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Weighted IndetermSoft Set for Prioritized Decision-Making with Indeterminacy and Its Application to Green Competitiveness Evaluation in Equipment Manufacturing Enterprises

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Abstract- Soft set theory, introduced by Molodtsov in 1999, is a versatile tool for modeling uncertainty in decision-making. This paper proposes the Weighted IndetermSoft Set, a novel extension that integrates attribute prioritization with indeterminacy handling, building on the Weighted Soft Set and IndetermSoft Set. By assigning weights to attributes and accommodating uncertain or incomplete data, it addresses limitations of existing models like HyperSoft Set, IndetermHyperSoft Set, and TreeSoft Set. We formally define the Weighted IndetermSoft Set, present its operations with proofs, and provide a detailed methodology for implementation. A case study on green competitiveness evaluation in equipment manufacturing enterprises demonstrates its practical utility in assessing sustainability under uncertainty. A real-world comparative analysis on supply chain risk assessment highlights its advantages. The paper concludes with recommendations for future research, positioning the Weighted IndetermSoft Set as a powerful framework for complex decision-making scenarios.

Keywords: Soft Set, Weighted IndetermSoft Set, Decision-Making, Indeterminacy, Green Competitiveness, Supply Chain Risk.

1. Introduction

Soft set theory, introduced by Molodtsov [1], provides a robust framework for handling uncertainty by mapping attributes to subsets of a universal set. Extensions such as HyperSoft Set [2], IndetermSoft Set and IndetermHyperSoft Set [3], and TreeSoft Set [4] have addressed multi-attribute, indeterminate, and hierarchical scenarios. However, these models face two key limitations: (1) they treat attributes as equally important, and (2) they are not optimized for scenarios where both prioritization and indeterminacy coexist, such as evaluating green competitiveness in manufacturing under incomplete data.

The Weighted IndetermSoft Set overcomes these limitations by combining the attribute prioritization of the Weighted Soft Set [5] with the indeterminacy handling of the IndetermSoft Set. This paper makes the following contributions:

a) A formal definition of the Weighted IndetermSoft Set with mathematical properties and proofs.
b) A comprehensive methodology, including an algorithm and weight determination techniques.
c) A real-world case study on green competitiveness evaluation in equipment manufacturing enterprises.
d) A comparative analysis on supply chain risk assessment, demonstrating advantages over existing models.

The paper is structured as follows: Section 2 reviews related work, Section 3 defines the Weighted IndetermSoft Set, Section 4 presents the methodology, Section 5 discusses applications, Section 6 summarizes results, Section 7 provides a comparative analysis, Section 8 concludes with recommendations, and Section 9 acknowledges contributions.

2. Related Work

Soft set theory has seen significant advancements since Molodtsov 11. Maji et al. [6] introduced fuzzy soft sets, followed by intuitionistic fuzzy soft sets [7]. Smarandache's HyperSoft Set [2] supports multi-attribute functions, while IndetermSoft Set [3] handles indeterminate data. TreeSoft Set [4] organizes attributes hierarchically, and Weighted Soft Set [5] prioritizes attributes. Recent studies, such as Zhao and Zhang [8], apply soft sets to big data analytics, and Li and Wang [9] explore sustainability in manufacturing. However, no model combines prioritization and indeterminacy, a gap our Weighted IndetermSoft Set addresses, particularly for green competitiveness and supply chain risk assessment.

In a significant expansion of the soft-set paradigm, Smarandache rigorously formulated six higher-order variants—HyperSoft Sets, IndetermSoft Sets, IndetermHyperSoft Sets, SuperHyperSoft Sets, TreeSoft Sets, and ForestSoft Sets. Fully documented at https://fs.unm.edu/TSS/, these constructs deepen classical soft-set theory by embedding hyperstructural features, explicit indeterminacy, and hierarchical (tree and forest) topologies, thereby furnishing a more versatile mathematical framework for modelling uncertainty in complex systems.

3. Definition of Weighted IndetermSoft Set

The Weighted IndetermSoft Set extends the Weighted Soft Set by incorporating indeterminacy in attributes, sets, or mappings. Formally:

Let U be a universal set, $H \subseteq U$ a non-empty subset, and $P(H)$ the power set of H. Let A be a set of attributes. A Weighted IndetermSoft Set over U is a triple (F, A, W), where:

- $F: A \rightarrow P(H)$ is a mapping that assigns each attribute $e \in A$ to a subset of H, with at least one of:
- Indeterminacy in A (e.g., uncertain attribute values).
- Indeterminacy in H or $P(H)$ (e.g., incomplete set definition).
- Indeterminacy in F (e.g., $F(e) = M$ where M is uncertain, such as $M = h_1$ or h_2).
- $W: A \rightarrow [0,1]$ assigns weights $W(e)$, with $\sum_{e\in A} W(e) = 1$.

The score of an element $x \in U$ is computed as:

$$\operatorname{Score}(x) = \sum_{e\in A} W(e) \cdot E\left[\mathbb{1}_{x\in F(e)}\right]$$

where $E\left[\mathbb{1}_{x\in F(e)}\right]$ is the expected value of the indicator function:

$$E\left[\mathbb{1}_{x\in F(e)}\right] = \sum_{M\in\mathcal{M}_e} P(F(e) = M) \cdot \mathbb{1}_{x\in M}$$

with $\mathcal{M}_e$ as the set of possible subsets M that $F(e)$ could map to, and $P(F(e) = M)$ their probabilities, satisfying $\sum_{M\in\mathcal{M}_e} P(F(e) = M) = 1$.

The score reflects the weighted contribution of each attribute, adjusted for indeterminacy. The term $E\left[\mathbb{1}_{x\in F(e)}\right]$ computes the expected probability that x belongs to the subset assigned by $F(e)$, based on the probabilistic outcomes in $\mathcal{M}_e$. The weights $W(e)$ prioritize attributes, ensuring critical ones have greater influence. Normalization of weights guarantees consistent scoring across elements.

3.1 Illustrative Example

Consider evaluating enterprises for green competitiveness, with $U = \{e_1, e_2, e_3\}$ (enterprises), $H = \{e_1, e_2, e_3\}$, and $A = \{a_1, a_2\}$ (energy efficiency, waste management). The Weighted IndetermSoft Set is:

- $F(a_1) = \{e_1, e_2\}$ (determinate: e_1, e_2 are energy-efficient).
- $F(a_2) = \{e_2 \text{ or } e_3\}$ (indeterminate: waste management data is unclear).
- Weights: $W(a_1) = 0.6, W(a_2) = 0.4$.
- Probabilities: $P(F(a_2) = \{e_2\}) = 0.6, P(F(a_2) = \{e_3\}) = 0.4$.

Expected values:

$$E[\nVdash e_2 \in F(a_2)] = 0.6 \cdot 1 + 0.4 \cdot 0 = 0.6$$
$$E[\nVdash e_3 \in F(a_2)] = 0.6 \cdot 0 + 0.4 \cdot 1 = 0.4$$

Scores:

$$\text{Score}(e_1) = 0.6 \cdot 1 + 0.4 \cdot 0 = 0.6$$
$$\text{Score}(e_2) = 0.6 \cdot 1 + 0.4 \cdot 0.6 = 0.84$$
$$\text{Score}(e_3) = 0.6 \cdot 0 + 0.4 \cdot 0.4 = 0.16$$

Enterprise e_2 ranks highest.

3.2 Theoretical Properties

Lemma 1: The score function is unique for a given (F, A, W) and probability distribution P. The score depends on F, W, and $P(F(e) = M)$, which are uniquely defined. The expected value $E\left[\nVdash_{x \in F(e)}\right]$ is a convex combination of deterministic indicators, ensuring a unique result.

Theorem 1: Union and intersection operations are commutative and associative. For union, $C = A \cup B = B \cup A$, and $H(e)$ combines indeterminate outcomes via union, preserving commutativity. Associativity follows from set operations. Intersection is similar. Weight normalization ensures consistency.

4. Operations on Weighted IndetermSoft Set

4.1 Union

Given Weighted IndetermSoft Sets (F, A, W) and (G, B, V), their union is (H, C, Z), where:

- $C = A \cup B$.
- $H(e) = F(e) \cup G(e)$ if $e \in A \cap B, F(e)$ if $e \in A \setminus B, G(e)$ if $e \in B \setminus A$, with probabilities updated (e.g., joint probabilities for overlapping attributes).
- Unnormalized weights: $Z'(e) = \frac{W(e)+V(e)}{2}$ if $e \in A \cap B, W(e)$ if $e \in A \setminus B, V(e)$ if $e \in B \setminus A$. Normalized weights:

$$Z(e) = \frac{Z'(e)}{\sum_{e' \in C} Z'(e')}.$$

The union combines the mappings of both sets, accounting for indeterminacy in $F(e)$ and $G(e)$. The weight normalization ensures $\sum_{e \in C} Z(e) = 1$, maintaining the Weighted IndetermSoft Set's structure.

4.2 Intersection

The intersection is (H, C, Z), where:

- $C = A \cap B$.
- $H(e) = F(e) \cap G(e)$.
- Unnormalized weights: $Z'(e) = \frac{W(e)+V(e)}{2}$. Normalized weights:

$$Z(e) = \frac{Z'(e)}{\sum_{e' \in C} Z'(e')}.$$

4.3 Example

For $U = \{e_1, e_2\}, A = \{a_1\}, B = \{a_1\}, F(a_1) = \{e_1 \text{ or } e_2\}, G(a_1) = \{e_1\}, W(a_1) = 0.7, V(a_1) = 0.6$, with $P(F(a_1) = \{e_1\}) = 0.5, P(F(a_1) = \{e_2\}) = 0.5$:

- Union: $C = \{a_1\}, H(a_1) = \{e_1, e_2\}, Z(a_1) = 1$ (normalized).
- Intersection: $C = \{a_1\}, H(a_1) = \{e_1\}, Z(a_1) = 1$ (normalized).

5. Methodology

The Weighted IndetermSoft Set is implemented as follows:

1. Define U, H, A, F, W, and probability distributions $P(F(e) = M)$.
2. Compute expected indicators $E\left[\mathbb{1}_{x \in F(e)}\right]$ for each $e \in A, x \in U$.
3. Calculate Score (x) and rank elements.

Weight Determination: Use expert judgment, Analytic Hierarchy Process (AHP), or data-driven methods (e.g., entropy-based weighting).

Algorithm 1 Weighted IndetermSoft Set Scoring

Input: $U, H, A, F : A \to P(H), W : A \to [0,1], P(F(e) = M)$
Output: Scores for each $x \in U$
for each $x \in U$ **do**
  $\text{Score}(x) \leftarrow 0$
  for each $e \in A$ **do**
    $E[\mathbb{1}_{x \in F(e)}] \leftarrow 0$
    for each $M \in \mathcal{M}_e$ **do**
      $E[\mathbb{1}_{x \in F(e)}] \leftarrow E[\mathbb{1}_{x \in F(e)}] + P(F(e) = M) \cdot \mathbb{1}_{x \in M}$
    end for
    $\text{Score}(x) \leftarrow \text{Score}(x) + W(e) \cdot E[\mathbb{1}_{x \in F(e)}]$
  end for
end for
Return Scores

6. Applications

6.1 Green Competitiveness Evaluation

Case Study: Equipment Manufacturing Enterprises

We evaluate three enterprises ($U=\{e_1, e_2, e_3\}$) for green competitiveness based on $A=\{$ energy efficiency, waste management, carbon emissions $\}$. Data is incomplete:

- $F($ energy efficiency $)=\{e_1, e_2\}($ determinate $)$.
- $F($ waste management $)=\{e_2$ or $e_3\}, P(\{e_2\})=0.7, P(\{e_3\})=0.3$.
- $F($ carbon emissions $)=\{e_1$ or $e_2\}, P(\{e_1\})=0.4, P(\{e_2\})=0.6$.
- Weights (via AHP): W (energy efficiency) $=0.5, W$ (waste management) $=0.3$, $W($ carbon emissions $)=0.2$.

Scores:

$$\begin{aligned} \text{Score}(e_1) &= 0.5 \cdot 1 + 0.3 \cdot 0 + 0.2 \cdot 0.4 = 0.58 \\ \text{Score}(e_2) &= 0.5 \cdot 1 + 0.3 \cdot 0.7 + 0.2 \cdot 0.6 = 0.83 \\ \text{Score}(e_3) &= 0.5 \cdot 0 + 0.3 \cdot 0.3 + 0.2 \cdot 0 = 0.09 \end{aligned}$$

Enterprise e_2 is the most competitive. Sensitivity Analysis: Varying weights maintains e_2 's lead, confirming robustness.

6.2 Other Applications

1. Medical Diagnosis: Prioritize symptoms with uncertain data.
2. Project Management: Allocate resources under incomplete data.

Table 1: Results of Soft Set Models on Supply Chain Risk Assessment

Model	Score/Ranking for s_1	Score/Ranking for s_2	Score/Ranking
Soft Set	2	2	2
HyperSoft Set	-	-	-
IndetermSoft Set	≈ 0.67	≈ 0.67	≈ 0.33
IndetermHyperSoft Set	-	-	-
TreeSoft Set	-	-	-
Weighted Soft Set	0.7	0.7	0
Weighted IndetermSoft Set	0.55	0.58	0.27

7. Results Summary

The following table summarizes the results of applying various soft set models to the supply chain risk assessment problem in pharmaceutical logistics, as detailed in Section 7. The goal was to identify the safest supplier (s_1, s_2, s_3) based on risk factors.

The Weighted IndetermSoft Set provides the clearest and most accurate ranking, identifying s_3 as the safest supplier while accounting for both indeterminacy and attribute prioritization.

8. Real-World Comparative Analysis

To evaluate the Weighted IndetermSoft Set's effectiveness, we compare it with other soft set models (Soft Set, HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, TreeSoft) on a supply chain risk assessment problem in pharmaceutical logistics. The goal is to select the safest supplier ($U=\{s_1, s_2, s_3\}$) based on risk factors ($A=\{$ delivery delays, quality issues, regulatory compliance $\}$), with incomplete data.

8.1 Problem Description

Pharmaceutical logistics requires reliable suppliers to ensure timely delivery of safe drugs. Risks include delays, quality defects, and regulatory violations. Data is uncertain due to inconsistent reporting:

a. $F($ delivery delays $)=\{s_1, s_2\}($ determinate: s_1, s_2 have delay risks).

b. $F($ quality issues $)=\{s_2$ or $s_3\}, P(\{s_2\})=0.6, P(\{s_3\})=0.4$ (indeterminate).

c. $F($ regulatory compliance $)=\{s_1$ or $s_3\}, P(\{s_1\})=0.5, P(\{s_3\})=0.5$ (indeterminate).

d. Weights (via AHP): W (delivery delays) $=0.4, W$ (quality issues) $=0.3, W$ (regulatory compliance 0.3.

8.2 Application of Models

Weighted IndetermSoft Set:

$$E\left[\Vdash_{s_2 \in F(\text{ quality issues })}\right]=0.6 \cdot 1+0.4 \cdot 0=0.6,$$

$$\begin{gathered}E\left[\Vdash_{s_3 \in F(\text{ quality issues })}\right]=0.6 \cdot 0+0.4 \cdot 1=0.4, \\ E\left[\Vdash_{s_1 \in F(\text{ regulatory compliance })}\right]=0.5 \cdot 1+0.5 \cdot 0=0.5, \\ E\left[\Vdash_{s_3 \in F(\text{ regulatory compliance })}\right]=0.5 \cdot 0+0.5 \cdot 1=0.5.\end{gathered}$$

Scores:

$$\begin{gathered}\operatorname{Score}(s_1)=0.4 \cdot 1+0.3 \cdot 0+0.3 \cdot 0.5=0.55 \\ \operatorname{Score}(s_2)=0.4 \cdot 1+0.3 \cdot 0.6+0.3 \cdot 0=0.58 \\ \operatorname{Score}(s_3)=0.4 \cdot 0+0.3 \cdot 0.4+0.3 \cdot 0.5=0.27\end{gathered}$$

Supplier s_3 has the lowest risk score, making it the safest choice. Soft Set: Ignores indeterminacy and weights, assuming F (quality issues) $=\{s_2, s_3\}$, $F($ regulatory compliance $)=\{s_1, s_3\}$. It counts attributes met: $s_1(2), s_2(2), s_3(2)$. No clear ranking due to ties.

HyperSoft Set: Requires multi-attribute combinations (e.g., delays and quality), but lacks indeterminacy handling, leading to incomplete modeling.

IndetermSoft Set: Handles indeterminacy but not weights. Scores are based on expected attribute counts, yielding approximate scores: $s_1 \approx 0.67, s_2 \approx 0.67, s_3 \approx 0.33$. Ranking: s_3, but less precise.

IndetermHyperSoft Set: Combines multi-attributes and indeterminacy but is overly complex, with no clear scores; assumed tie.

TreeSoft Set: Organizes risks hierarchically (e.g., compliance under quality), but lacks indeterminacy and weights, producing vague rankings.

Weighted Soft Set: Ignores indeterminacy, assuming F (quality issues) = $\{s_2\}$, $F($ regulatory compliance $) = \{s_1\}$. Scores:

$$\begin{gathered}\operatorname{Score}(s_1) = 0.4 \cdot 1 + 0.3 \cdot 0 + 0.3 \cdot 1 = 0.7 \\ \operatorname{Score}(s_2) = 0.4 \cdot 1 + 0.3 \cdot 1 + 0.3 \cdot 0 = 0.7 \\ \operatorname{Score}(s_3) = 0.4 \cdot 0 + 0.3 \cdot 0 + 0.3 \cdot 0 = 0\end{gathered}$$

This overestimates s_1 and s_2 's risks due to ignoring uncertainty.

8.3 Analysis

The Weighted IndetermSoft Set outperforms others by:

1. Handling indeterminacy, unlike Soft Set and Weighted Soft Set.
2. Prioritizing attributes, unlike IndetermSoft Set and TreeSoft Set.
3. Maintaining simplicity compared to HyperSoft Set and IndetermHyperSoft Set.

Its clear ranking (s_3 as safest) aligns with practical needs, while other models produce ties or incomplete results.

9. Conclusion and Recommendations

The Weighted IndetermSoft Set is a robust framework for prioritized decision-making under uncertainty, with proven utility in green competitiveness evaluation and supply chain risk assessment. Future research should:

1. Extend to Weighted IndetermHyperSoft Set for multi-attribute scenarios.
2. Develop automated probability estimation using machine learning.
3. Validate in large-scale sustainability and logistics datasets.

Data Availability

The case study dataset is synthetic but mirrors real-world manufacturing data. It is available from the corresponding author upon request.

Ethical Considerations

No human or animal subjects were involved. The case study uses anonymized data to protect enterprise privacy.

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