



Neutrosophic Magnetic Field – An Introduction

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Abstract: This paper recalls the concept of a Neutrosophic Magnetic Field introduced by the author in 2016 [4], extending the traditional understanding of magnetic fields by incorporating principles from neutrosophy. Departing from a crisp binary distinction between the presence and absence of magnetic force, the Neutrosophic Magnetic Field proposes a tripartite structure: an inner-zone where the magnetic force acts completely, an outer-zone where it does not act at all, and a crucial neutro-zone — a buffer or indeterminate region where the magnetic force is vague and unclear. This neutro-zone acknowledges the inherent imprecision at the boundaries of physical phenomena and offers a more comprehensive framework for analysing magnetic interactions, particularly in complex or ill-defined environments. Proposed conceptual formulae are presented to illustrate the varying degrees of magnetic influence across these distinct zones, providing a foundation for further exploration of neutrosophic applications in electromagnetism.

Keywords: Neutrosophic Magnetic Field; Magnetic Field; Neutrosophy; Indeterminate Zone; Inner-Zone; Outer-Zone; Magnetic Force; Boundary; Electromagnetism; Fuzzy Boundary.

1. Introduction

The concept of a magnetic field is fundamental to physics and technology, describing the region around a magnetic material or a moving electric charge within which the force of magnetism acts. Classically, a magnetic field is often perceived as having clear boundaries, where the magnetic force either exists or does not.

However, real-world phenomena often exhibit degrees of presence, absence, and indeterminacy, which are not fully captured by binary representations. This limitation becomes particularly apparent when considering the transitional regions surrounding magnetic sources, where the influence may not be definitively "on" or "off."

Smarandache's neutrosophy, a philosophical and mathematical framework, offers a powerful lens through which to examine such indeterminate systems. Neutrosophy postulates that every idea or proposition A has a degree of truth T(A), a degree of indeterminacy I(A), and a degree of falsehood F(A), where T(A) + I(A) + F(A) is not necessarily 1. This framework is ideally suited for modelling situations where clarity is elusive and where a neutral or indeterminate state coexists with opposing states.

This paper proposes the application of neutrosophic principles to the understanding of magnetic fields, introducing the notion of a "Neutrosophic Magnetic Field." We posit that the field of force surrounding a magnetic pole, or a current-carrying conductor, is not a simple binary system but rather a complex region characterized by three distinct zones: a fully active magnetic field, a completely inactive region, and crucially, a neutral or indeterminate zone. This neutro-zone represents a buffer where the magnetic force is vague and its influence is not clearly defined, offering a more nuanced and realistic portrayal of magnetic phenomena.

2. Neutrosophic Magnetic Field, an Introduction

Let Ψ be a magnetic pole or a conductor throughout which a current flows. The field of force surrounding Ψ , where a magnetic flux exists, is actually a Neutrosophic Magnetic Field. This is because it is formed by three main zones, analogous to the neutrosophic components { $\langle \Psi \rangle$, <indeterminate Ψ >, and <non Ψ >}.



Figure 1. Differences of Classical Magnetic Field and Neutrosophic Magnetic Field.

Left Side: *Classical Magnetic Field* — Clean, solid lines going from the North (N) to the South (S) pole of a magnet. These lines represent the traditional idea of magnetic field lines: smooth, continuous, predictable. Every point around the magnet has a precisely known field direction and strength. There's no fuzziness or uncertainty in the classical model — it's sharp and exact.

Right Side: *Neutrosophic Magnetic Field* — Dashed, scattered lines and coloured mist (fuzzy areas) around the magnet. These lines show that at each point, the magnetic field isn't purely known: dashed lines suggest uncertainty: the field might bend, fluctuate, or be weaker/stronger than expected; coloured mist (yellow-orange and blue-grey tones) visually represents indeterminacy — places where the field's exact value is unclear. In this neutrosophic model, at each point near the magnet, the field has degrees of:

- o *Truth* (how much the expected field is present),
- o Indeterminacy (how much uncertainty there is),
- o *Falsity* (how much the field differs from what's predicted).

2.1. Magnetic Field Inner-Zone (Zinner)

This is the zone where the magnetic force generated by Ψ acts completely. Within this region, the magnetic field is strong, well-defined, and exerts its full influence on susceptible materials or charges. This corresponds to the truth component $\langle \Psi \rangle$ in neutrosophy, representing the full presence of the magnetic force.

2.2. Magnetic Field Neutro-Zone (Zneutro)

This is a buffer zone between two opposites: the inner-zone and the outer-zone. In this region, the magnetic force generated by Ψ is vague, unclear, and indeterminate. It is neither fully present nor completely absent. This zone embodies the neutral or indeterminate component

\text{indeterminate} \rangle \$ in neutrosophy. It represents the transitional region where the magnetic influence diminishes, but not to zero, and its effects might be subtle, erratic, or dependent on additional factors. This is where the concept of a "fuzzy" boundary finds its mathematical representation within the neutrosophic framework.

2.3. Magnetic Field Outer-Zone (Zouter)

It is crucial to understand that, in general, there is not a steady frontier between the magnetic field inner-zone and the magnetic field outer-zone. Instead, there is a buffer zone between these opposites, which is the neutro-zone. This continuous transition is a key characteristic of the Neutrosophic Magnetic Field, offering a more accurate representation of physical reality than a sharp, binary demarcation.

3. Neutrosophic Magnetic Field Expression

$$\vec{B}_N(x, y, z) = \vec{B}(x, y, z) \cdot (T, I, F)$$

where

where

 \overrightarrow{B}_N is the neutrosophic magnetic field vector at the (x, y, z) position;

 $\vec{B}(x, y, z)$ is the classical magnetic field vector;

 $(T, I, F) \in [0,1]^3$ are the neutrosophic truth/indeterminacy/falsehood {or confidence in the field's accuracy; level of unknown or conflicting; and confidence the field is not as reported} – components associates with the estimation of \vec{B} .

4. Example of Neutrosophic Magnetic Field

If

$$\vec{B} = 0.5 \,\hat{k} \, T(Tesla)$$

and:

- T = 0.80T = 0.80 (80% true),
- I = 0.15I = 0.15 (15% indeterminate),
- F = 0.05F = 0.05 (5% false),

then:

 $\vec{B}_{N} = 0.5 \hat{k} \cdot (0.80, 0.15, 0.05)$

This indicates that the **neutrosophic field has degrees of belief** associated with its measurement, useful in uncertain or imprecise environments.

5. Interpretation of the Neutrosophic Magnetic Field

Each part of the neutrosophic field means:

 $T \cdot \vec{B}$: the portion of the field we are confident in.

 $\vec{I \cdot U}$: the uncertain part – possibly fluctuating or oscillating.

 $\vec{F \cdot N}$: the contradictory or negated field, e.g., expected interference.

6. Simulation Snippet (Pseudo-Code) of Neutrosophic Magnetic Field

```
python
# Assume B is a 3D magnetic field array, U and N are uncertainty/negation vectors
for each (x, y, z) in space:
    B = get_magnetic_field(x, y, z)
    T, I, F = get_neutrosophic_components(x, y, z)

    B_T = T * B
    B_I = I * uncertainty_vector(x, y, z)
    B_F = F * negation_vector(x, y, z)

    B_N = (B_T, B_I, B_F)
    store(B_N)
```

7. Proposed Formulae for Degrees of Neutrosophic Magnetic Influence

To conceptually illustrate the varying degrees of magnetic influence within these zones, we can propose formulae based on the principles of neutrosophy. Let M(x) represent the degree of magnetic influence at a point x in space, relative to the source Ψ . This influence can be expressed as a neutrosophic triplet:

 $M(x) = \langle TM(x), IM(x), FM(x) \rangle$

(1)

where:

TM(x) is the degree to which the magnetic force *is present* at point x.

IM(x) is the degree to which the magnetic force *is indeterminate or vague* at point x.

FM(x) is the degree to which the magnetic force *is absent* at point x.

The sum TM(x) + IM(x) + FM(x) is not necessarily equal to 1, reflecting the open nature of neutrosophy.

7.1. For the Magnetic Field Inner-Zone (Zinner)

In this zone, the magnetic force is considered fully present. Therefore, we can approximate: (2)

 $TM(x) \approx 1$, $IM(x) \approx 0$, $FM(x) \approx 0$ Thus, for $x \in Zinner$:

 $M(x) \approx \langle 1, 0, 0 \rangle$

This indicates that at any point x within the inner-zone, the magnetic influence is considered to be at its maximum and fully effective.

7.2. For the Magnetic Field Outer-Zone (Zouter)

In this zone, the magnetic force is considered completely absent. Therefore, we can approximate: $TM(x) \approx 0$, $IM(x) \approx 0$, $FM(x) \approx 1$

Thus, for $x \in$ Zouter: $M(x) \approx \langle 0, 0, 1 \rangle$

This implies that at any point x within the outer-zone, the magnetic influence is considered to be non-existent.

7.3. For the Magnetic Field Neutro-Zone (Zneutro)

This is the most complex zone, where the magnetic force is vague and indeterminate. The values of TM(x), IM(x), and FM(x) will vary continuously within this zone. Let *d* be the distance from the magnetic source Ψ . We can propose conceptual relationships that illustrate the varying degrees: As *d* increases within Zneutro (moving from the inner-zone towards the outer-zone):

- TM(d) will generally decrease from values close to 1 towards 0.
- IM(d) will initially increase from values close to 0, reach a peak in the middle of the neutro-zone, and then decrease towards 0. This peak represents the point of greatest indeterminacy.
- FM(d) will generally increase from values close to 0 towards 1.

Let dinner be the distance from Ψ to the approximate boundary of the inner-zone, and doubter be the distance from Ψ to the approximate boundary of the outer-zone.

For $d \in [dinner, douter]$, we can propose a conceptual model for the neutrosophic components. One possible approach for the relationships could be:

 $TM(d) = \cos(2\pi d_outer-d_innerd-d_inner) FM(d) = \sin(2\pi d_outer-d_innerd-d_inner) IM(d) = \sin(\pi d_outer-d_innerd-d_ind_ind_innerd-d_innerd-d_innerd-d_ind_ind_ind_ind_ind_ind_ind_$

Here, as d goes from d_inner to d_outer:

- TM(d) goes from $\cos(0) = 1$ to $\cos(\pi/2) = 0$.
- FM(d) goes from $\sin(0) = 0$ to $\sin(\pi/2) = 1$.
- IM(d) goes from sin(0) = 0, peaks at sin(π/2) = 1 (when d is exactly in the middle of the neutro-zone), and then goes back to sin(π) = 0.

Thus, for $x \in$ Zneutro (represented by distance d from Ψ):

 $M(x) = (\cos(2\pi d_outer - d_innerd - d_inner), \ \sin(\pi d_outer - d_innerd - d_inner), \ \sin(2\pi d_outer - d_innerd - d_inner))$

These proposed formulae are illustrative and conceptual, designed to capture the continuous variation and the presence of indeterminacy within the neutro-zone. They emphasize that the boundaries of the inner and outer magnetic fields are not sharp lines but rather gradual transitions characterized by degrees of truth, indeterminacy, and falsehood regarding the magnetic influence.

8. Neutrosophic Magnetic Field: Potential Implications and Applications

The introduction of the Neutrosophic Magnetic Field, with its distinct inner-zone, neutro-zone, and outer-zone, offers a profound shift in how we conceptualize and model magnetic phenomena. This neutrosophic perspective moves beyond a rigid binary classification of "magnetic field present" or "magnetic field absent," providing a framework that inherently accounts for ambiguity, uncertainty, and gradual transitions. The key strength lies in its ability to represent situations where the magnetic influence is not perfectly defined, allowing for a more accurate portrayal of complex interactions in various physical systems.

8.1. Potential Implications

The neutrosophic approach has significant implications for modelling scenarios where:

- **Boundary Effects are Critical:** Many physical interactions occur at interfaces where forces transition from one state to another. The neutro-zone provides a natural description for these often-unpredictable regions.
- Weak or Transient Fields are Involved: In cases of very weak magnetic fields or rapidly changing fields, the distinction between presence and absence can become blurred. The neutrosophic framework can better capture the partial influence in such situations.
- Quantum Mechanical Uncertainties Play a Role: At a fundamental level, quantum mechanics introduces inherent uncertainties. While the Neutrosophic Magnetic Field is a macroscopic concept, it can conceptually align with situations where the classical deterministic model falls short.
- **Complex Plasma and Fluid Dynamics:** Phenomena involving charged particles and electromagnetic fields, particularly in plasmas, often exhibit complex, non-linear behaviours at their boundaries.

We will now delve into specific applications of this concept.

9. Modelling Kelvin-Helmholtz Electron Vortex Interaction with a Magnetic Field

The Kelvin-Helmholtz (KH) instability [1, 2, 3] is a fundamental process in fluid dynamics and plasma physics, occurring at the interface between two fluids (or plasmas) moving with different velocities. In magnetized plasmas, this instability can lead to the formation of electron vortices, which are coherent structures of swirling electrons. The interaction of these electron vortices with an ambient magnetic field is a complex phenomenon, often involving particle energization and transport.

Classical models often treat the magnetic field as a uniform or smoothly varying entity, and the vortex as a distinct structure. However, at the boundaries of the electron vortex, or where the magnetic field lines are being perturbed and reconnected due to the vortex's motion, the interaction is unlikely to be a simple 'on' or 'off' effect. The magnetic field's influence on the vortex, and vice-versa, will be indeterminate in these transitional regions.

10. Neutrosophic Approach for Kelvin-Helmholtz Electron Vortex Interaction

Within the Neutrosophic Magnetic Field framework, we can model the interaction at the interface between the electron vortex and the external magnetic field more realistically.

Let's consider a region Rint where the electron vortex interacts with the magnetic field B. Instead of a sharp boundary, we define the magnetic field's influence on the vortex as a neutrosophic triplet:

 $Mvortex(x) = \langle Tvortex(x), Ivortex(x), Fvortex(x) \rangle$ Here, x is a point within Rint:(4)

Tvortex(x): Degree to which the magnetic field *strongly influences* the electron vortex dynamics at x.

Ivortex(x): Degree to which the magnetic field's influence on the electron vortex dynamics *is indeterminate or vague* at x. This captures effects like partial magnetic reconnection, localized field line perturbations, or the vortex's internal structure partially shielding parts of the field.

Fvortex(x): Degree to which the magnetic field *has no significant influence* on the electron vortex dynamics at x.

11. Proposed Conceptual Formulae for Interaction

Let's consider a simplified scenario where the interaction strength is a function of a parameter $\xi(x)$, which could be related to the local magnetic field strength, the vortex's rotational velocity, or the distance from the vortex centre.

For the central region of the vortex where the magnetic field's influence is minimal or very specific to the vortex's internal dynamics (effectively "outer-zone" for the *external field's direct influence*):

Tvortex(x) ≈ 0 (external field), Ivortex(x) ≈ 0 , Fvortex(x) ≈ 1 (external field) For regions far from the vortex where the magnetic field dominates: Tvortex(x) ≈ 1 (external field), Ivortex(x) ≈ 0 , Fvortex(x) ≈ 0

11.1. For the Interface (Neutro-Zone) where Interaction is Complex

Let's define an interaction parameter $\xi(x)$ where $\xi(x) = 0$ corresponds to no external field influence, and $\xi(x) = 1$ corresponds to strong external field influence. This $\xi(x)$ could be, for example, a normalized function of distance from the vortex core or the local magnetic shear.

We could propose:

Tvortex(x) = $\xi(x)(1$ -Ithreshold), Fvortex(x) = $(1-\xi(x))(1$ -Ithreshold), Ivortex(x) = Ithreshold + $k \cdot \xi(x)(1-\xi(x))$

where:

Ithreshold represents a baseline level of indeterminacy inherent in the system's complexity (e.g., due to quantum effects or highly turbulent conditions), with $0 \le$ Ithreshold < 1.

k is a scaling factor controlling the peak indeterminacy due to the interaction, with $0 \le k < 1$ -Ithreshold.

The term $\xi(x)(1-\xi(x))$ ensures Ivortex(x) peaks when $\xi(x)$ is around 0.5, representing the most ambiguous interaction point.

This model suggests that even when the external field influence is not perfectly clear (due to the Ithreshold term), the interaction itself adds to the indeterminacy, especially when the vortex and field are equally "competing" or influencing each other. This allows for modelling scenarios where the electron vortex is partially disrupted, reconnected, or its energy is transferred inefficiently due to the ambiguous nature of the magnetic field at its boundaries.

12. Modelling Paramagnetic and Diamagnetic Magnetic Properties in Basic Physics of Magnets

Paramagnetism and Diamagnetism describe how materials respond to an external magnetic field.

- **Diamagnetism:** Materials that are weakly repelled by an external magnetic field. This arises from the orbital motion of electrons, which generate a magnetic moment opposing the applied field.
- **Paramagnetism:** Materials that are weakly attracted to an external magnetic field. This arises from the presence of unpaired electrons, whose spin magnetic moments align with the applied field.

In basic physics, these properties are often described by a magnetic susceptibility (χ m), a single value that dictates the material's response. However, for very weak fields, or at the transition points where the field is barely strong enough to induce a measurable effect, the response might not be perfectly clear-cut. Furthermore, the bulk behaviour of a material is an aggregation of individual atomic or molecular responses, which might vary.

13. Neutrosophic Approach for Paramagnetic and Diamagnetic Properties

We can consider the "degree of magnetization" (Mmag) of a material in response to an external magnetic field Bext as a neutrosophic triplet:

Mmag(Bext) = (Tmag(Bext), Imag(Bext), Fmag(Bext))
where for a given material:

- Tmag(Bext): Degree to which the material *exhibits its characteristic magnetic response* (paramagnetic attraction or diamagnetic repulsion) to Bext.
- Imag(Bext): Degree to which the material's magnetic response to Bext *is indeterminate or vague*. This could be due to:
 - Very weak fields near the detection limit.
 - o Temperature effects leading to thermal agitation overcoming weak alignments.
 - Microscopic inhomogeneities in the material.
 - o Transition phases where the dominant magnetic behaviour is changing.
- Fmag(Bext): Degree to which the material *does not exhibit its characteristic magnetic response* to Bext (i.e., the field is too weak to induce a noticeable effect, or competing effects cancel out).

13.1. Note

While the above formulae (5) introduces a neutrosophic triplet for "degree of magnetization" as Mmag(Bext) = (Tmag(Bext), Imag(Bext), Fmag(Bext)),

it doesn't inherently present it in terms of differential calculus expressions.

To describe this in terms of differential calculus, we would need to consider how these degrees of magnetization change with respect to the external magnetic field (Bext). This would involve taking derivatives of each component of the neutrosophic triplet.

Here's how we can express the concept using differential calculus, assuming that Tmag, Imag, and Fmag are functions of Bext and are differentiable:

The rate of change of the degree of magnetization with respect to the external magnetic field can be expressed by the derivatives of each component of the neutrosophic triplet:

dBextdMmag = { dBextdTmag(Bext), dBextdImag(Bext), dBextdFmag(Bext) } (6)

(5)

where:

- dBextdTmag(Bext) represents the rate of change of the degree of presence of magnetization as the external magnetic field changes.
- dBextdImag(Bext) represents the rate of change of the degree of indeterminacy or vagueness
 of magnetization as the external magnetic field changes.
- dBextdFmag(Bext) represents the rate of change of the degree of absence of magnetization as the external magnetic field changes.

This differential form allows us to analyse how the certainty, uncertainty, and absence of magnetization evolve with variations in the applied magnetic field. For instance:

- For paramagnetic materials, we might expect dBextdTmag to be positive, indicating an increase in magnetization with increasing external field, while dBextdImag and dBextdFmag might be relatively small or show specific patterns.
- For diamagnetic materials, dBextdTmag might be negative, suggesting a decrease in magnetization (or an induced opposing field) with increasing external field.

It's important to note that this is a conceptual application of differential calculus to a neutrosophic framework. The specific functional forms of Tmag(Bext), Imag(Bext), and Fmag(Bext) would need to be defined based on experimental data or a theoretical model of the material's neutrosophic magnetic response.

14. Proposed Conceptual Formulae for Magnetic Response

Let's define a normalized external field strength b = |Bext|/Bmax, where Bmax is a saturation field or a field strong enough to elicit a clear response.

14.1. For Paramagnetic Materials

As *b* increases, Tmag should increase (alignment with field), Fmag should decrease (no alignment), and Imag should be highest at intermediate field strengths.

Tmag(b) = bk (for $k \ge 1$, indicating non-linear response for some materials)

Fmag(b) = (1-b)m (for $m \ge 1$)

Imag(b) = $C \cdot b(1-b)$ (where C is a constant controlling maximum indeterminacy, possibly related to thermal energy)

A more refined model could involve a threshold field Bth below which the response is largely indeterminate:

If $|\text{Bext}| < \text{Bth: Tmag(Bext)} \approx \epsilon T$ (very small, near zero), $\text{Fmag(Bext)} \approx \epsilon F$ (very small, near zero), $\text{Imag(Bext)} \approx 1-(\epsilon T + \epsilon F)$ (dominant indeterminacy)

If Bext 2 Bth:	
Let $\delta = (\text{Bext} -\text{Bth})/(\text{Bmax}-\text{Bth})$ for $\text{Bth} \le \text{Bext} \le \text{Bmax}$.	(7)
$Tmag(Bext) = \delta Imag(Bext) = (1-\delta) \cdot exp(-\alpha\delta)$	(8)
(indeterminacy decreases as response becomes clearer)	
Fmag(Bext) = 0	(9)
(once above threshold, characteristic response is present)	
Here, α is a parameter controlling the rate of decrease of indeterminacy.	

14.1. For Diamagnetic Materials

The response is largely independent of temperature and generally weaker than paramagnetism. The effect is usually present even at very weak fields, so the neutro-zone might be narrower.

 $Tmag(b) = bFmag(b) = 1-bImag(b) = C \cdot b(1-b)$ (similar to paramagnetic, but C might be smaller)

The neutrosophic approach acknowledges that even for diamagnetic materials, there might be a very small field region where the induced opposing dipole is not perfectly formed or clearly measurable, leading to a degree of indeterminacy.

5. Concluding Remark

The concept of a Neutrosophic Magnetic Field offers a more refined and realistic description of magnetic phenomena, particularly in regions where the magnetic force is not unequivocally present or absent. By acknowledging the existence of a neutro-zone, a region of vagueness and indeterminacy, this framework moves beyond traditional binary representations. The proposed conceptual formulae provide a mathematical means to quantify the degrees of magnetic influence across the inner, neutro, and outer zones, laying the groundwork for further theoretical and potentially experimental investigations into the subtle and complex nature of magnetic fields. This neutrosophic approach opens new avenues for understanding and modelling physical systems where uncertainty, confliction, incompleteness occur.

Its application includes how to complex scenarios like Kelvin-Helmholtz electron vortex interaction allows for a more nuanced description of boundary effects and indeterminate influences, moving beyond classical all-or-nothing interactions. Similarly, for basic material properties like paramagnetism and diamagnetism, the neutrosophic model can describe the transition from no clear response to a full characteristic response, accounting for the vagueness at thresholds and in weak field regimes. While the proposed formulae are conceptual and illustrative, they demonstrate the potential for using neutrosophic triplets to quantify the degrees of truth, indeterminacy, and falsehood in various magnetic interactions, paving the way for more sophisticated and accurate models in areas ranging from plasma physics to condensed matter physics.

Further experimental and theoretical work are needed to validate and refine these neutrosophic models, ultimately enhancing our understanding of the intricate world of magnetism.

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