QuantumSoft Set: A Quantum-Inspired Extension of Soft Sets for Evaluating Multimedia Teaching Effectiveness in College English

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Abstract- In real-world decision-making, information is rarely complete, precise, or static. Traditional soft set extensions including HyperSoft Sets and TreeSoft Sets have improved the capacity of modeling uncertain or hierarchical data. However, none of these models effectively capture situations where elements simultaneously exhibit overlapping or probabilistic behaviors across multiple states. This paper introduces the QuantumSoft Set, a new model based on principles from quantum mechanics, such as superposition and probabilistic state collapse. We define the mathematical structure of QuantumSoft Sets, compare them to existing models, and apply them to evaluate multimedia teaching effectiveness in college-level English. A detailed case study with analytical results and visual representations demonstrates the model's advantages in capturing complex, uncertain, and subjective evaluations.

Keywords:QuantumSoft Set, HyperSoft Set, TreeSoft Set, Multimedia Teaching, Teaching Effectiveness, Soft Set Theory, Quantum Decision, Probabilistic Modeling

1. Introduction

In the modern educational landscape, especially in language teaching, evaluating teaching effectiveness is becoming increasingly complex. Multimedia tools such as videos, interactive slides, and digital whiteboards bring diverse experiences to students. However, different students perceive effectiveness differently, and even a single student might have mixed feelings at the same time. Traditional models such as classical Soft Sets fail to capture this overlapping perception.

Between 2018 and 2024,[7-9] Smarandache introduced several novel extensions to the theory of soft sets, significantly enriching the mathematical framework used to handle uncertainty and indeterminacy. These new types include the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set, and ForestSoft Set. Each of these variations builds upon the foundational principles of soft set theory, integrating elements such as hierarchical structure (as in TreeSoft and ForestSoft Sets), higher-order relations (as in HyperSoft and SuperHyperSoft Sets), and various forms of indeterminacy (notably in IndetermSoft and IndetermHyperSoft Sets). Smarandache also explored algebraic operations within these systems, particularly focusing on how indeterminate soft operators interact within the broader framework of soft algebra. These developments provide valuable tools for modeling complex systems where classical binary logic falls short, and they open new directions for research in areas such as decision-making, data analysis, and artificial intelligence.

Although improved versions like HyperSoft Sets and TreeSoft Sets provide multi-attribute or hierarchical structures, they still assume that a student or a group can be clearly classified under one evaluation category. Inspired by quantum mechanics, we propose a new model, the QuantumSoft Set, to address such cases. This model captures simultaneous and probabilistic evaluation states, enabling more accurate modeling of real student feedback.

2. Literature Review

Soft Set Theory was first introduced by Molodtsov in 1999 to handle uncertainty without requiring exact parameterization [1]. Since then, multiple extensions have emerged:

HyperSoft Set Introduced by Smarandache in 2018, this model expanded Soft Sets by using multi-attribute functions that map combinations of attributes to subsets of a universe [2].

IndetermSoft Set This model was proposed to manage indeterminate information where values are unclear, incomplete, or conflicting [3].

IndetermHyperSoft Set Combines the features of both HyperSoft and IndetermSoft Sets, allowing multiple attributes and uncertainty simultaneously [4].

TreeSoft Set Presented by Smarandache in 2022, TreeSoft Sets extend attribute hierarchies into tree-like structures, supporting sub-attributes across multiple levels [5].

MultiSoft Set Developed by Alkhazaleh et al. (2010), this generalizes Soft Sets for simultaneous multiple parameters [6].

The HyperSoft Set is a generalization of the classical soft set that supports multi-parameter decision modeling. It allows attributes to be combined as tuples and maps them to subsets of the universe, which proves useful in problems requiring multi-dimensional reasoning [5].

Saqlain et al. applied the neutrosophic hypersoft set to a player selection problem using similarity measures and showed its advantages in real-life decision-making [10]. In further research, the same authors proposed a hypersoft-based TOPSIS method for solving complex ranking problems [11]. Khan et al. introduced a probabilistic hypersoft model for supply chain risk management [12]. In healthcare, hypersoft sets have been used for diagnosing diseases using interval-valued neutrosophic logic [13]. Smarandache also provided foundational work by integrating HSS with plithogenic theory for applications in mathematical modeling [14].

The IndetermSoft Set (ISS) and IndetermHyperSoft Set (IHSS) handle uncertain or incomplete data more robustly than traditional soft models. These models allow handling indeterminacy directly within decision-making environments [5].

Applications of ISS and IHSS include digital marketing effectiveness evaluation [15], uncertain expert system modeling [16], patient prioritization systems [17], and climate risk assessment [18]. Additionally, Smarandache proposed algebraic operators acting on these sets, enabling logical operations under indeterminate frameworks [1]. Also, ISS has been integrated with rough sets for decision support in security screening contexts [19].

TreeSoft Set (TSS) structures parameter sets in hierarchical tree form, allowing better modeling of multi-level decisions [1]. Dhamodharan et al. applied TSS in desalination process selection using multi-criteria decision-making techniques [20]. Another application used TreeSoft sets for evaluating university performance [21]. TSS has been adapted for medical treatment planning [22], environmental sustainability analysis [23], and optimal staff recruitment [24]. These hierarchical structures allow layered decision frameworks that are closer to real-world conditions.

Despite these advancements, none of the existing models allow an entity to exist in a superposition of attribute values, nor do they integrate probabilistic decision theory based on quantum principles. The QuantumSoft Set addresses this unique gap by modeling uncertainty not just as "unknown" or "uncertain" but as co-existing probabilistic states.

3. QuantumSoft Set

3.1 Mathematical Definition

Let:

— $U = \{u_1, u_2, \dots, u_n\}$ be the universe of discourse.

— $A = \{a_1, a_2, \dots, a_m\}$ be a finite set of attributes.

— Each element $u_i \in U$ is associated with a quantum state vector $|u_i\rangle$.

— Define a mapping:

$$F: A \to \mathcal{H}(U)$$

where $\mathcal{H}(U)$ is a complex Hilbert space over U. Each mapping produces a quantum superposition of the elements in U, such that:

$$F(a_j) = \sum_{i=1}^{n} \alpha_{ij} |u_i\rangle, \text{ where } \sum_{i=1}^{n} \left|\alpha_{ij}\right|^2 = 1$$

Here, α_{ij} is the amplitude associated with the likelihood of element u_i having attribute a_j.

3.2 Measurement and Collapse

When a query is made-i.e., when we ask: "Which element has attribute a ?"-the quantum state collapses to a definite outcome with probability:

$$P(u_i) = |\alpha_{ij}|^2$$

This process mimics quantum measurement, where the uncertain state is resolved into a single classical result.

4. Case Study

We consider four student groups:

1. s_1 : Group A
2. s_2 : Group B
3. s_3 : Group C
4. s_4 : Group D

4.1 Attribute: Teaching Effectiveness

The possible evaluation values are:

a) Very Effective
b) Neutral
c) Confusing

Using student surveys, we derive the QuantumSoft mapping as follows:

a) Very Effective

$F(\text{ Very Effective }) = 0.8|s_1\rangle + 0.4|s_2\rangle + 0.1|s_3\rangle$

$P(s_1) = 0.64, P(s_2) = 0.16, P(s_3) = 0.01$

b) Neutral

$F(\text{ Neutral }) = 0.1|s_1\rangle + 0.2|s_2\rangle + 0.5|s_3\rangle + 0.1|s_4\rangle$

$P(s_3) = 0.25$ (most likely)

c) Confusing

$F(\text{ Confusing }) = 0.3|s_2\rangle + 0.6|s_3\rangle + 0.5|s_4\rangle$

$P(s_3) = 0.36, \qquad P(s_4) = 0.25$

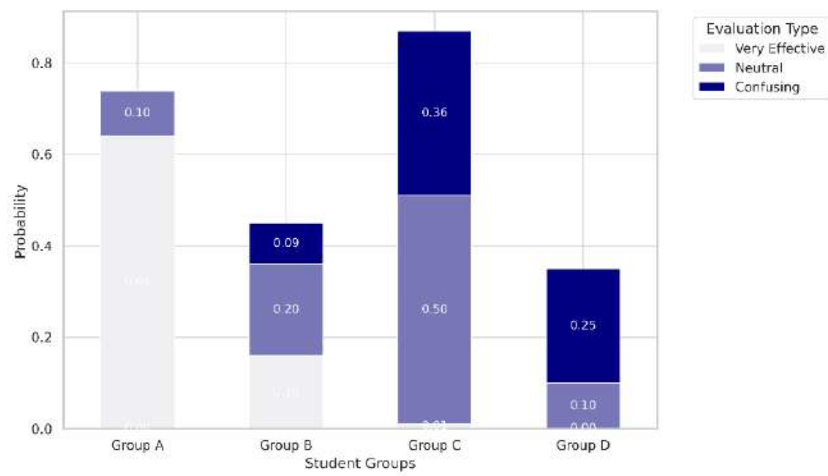

Figure 1 showing the probabilities for each group across evaluation types.

To help picture how different student groups responded to multimedia teaching, we created a bar chart showing the probabilities of each group under the three evaluation categories. Figure 1 clearly highlights the differences in perception. Group A shows the highest confidence in the teaching being very effective, while Group C has a mixed response, leaning more toward "confusing" and "neutral." Group D primarily viewed the teaching as confusing.

5. Comparison with HyperSoft and TreeSoft Models

To better understand the effectiveness of the proposed model, it is essential to compare it with existing extensions such as the HyperSoft and TreeSoft sets. These models offer unique structural advantages that address multi-criteria and hierarchical decision-making under uncertainty.

5.1 HyperSoft Set Approach

Attributes:

1. $A_1 = \{$ Tool Type: Video, Slides, Interactive $\}$
2. $A_2 = \{$ Student Feedback: Effective, Neutral, Confusing$\}$

Function:

$$F: A_1 \times A_2 \rightarrow P(U)$$

Example:

$$F(\text{ Video , Effective }) = \{s_1, s_2\}, F(\text{ Interactive , Confusing }) = \{s_4\}$$

This provides a crisp categorization, but no overlap or uncertainty modeling.

5.2 TreeSoft Set Approach

Attributes are structured hierarchically:
1. Level 0: Effectiveness
2. Level 1: Tool Type
3. Level 2: Student Perception

Example mapping:

$$F(\text{ Slides } \rightarrow \text{ Neutral }) = \{s_3\}, F(\text{ Interactive } \rightarrow \text{ Confusing }) = \{s_4\}$$

Useful for structured feedback but lacks probabilistic representation or co-existing states.

5.3 QuantumSoft Set Advantage

Unlike the above models, QuantumSoft:
1. Allows overlapping group membership
2. Models mixed or uncertain perceptions
3. Reflects probabilistic decision-making

Mimics real student feedback, where opinions are not always clear or exclusive

Table 1 provides a clear comparison of the five models discussed. It highlights how each model handles key features such as multi-attribute input, indeterminate data, hierarchical structure, and probabilistic or overlapping responses. This helps demonstrate why the QuantumSoft Set is uniquely positioned for modeling subjective and complex evaluations in education and beyond.

Table 1. Comparison of Soft Set Models

Feature	**Soft Set**	**HyperSoft Set**	**IndetermSoft Set**	**TreeSoft Set**	**QuantumSoft Set**
Handles Multiple Attributes	X	√	X	√	X
Supports Indeterminate Data	X	X	√	X	√
Hierarchical Structure	X	X	X	√	X
Handles Overlapping States	X	X	X	X	√
Based on Probabilities	X	X	X	X	√
Simulates Real Decision Uncertainty	X	√	√	√	√
Requires Complex Structure	X	X	X	√	X

6. Discussion

As we compare the existing soft set models with the newly proposed QuantumSoft Set, we begin to see a clear shift in how uncertainty and human judgment can be represented in mathematical models.

Traditional models like the Soft Set and HyperSoft Set work well when the information is structured and the answers are clear. They allow us to handle multiple factors at once, but they assume that each situation has a definite, fixed outcome. This doesn't always reflect how people actually think or feel especially in areas like education, where opinions are often mixed or unclear.

The IndetermSoft Set and TreeSoft Set improved things by allowing us to handle unknown or layered information. For example, TreeSoft Sets help when feedback can be organized in levels, such as from general impressions to specific issues. But even these models assume that once we ask a question, we get one solid answer. They don't allow for a person or group to feel multiple things at once like finding a lecture both helpful and confusing.

That's where the QuantumSoft Set offers something new. It accepts that people can hold different opinions at the same time, and that those opinions can shift depending on how we ask or interpret the question. This reflects real student feedback more accurately, because emotions and judgments are rarely black and white. They're usually a mix.

This kind of thinking wasn't possible without the deep groundwork laid by Florentin Smarandache, who made major contributions to this field. Between 2018 and 2024, he introduced several powerful models that helped expand soft set theory in important ways. His work made it possible to deal with complex relationships, incomplete data, and structured feedback in a much more flexible way. These models including the HyperSoft, IndetermSoft, IndetermHyperSoft, SuperHyperSoft, TreeSoft, and ForestSoft Sets have become key tools in many modern decision-making systems.

Building on that foundation, the QuantumSoft Set takes the next step. It combines mathematics with the way people actually think and feel. Instead of forcing answers into categories, it lets us explore the gray areas those in-between states where real understanding happens. This makes it a better fit for fields like education, psychology, and human-centered technology, where clear answers are often hard to find.

In short, the QuantumSoft Set doesn't just add another layer to soft set theory it brings the theory closer to real life.

7. Limitations and Future Work

While the QuantumSoft Set offers a more realistic way to model uncertain and mixed opinions, it is not without challenges. One limitation is that it requires estimating probability values for each possible opinion, which may be difficult to gather in real settings without detailed data or expert judgment.

Additionally, the model currently focuses on flat structures and does not support multi-level or hierarchical feedback like the TreeSoft Set. Combining the quantum approach with tree-like or fuzzy structures could make it even more flexible and powerful.

Future research can explore ways to simplify the model's use in practical systems, create automated tools for estimating probabilities, and apply it in other areas such as healthcare, psychology, or smart recommendation systems. Developing algebraic operations for QuantumSoft Sets like union and intersection would also help expand their use in more complex decision-making environments.

6. Conclusion

This research introduced the QuantumSoft Set as a novel and powerful extension of Soft Set theory. Rooted in the principles of quantum mechanics, it handles complex, subjective, and overlapping evaluations more effectively than classical models. In the case study on college English teaching, QuantumSoft Sets demonstrated clear advantages in capturing mixed student responses toward multimedia teaching tools. By integrating probabilistic modeling and superposition, this model enhances our ability to process fuzzy feedback and supports more nuanced educational decision-making.

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