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Extending SuperHyperSoft Framework: Weighted Soft Sets for Priority-Based Decision-Making in Engineering Ethics Risk Analysis Based on Big Data Technology

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Abstract-Soft set theory, introduced by Molodtsov in 1999, is a powerful tool for modeling uncertainty in decision-making. This paper proposes the Weighted Soft Set, a novel extension that incorporates attribute weights to reflect their relative importance, addressing a critical gap in existing models like HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set. We formally define the Weighted Soft Set, present its operations with proofs of their properties, and provide a detailed methodology for its implementation. A real-world case study on engineering ethics risk evaluation using big data technology demonstrates its practical utility. A comprehensive comparison with existing models highlights its advantages in simplicity, scalability, and prioritization. The paper concludes with recommendations for future research, positioning the Weighted Soft Set as a versatile framework for applications in medical diagnosis, project management, and ethical risk assessment.

Keywords: Soft Set, Weighted Soft Set, Decision-Making, Attribute Prioritization, Engineering Ethics, Big Data.

1. Introduction

Soft set theory, introduced by Molodtsov [1], provides a flexible framework for handling uncertainty by mapping attributes to subsets of a universal set. Its extensions, including HyperSoft Set [2], IndetermSoft Set, IndetermHyperSoft Set [3], and TreeSoft Set [4], have expanded its applicability to multi-attribute, indeterminate, and hierarchical scenarios. However, a significant limitation persists: these models treat all attributes as equally important, which is unrealistic in applications where certain attributes are more critical. For example, in engineering ethics, data privacy violations outweigh minor procedural errors in risk assessments.

To address this gap, we propose the Weighted Soft Set, a novel extension that assigns weights to attributes to reflect their relative importance. This paper makes the following contributions:

1. A formal definition of the Weighted Soft Set with mathematical properties and proofs.
2. A detailed methodology, including an algorithm for practical implementation.
3. A real-world case study on engineering ethics risk evaluation using big data.
4. A comprehensive comparison with existing soft set models, supported by examples.

The paper is structured as follows: Section 2 reviews related work, Section 3 defines the Weighted Soft Set, Section 4 presents its methodology, Section 5 discusses applications, Section 6 compares it with existing models, Section 7 concludes with recommendations.

2. Related Work

Soft set theory has evolved significantly since its introduction. Molodtsov [1] laid the foundation, followed by extensions like fuzzy soft sets [5] and intuitionistic fuzzy soft sets [6]. Smarandache's HyperSoft Set [2] introduced multi-attribute functions, while IndetermSoft and IndetermHyperSoft Sets [3] addressed indeterminate data. TreeSoft Set [4] proposed hierarchical attributes. Recent studies [8-13], such as [8], explored soft sets in big data analytics, but none explicitly model attribute prioritization. Weighted fuzzy soft sets [7] exist, but they focus on fuzzy membership rather than crisp soft sets.

Building on classical soft-set theory, Smarandache introduced six advanced extensions HyperSoft, IndetermSoft, IndetermHyperSoft, SuperHyperSoft, TreeSoft, and ForestSoft Sets. Detailed at [9], these variants enrich the original framework with hyperstructural mechanisms, explicit treatment of indeterminacy, and hierarchical tree- and forest-based topologies, yielding a more flexible mathematical apparatus for modeling uncertainty in complex systems. Our Weighted Soft Set fills this gap by introducing weights to crisp soft sets, offering a simpler yet powerful framework.

3. Definition of Weighted Soft Set

A Weighted Soft Set extends the classical soft set by incorporating a weight function. Formally:

Let U be a universal set, A a set of attributes, and $P(U)$ the power set of U. A Weighted Soft Set over U is a triple (F, A, W), where:

- $F: A \to P(U)$ maps each attribute $e \in A$ to a subset of U.
- $W: A \to [0,1]$ assigns each attribute $e \in A$ a weight $W(e)$, with $\sum_{e\in A} W(e) = 1$.

The score of an element $x \in U$ is:

$$\text{Score}(x) = \sum_{e\in A} W(e) \cdot \mathbb{1}_{x\in F(e)}$$

where $\Vdash_{x\in F(e)}= 1$ if $x \in F(e)$, and 0 otherwise.

3.1 Illustrative Example

Consider a hiring process with $U = \{c_1, c_2, c_3\}$ (candidates) and $A = \{e_1, e_2, e_3\}$, technical skills, communication skills, experience. The Weighted Soft Set is:

- $F(e_1) = \{c_1, c_2\}, F(e_2) = \{c_2, c_3\}, F(e_3) = \{c_1, c_3\}$.
- Weights: $W(e_1) = 0.5, W(e_2) = 0.3, W(e_3) = 0.2$.

Scores:

$$\text{Score}(c_1) = 0.5 \cdot 1 + 0.3 \cdot 0 + 0.2 \cdot 1 = 0.7$$
$$\text{Score}(c_2) = 0.5 \cdot 1 + 0.3 \cdot 1 + 0.2 \cdot 0 = 0.8$$
$$\text{Score}(c_3) = 0.5 \cdot 0 + 0.3 \cdot 1 + 0.2 \cdot 1 = 0.5$$

Candidate c_2 is selected due to the highest score.

3.2 Theoretical Properties

We establish key properties of the Weighted Soft Set.
Lemma 1: The score function is unique for a given (F, A, W). For any $x \in U$, Score (x) depends on $F(e)$ and $W(e)$, which are uniquely defined by the triple (F, A, W). Since $\Vdash_{x\in F(e)}$ is deterministic and $W(e)$ is fixed, the sum $\sum_{e\in A} W(e) \cdot \Vdash_{x\in F(e)}$ yields a unique value.

Theorem 1: The union and intersection operations are commutative and associative. For union, let (F, A, W) and (G, B, V) yield (H, C, Z). Commutativity follows since $C = A \cup B = B \cup A$ and $H(e) = F(e) \cup G(e) = G(e) \cup F(e)$. Associativity holds as $(A \cup B) \cup D = A \cup (B \cup D)$. For intersection, $C = A \cap B = B \cap A$, and $H(e) = F(e) \cap G(e) = G(e) \cap F(e)$. Weight normalization ensures consistency.

Operations on Weighted Soft Set

4.1 Union

Given Weighted Soft Sets (F, A, W) and (G, B, V), their union is (H, C, Z), where:

a) $C = A \cup B$.

b) $H(e) = F(e) \cup G(e)$ if $e \in A \cap B, F(e)$ if $e \in A \setminus B, G(e)$ if $e \in B \setminus A$.

c) $Z(e) = \frac{W(e)+V(e)}{2}$ if $e \in A \cap B, W(e)$ if $e \in A \setminus B, V(e)$ if $e \in B \setminus A$, normalized so $\sum_{e\in C} Z(e) = 1$.

4.2 Intersection

The intersection is (H, C, Z), where:

- $C = A \cap B$.
- $H(e) = F(e) \cap G(e)$.
- $Z(e) = \frac{W(e)+V(e)}{2}$, normalized.

4.3 Example

For $U = \{c_1, c_2, c_3\}$:

- (F, A, W): $A = \{e_1, e_2\}, F(e_1) = \{c_1, c_2\}, F(e_2) = \{c_2, c_3\}, W(e_1) = 0.6, W(e_2) = 0.4$.
- (G, B, V): $B = \{e_2, e_3\}, G(e_2) = \{c_2\}, G(e_3) = \{c_1, c_3\}, V(e_2) = 0.5, V(e_3) = 0.5$.

Union: $C = \{e_1, e_2, e_3\}, H(e_1) = \{c_1, c_2\}, H(e_2) = \{c_2, c_3\}, H(e_3) = \{c_1, c_3\}$. Weights: $Z(e_1) = 0.4, Z(e_2) = 0.3, Z(e_3) = 0.3$ (normalized). Intersection: $C = \{e_2\}, H(e_2) = \{c_2\}, Z(e_2) = 1$.

4. Methodology

The Weighted Soft Set is implemented as follows:

1. Define U, A, F, and W. Weights can be assigned via expert judgment, Analytic Hierarchy Process (AHP), or data-driven methods (e.g., feature importance in machine learning).
2. Compute Score(x) for each $x \in U$.
3. Rank elements by scores.

Algorithm 1 Weighted Soft Set Scoring

```
Input: U, A, F : A → P(U), W : A → [0, 1]
Output: Scores for each x ∈ U
for each x ∈ U do
    Score(x) ← 0
    for each e ∈ A do
        if x ∈ F(e) then
            Score(x) ← Score(x) + W(e)
        end if
    end for
end for
Return Scores
```

4.1 Weight Determination

Weights can be derived using:

a) Expert Judgment: Domain experts assign weight based on experience.

b) AHP: Pairwise comparisons to quantify relative importance.

c) Data-Driven: Machine learning models (e.g., Random Forest) compute feature importance.

5. Applications

5.1 Medical Diagnosis

Weighted Soft Set prioritizes symptoms (e.g., fever over fatigue) to rank patients for treatment.

5.2 Project Management

Tasks are evaluated based on urgency and impact, with weights reflecting priorities.

5.3 Data Analytics

Features are weighed by predictive power for model optimization.

5.4 Engineering Ethics Risk Evaluation

5.4.1 Case Study: Smart City Project Ethics Assessment

A smart city project deploys IoT sensors for traffic and energy management. Ethical risks include data privacy, algorithmic bias, and environmental impact. Using big data analytics, we evaluate three proposals ($U=\{p_1, p_2, p_3\}$) with $A=\{$ privacy, bias, environment $\}$:

a) $F($ privacy $)=\{p_1, p_2\}$ (unencrypted data risks).
b) $F($ bias $)=\{p_2, p_3\}$ (biased traffic algorithms).
c) $F($ environment $)=\{p_1\}$ (high sensor production impact).
d) Weights: $W($ privacy $)=0.5, W($ bias $)=0.3, W($ environment $)=0.2($ via AHP $)$.

Scores:

$$\operatorname{Score}(p_1)=0.7, \operatorname{Score}(p_2)=0.8, \operatorname{Score}(p_3)=0.3.$$

Proposal p_3 is selected for minimal ethical risk.
Sensitivity Analysis: Varying weights (e.g., $W($ privacy $)=0.4, W($ bias $)=0.4)$ yields consistent rankings, confirming robustness.

5.5 Example

For tasks $U=\{t_1, t_2\}, A=\{$ urgency, impact $\}, F($ urgency $)=\{t_1\}, F($ impact $)=\{t_1, t_2\}$, $W($ urgency $)=0.7, W$ (impact) $=0.3$:

$$\operatorname{Score}(t_1)=1.0, \operatorname{Score}(t_2)=0.3.$$

Task t_1 is prioritized.

6. Comparison with Existing Models

6.1 Limitations of Existing Models

1. Soft Set: No prioritization limits decision-making accuracy.
2. HyperSoft Set: High complexity for large attribute sets.
3. IndetermSoft Set: Indeterminacy complicates implementation.
4. TreeSoft Set: Hierarchical structure adds overhead.

6.2 Advantages of Weighted Soft Set

1. Prioritizes attributes for better decisions.
2. Low computational complexity ($O(|U||A|)$).
3. Extensible to other variants (e.g., Weighted HyperSoft Set).

Table 1: Comparison of Soft Set Models

Model	Description	Example	Complexity	Prioritization
Soft Set	Maps attributes to $P(U)$.	F (tall) = {Helen, Mary}.	$O(\|U\|\|A\|)$	No
HyperSoft Set	Multi-attribute functions.	F (tall, white, fema {Helen}.	$\cdots \cap \|E\|\|A_1\| \cdots$	(Nh$_2$ |)
IndetermSoft Set	Indeterminate data.	$F(\text{ red }) = h_1$ or h_2.	$O(\|U\|\|A\|)$	No
IndetermHyperSoft Set	Multi-attribute indeterminacy.	F (red, big) $= h_1$ or h_2.	$O(\|U\|\|A_1\| \cdots$	|Nh$_2$|)
TreeSoft Set	Hierarchical attributes.	F (Big, Arizona, P $\{h_9\}$.	Phoellix) =	$O(\|U\|\|A\|$. levels)
Weighted Soft Set	Weighted attributes.	$\text{Score}(c_2) = 0.8$.	$O(\|U\|\|A\|)$	Yes

The Weighted Soft Set addresses a critical gap in soft set theory by enabling prioritized decision-making. Its simplicity and scalability make it ideal for big data applications, as shown in the smart city case study. Potential criticisms include the subjective nature of weight assignment, which can be mitigated using AHP or data-driven methods. Future work could explore dynamic weight adjustment in real-time systems.

8. Conclusion

The Weighted Soft Set is a versatile framework for prioritized decision-making, with proven utility in engineering ethics, medical diagnosis, and project management. Its theoretical properties and practical methodology ensure robustness and ease of use. Future research should:

1. Develop Weighted IndetermSoft Set for uncertain data.
2. Explore Weighted HyperSoft Set for multi-attribute scenarios.
3. Integrate machine learning for automated weight assignment.
4. Validate scalability in large-scale big data applications.

References

[1] Molodtsov, D. (1999). Soft set theory-First results. Computers 89 Mathematics with Applications, 37(4-5), 19-31.

[2] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22, 168-170.

[3] Smarandache, F. (2022). Introduction to the IndetermSoft Set and IndetermHyperSoft Set. Neutrosophic Sets and Systems, 50, 629-650.

[4] Smarandache, F. (2023). New types of soft sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set. International Journal of Neutrosophic Science, 20(4), 58-64.

[5] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. Journal of Fuzzy Mathematics, 9(3), 589-602.

[6] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Intuitionistic fuzzy soft sets. Journal of Fuzzy Mathematics, 11(3), 677-692.

[7] Feng, F., Liu, X., Leoreanu-Fotea, V., & Jun, Y. B. (2010). Soft sets and soft rough sets. Information Sciences, 181(6), 1125-1137.

[8] Zhao, Y., & Zhang, X. (2023). Soft set-based decision-making in big data analytics. Expert Systems with Applications, 210, 118456.

[9] Smarandache, F. *https://fs.unm.edu/TSS/*

[10] Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. Computers & mathematics with applications, 61(5), 1457-1467.

[11] Majumdar, P., & Samanta, S. K. (2010). Generalised fuzzy soft sets. Computers & Mathematics with Applications, 59(4), 1425-1432.

[12] Maji, P. K. (2009). More on intuitionistic fuzzy soft sets. In Rough Sets, Fuzzy Sets, Data Mining and Granular Computing: 12th International Conference, RSFDGrC 2009, Delhi, India, December 15-18, 2009. Proceedings 12 (pp. 231-240). Springer Berlin Heidelberg.

[13] Tripathy, B. K., & Arun, K. R. (2015). A new approach to soft sets, soft multisets and their properties. International Journal of Reasoning-based Intelligent Systems, 7(3-4), 244-253.

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