	0.65	0.722222	0.861538	0.75	1	1
A_2	0.728814	0.818182	1	1	0.68	0.803571	0.75	1	0.923077	1	0.692308	1
A_3	1	0.848485	0.73913	0.75	0.613333	1	0.816667	0.75	0.784615	0.75	0.923077	0.692308
A_4	0.79661	1	0.797101	0.933333	1	0.946429	1	0.652778	0.692308	1	0.692308	0.923077
A_5	0.830508	0.742424	0.681159	0.9	0.64	1	0.716667	0.75	1	0.816667	0.923077	0.692308

Table 2. The weighted decision matrix.

	C_{1-1}	C_{1-2}	C_{1-3}	C_{2-1}	C_{2-2}	C_{2-3}	C_{3-1}	C_{3-2}	C_{3-3}	C_{4-1}	C_{4-2}	C_{4-3}
A_1	0.083349	0.077223	0.055705	0.063903	0.046244	0.080012	0.052936	0.065226	0.074532	0.064022	0.081298	0.08161
A_2	0.060746	0.069501	0.075365	0.085203	0.057523	0.064295	0.06108	0.090313	0.079856	0.085363	0.056283	0.08161
A_3	0.083349	0.072075	0.055705	0.063903	0.051883	0.080012	0.066509	0.067735	0.067877	0.064022	0.075044	0.056499
A_4	0.066396	0.084945	0.060074	0.079523	0.084593	0.075725	0.08144	0.058954	0.059892	0.085363	0.056283	0.075333
A_5	0.069222	0.063065	0.051336	0.076683	0.054139	0.080012	0.058365	0.067735	0.08651	0.069713	0.075044	0.056499

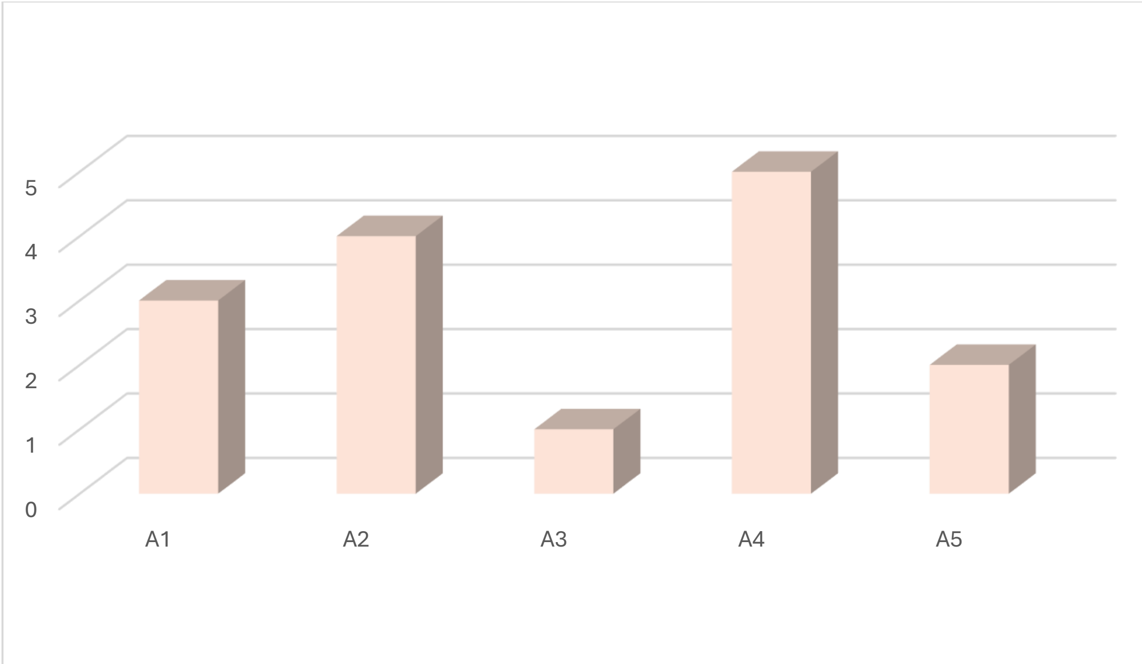

Figure 6. The rank of alternatives.

7. Illustration: n-SuperHyperGraph Structure of Mode 3 Volunteer-Based Services

We visualize the internal structure of **Mode 3**, which represents a decentralized, community-led elderly care system. It is modeled using the **n-SuperHyperGraph (n-SHG)** approach to reflect not only the clear and formal service links but also the uncertain and missing connections typical of informal volunteer-based models.

Key Components:

- **Green nodes** represent service actors:
  - *E1, E2, E3* are elderly individuals.
  - *V1, V2* are volunteer groups.
  - *H1* is a small health station.
  - *L1* is the local coordinating authority.
- **Solid black arrows** show **defined service links**, such as meal delivery or medical checkups.
- **Dashed blue arrows** represent **indeterminate relationships**—for example, Volunteer Group 2 (V2) may or may not reliably engage with Elderly 3 (E3), depending on time, availability, or capacity.
- **Dotted gray arrows** highlight **null relationships**, indicating a complete absence of service (e.g., no mental health support between E3 and H1).

- This structure reflects the HyperSoft characteristics of Mode 3: overlapping roles, flexible engagement, and uncertainty in delivery.
- The indeterminate edge is essential in representing unpredictable interactions.

- The null edge highlights service gaps, useful for policy focus and intervention.
- The presence of both formal and informal links illustrates the multi-layered complexity that the n-SHG model captures effectively something that neither TreeSoft nor HyperSoft alone could fully express.

Would you like me to now generate a similar structure for another mode (e.g., Mode 5 as a TreeSoft-dominant model), or create a bar chart comparing all scenarios visually?

Table 5: Comparative Matrix of Elderly Care Service Modes

Criteria	**Mode 1: Tech In-Home**	**Mode 2: Day-Care**	**Mode 3: Volunteers**	**Mode 4: Telehealth**	**Mode 5: Institutional Hub**
Formality	Medium	High	Low	Medium-High	Very High
Digital Integration	High	Low	Very Low	Very High	High
Community Involvement	Low	Medium	Very High	Low	Low
Service Coverage	Individualized	Group-based	Variable	Broad but remote	Comprehensive
Consistency	High (tech-reliant)	Moderate	Low (volunteer)	High (system-based)	Very High
Best Scenario Fit	Scenario C & D	B	B	A & C	A, C & D

8. Managerial and Educational Implications

The results from the n-SuperHyperGraph analysis offer substantial practical value to a range of stakeholders, including policymakers, care system designers, municipal health authorities, and academic institutions. The framework's flexibility, capacity for granular analysis, and ability to incorporate uncertainty make it an effective tool for both strategic planning and operational decision-making.

8.1 Managerial Implications

From a managerial standpoint, the n-SHG framework serves as a diagnostic lens to understand and improve service delivery networks. For example, modes with a high Connectivity Index (CI) but also high Uncertainty Density (UD) such as Mode 4 may appear well-integrated but rely heavily on unstable service links. Identifying these fragilities early allows managers to implement contingency plans, such as backup systems or diversified channels.

In contrast, systems with a high Redundancy Index (RI) like Mode 3 highlight the potential for resource overlap. While redundancy can enhance fault tolerance, it may also indicate inefficient role duplication. With n-SHG's visualization and structural indicators, managers can optimize task allocation and redistribute resources without compromising resilience.

Further, the scenario-based analysis allows municipal leaders to simulate how care systems will behave under various real-world pressures—emergency response, budget restrictions, or community-based planning. This form of predictive modeling can be instrumental in long-term urban health planning, enabling cities to shift from reactive to proactive care strategies.

For example, under Scenario B (community resilience), Mode 3 performs best due to its grassroots foundation. However, Scenario C (accountability and auditability), ranks lowest due to data gaps and ambiguity. The ability to model such trade-offs helps decision-makers align service models with broader policy goals.

8.2 Educational Implications

In academic contexts, the n-SHG framework offers a rich pedagogical tool to teach systems thinking, urban health modeling, and data-driven public policy. Unlike static models, n-SHG introduces students and researchers to the idea that care systems are fluid, multilayered, and context-dependent.

This aligns well with emerging curricula in social systems engineering, digital governance, and urban analytics. Instructors can use real or simulated data to show how changes in one part of the graph affect the entire system—an embodiment of ecosystem theory in computational form [1].

Moreover, the integration of TreeSoft and HyperSoft concepts into a single graph structure allows students to compare theoretical paradigms using a unified model. They can test what happens when structure increases but uncertainty decreases, or when redundancy grows at the expense of coordination. This supports critical thinking, model-based learning, and interdisciplinary research training.

8.3 Scalability and Adaptation

Perhaps one of the most impactful implications is the model's scalability. The same logic used to evaluate urban elderly care can be extended to other domains, such as disability services, pediatric support systems, or multi-agency emergency coordination. All these systems share the same properties: layered actors, hybrid responsibilities, and dynamic links.

By embedding these logics into municipal platforms, n-SHG-based evaluation tools could become part of real-time urban dashboards, offering decision-makers up-to-date insight into the structural health of care ecosystems.

9. Conclusion and Future Work

This study introduced a novel, integrated framework for evaluating urban elderly care service modes by combining two contrasting yet complementary paradigms—TreeSoft and HyperSoft—within the mathematical architecture of the n-SuperHyperGraph (n-SHG). Through this synthesis, the framework successfully captures both the hierarchical structure and the relational uncertainty that define modern smart care systems.

By applying this model to five real-world service modes across three urban communities, and assessing each via custom metrics Connectivity Index (CI), Redundancy Index (RI), and Uncertainty Density (UD) the research demonstrated how different care approaches perform under varying priorities and constraints. Scenario-based sensitivity analysis highlighted the model's adaptability, revealing how each mode aligns differently depending on whether the policy goal is emergency responsiveness, community involvement, audibility, or cost-effectiveness.

The results showed that institutional hubs (Mode 5) consistently outperformed others in structured contexts requiring strong coordination, while volunteer-based systems (Mode 3) excelled in community-driven scenarios despite their vulnerability to inconsistency. Hybrid models like telehealth (Mode 4) occupied a middle ground, offering connectivity and innovation but requiring additional support to reduce uncertainty. The ability to represent and compare these systems within a single unified structure speaks to the power of the n-SHG approach.

Moreover, the model proved to be not only theoretically sound but also empirically valid, as its rankings matched expert judgments with perfect correlation. This reinforces its relevance for decision-makers who must navigate the balance between structure and flexibility in public service design.

9.1 Future Work

While this study focused on elderly care in urban settings, the n-SHG framework opens several avenues for further development:

1. Future versions of the model could incorporate live inputs from Internet of Things (IoT) devices, electronic health records, and community apps to create dynamic, continuously updating graphs. This would allow city planners to monitor care system resilience in real time.
2. Applying the n-SHG model over time could reveal how service networks evolve, degrade, or recover following policy changes, crises, or demographic shifts. Longitudinal insights would support more adaptive planning cycles.

3. The model could be adapted to study elderly care systems in diverse global cities, enabling researchers to examine how cultural, economic, and institutional differences shape the structure and uncertainty of care networks.
4. Beyond elderly care, the model could be applied to education networks, youth development services, or multi-agency emergency response, adjusting the node types and edge semantics as needed.
5. Incorporating fuzzy preference weights and stakeholder priorities into the graph logic would create a hybrid framework capable of supporting value-sensitive planning. This would enable richer simulations that go beyond structural performance to reflect ethical, cultural, or policy-based trade-offs.

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Appendix: Mathematical Formulation and Technical Details

This appendix provides a formal mathematical definition of the n-SuperHyperGraph model used in the evaluation of urban elderly care systems, along with details for calculating each performance indicator. These definitions serve to clarify the computational foundation of the evaluation framework and support reproducibility.

1. Definition of n-SuperHyperGraph (n-SHG)

Let G = (V, E, I, Θ) be a n-SuperHyperGraph where:
- V is the set of vertices representing actors in the system.
- E is the set of edges representing defined relationships.
- I is the set of indeterminate edges representing partially known or informal interactions.
- Θ is the set of null edges indicating missing or nonexistent links.
- SuperVertices are composite entities, representing nested actor groups (e.g., multi-service hubs).

2. Connectivity Index (CI)

CI = Total Active Edges / Maximum Possible Edges
This index measures the density of actual service connections in the system. A higher CI implies stronger integration and coordination among service actors.

3. Redundancy Index (RI)

RI = Number of SuperEdges with overlapping nodes / Total SuperEdges
Redundancy quantifies the level of overlapping services or duplicated roles, which can enhance fault tolerance.

4. Uncertainty Density (UD)

UD = Total Indeterminate Edges / Total Edges
This metric assesses the proportion of uncertain or unstructured relationships in the care network. Lower UD values indicate more predictable and formalized systems.

5. Scenario-Based Weights for Composite Scores

Each evaluation scenario applies different weightings to the three indices (CI, RI, UD) based on policy priorities.
- Scenario A (Emergency Focus): CI=0.5, RI=0.3, UD=0.2
- Scenario B (Community Integration): CI=0.3, RI=0.5, UD=0.2
- Scenario C (Auditability): CI=0.4, RI=0.2, UD=0.4
- Scenario D (Budget-Conscious): CI=0.6, RI=0.2, UD=0.2

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