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Evaluation of Practice-Based Curriculum Objectives Achievement Degree Using Einstein Aggregation Operators of Single-Valued Neutrosophic Credibility Numbers

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Abstract: Currently, student-centered and outcome-based engineering education accreditation is being actively promoted in universities across China. Evaluation of the curriculum objectives achievement degree serves as an effective method after OBE-based courses teaching to assess the improvement of students' abilities, evaluate education quality, and facilitate self-reflection of teaching by instructors. The process of evaluating curriculum objectives involves a complex multiple-attribute decision-making (MADM) scenario, often accompanied by elements of vagueness, uncertainty, and inconsistency. The application of single-valued neutrosophic credibility numbers sets (SvNCNs) offers a robust approach to handle and represent uncertain information throughout this evaluation process. Therefore, to enhance the accuracy of course achievement evaluation, this paper proposes a MADM framework based on SvNCNs, integrated with improved Einstein aggregation operators, to the achievement degree of practice-based curriculum objectives evaluation. The method is applied to the practice-based curriculum and is further compared and analyzed with other classical methods to show the efficiency of the proposed method, which will assist decision-makers in making better decisions when dealing with similar MADM assessment problems.

Keywords: single-valued neutrosophic credibility number; weighted averaging operator of Einstein; weighted geometric operator of Einstein; curriculum objectives achievement evaluation

1. Introduction

Multiple attribute decision-making (MADM) problems are inevitable aspect of daily life. Due to the inherent fuzziness and uncertainty in MADM, researchers have been actively exploring methods to achieve optimal decisions. Fuzzy theory [1] was first proposed by Zadeh, which was described by a membership function. Based on this theory, its applications have obtained a lot of advancements. In the domain of fuzzy decision-making (DM), Greco and Matarazzo combined rough sets and fuzzy sets in solving MADM problems [2]. Merigo et al. introduced generalized fuzzy aggregation operators [3] and applied them in multi-person decision-making scenarios. Kirisci proposed the Fermatean hesitant fuzzy set [4]. Ajay et al. introduced a Spherical Fuzzy Weighted Exponential Average (SFWEA) aggregation operator [5], which was applied in MADM for psychotherapy. Atanassov and Stoeva extended fuzzy sets by proposing the intuitionistic fuzzy set (IFS) model [6], which characterized membership in terms of support, opposition, and neutrality.

Based on IFS, scholars have proposed various aggregation operators and improved DM methods such as trapezoidal IFS [7-9], intuitionistic fuzzy rough sets [10-12], generalized Pythagorean fuzzy sets [13-15], optimized Aggregation operators of Einstein [16-17] and Dombi [18-19] for MADM. However, IFS is limited in that it cannot independently express truth, false, and uncertainty. This limitation was addressed by Smarandache's introduction of neutrosophic sets (NSs) [20]，and further extend the application of this theory[21-22]. Building upon the theory of neutrosophic sets, Muniba K, et al. proposed a climate change prediction framework that integrates neutrosophic soft functions [23], significantly enhancing the accuracy of climate change prediction models. Additionally, the theory of neutrosophic topological spaces based on NSs has seen significant progress. A. A. Salama, et al. investigated the application of Non-Standard Topology Sets (NTS) in uncertainty modeling within the field of computer science [24]. Meanwhile, G. Vetrivel, et al., explored the forgotten topological index (ToI) and the edge forgetting index within the context of three-valued logic intelligence graphs, deriving important theoretical results and applications [25].

Ye et al. presented fuzzy credibility sets (FCSs) and its operational laws [26]. On the basis of FCSs, Ye et al. further proposed a single-value neutrosophic credibility numbers sets (SvNCNs) and the trigonometric aggregation operators for SvNCNs [27], which defined credibility levels of the true, false, and uncertainty component of fuzzy values, enhancing the reliability of fuzzy values in MADM processes. This paper presents an aggregation operator method based on Einstein's t-conorm and t-norm for geometric weighted and arithmetic weighted aggregation of SvNCN sets, and the methods are applied to the achievement degree of practice-based curriculum objectives evaluation. The organization of this article is as below.

The Outcome-Based Education teaching model is being increasingly adopted by universities in China, and course achievement assessment serves as an effective method for evaluating the effectiveness of the OBE teaching model. However, during the achievement assessment process, particularly in engineering practice-based courses, there are significant inconsistencies and uncertainties in the attributes of the data being evaluated. As a result, the assessment outcomes tend to be influenced by the evaluator's subjective opinions, which makes it difficult to accurately reflect the teaching effectiveness. In such uncertain and inconsistent environments SvNCNs offer an advantage, as they not only represent true, false, and uncertain information but also express their credibility. In contrast, methods such as rough sets, IVIFS, and QROF are unable to express these types of information.

The Einstein aggregation operator is one of the most fundamental and widely recognized algebraic operations. It is a norm characterized by its nonlinear properties, which provides significant advantages in its strong ability to handle nonlinear processing and adaptability. This makes it especially effective for solving complex fuzzy problems, particularly excelling in MADM. In comparison, the Aczel-Alzina, Frank, and Dombi norms also offer advantages in specific applications. However, when dealing with more complex nonlinear relationships and multi-dimensional fuzzy data, the Einstein norm typically yields better results. Therefore, this paper proposes an improved aggregation operator method based on Einstein's t-conorm and t-norm, built upon SvNCNs. This method is then applied to evaluate the achievement of practical course objectives in the Electrical Engineering program at Shaoxing University. The structure of this paper is outlined as follows.

Section2 introduces the theory of SvNCNs. Section3 shows the operational laws and proof processes of Einstein t-conorm and t-norm aggregation operators on SvNCNs. Section 4 elaborates on the application steps in which SvNCNEWG (Einstein weighted geometric operator of SvNCNs) and SvNCNAWG (Einstein weighted arithmetic operator of SvNCNs) are applied to MADM. Section 5 gives an application example of the presented method in curriculum objective achievement degree evaluation and a comparative analysis with other techniques. The conclusion is provided in the final section.

2. SvNCNs

Ye et al. presented SvNCNs to enhance the credibility levels of SvNSs, along with the definition and basic operational rules of SvNCNs.

Definition 1. Set $V=\{v_1,v_2,...,v_n\}$ as a finite universe set, then a SvNCN N_c in V is expressed below:

$$N_c=\{\langle v,(a_r(v),c_r(v)),(a_i(v),c_i(v)),(a_e(v),c_e(v))\rangle|v\in V\} \tag{1}$$

In the above equation $V\to[0,1]^2$, $(a_r(v),c_r(v)),(a_i(v),c_i(v)),(a_e(v),c_e(v))$ represent the true FCN, uncertain FCN, and false FCN, respectively. Int the three ordered pairs, the first component $a_j(v)\{j=1,2,3\}$ represents the fuzzy value, and the second component $c_i(v)\{i=1,2,3\}$ represents the credibility level closely associated with the first component, which is used to ensure the reliable measurement of the first component. Furthermore, N_c satisfies the conditions $0\le a_r(v)+a_i(v)+a_e(v)\le 3$ and $0\le c_r(v)+c_i(v)+c_e(v)\le 3$.

The simplified representation of the formula (1) is expressed as:

$$n_c=\langle(a_r,c_r),(a_i,c_i),(a_e,c_e)\rangle \tag{2}$$

Definition 2. Set two SvNCN, $n_{c1}=\langle(a_{r1},c_{r1}),(a_{i1},c_{i1}),(a_{e1},c_{e1})\rangle$ and $n_{c2}=\langle(a_{r2},c_{r2}),(a_{i2},c_{i2}),(a_{e2},c_{e2})\rangle$. Their mutual relationships and operational rules between them are defined as below:

(1) $n_{c1}\supseteq n_{c2}\Leftrightarrow a_{r1}>a_{r2},c_{r1}>c_{r2},a_{i2}<a_{i2},c_{i1}<c_{i2},a_{e2}<a_{e2},c_{e1}<c_{e2}$;

(2) $n_{c1}=n_{c2}\Leftrightarrow n_{c1}\supseteq n_{c2}$ and $n_{c2}\supseteq n_{c1}$;

(3) $n_{c1}\bigcup n_{c2}=\langle(a_{r1}\vee a_{r2},c_{r1}\vee c_{r2}),(a_{i1}\wedge a_{i2},c_{i1}\wedge c_{i2}),(a_{e1}\wedge a_{e2},c_{e1}\wedge c_{e2})\rangle$;

(4) $n_{c1}\bigcap n_{c2}=\langle(a_{r1}\wedge a_{r2},c_{r1}\wedge c_{r2}),(a_{i1}\vee a_{i2},c_{i1}\vee c_{i2}),(a_{e1}\vee a_{e2},c_{e1}\vee c_{e2})\rangle$;

Definition 3. For two SvNCN $n_{cj}=\langle(a_{rj}a_{rj},c_{rj}c_{rj}),(a_{ij}a_{ij},c_{ij}c_{ij}),(a_{ej}a_{ej},c_{ej}c_{ej})\rangle$ $j=1,2$, their sorting is determined by the score function and accuracy function, which are defined below:

$$S(n_{cj})=\frac{2+a_{rj}c_{rj}-a_{ij}c_{ij}-a_{ej}c_{ej}}{3}\ for\ S(n_{cj})\in[0,1] \tag{3}$$

$$A(n_{cj})=\alpha_{rj}\beta_{rj}-\alpha_{ej}\beta_{ej}\ for\ A(n_{cj})\in[-1,1] \tag{4}$$

The score values $S(n_{cj})$ and accuracy values $A(n_{cj})$ calculated using the above two formulas are ranked according to the following criteria:

(i) If $S(n_{c1})>S(n_{c2})$, the sorting oder is $n_{c1}>n_{c2}$;

(ii) If $S(n_{c1})=S(n_{c2})$ & $A(n_{c1})>A(n_{c2})$, the oder is $n_{c1}>n_{c2}$;

(iii) If $S(n_{c1})=S(n_{c2})$ & $A(n_{c1})=A(n_{c2})$, both is $n_{c1}\cong n_{c2}$.

3. Modified Aggregation Operators of Einstein for SvNCNs

3.1 Einstein paradigm operations of SvNCNs

Definition 4. The Einstein t-norm function $\psi(\upsilon,\omega)$ and t-conorm function $\psi^C(\upsilon,\omega)$ are defined as below, which υ and ω are real numbers within the range [0, 1].

$$\psi(\upsilon,\omega)=\frac{\upsilon\omega}{1+(1-\upsilon)(1-\omega)} \tag{5}$$

$$\psi^C(\upsilon,\omega)=\frac{\upsilon+\omega}{1+\upsilon\omega} \tag{6}$$

Both $\psi(\upsilon,\omega)$ and $\psi^C(\upsilon,\omega)$ are monotone increasing functions, with their values ranging within [0, 1]. Based on the formulas (5) and (6), the operational rules for SvNCNs are defined as below.

Definition 5. Set $\rho_1=\langle(a_{r1},c_{r1}),(a_{i1},c_{i1}),(a_{e1},c_{e1})\rangle$ and $\rho_2=\langle(a_{r2},c_{r2}),(a_{i2},c_{i2}),(a_{e2},c_{e2})\rangle$ are two SvNCNs, ρ_1 and $\rho_2\in[0,1]$, φ is the weight value. The operations and relationships between ρ_1 and ρ_2 are defined as follows:

$$\rho_1\oplus\rho_2=\left\langle\begin{array}{l}\left(\frac{a_{r1}+a_{r2}}{1+a_{r1}a_{r2}}-\frac{a_{r1}a_{r2}}{1+(1-a_{r1})(1-a_{r2})},\frac{c_{r1}+c_{r2}}{1+c_{r1}c_{r2}}-\frac{c_{r1}c_{r2}}{1+(1-c_{r1})(1-c_{r2})}\right),\left(\frac{a_{i1}a_{i2}}{1+(1-a_{i1})(1-a_{i2})},\frac{c_{i1}c_{i2}}{1+(1-c_{i1})(1-c_{i2})}\right),\\ \left(\frac{a_{e1}a_{e2}}{1+(1-a_{e1})(1-a_{e2})},\frac{c_{e1}c_{e2}}{1+(1-c_{e1})(1-c_{e2})}\right)\end{array}\right\rangle \tag{7}$$

$$\rho_1\otimes\rho_2=\left\langle\begin{array}{l}\left(\frac{a_{r1}a_{r2}}{1+(1-a_{r1})(1-a_{r2})},\frac{c_{r1}c_{r2}}{1+(1-c_{r1})(1-c_{r2})}\right),\left(\frac{a_{i1}+a_{i2}}{1+a_{i1}a_{i2}}-\frac{a_{i1}a_{i2}}{1+(1-a_{i1})(1-a_{i2})},\frac{b_{i1}+b_{i2}}{1+b_{i1}b_{i2}}-\frac{b_{i1}b_{i2}}{1+(1-b_{i1})(1-b_{i2})}\right),\\ \left(\frac{a_{e1}+a_{e2}}{1+a_{e1}a_{e2}}-\frac{a_{e1}a_{e2}}{1+(1-a_{e1})(1-a_{e2})},\frac{c_{e1}+c_{e2}}{1+c_{e1}c_{e2}}-\frac{c_{e1}c_{e2}}{1+(1-c_{e1})(1-c_{e2})}\right)\end{array}\right\rangle \tag{8}$$

$$\varphi\rho_1=\left\langle\begin{array}{l}\left(\frac{(1+a_{r1})^{\varphi}-(1-a_{r1})^{\varphi}}{(1+a_{r1})^{\varphi}+(1-a_{r1})^{\varphi}},\frac{(1+c_{r1})^{\varphi}-(1-c_{r1})^{\varphi}}{(1+c_{r1})^{\varphi}+(1-c_{r1})^{\varphi}}\right),\left(\frac{2(a_{i1})^{\varphi}}{(2-a_{i1})^{\varphi}+(a_{i1})^{\varphi}},\frac{2(c_{i1})^{\varphi}}{(2-c_{i1})^{\varphi}+(c_{i1})^{\varphi}}\right),\\ \left(\frac{2(a_{e1})^{\varphi}}{(2-a_{e1})^{\varphi}+(a_{e1})^{\varphi}},\frac{2(c_{e1})^{\varphi}}{(2-c_{e1})^{\varphi}+(c_{e1})^{\varphi}}\right)\end{array}\right\rangle \tag{9}$$

$$\rho_1^{\varphi}=\left\langle\begin{array}{l}\left(\frac{2(a_{r1})^{\varphi}}{(2-a_{r1})^{\varphi}+(a_{r1})^{\varphi}},\frac{2(c_{r1})^{\varphi}}{(2-c_{r1})^{\varphi}+(c_{r1})^{\varphi}}\right),\left(\frac{(1+a_{i1})^{\varphi}-(1-a_{i1})^{\varphi}}{(1+a_{i1})^{\varphi}+(1-a_{i1})^{\varphi}},\frac{(1+c_{i1})^{\varphi}-(1-c_{i1})^{\varphi}}{(1+c_{i1})^{\varphi}+(1-c_{i1})^{\varphi}}\right),\\ \left(\frac{(1+a_{e1})^{\varphi}-(1-a_{e1})^{\varphi}}{(1+a_{e1})^{\varphi}+(1-a_{e1})^{\varphi}},\frac{(1+c_{e1})^{\varphi}-(1-c_{e1})^{\varphi}}{(1+c_{e1})^{\varphi}+(1-c_{e1})^{\varphi}}\right)\end{array}\right\rangle \tag{10}$$

3.2 Operator of Weighted Arithmetic Average and Weighted Geometric Average of Einstein for SvNCNs

Definition 6. Set $\rho=\{\rho_1,\rho_2,\rho_3\cdots\rho_z\}$ is a SvNCNs. This weighted arithmetic average operator of Einstein for SvNCNs (SvNCNEWA) is expressed as below:

$$SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_\kappa)=\overset{z}{\underset{\kappa=1}{\oplus}}\varphi_\kappa\rho_\kappa \tag{11}$$

Where φ_κ is the weight of ρ_κ, with a range of [0,1], and satisfies the criteria $\sum_{\kappa=1}^{z}\varphi_\kappa=1$.

Theorem 1. Let $\rho_\kappa=\langle R_\kappa,I_\kappa,E_\kappa\rangle=\langle(a_{r\kappa},c_{r\kappa}),(a_{i\kappa},c_{i\kappa}),(a_{e\kappa},c_{e\kappa})\rangle$ (κ=1,2,3$\cdots z$) be a group of SvNCNs, where the weight coefficients of each SvNCN satisfy $\varphi_\kappa\in[0,1]$ and $\sum_{\kappa=1}^{z}\varphi_\kappa$. According to equation (11), the result of SvNCNEWA can be calculated as follows:

$$SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_\kappa)=$$
$$\left\langle \begin{array}{l} \left(\frac{\prod_{\kappa=1}^{z}(1+a_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-a_{r\kappa})^{\varphi_\kappa}},\frac{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}}\right),\left(\frac{2\prod_{\kappa=1}^{z}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(a_{i\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}}\right), \\ \left(\frac{2\prod_{g=1}^{z}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(a_{e\kappa})^{\varphi_\kappa}},\frac{2\prod_{g=1}^{z}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(c_{e\kappa})^{\varphi_\kappa}}\right) \end{array}\right\rangle \quad (12)$$

Proof:

(1) If κ =2, the SvNCNEWA can be calculated using the formula(7) and (9) as follows:

$$SvNCNEWA(\rho_1,\rho_2)=\varphi_1\rho_1\oplus\varphi_2\rho_2$$

$$=\left\langle \begin{array}{l} \left(\frac{(1+a_{r1})^{\varphi}-(1-a_{r1})^{\varphi}}{(1+a_{r1})^{\varphi}+(1-a_{r1})^{\varphi}},\frac{(1+c_{r1})^{\varphi}-(1-c_{r1})^{\varphi}}{(1+c_{r1})^{\varphi}+(1-c_{r1})^{\varphi}}\right),\left(\frac{2(a_{i1})^{\varphi}}{(2-a_{i1})^{\varphi}+(a_{i1})^{\varphi}},\frac{2(c_{i1})^{\varphi}}{(2-c_{i1})^{\varphi}+(c_{i1})^{\varphi}}\right), \\ \left(\frac{2(a_{e1})^{\varphi}}{(2-a_{e1})^{\varphi}+(a_{e1})^{\varphi}},\frac{2(c_{e1})^{\varphi}}{(2-c_{e1})^{\varphi}+(c_{e1})^{\varphi}}\right) \end{array}\right\rangle$$

$$\oplus\left\langle \begin{array}{l} \left(\frac{(1+a_{r2})^{\varphi_2}-(1-a_{r2})^{\varphi_2}}{(1+a_{r2})^{\varphi_2}+(1-a_{r2})^{\varphi_2}},\frac{(1+c_{r2})^{\varphi_2}-(1-c_{r2})^{\varphi_2}}{(1+c_{r2})^{\varphi_2}+(1-c_{r2})^{\varphi_2}}\right),\left(\frac{2(a_{i2})^{\varphi_2}}{(2-a_{i2})^{\varphi_2}+(a_{i2})^{\varphi_2}},\frac{2(c_{i2})^{\varphi_2}}{(2-c_{i2})^{\varphi_2}+(c_{i2})^{\varphi_2}}\right), \\ \left(\frac{2(a_{e2})^{\varphi_2}}{(2-a_{e2})^{\varphi_2}+(a_{e2})^{\varphi_2}},\frac{2(c_{e2})^{\varphi_2}}{(2-c_{e2})^{\varphi_2}+(c_{e2})^{\varphi_2}}\right) \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \left(\frac{(1+a_{r1})^{\varphi_1}(1+a_{r2})^{\varphi_2}-(1-a_{r2})^{\varphi 1}(1-a_{r2})^{\varphi_2}}{(1+a_{r1})^{\varphi_1}(1+a_{r2})^{\varphi_2}+(1-a_{r2})^{\varphi_1}(1-a_{r2})^{\varphi_2}},\frac{(1+c_{r1})^{\varphi_1}(1+c_{r2})^{\varphi_2}-(1-c_{r2})^{\varphi_1}(1-c_{r2})^{\varphi_2}}{(1+c_{r1})^{\varphi_1}(1+c_{r2})^{\varphi_2}+(1-c_{r2})^{\varphi_1}(1-c_{r2})^{\varphi_2}}\right) \\ \left(\frac{2(a_{i1})^{\varphi_1}(a_{i2})^{\varphi_2}}{(2-a_{i1})^{\varphi_1}(2-a_{i2})^{\varphi_2}+(a_{i1})^{\varphi_1}(a_{i2})^{\varphi_2}},\frac{2(c_{i1})^{\varphi_1}(c_{i2})^{\varphi_2}}{(2-c_{i1})^{\varphi_1}(2-c_{i2})^{\varphi_2}+(c_{i1})^{\varphi_1}(c_{i2})^{\varphi_2}}\right) \\ \left(\frac{2(a_{e1})^{\varphi_1}(a_{e2})^{\varphi_2}}{(2-a_{e1})^{\varphi_1}(2-a_{e2})^{\varphi_2}+(a_{e1})^{\varphi_1}(a_{e2})^{\varphi_2}},\frac{2(c_{e1})^{\varphi_1}(c_{e2})^{\varphi_2}}{(2-c_{e1})^{\varphi_1}(2-c_{e2})^{\varphi_2}+(c_{e1})^{\varphi_1}(c_{e2})^{\varphi_2}}\right) \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \left(\frac{\prod_{\kappa=1}^{2}(1+a_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{2}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(1+a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(1-a_{r\kappa})^{\varphi_\kappa}},\frac{\prod_{\kappa=1}^{2}(1+c_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{2}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(1+c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(1-c_{r\kappa})^{\varphi_\kappa}}\right),\left(\frac{2\prod_{\kappa=1}^{2}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(2-a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(a_{i\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{2}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(2-c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(c_{i\kappa})^{\varphi_\kappa}}\right), \\ \left(\frac{2\prod_{\kappa=1}^{2}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(2-a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(a_{e\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{2}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{2}(2-c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{2}(c_{e\kappa})^{\varphi_\kappa}}\right) \end{array}\right\rangle$$

(2) Let κ =W, then the SvNCNEWA can be expressed as:

$$SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_W)=$$
$$\left\langle \begin{array}{l} \left(\frac{\prod_{\kappa=1}^{W}(1+a_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(1+a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(1-a_{r\kappa})^{\varphi_\kappa}},\frac{\prod_{\kappa=1}^{W}(1+c_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(1+c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(1-c_{r\kappa})^{\varphi_\kappa}}\right),\left(\frac{2\prod_{\kappa=1}^{W}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(a_{i\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(c_{i\kappa})^{\varphi_\kappa}}\right), \\ \left(\frac{2\prod_{\kappa=1}^{W}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(a_{e\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(c_{e\kappa})^{\varphi_\kappa}}\right) \end{array}\right\rangle$$

(3) Let κ =W+1, then SvNCNEWA is expressed as:

$$SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_{W+1})=SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_W)\ \oplus\ \varphi_{W+1}*\rho_{W+1}$$

$$=\left\langle \begin{array}{l} \left(\frac{\prod_{\kappa=1}^{W}(1+a_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(1+a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(1-a_{r\kappa})^{\varphi_\kappa}},\frac{\prod_{\kappa=1}^{W}(1+c_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(1+c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(1-c_{r\kappa})^{\varphi_\kappa}}\right),\left(\frac{2\prod_{\kappa=1}^{W}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(a_{i\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(c_{i\kappa})^{\varphi_\kappa}}\right), \\ \left(\frac{2\prod_{\kappa=1}^{W}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(a_{e\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W}(2-c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W}(c_{e\kappa})^{\varphi_\kappa}}\right) \end{array}\right\rangle$$

$$\oplus\left\langle \begin{array}{l} \left(\frac{(1+a_{r1})^{\varphi_{W+1}}-(1-a_{r1})^{\varphi_{W+1}}}{(1+a_{r1})^{\varphi_{W+1}}+(1-a_{r1})^{\varphi_{W+1}}},\frac{(1+c_{r1})^{\varphi_{W+1}}-(1-c_{r1})^{\varphi_{W+1}}}{(1+c_{r1})^{\varphi_{W+1}}+(1-c_{r1})^{\varphi_{W+1}}}\right),\left(\frac{2(a_{i1})^{\varphi_{W+1}}}{(2-a_{i1})^{\varphi_{W+1}}+(a_{i1})^{\varphi_{W+1}}},\frac{2(c_{i1})^{\varphi_{W+1}}}{(2-c_{i1})^{\varphi_{W+1}}+(c_{i1})^{\varphi_{W+1}}}\right), \\ \left(\frac{2(a_{e1})^{\varphi_{W+1}}}{(2-a_{e1})^{\varphi_{W+1}}+(a_{e1})^{\varphi_{W+1}}},\frac{2(c_{e1})^{\varphi_{W+1}}}{(2-c_{e1})^{\varphi_{W+1}}+(c_{e1})^{\varphi_{W+1}}}\right) \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \left(\frac{\prod_{\kappa=1}^{W+1}(1+a_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W+1}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{p+1}(1+a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{p+1}(1-a_{r\kappa})^{\varphi_\kappa}},\frac{\prod_{\kappa=1}^{W+1}(1+c_{r\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{W+1}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{p+1}(1+c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{p+1}(1-c_{r\kappa})^{\varphi_\kappa}}\right),\left(\frac{2\prod_{\kappa=1}^{W+1}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W+1}(2-a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W+1}(a_{i\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W+1}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W+1}(2-c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W+1}(c_{i\kappa})^{\varphi_\kappa}}\right), \\ \left(\frac{2\prod_{\kappa=1}^{W+1}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W+1}(2-a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W+1}(a_{e\kappa})^{\varphi_\kappa}},\frac{2\prod_{\kappa=1}^{W+1}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{W+1}(2-c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{W+1}(c_{e\kappa})^{\varphi_\kappa}}\right) \end{array}\right\rangle$$

From the above formula, it can be concluded that the equation holds for any value of κ, and the SvNCNEWA satisfies the below properties.

(i) **Idempotency**. Let $\rho_\kappa=\langle(a_{r\kappa},c_{r\kappa}),(a_{i\kappa},c_{i\kappa}),(a_{e\kappa},c_{e\kappa})\rangle$ $(\kappa=1,2,3\cdots z)$ be a group of SvNCNs. When $\rho_\kappa=\rho$, it satisfies $SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_z)=\rho$.

(ii) **Boundedness**. Let $\rho_\kappa = \langle (a_{r\kappa}, c_{r\kappa}), (a_{i\kappa}, c_{i\kappa}), (a_{e\kappa}, c_{e\kappa}) \rangle$ (κ=1,2,3$\cdots z$) be a group of SvNCNs, then maximum values and the minimum are given by:

$\rho_{\min} = \langle \min(a_{r\kappa}, c_{r\kappa}), \max(a_{i\kappa}, c_{i\kappa}), \max(a_{e\kappa}, c_{e\kappa}) \rangle$, $\rho_{\max} = \langle \max(a_{r\kappa}, c_{r\kappa}), \min(a_{i\kappa}, c_{i\kappa}), \min(a_{e\kappa}, c_{e\kappa}) \rangle$.

(iii)**Monotonicity**. Let $\rho^1_\kappa = \langle (a^1_{r\kappa}, c^1_{r\kappa}), (a^1_{i\kappa}, c^1_{i\kappa}), (a^1_{e\kappa}, c^1_{e\kappa}) \rangle$ and $\rho^2_\kappa = \langle (a^2_{r\kappa}, c^2_{r\kappa}), (a^2_{i\kappa}, c^2_{i\kappa}), (a^2_{e\kappa}, c^2_{e\kappa}) \rangle$ (κ=1,2,$\cdots z$) be two group of SvNCNs. If $\rho^1_\kappa \subseteq \rho^2_\kappa$, then it satisfies $SvNCNEWA(\rho^1_1, \rho^1_2, \rho^1_3 \cdots \rho^1_z) \subseteq SvNCNEWA(\rho^2_1, \rho^2_2, \rho^2_3 \cdots \rho^2_z)$.

Proof:

(1) Let $\rho_\kappa = \langle (a_{r\kappa}, c_{r\kappa}), (a_{i\kappa}, c_{i\kappa}), (a_{e\kappa}, c_{e\kappa}) \rangle = \rho = \langle (a_r, c_r), (a_i, c_i), (a_e, c_e) \rangle$ be a group of SvNCNs, where the weight coefficients φ_κ represent the weights of ρ_κ, which are constrained within the range [0, 1]. Furthermore, it satisfies $\sum_{\kappa=1}^{z} \varphi_\kappa = 1$.

$$SvNCNEWA(\rho_1, \rho_2, \rho_3 \cdots \rho_z)$$

$$= \left\langle \begin{array}{l} \left(\dfrac{\prod_{\kappa=1}^{z}(1+a_{r\kappa})^{\varphi_\kappa} - \prod_{\kappa=1}^{z}(1-a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{q}(1+a_{r\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(1-a_{r\kappa})^{\varphi_\kappa}}, \dfrac{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa} - \prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}} \right), \left(\dfrac{2\prod_{\kappa=1}^{z}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{i\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(\alpha_{i\kappa})^{\varphi_\kappa}}, \dfrac{2\prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{i\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}} \right), \\ \left(\dfrac{2\prod_{\kappa=1}^{z}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{e\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(a_{e\kappa})^{\varphi_\kappa}}, \dfrac{2\prod_{\kappa=1}^{z}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{e\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(c_{e\kappa})^{\varphi_\kappa}} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(\dfrac{(1+a_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa} - (1-a_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(1+a_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (1-a_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}, \dfrac{(1+c_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa} - (1-c_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(1+c_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (1-c_r)^{\sum_{\kappa=1}^{z}\varphi_\kappa}} \right), \left(\dfrac{2(a_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(2-a_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (a_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}, \dfrac{2(c_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(2-c_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (c_i)^{\sum_{\kappa=1}^{z}\varphi_\kappa}} \right), \\ \left(\dfrac{2(a_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(2-a_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (a_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}, \dfrac{2(c_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa}}{(2-c_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa} + (c_e)^{\sum_{\kappa=1}^{z}\varphi_\kappa}} \right) \end{array} \right\rangle$$

$$= \langle (a_r, c_r), (a_i, c_i), (a_e, c_e) \rangle = \rho$$

(2) Set $SvNCNEWA(\rho_1, \rho_2, \rho_3 \cdots \rho_z)$ be given. According to the formula (12), it be expressed below:

$$\min(\alpha_{r\kappa}) \le \frac{\prod_{\kappa=1}^{z}(1+\alpha_{r\kappa})^{\varphi_\kappa} - \prod_{\kappa=1}^{z}(1-\alpha_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+\alpha_{r\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(1-\alpha_{r\kappa})^{\varphi_\kappa}} \le \max(a_{r\kappa}) \text{ and } \min(c_{r\kappa}) \le \frac{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa} - \prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+c_{r\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(1-c_{r\kappa})^{\varphi_\kappa}} \le \max(c_{r\kappa})$$

$$\min(a_{i\kappa}) \le \frac{2\cdot\prod_{\kappa=1}^{z}(a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{i\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(a_{i\kappa})^{\varphi_\kappa}} \le \max(a_{i\kappa}) \text{ and } \min(c_{i\kappa}) \le \frac{2\cdot\prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{i\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(c_{i\kappa})^{\varphi_\kappa}} \le \max(c_{i\kappa})$$

$$\min(a_{e\kappa}) \le \frac{2\cdot\prod_{\kappa=1}^{z}(a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{e\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(a_{e\kappa})^{\varphi_\kappa}} \le \max(a_{e\kappa}) \text{ and } \min(c_{e\kappa}) \le \frac{2\cdot\prod_{\kappa=1}^{z}(c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{e\kappa})^{\varphi_\kappa} + \prod_{\kappa=1}^{z}(c_{e\kappa})^{\varphi_\kappa}} \le \max(c_{e\kappa})$$

According to the formula (3), $SvNCNEWA(\rho_1, \rho_2, \rho_3 \cdots \rho_z)(\kappa$=1, 2, 3$\cdots z)$, $\rho_{\min}$, $\rho_{\max}$ are defined see below:

$$S(SvNCNEWA(\rho_1, \rho_2, \rho_3 \cdots \rho_z)) = \frac{2 + a_r c_r - a_i c_i - a_e c_e}{3}$$

$$S(\rho_{\min}) = \{(2 + \min(a_{r\kappa})\min(c_{r\kappa}) - \max(a_{i\kappa})\max(c_{i\kappa}) - \max(a_{e\kappa})\max(c_{e\kappa}))/3\},$$

$$S(\rho_{\max}) = \{(2 + \max(a_{r\kappa})\max(c_{r\kappa}) - \min(a_{i\kappa})\min(c_{i\kappa}) - \min(a_{e\kappa})\min(c_{e\kappa}))/3\}.$$

From the above formula, $S(\rho_{\min}) \le S(SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_z)) \le S(\rho_{\max})$ can be obtained. According to the (i)-th properties of the score function, $\rho_{\min} \le SvNCNEWA(\rho_1,\rho_2,\rho_3\cdots\rho_z) \le \rho_{\max}$ can be derived.

(3) Set two groups SvNCNs, their relationship can be expressed as follows: $\rho_\kappa^1 = \langle (a^1_{r\kappa}, c^1_{r\kappa}),\ (a^1_{i\kappa}, c^1_{i\kappa}),\ (a^1_{e\kappa}, c^1_{e\kappa}) \rangle$, $\rho_\kappa^2 = \langle (a^2_{r\kappa}, c^2_{r\kappa}),\ (a^2_{i\kappa}, c^2_{i\kappa}),\ (a^2_{e\kappa}, c^2_{e\kappa}) \rangle$ then $\rho_\kappa^1 \subseteq \rho_\kappa^2$. Then they satisfy the following constraints: $a^1_{r\kappa} \le a^2_{r\kappa}$, $c^1_{r\kappa} \le c^2_{r\kappa}$, $a^1_{i\kappa} \ge a^2_{i\kappa}$, $c^1_{i\kappa} \ge c^2_{i\kappa}$, $a^1_{e\kappa} \ge a^2_{e\kappa}$, $c^1_{e\kappa} \ge c^2_{e\kappa}$.

Using the equation (12), it can be derived as: $SvNCNEWA(\rho^1_1, \rho^1_2, \rho^1_3 \cdots \rho^1_z) = \langle (a^1_r, c^1_r), (a^1_i, c^1_i), (a^1_e, c^1_e) \rangle$ and $SvNCNEWA(\rho^2_1, \rho^2_2, \rho^2_3 \cdots \rho^2_z) = \langle (a^2_r, c^2_r), (a^2_i, c^2_i), (a^2_e, c^2_e) \rangle$.

According to the above formula, it is evident that $a^1_r \le a^2_r, c^1_r \le c^2_r, a^1_i \ge a^2_i, c^1_i \ge c^2_i, a^1_e \ge a^2_e, c^1_e \ge c^2_e$ can be obtained. Consequently, $SvNCNEWA(\rho^1_1, \rho^1_2, \cdots \rho^1_z) \subseteq SvNCNEWA(\rho^2_1, \rho^2_2, \cdots \rho^2_z)$ can be deduced.

Definition 7. Let $\rho = \{\rho_1, \rho_2, \rho_3 \cdots \rho_z\}$ be a group of SvNCNs. The weighted geometric average operator of Einstein for SvNCNs (SvNCNEWG) is expressed as follows:

$$SvNCNEWG(\rho_1, \rho_2, \rho_3 \cdots \rho_z) = \bigotimes_{\kappa=1}^{z} \rho_\kappa^{\varphi_\kappa} \tag{13}$$

Where φ_κ represents the weight of ρ_κ, with a range of [0, 1], and it satisfies $\sum_{\kappa=1}^{z} \varphi_\kappa = 1$.

Theorem 2. Let $\rho_\kappa = \langle R_\kappa, I_\kappa, E_\kappa \rangle = \langle (a_{r\kappa}, c_{r\kappa}), (a_{i\kappa}, c_{i\kappa}), (a_{e\kappa}, c_{e\kappa}) \rangle$ (κ=1,2,3,$\cdots z$) be a group of SvNCNs, where weight coefficients of each SvNCN satisfy $\varphi_\kappa \in [0,1]$ and $\sum_{\kappa=1}^{z} \varphi_\kappa$. On basis of the operational rules, the result is calculated as follows:

$$SvNCNEWG(\rho_1,\ \rho_2,\ \rho_3 \cdots \rho_\kappa) =$$

$$\left\langle \begin{array}{l} \left(\dfrac{2\cdot\prod_{\kappa=1}^{z}(a_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-a_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(a_{r\kappa})^{\varphi_\kappa}}, \dfrac{2\cdot\prod_{\kappa=1}^{z}(c_{r\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(2-c_{r\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(c_{r\kappa})^{\varphi_\kappa}} \right), \left(\dfrac{\prod_{\kappa=1}^{z}(1+a_{i\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-a_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+a_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-a_{i\kappa})^{\varphi_\kappa}}, \dfrac{\prod_{\kappa=1}^{z}(1+c_{i\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-c_{i\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+c_{i\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-c_{i\kappa})^{\varphi_\kappa}} \right), \\ \left(\dfrac{\prod_{\kappa=1}^{z}(1+a_{e\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-a_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+a_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-a_{e\kappa})^{\varphi_\kappa}}, \dfrac{\prod_{\kappa=1}^{z}(1+c_{e\kappa})^{\varphi_\kappa}-\prod_{\kappa=1}^{z}(1-c_{e\kappa})^{\varphi_\kappa}}{\prod_{\kappa=1}^{z}(1+c_{e\kappa})^{\varphi_\kappa}+\prod_{\kappa=1}^{z}(1-c_{e\kappa})^{\varphi_\kappa}} \right) \end{array} \right\rangle \tag{14}$$

The proof process for SvNCNEWG is consistent with that of SvNCNAWG.

4. Application of SvNCNEWG and SvNCNAWG in MADM

In order to make optimal decisions, this section introduces the MADM based on SvNCNs. In the DM problem, it is assumed that a decision-maker needs to evaluate *H* alternatives, with the set of alternatives expressed as $A = \{A_1, A_2,\ A_3 \cdots,\ A_H\}$. Each alternative has *L* attributes which expressed as $C = \{C_1, C_2,\ C_3 \cdots,\ C_L\}$. The weight coefficient corresponding to each attribute in the decision process is denoted as $\varphi = \{\varphi_1, \varphi_2, \varphi_3 \cdots, \varphi_L\}$. Thus, the SvNCN of the $\xi-$th attribute of the $\varsigma-$th alternative is expressed as $\rho_{\varsigma,\xi} = \langle (a_{t_{\varsigma,\xi}}, c_{t_{\varsigma,\xi}}), (a_{i_{\varsigma,\xi}}, c_{i_{\varsigma,\xi}}), (a_{f_{\varsigma,\xi}}, c_{f_{\varsigma,\xi}}) \rangle$ $\varsigma = 1, 2, \cdots, H$ and $\xi = 1, 2, \cdots, L$.

For all alternatives with *L* attributes, the expression is as below matrix: $\rho = \{(\rho_{\varsigma,\xi})\}_{H\times L}$. The steps for applying the SvNCNs method in MADM are see below:

Step 1: Calculate the aggregated value of SvNCNs for *H* alternatives and *L* attributes using the given aggregation formulas (12) and (14), each attribute of each candidate is composed of truth, false, uncertainty values and their corresponding credibility values.

$$\rho_\varsigma = SvNCNEWA(\rho_{\varsigma 1}, \rho_{\varsigma 2}, \rho_{\varsigma 3} \cdots \rho_{\varsigma H}) = \bigoplus_{\xi=1}^{L} \varphi_\xi \rho_{\varsigma\xi}$$

$$= \left\langle \begin{array}{l} \left(\dfrac{\prod_{\xi=1}^{L}(1+a_{r\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-a_{r\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+a_{r\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-a_{r\varsigma\xi})^{\varphi_\xi}}, \dfrac{\prod_{\xi=1}^{L}(1+c_{r\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-c_{r\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+c_{r\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-c_{r\varsigma\xi})^{\varphi_\xi}} \right), \left(\dfrac{2\prod_{\xi=1}^{L}(a_{i\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(2-a_{i\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(a_{i\varsigma\xi})^{\varphi_\xi}}, \dfrac{2\prod_{\xi=1}^{L}(c_{i\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(2-c_{i\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(c_{i\varsigma\xi})^{\varphi_\xi}} \right), \\ \left(\dfrac{2\prod_{\xi=1}^{L}(a_{e\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(2-a_{e\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(a_{e\varsigma\xi})^{\varphi_\xi}}, \dfrac{2\prod_{\xi=1}^{L}(c_{e\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(2-c_{e\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(c_{e\varsigma\xi})^{\varphi_\xi}} \right) \end{array} \right\rangle$$

or $\rho_\varsigma = SvNCNEWG(\rho_{\varsigma 1}, \rho_{\varsigma 2}, \rho_{\varsigma 3} \cdots \rho_{\varsigma H}) = \bigotimes_{\xi=1}^{L} \rho_{\varsigma\xi}^{\varphi_\xi}$

$$= \left\langle \begin{array}{l} \left(\dfrac{2\cdot\prod_{\xi=1}^{L}(a_{r\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{l}(2-a_{r\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{l}(a_{r\varsigma\xi})^{\varphi_\xi}}, \dfrac{2\cdot\prod_{\xi=1}^{L}(c_{r\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{n}(2-c_{r\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{n}(c_{r\varsigma\xi})^{\varphi_\xi}} \right), \left(\dfrac{\prod_{\xi=1}^{L}(1+a_{i\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-a_{i\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+a_{i\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-a_{i\varsigma\xi})^{\varphi_\xi}}, \dfrac{\prod_{\xi=1}^{L}(1+c_{i\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-c_{i\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+c_{i\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-c_{i\varsigma\xi})^{\varphi_\xi}} \right), \\ \left(\left(\dfrac{\prod_{\xi=1}^{L}(1+a_{e\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-a_{e\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+a_{e\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-a_{e\varsigma\xi})^{\varphi_\xi}}, \dfrac{\prod_{\xi=1}^{L}(1+c_{e\varsigma\xi})^{\varphi_\xi} - \prod_{\xi=1}^{L}(1-c_{e\varsigma\xi})^{\varphi_\xi}}{\prod_{\xi=1}^{L}(1+c_{e\varsigma\xi})^{\varphi_\xi} + \prod_{\xi=1}^{L}(1-c_{e\varsigma\xi})^{\varphi_\xi}} \right) \right) \end{array} \right\rangle$$

Step 2: Substitute the aggregated values of SvNCN obtained from Step 1 into the scoring formula (3) and (4), then calculate the values of score and accuracy for each alternative.

Step 3: Rank alternatives on basis of their score values and make the optimal choice among the multiple alternatives.

5. Application Instance and Algorithm Comparison

This section presents an application instance of Einstein aggregation operators based on SvNCNs in MADM and provides a comparative analysis with other DM methods.

5.1 Application Example

The the achievement degree of curriculum objectives evaluation is an effective method to construct the evaluation system of students' learning quality and measure the effectiveness of OBE teaching. The evaluation carried out in the practical curriculums of electrical engineering major in Shaoxing University，and its evaluation results consists of four components: regular performance, design report, project demonstration, and teamwork. However, there is fuzziness and uncertainty in the evaluation of each part of the practical curriculum，so we set the four curriculums of Embedded System, Single Chip Microcomputer Principle, Electronic Circuit Design and Sensor as the alternatives, denoted as $\chi=\{\chi_1, \chi_2, \chi_3, \chi_4\}$.The attributes of the four evaluation parts are defined as $\psi=\{\psi_1,\psi_2,\psi_3,\psi_4\}$. The known weight vector corresponding to each attribute is defined as: $\tau=\{\tau_1=0.2, \tau_2=0.3, \tau_3=0.4, \tau_4=0.1\}$. Thus, the achievement degree evaluation of these four curriculums can be expressed as a SvNCNs matrix:

$$\chi = \begin{bmatrix} \langle(0.7,0.8),(0.2,0.7),(0.2,0.8)\rangle & \langle(0.7,0.9),(0.1,0.6),(0.1,0.7)\rangle & \langle(0.8,0.8),(0.2,0.7),(0.2,0.8)\rangle & \langle(0.8,0.7),(0.1,0.7),(0.2,0.8)\rangle \\ \langle(0.8,0.8),(0.1,0.6),(0.1,0.7)\rangle & \langle(0.8,0.7),(0.1,0.8),(0.2,0.6)\rangle & \langle(0.9,0.7),(0.2,0.7),(0.1,0.8)\rangle & \langle(0.7,0.7),(0.2,0.7),(0.2,0.8)\rangle \\ \langle(0.8,0.9),(0.2,0.8),(0.1,0.7)\rangle & \langle(0.8,0.8),(0.1,0.9),(0.1,0.7)\rangle & \langle(0.9,0.8),(0.2,0.6),(0.1,0.8)\rangle & \langle(0.8,0.6),(0.2,0.7),(0.1,0.9)\rangle \\ \langle(0.7,0.7),(0.1,0.7),(0.2,0.8)\rangle & \langle(0.8,0.8),(0.2,0.7),(0.1,0.8)\rangle & \langle(0.7,0.8),(0.2,0.8),(0.1,0.9)\rangle & \langle(0.7,0.6),(0.1,0.8),(0.1,0.8)\rangle \end{bmatrix}$$

According to the application of the Einstein aggregation operator based on SvNCNs in MADM, the calculation steps see below:

Step 1: Calculate aggregation values of SvNCNEWA and SvNCNEWG using the formulas (12) and (14). The aggregated value of each course consists of three parts: truth, uncertainty and false, with each part represented as a SvNCN. The aggregate values are shown in the Table 1 below:

Table 1. Calculation of Two Aggregation Operators

Aggregate operations	Aggregate values
SvNCNEWA	$\chi_1 = \langle (0.75, 0.83), (0.15, 0.66), (0.16, 0.76) \rangle$
	$\chi_2 = \langle (0.84, 0.72), (0.14, 0.70), (0.13, 0.72) \rangle$
	$\chi_3 = \langle (0.84, 0.81), (0.16, 0.73), (0.10, 0.76) \rangle$
	$\chi_4 = \langle (0.73, 0.76), (0.16, 0.74), (0.11, 0.84) \rangle$
SvNCNEWG	$\chi_1 = \langle (0.75, 0.82), (0.16, 0.67), (0.17, 0.77) \rangle$
	$\chi_2 = \langle (0.83, 0.72), (0.15, 0.72), (0.14, 0.73) \rangle$
	$\chi_3 = \langle (0.84, 0.79), (0.17, 0.77), (0.10, 0.77) \rangle$
	$\chi_4 = \langle (0.72, 0.75), (0.17, 0.75), (0.12, 0.85) \rangle$

Step 2: In accordance with the scoring formula (3) and (4), the results of the four curriculums are ranked as shown in the Table 2 below:

Table 2. Scores and Rankings of Two Aggregation Operators

Aggregation Operators	Score (χ_δ (δ = 1, 2, 3, 4))	Rank	Optimal Value
SvNCNEWA	0.801,0.806,0.834,0.783	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3
SvNCNEWG	0.791,0.795,0.820,0.774	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3

Step 3: After sorting the results of the two aggregation operators from Table 1 and Table 2, the curriculum with the highest score is is identified as having the optimal achievement.

5.2 Comparative Analysis

Ye et al. proposed a MCDM model on basis trigonometric weighted average operators and trigonometric geometry operators for SvNCNs. This model utilizes the operational rules of triangular t-norm and triangular t-conorm to define aggregation operations, including triangular weighted averaging (SvNCNTWA) and triangular weighted geometric (SvNCNTWG) operators for SvNCNs matrices, and has been successfully applied to slope protection in mountainous terrain. For comparison, this paper presents the two aggregation operations based on the Einstein aggregation operator (SvNCNEWG and SvNCNEWA) and compares them with Ye's SvNCNTWA and SvNCNTWG, as well as the classic SvNNWA and SvNNWG. All six methods use the same SvNCNs matrix and are compared in Table 3.

Table 3. Comparison table of the four methods

Aggregation Operators	**Score** χ_δ (δ = 1, 2, 3, 4)	**Rank**	**Optimal Value**
SvNNWA	0.813, 0.856, 0.862, 0.819	$\chi_3 > \chi_2 > \chi_4 > \chi_1$	χ_3
SVNNWG	0.846, 0.889, 0.900, 0.863	$\chi_3 > \chi_2 > \chi_4 > \chi_1$	χ_3
SvNCNTWA	0.799, 0.804, 0.830, 0.781	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3
SvNCNTWG	0.784, 0.788, 0.813, 0.768	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3
SvNCNEWA	0.801, 0.806, 0.834, 0.783	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3
SvNCNEWG	0.791, 0.795, 0.820, 0.774	$\chi_3 > \chi_2 > \chi_1 > \chi_4$	χ_3

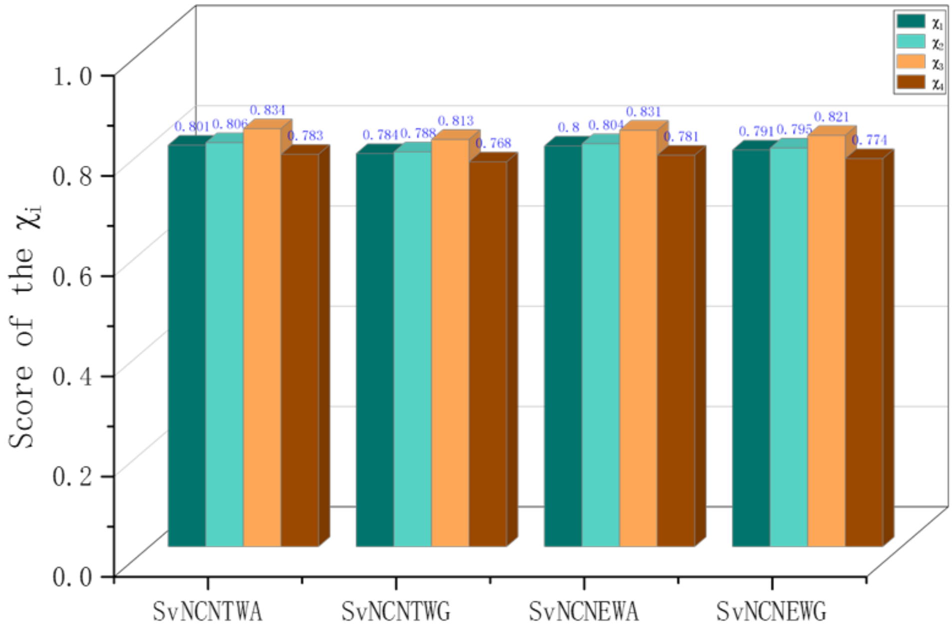

Figure 1. Comparison of aggregation operator

As shown in Table 3, the classic SvNNWA and SvNNWG methods yield different ranking results when assessing course achievement, compared to the operators based on the SvNCNs. However, the final optimal decision remains the same. This indicates that the classic methods lack the credibility measure of SvNCNs, which is crucial when determining the optimal ranking for the same multi-attribute problem. In contrast, the SvNCNs-based aggregation operators effectively address this gap. Among the four SvNCNs-based methods, the two aggregation operations proposed in this paper SvNCNEWA and SvNCNEWG produce the same ranking results as the two previously proposed SvNCNTWA and SvNCNTWG algorithms, demonstrating the effectiveness of the methods introduced in this study.

As shown in Figure 1, when the four SvNCNs-based methods are used to evaluate the four courses, the SvNCNEWA method proposed in this paper consistently achieves higher scores than the other three methods. This highlights the ability of the improved Einstein operator based on SvNCNs to help decision-makers make the optimal choice in MADM problems.

6. Conclusions

This paper first presents the SvNCNEWA and SvNCNEWG aggregation algorithms, based on the improved Einstein paradigm within SvNCNs. These algorithms were applied to assess the achievement levels of four practice-based courses in the Electrical Engineering program at Shaoxing University. By comparing the proposed methods with four existing aggregation algorithms, the effectiveness of the approach in addressing MADM problems is demonstrated. Looking ahead, future research will focus on extending SvNCNs to topological spaces and exploring the application of neutrosophic topological spaces in areas such as data analysis, pattern recognition, and robot control.

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