Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS)

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Abstract: Technique for order performance by similarity to ideal solution (TOPSIS) is a Multi-Criteria Decision-Making method (MCDM), that consists on handling real complex problems of decision-making. However, real MCDM problems are often involves imperfect information such as uncertainty and inconsistency. The imperfect information is often manipulated through Neutrosophics theory, using certain degree of truth (T), falsity degree (F) and indeterminacy degree(I). and thus single-valued neutrosophic set (SVNs) had prodded a strong capacity to model such complex information. To overcome that kind of problems, In this paper, first, the authors simplify the popular TOPSIS method to a lite TOPSIS (S-TOPSIS), that gives the same result as standard version. Second, mapping S-TOPSIS to Neutrosophics Environment, investigating SVNS, called nS-TOPSIS, to deal with imperfect information in the real decision-making problems. Numerical examples show the contributions of proposed S-TOPSIS method to get the same results with standard TOPSIS with simple way of calculus, and how Neutrosophic environment manage the uncertain information using SVN.

Keywords: Technique for order performance by similarity to ideal solution (TOPSIS), MCDM, Single-Valued Neutrosophic set(SVNs), Neutrosophic Simplified TOPSIS(nS-TOPSIS).

1 Introduction

Technique for Order Preference by Similarity to Ideal Solution(TOPSIS) is a popular Multicriteria Decision Making (MCDM). TOPSIS was first introduced by Hwang and Yoon ([1]) to deal with structuring Multicriteria issues with crisp numerical values in real situation. However, real MCDM problems are often formulated under as set of indeterminate or inconsistent information. Thus, TOPSIS consists on many complicate steps of calculation. To deal with thoses problems, First, we introduce a lite version of TOPSIS method (S-TOPSIS) with guaranty of obtention of the same results simplifying many complicated steps of calculation. Thus, we introduce single valued neutrosophic set (SVNs) modifications of Simplified TOPSIS (nS-TOPSIS).

To manage information outcome from real problem, that are usually endowed with imperfection such as uncertainty, fuzziness and inconsistency, Smarandache ([2,3]) initiated a new notion, which is a generalization of the Intuitionistic $Fuzzy$ Set (IFS), called Neutrosophics Set (NS), which based on three values (truth (T), indeterminacy (I), and falsity (F) membership degrees). The main propriety of NS is that the sum of three values is 3 instead of 1 in the case of IFS. Although, the NS as introduced by Smarandache was a philosophical concept, unable to be used in real study cases. Many researchers are working on to produce mathematical property, theories, Arithmetic Operations, etc. On the one hand, Wang and al. ([5]) embodied Neutrosophic concept in a metric, called single-valued neutrosophic set (SVNs) as three values in one ($truth-membership\quad degree$, $indeterminacy-membership\quad degree$, and $falsity-membership\quad degree$). In addition, Broumi and al. ([4,6,7]) defined, in Neutrosophic space, similarity mesure and distances metric between SVNS values. the defined SVNS show stronge power to modelize imperfect information, such as uncertainty, imprecise, incomplete, and inconsistent information.

On the other hand, Other researchers are working on deploying Neutrosophic in MCDM field. Biswas ([8]) proposed extended TOPSIS Method to deal with real MCDM problems based on weighted Neutrosophic and aggregated SVNS operators

Ye [9,10] introduced two concepts, single valued neutrosophic cross-entropy of single valued neutrosophic and weighted correlation coefficient of SVNSs into multicriteria decision-making problems. Deli et al. [11] studied deploying Bipolar Neutrosophic Sets in Multi-Criteria Decision Making field

The remainder of the paper presents the preliminaries to build our Method, TOPSIS method and single valued neutrosophic set (SVNs). next Simplified-TOPSIS as first contribution was introduced. Then, hybrid methods Neutrosophic-TOPSIS and Neutrosophic-Simplified-TOPSIS are proposed to deal with real example. Results and discussions are presented at the end of this paper.

2 TOPSIS method

Consider a multi-attribute decision making problem that could be formulated as follow, $A = \{A_1, A_2, \cdots, A_n\}$ a set of m preferences, and $C = \{C_1, C_2, \cdots, C_n\}$ a set of n criteria. The relationships between preferences A_i and criteria C_j quantified by rating a_{ij} provided by decision maker. Weight vector W is a set of weights ω_i associated to criteria C_j. The all details described above could be reshaped on decision matrix bellow, denoted by D.

$$D = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \text{(Decision Matrix)} \tag{2.1}$$

Technique for order performance by similarity to ideal solution (TOPSIS) method summarized as follow:

Step 1: Calculate normalized form of decision matrix r_{ij} dividing each element a_{ij} on the sum of whole column.

$$r_{ij} = a_{ij} / \left(\sum_{i=1}^{m} a_{ij}^2 \right)^{0.5} ;\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{2.2}$$

Step 2: Calculate also weighted form v_{ij} of matrix r_{ij} obtained from previous step, multiplying each element r_{ij} by its associated weight w_j.

$$v_{ij} = w_j r_{ij};\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{2.3}$$

Step 3: Based on the weighted decision matrix, we calculate positive ideal solution (POS) and negative ideal solution(NIS).

$$A^+ = (v_1^+, v_2^+, \cdots, v_n^+) = \left\{ \begin{array}{l} (max_i \{v_{ij} | j \in B\}), \\ (min_i \{v_{ij} | j \in C\}) \end{array} \right\} \quad (2.4)$$

$$A^- = (v_1^-, v_2^-, \cdots, v_n^-) = \left\{ \begin{array}{l} (min_i \{v_{ij} | j \in B\}), \\ (max_i \{v_{ij} | j \in C\}) \end{array} \right\} \quad (2.5)$$

B quantify the benefit set, and C is the cost attribute set. **Step 4:** By subtracting each weighted element v_{ij} From POS and NIS, we got tow vectors of separation measures cited below.

$$S_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \quad (2.6)$$

$$S_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \quad (2.7)$$

Step 5: Using the both measures calculated in the previous step, we calculate the rating metric.

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)}; \; i = 1, 2 \cdots, m \quad (2.8)$$

Once we calculate T_i that will be used to rank set of alternatives A_i.

2.1 Numerical example

Let consider the numerical example summarized by table Table-1. below, that contains alternatives with respect of criteria weights.

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

Table 1: Decision Matrix.

Table Table-2. is result of application of this formula $\sum_{i=1}^{n} a_{ij}$ on each column.

To determine Normalized matrix r_{ij} Table-3. each value is divide by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$:

Weighted Decision matrix v_{ij} Table-4 is the multiplication of each column by w_j.

The table Table-5. below figure out the solution of the above MCDM problem listing furthermore, final rankings for decision matrix, separation metric from POS and NIS.

Preferences, in descending preference order, are ranked as $A_3 > A_1 > A_4 > A_2$ as showed in Table-5.

a^2_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	49	81	81
A_2	64	49	64
A_3	81	36	64
A_4	36	49	64
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Table 2: Multiple decision matrix.

r_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	0.4616	0.6138	0.5447
A_2	0.5275	0.4774	0.4842
A_3	0.5934	0.4092	0.4842
A_4	0.3956	0.4774	0.4842
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Table 3: Normalized decision matrix.

3 Simplified-TOPSIS method (our proposed method)

The Simplified-TOPSIS algorithmic consists on steps bellow :

Step 1: Structure the criteria of the decision-making problem under a hierarchy.

Let considere $C = \{C_1, C_2, \cdots, C_n\}$ is a set of Criteria, with $n \geq 2$, $A = \{A_1, A_2, \cdots, A_n\}$ is the set of Preferences (Alternatives), with $m \geq 1$, a_{ij} the score of preference i with respect to criterion j, and let ω_i weight of criteria C_i.

$$D = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \quad \text{(Decision Matrix)} \tag{3.1}$$

Step 2: Calculation of the Weighted Decision Matrix v_{ij}.

Let v_{ij} Weighted Decision Matrix (WDM) that is obtained by multiplication of each column by its weight.

$$v_{ij} = w_j a_{ij};\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{3.2}$$

The difference between proposed method and standard TOPSIS section 2), the normalized step is ignored and WDM v_{ij} is calculated directly without normalization by multiplying a_{ij} with w_j.

Step 3: Determination of LIS and SIS.

The maximum (largest) ideal solution (LIS), as its name indicate, is the the set of maximums raws and smallest ideal solution (SIS) is the set of minimums raws.

$$A^+ = (v_1^+, v_2^+, \cdots, v_m^+) = (max_i \{v_{ij} | j = 1, 2, \cdots, n\}) \tag{3.3}$$

v_{ij}	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$
ω_i	12/16	3/16	1/16
A_1	0.3462	0.1151	0.0340
A_2	0.3956	0.0895	0.0303
A_3	0.4451	0.0767	0.0303
A_4	0.2967	0.0895	0.0303
v_{max}	0.4451	0.1151	0.0340
v_{min}	0.2967	0.0767	0.0303

Table 4: Weighted decision matrix.

Alternative	S_i^+	S_i^-	T_i
A_1	0.0989	0.0627	0.3880
A_2	0.0558	0.0997	0.6412
A_3	0.0385	0.1484	**0.7938**
A_4	0.1506	0.0128	0.0783

Table 5: Distance measure and ranking coefficient.

$$A^+ = (v_1^-, v_2^-, \cdots, v_m^-) = \left(\min_i \{v_{ij} | j = 1, 2, \cdots, n\}\right) \tag{3.4}$$

Step 4: Calculation of positive and negative solutions.

The positive and negative solution are the entropies of orders two of calculated using the formulas below respectively:

$$S_i^+ = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^+)^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \tag{3.5}$$

$$S_i^- = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^-)^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \tag{3.6}$$

Arrange preferences (set of alternatives A) based on value of sums of either alternative solutions $\left(S_i^+\right)$ or $\left(S_i^-\right)$. The choice of minimum or maximum depend on nature of problem, if the problem to be minimized or maximized

Step 5 (optional): Another step is missed in our Simplified TOPSIS is calculation of ranking measure T_i (relative closeness to the ideal solution), because of many reasons : first preferences can classified according to many aggregated measures calculated before, second, it's a way of normalization that can be changed by any form of normalization dividing by max, or normalized to $[0, 1]$ range, etc.

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)}; \; i = 1, 2 \cdots, m \tag{3.7}$$

3.1 Numerical example

In order to check the consistency of our proposed method, the Simplified-TOPSIS method is applied on the same example (Decision Matrix presented in Table-1.) as classical TOPSIS.

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

Table 6: Decision matrix.

Weighed Decision Matrix is gotten (Table-2.).

$\omega_j a_{ij}$	C_1	C_2	C_3
ω_i	*12/16*	*3/16*	*1/16*
A_1	*84/16*	*27/16*	*9/16*
A_2	*96/16*	*21/16*	*8/16*
A_3	*108/16*	*18/16*	*8/16*
A_4	*72/16*	*21/16*	*8/16*

Table 7: Weighted decision matrix.

Next, we calculate the positive and negative solutions as follow :
S1+ = |84/16-108/16| + |27/16-27/16| + |9/16-9/16| = 1.5000
S2+ = |96/16-108/16| + |21/16-27/16| + |8/16-9/16| = 1.1875
S3+ = |108/16-108/16| + |18/16-27/16| + |8/16-9/16| = 0.6250
S4+ = |72/16-108/16| + |21/16-27/16| + |8/16-9/16| =2.6875
S1- = |84/16-72/16| + |27/16-18/16| + |9/16-8/16| =1.3750
S2- = |96/16-72/16| + |21/16-18/16| + |8/16-8/16| =1.6875
S3- = |108/16-72/16| + |18/16-18/16| + |8/16-8/16| =2.2500
S4- = |72/16-72/16| + |21/16-18/16| + |8/16-8/16| = 0.1875
By the end we got both sets of negative and positive solutions $(S3-, S2-, S1-, S4-)$ and $(S3+, S2+, S1+, S4+)$, before arranging preferences, we need to determine which solutions to use, that decision tacked based on the nature of problem, if we seek to minimize or maximize. The minimization of the solution, such as cost to pay, consists on the solution closer to the negative solution, while he maximization of the solution, such as price to sale, consists on the solution closer to the positive solution.

The optional ranking measure T_i confirm the same result.

$$T1 = (S1-)/[(S1-) + (S1+)] = 0.478261 \tag{3.8}$$

$$T2 = (S2-)/[(S2-) + (S2+)] = 0.586957 \tag{3.9}$$

$$T3 = (S3-)/[(S3-) + (S3+)] = 0.782609 \tag{3.10}$$

$$T4 = (S4-)/[(S4-) + (S4+)] = 0.065217 \tag{3.11}$$

The table (Table-8.) figure out all calculus did before

Alternative	S_i^+	S_i^-	T_i
A_1	1.5000	1.3750	0.478261
A_2	1.1875	1.6875	0.586957
A_3	0.6250	2.2500	0.782609
A_4	2.6875	0.1875	0.065217

Table 8: Distance measure and ranking coefficient.

By applying Simplified-TOPSIS, we get for T_3 (0.782609), T_2(0.586957), T_1(0.478261) and T_4(0.065217), and we got with classical TOPSIS T_3(0.7938), T_2(0.6412), T_1(0.3880) and T_4(0.0783). Hence the order obtained with our approach simplified-TOPSIS is the same of classical TOPSIS: T_3, T_2, T_1 and T_4, with little change in values between both approaches.

The both methods our simplified-TOPSIS and Standard TOPSIS produce the same results witt the same ranking $(T3, T2, T1andthenT4)$, with a little differences of ranking measures. For example, with Simplified-TOPSIS T_3 is 0.782609, and with TOPSIS T_3 is 0.7938, the same for all others(Simplified-TOPSIS : $T_2(0.586957)$, $T_1(0.478261)$ and $T_4(0.065217)$ and with TOPSIS : $T_2(0.6412)$, $T_1(0.3880)$ and $T_4(0.0783)$.

4 Standard TOPSIS in Neutrosophic [12]

Standard TOPSIS in Neutrosophic procedure can be summarized as follow :
Step 1: In order to apply neutrosophic TOPSIS algorithm, crisp number Decision Matrix need to be mapped to single valued neutrosophic environment, then, we got neutrosophic decision matrix

$$D = (d_{ij})_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} \quad \text{(Neutrosophic Decision Matrix)} \tag{4.1}$$

Where T_{ij}, I_{ij} and F_{ij} are truth, indeterminacy and falsity membership scores respectively. i refer to preference A_i and j to creterion C_j.

And $w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

Example 1:

To compare our method Neutrosophic Simplified TOPSIS (nS-TOPSIS : section 5) and standard NeutrosophicTOPSIS proposed by Biswas ([11]). we use Biswas's numercal example.

Let (DM_1, DM_2, DM_3, DM_4) fours decisions makers aims to select an alternative A_i $(A_1, A_2, A_3, A_4)$with respect six criteria$(C_1, C_2, C_3, C_4, C_5, C_6)$. The mapped weights of criteria and decision matrix in Neutrosophic environment are presented in tables Table-9. and Table-10. respectively.

	C_1	C_2	C_3
ω_i	$(0.755, 0.222, 0.217)$	$(0.887, 0.113, 0.107)$	$(0.765, 0.226, 0.182)$
	C_4	C_5	C_6
ω_i	$(0.692, 0.277, 0.251)$	$(0.788, 0.200, 0.180)$	$(0.700, 0.272, 0.244)$

Table 9: Criteria weights.

	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.: 092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$
	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_2	$(0.744, 0.256, 0.204)$	$(0.652, 0.348, 0.293)$	$(0.608, 0.392, 0.336)$
A_3	$(0.779, 0.256, 0.204)$	$(0.811, 0.189, 0.109)$	$(0.850, 0.150, 0.092)$
A_4	$(0.727, 0.273, 0.221)$	$(0.791, 0.209, 0.148)$	$(0.808, 0.192, 0.127)$

Table 10: Neutrosophic Decision Matrix.

Step 2: Weighted decision matrix in neutrosophic is gotten by applying aggregation operator of multiplication i. e. application of generalization of multiplication operator in Neutrosophic space.

$$D^w = D \otimes W = (d_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{4.2}$$

Step 3: Calculate of POS-SVNs (positive ideal solution in SVNs) and NIS-SVNs (negative ideal solution in SVNS) measures.

$$T_j^{w+} = \left\{ \left(max_i \left\{ T_{ij}^{w_i} \left| j \in B \right. \right\} \right), \left(min_i \left\{ T_{ij}^{w_i} \left| j \in C \right. \right\} \right) \right\} \tag{4.3}$$

$$Q_N^+ = (d_1^{w+}, d_2^{w+}, \cdots, d_n^{w+}) \tag{4.4}$$

$$T_j^{w+} = \left\{ \left(max_i \left\{ T_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(min_i \left\{ T_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.5}$$

$$I_j^{w+} = \left\{ \left(min_i \left\{ I_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(max_i \left\{ I_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.6}$$

$$F_j^{w+} = \left\{ \left(min_i \left\{ F_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(max_i \left\{ F_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.7}$$

$$Q_N^- = (d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-}) \tag{4.8}$$

$$T_j^{w-} = \left\{ \left(min_i \left\{ T_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(max_i \left\{ T_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.9}$$

$$I_j^{w-} = \left\{ \left(max_i \left\{ I_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(min_i \left\{ I_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.10}$$

$$F_j^{w-} = \left\{ \left(max_i \left\{ F_{ij}^{w_j} \left| j \in B \right. \right\} \right), \left(min_i \left\{ F_{ij}^{w_j} \left| j \in C \right. \right\} \right) \right\} \tag{4.11}$$

Where B represents the benefit and C quantify the cost.

Step 4: Calculate length of each alternative from the POS-SVNs and NIS-SVNs calculated in previous step.

$$D_{Eu}^{i+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right)^2\end{array}\right\}} \tag{4.12}$$

$$D_{Eu}^{i-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right)^2\end{array}\right\}} \tag{4.13}$$

With $i = 1, 2\cdots, m$

Step 5: Calculate the aggregated coefficient of closeness in Neutrosophic.

$$C_i^* = \frac{NS_i^-}{(NS_i^+ + NS_i^-)};\ i = 1, 2\cdots, m \tag{4.14}$$

All values of aggregated coefficient of closeness are shown in the table Table-11. below.

Alternative	C_i^*
A_1	0.8190
A_2	0.1158
A_3	**0.8605**
A_4	0.4801

Table 11: Closeness Coefficient.

Using the associate values of aggregated coefficient of closeness C_i^* to preference A_i, in descending order, to rank alternatives. Hence, preferences could be ordered as follow $A_3 > A_1 > A_4 > A_2$. Then, the alternative A_3 is the best solution.

5 Neutrosophic-Simplified-TOPSIS (our proposed method)

Step 1: Construct Neutrosophic decision matrix.

As made for Standard Neutrosophic TOPSIS, let consider neutrosophic decision matrix and SVNs weighted criteria.

$$D = (d_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{5.1}$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & \cdots & \cdots & d_{mn} \end{pmatrix}\end{array}$$

Where T_{ij} denote truth, I_{ij} indeterminacy and N_{ij} falsity membership score of preference i knowing j in neutrosophic environment.

$w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

Step 2: Calculate SVNs weighted decision matrix.

$$D^w = D \otimes W = (d_{ij}^w)_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} = \omega_j \otimes d_{ij}^w = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} \tag{5.2}$$

$$\omega_j \otimes d_{ij} = \left(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij}\right) \tag{5.3}$$

Step 3: Calculate LNIS and SNIS metrics.

LNIS and SNIS are maximum (larger) and minimum (smaller) neutrosophic ideal solution respectively.

$$A_N^+ = (d_1^{w+}, d_2^{w+}, \cdots, d_n^{w+}) \tag{5.4}$$

$$d_j^{\omega+} = \left(T_j^{\omega+}, I_j^{\omega+}, F_j^{\omega+}\right) \tag{5.5}$$

$$T_j^{w+} = \left\{\left(max_i \left\{T_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.6}$$

$$I_j^{w+} = \left\{\left(min_i \left\{I_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.7}$$

$$F_j^{w+} = \left\{\left(min_i \left\{F_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.8}$$

$$A_N^- = (d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-}) \tag{5.9}$$

$$d_j^{\omega-} = \left(T_j^{\omega-}, I_j^{\omega-}, F_j^{\omega-}\right) \tag{5.10}$$

$$T_j^{w-} = \left\{\left(min_i \left\{T_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.11}$$

$$I_j^{w-} = \left\{\left(max_i \left\{I_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.12}$$

$$F_j^{w-} = \left\{\left(max_i \left\{F_{ij}^{w_j} \left| j = 1, \cdots, n\right.\right\}\right)\right\} \tag{5.13}$$

Step 4: Determination of the distance measure of every alternative from the RNPIS and the RNNIS for SVNSs.

To perform that calculus, we need to introduce a new distance measure, in this paper we mapped Manhattan distance ([13]) to Neutrosophic environment (definition 1). The new proposed distance called Neutrosophic Manhattan distance that perform the difference between two single-valued neutrosophic(SVNs) measures.

Definition 1. Let $X_1 = (x_1, y_1, z_1)$ and $X_2 = (x_2, y_2, z_2)$ be a SVN numbers. Then the separation measure between X_1 and X_2 based on Manhattan distance is defined as follows:

$$D_{Manh}(X_1, X_2) = |x_1 - x_2| + |y_1 - y_2| + |z_1 - x_2| \tag{5.14}$$

The application of Neutrosophic Manhattan distance to calculate the separation from the maximum and minimum Neutrosophic ideal solution respectively are :

$$D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \left\{ \begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right| \end{array} \right\} \tag{5.15}$$

with $j = 1, 2 \cdots, n$

$$NS_i^+ = \sum_{j=1}^{n} D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) \tag{5.16}$$

with $i = 1, 2 \cdots, m$

Similarly, the separation from the minimum neutrosophic ideal solution is:

$$D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \left\{ \begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right| \end{array} \right\} \tag{5.17}$$

with $j = 1, 2 \cdots, n$

$$NS_i^- = \sum_{j=1}^{n} D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) \tag{5.18}$$

with $i = 1, 2 \cdots, m$

Preferences are ordered regarding to the values of NS_i^- or according to $1/NS_i^+$. In other words, the alternatives with the highest appraisal score is the best solution.

Step 5: Rank the alternatives according to Ranking coefficient NT_i.

Ranking coefficient is formulated as :

$$NT_i = \frac{NS_i^-}{\left(NS_i^+ + NS_i^-\right)};\ i = 1, 2 \cdots, m \tag{5.19}$$

A set of alternatives can now be ranked according to the descending order of the value of NT_i

5.1 Numerical example

Step 1. Formulate the MCDM problem in neutrosophic by building Neutrosophic decision matrix decision matrix and SVNs weights of criteria.

Let $A_i\,(A_1, A_2, A_3, A_4)$a set of alternative and $C_i\,(C_1, C_2, C, C_4, C_5, C_6)$ a set of criteria. Let considers the following neutrosophic weights of criteria (Table-12.) and neutrosophic decision matrix (Table-13.) respectively (used in above example 1).

Step 2: Calculation of SVNs Weighted Decision Matrix

$$D^w = (d_{ij}^w)_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \leq i \leq n \\ 1 \leq j \leq m}} \tag{5.20}$$

	C_1	C_2	C_3
ω_i	$(0.755, 0.222, 0.217)$	$(0.887, 0.113, 0.107)$	$(0.765, 0.226, 0.182)$
	C_4	C_5	C_6
ω_i	$(0.692, 0.277, 0.251)$	$(0.788, 0.200, 0.180)$	$(0.700, 0.272, 0.244)$

Table 12: Criteria neutrosophic weights.

d_{ij}	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$
	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_2	$(0.744, 0.256, 0.204)$	$(0.652, 0.348, 0.293)$	$(0.608, 0.392, 0.336)$
A_3	$(0.779, 0.256, 0.204)$	$(0.811, 0.189, 0.109)$	$(0.850, 0.150, 0.092)$
A_4	$(0.727, 0.273, 0.221)$	$(0.791, 0.209, 0.148)$	$(0.808, 0.192, 0.127)$

Table 13: Neutrosophic Decision Matrix.

$$d_{ij}^{w} = \Big(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij}\Big) \tag{5.21}$$

SVNs Weighted Decision Matrix is obtained by multiplication of weights of criteria with its associated column of neutrosophic decision matrix:

$$T_{11}^{\omega} = 0.864 \times 0.755 = 0.6523$$

$$I_{11}^{\omega} = 0.136 + 0.222 - 0.136 \times 0.222 = 0.328$$

$$F_{11}^{\omega} = 0.081 + 0.217 - 0.081 \times 0.217 = 0.280$$

d_{ij}^{w}	C_1	C_2	C_3
A_1	$(0.6523, 0.328, 0.28)$	$(0.7566, 0.2434, 0.1892)$	$(0.612, 0.381, 0.305)$
A_2	$(0.5036, 0.481, 0.434)$	$(0.6448, 0.3552, 0.3026)$	$(0.510, 0.484, 0.409)$
A_3	$(0.6644, 0.315, 0.269)$	$(0.787, 0.2132, 0.1642)$	$(0.638, 0.354, 0.274)$
A_4	$(0.5036, 0.481, 0.434)$	$(0.6519, 0.3481, 0.2811)$	$(0.588, 0.406, 0.329)$
	C_4	C_5	C_6
A_1	$(0.487, 0.491, 0.432)$	$(0.649, 0.342, 0.281)$	$(0.605, 0.371, 0.305)$
A_2	$(0.515, 0.462, 0.404)$	$(0.514, 0.478, 0.420)$	$(0.426, 0.557, 0.498)$
A_3	$(0.539, 0.462, 0.404)$	$(0.639, 0.351, 0.269)$	$(0.595, 0.381, 0.314)$
A_4	$(0.503, 0.474, 0.417)$	$(0.623, 0.367, 0.301)$	$(0.566, 0.412, 0.34)$

Table 14: Weighted Neutrosophic decision matrix.

Step 3: Determination of LNIS and SNIS.

	C_1	C_2	C_3
$d_j^{\omega+}$	$(0.664, 0.315, 0.269)$	$(0.887, 0.213, 0.264)$	$(0.638, 0.354, 0.274)$
	C_4	C_5	C_6
$d_j^{\omega+}$	$(0.539, 0.462, 0.404)$	$(0.649, 0.341, 0.294)$	$(0.605, 0.371, 0.305)$

Table 15: Maximum (large) Neutrosophic Ideal Solution(LNIS).

	C_1	C_2	C_3
$d_j^{\omega-}$	$(0.504, 0.481, 0.434)$	$(0.645, 0.355, 0.303)$	$(0.510, 0.484, 0.409)$
	C_4	C_5	C_6
$d_j^{\omega-}$	$(0.487, 0.491, 0.432)$	$(0.514, 0.478, 0.420)$	$(0.426, 0.557, 0.498)$

Table 16: Minimum (smaller) Neutrosophic Ideal Solution (SNIS).

	NS_i^+	NS_i^-	NT_i
A_1	*0,324*	*2,07*	*0,86459295*
A_2	*2,31*	*0,084*	*0,03521102*
A_3	*0,047*	*2,347*	***0,98021972***
A_4	*1,293*	*1,101*	*0,45987356*

Table 17: Neutrosophic Separation Measures and Neutrosophic Measure Ranking.

Step 4: Calculation of NS_i^+ and NS_i^- To calculate NS_i^+ and NS_i^-, we calculate sum of each line, and then subtracting from the LNIS and from SNIS respectively.

According to the obtained result (Table-17.), alternatives can be ranked as follow $A_3 > A_1 > A_4 > A_2$. Then the best preference is A_3. Using the same example, our proposed method neutrosophic-simplified-TOPSIS(nTOPSIS), we get similar result as neutrosophic-TOPSIS.

6 Conclusion

This paper aims to present tow new TOPSIS based approaches for MCDM. First one is Simplified TOPSIS (sTOPSIS) that simplify the TOPSIS calculation procedure. Second one, neutrosophic simplified-TOPSIS (nTOPSIS) extend the proposed method to neutrosophic environment, that use, instead of crisp number, the single valued neutrosophic(SVN). To formulate the both proposed method, many measures are defined such as Neutrosophic Manhattan Distance measure, that is used to calculate, distances from Maximum (larger) Neutrosophic Ideal Solution (LNIS) minimum neutrosophic ideal solutions, as two new defined measures.

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