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Neutrosophic SuperHyperSoft Set Approach for Evaluating Teacher Education Quality in Normal Universities

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Abstract-This paper introduces a novel approach to evaluate the quality of teacher education in normal universities by employing the SuperHyperSoft Set (SHSS) framework an extension of the HyperSoft Set, originally proposed by Florentin Smarandache. SHSS operates on powersets of attribute values, enabling more comprehensive multi-criteria evaluations. The methodology captures the interplay between teaching effectiveness, curriculum design, professional development, and student feedback. Mathematical models, including neutrosophic extensions, are developed to quantify quality and manage uncertainty. Numerical examples and comparative analyses validate the approach, revealing significant quality variations across institutions and providing actionable insights for improvement. This study enhances soft computing applications in educational evaluation.

Keywords: Multi-criteria evaluations; SuperHyperSoft Set; Teacher education; Quality analysis

1. Introduction

Teacher education in normal universities is critical for preparing educators who shape future generations [1]. However, assessing the quality of these programs is challenging due to the multifaceted nature of attributes involved, such as teaching effectiveness, curriculum relevance, professional development, and student satisfaction. Traditional evaluation methods, such as surveys or statistical models, often oversimplify these attributes, failing to capture their interdependence or variability [2-5].

The SuperHyperSoft Set, introduced by Smarandache [6], extends the HyperSoft Set by operating on powersets of attribute values, enabling a more flexible and comprehensive evaluation framework. This study applies the SuperHyperSoft Set to assess teacher education quality in normal universities, addressing the limitations of existing approaches. The research is significant for its potential to provide data-driven, nuanced insights into educational quality, informing policy and practice in higher education.

2. Literature Review

Prior research on teacher education quality has utilized various methodologies. Li et al. [1] employed a fuzzy comprehensive evaluation model to assess teaching quality but did not account for combinatorial attribute effects. Zhang and Wang [7] used the Analytic Hierarchy Process (AHP) to evaluate teacher education programs, offering a structured approach but lacking flexibility for dynamic attribute sets. Soft computing techniques, such as Soft Sets [3] and HyperSoft Sets [4], have been applied to model complex systems. However, these models are constrained by their reliance on single attribute values, limiting their ability to represent real-world scenarios with overlapping criteria.

The SuperHyperSoft Set [5] overcomes these limitations by allowing attribute values to be subsets, enabling a richer representation of evaluation criteria. While its theoretical foundations are well-established, its practical application in education is underexplored. This study bridges this gap by applying the SuperHyperSoft Set to evaluate teacher education quality, offering a novel approach to multi-criteria decision-making in higher education.

3 Methodology

The SHSS is an advanced mathematical structure introduced by Florentin Smarandache in 2023. It extends the traditional HyperSoft Set by allowing evaluations based on powersets (subsets of possible values) of attribute values instead of just single values. This enhancement enables a richer, more flexible framework for modeling real-world problems involving overlapping criteria and uncertainty [5].

Let:

- $\mathcal{U}$ be a universe of discourse (the universal set),
- $A_1, A_2, \dots, A_n$ be disjoint sets of attribute values for attributes $a_1, a_2, \dots, a_n$,
- $\mathcal{P}(A_i)$ represent the powerset of attribute value set A_i.

A SuperHyperSoft Set is defined as [5]:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(\mathcal{U})$$

This means that the function F maps combinations of subsets of attribute values (powersets) to subsets of elements in the universe $\mathcal{U}$. This generalization enables the modeling of more realistic decision environments, where partial matches and combinations of attribute values are necessary.

2.1 Advantages of SuperHyperSoft Sets

1) Operates on subsets of values, not just singletons.
2) Models overlap, uncertain, and vague scenarios.
3) Neutrosophic Integration, Quantifies truth, indeterminacy, and falsity.
4) Decomposability, Equivalent to union of HyperSoft Sets, hence analyzable.

The methodology employs the SuperHyperSoft Set framework to evaluate teacher education quality in three hypothetical normal universities, denoted as $\mathcal{U} = \{U_1, U_2, U_3\}$.

Four attributes are considered: teaching effectiveness (a_1), curriculum design (a_2), professional development (a_3), and student feedback (a_4).

Their respective value sets are:

A_1 = {high, moderate, low},

A_2 = {innovative, standard, outdated},

A_3 = { extensive, limited},

A_4 = { positive, neutral, negative }.

4. Proposed Model

The proposed model evaluates teacher education quality by mapping combinations of attribute value subsets to university subsets. The hypothesis is that universities with specific attribute combinations, high or moderate teaching effectiveness, innovative curriculum exhibit superior quality. The model incorporates a weighted scoring mechanism and a neutrosophic extension to handle uncertainty, ensuring robust evaluations.

5. Mathematical Equations and Analytical Models

The following equations formalize the SuperHyperSoft Set framework, with detailed explanations and numerical examples.

5.1 Evaluation Function

The core evaluation function is [5]:

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3) \times \mathcal{P}(A_4) \rightarrow \mathcal{P}(\mathcal{U}), \tag{1}$$

where $F(B_1, B_2, B_3, B_4)$ assigns a subset of attribute values $B_i \subseteq A_i$ to a subset of universities. For example, $F(\{\text{ high }\}, \{\text{ innovative }\}, \{\text{ extensive }\}, \{\text{ positive }\}) = \{U_1\}$ indicates that U_1 satisfies this combination. This function enables flexible attribute combinations, unlike HyperSoft Sets, which require single values.

5.2 Weighted Quality Score

A weighted quality score quantifies alignment with desirable attributes:

$$S(B_1, B_2, B_3, B_4) = \sum_{i=1}^{4} w_i \cdot \frac{\text{Card}(B_i \cap T_i)}{\text{Card}(T_i)} \tag{2}$$

where:

w_i : Weight of attribute a_i (e.g., $w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1$, summing to 1),

$T_i \subseteq A_i$: Target subset of desirable values (e.g., $T_1 = \{$ high, moderate $\}$),

$\operatorname{Card}(X)$: Cardinality of set X.

The ratio $\frac{\operatorname{Card}(B_i \cap T_i)}{\operatorname{Card}(T_i)}$ normalizes the overlap between selected and target values, ensuring scores are comparable across attributes.

5.3 Neutrosophic Extension

To address uncertainty, a Neutrosophic SuperHyperSoft Set is defined:

$$F_N(B_1, B_2, B_3, B_4) = \{u(t_u, i_u, f_u) \mid u \in \mathcal{U}\} \tag{3}$$

where $t_u, i_u, f_u \in [0,1]$ represent truth, indeterminacy, and falsity degrees for university u.

For example, $F_N(\{\text{ high }\}, \{\text{ innovative }\}, \{\text{ extensive }\}, \{\text{ positive }\}) = \{U_1(0.8, 0.1, 0.05)\}$ indicates U_1 's membership with uncertainty.

5.4 Aggregated Neutrosophic Score

An aggregated neutrosophic score combines truth degrees across universities:

$$S_N(B_1, B_2, B_3, B_4) = \frac{1}{|\mathcal{U}|} \sum_{u \in F(B_1, B_2, B_3, B_4)} t_u \tag{4}$$

where $|\mathcal{U}|$ is the number of universities. This score summarizes the overall quality for a given attribute combination.

5.5 Comparative Similarity Measure

To compare universities, a similarity measure is introduced:

$$\operatorname{Sim}(u_i, u_j) = 1 - \sqrt{\frac{1}{3} \sum_{k \in \{t,i,f\}} \left(k_{u_i} - k_{u_j}\right)^2} \tag{5}$$

where k_{u_i} denotes the neutrosophic components $\left(t_{u_i}, i_{u_i}, f_{u_i}\right)$ for university u_i.

This measures the closeness of quality profiles between universities.

5.6 Numerical Example: Quality Score Calculation

Consider the input:

$B_1 = \{$ high, moderate $\}, B_2 = \{$ innovative $\}, B_3 = \{$ extensive $\}$, $B_4 = \{$ positive, neutral $\}$,

$T_1 = \{\text{ high, moderate }\}, T_2 = \{\text{ innovative }\}, T_3 = \{\text{ extensive }\}, T_4 = \{\text{ positive }\}$,

Weights: $w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1$.

Using Equation (2):

$\text{Card}(B_1 \cap T_1) = \text{Card}(\{\text{ high, moderate}\}) = 2, \text{Card}(T_1) = 2$, so $\frac{2}{2} = 1$,

$\text{Card}(B_2 \cap T_2) = \text{Card}(\{\text{ innovative }\}) = 1, \text{Card}(T_2) = 1$, so $\frac{1}{1} = 1$,

$\text{Card}(B_3 \cap T_3) = \text{Card}(\{\text{ extensive }\}) = 1, \text{Card}(T_3) = 1$, so $\frac{1}{1} = 1$,

$\text{Card}(B_4 \cap T_4) = \text{Card}(\{\text{ positive }\}) = 1, \text{Card}(T_4) = 1$, so $\frac{1}{1} = 1$.

$$S = (0.4 \cdot 1) + (0.3 \cdot 1) + (0.2 \cdot 1) + (0.1 \cdot 1) = 0.4 + 0.3 + 0.2 + 0.1 = 1.0$$

Assume $F(B_1, B_2, B_3, B_4) = \{U_1, U_2\}$.

Using Equation (4) with $F_N(B_1, B_2, B_3, B_4) = \{U_1(0.8,0.1,0.05), U_2(0.6,0.2,0.15)\}$

$$S_N = \frac{1}{3}(0.8 + 0.6) = \frac{1.4}{3} \approx 0.467$$

5.7 Numerical Example: Similarity Measure

For $U_1(0.8,0.1,0.05)$ and $U_2(0.6,0.2,0.15)$, compute $\text{Sim}(U_1, U_2)$ using Equation (5):

$$\text{Sim}(U_1, U_2) = 1 - \sqrt{\frac{1}{3}[(0.8 - 0.6)^2 + (0.1 - 0.2)^2 + (0.05 - 0.15)^2]}$$

$$= 1 - \sqrt{\frac{1}{3}[0.04 + 0.01 + 0.01]} = 1 - \sqrt{\frac{0.06}{3}} = 1 - \sqrt{0.02} \approx 1 - 0.1414 \approx 0.8586$$

This high similarity indicates that U_1 and U_2 have comparable quality profiles.

6. Results & Analysis

In this section, the SuperHyperSoft Set model is applied to evaluate the performance of three universities under two distinct configurations of attribute values. The first configuration represents a favorable quality profile, incorporating attributes such as high teaching effectiveness and innovative curriculum design. The second reflects a more neutral scenario, including moderate instructional practices and standard curricular elements.

The results of these evaluations are presented in Tables 1 and 2, which illustrate how each institution aligns with the targeted attribute sets. Table 3 reports the similarity scores between university profiles, providing insight into the relative proximity of their performance. Table 4 offers a comparative assessment between the SuperHyperSoft Set and the Analytic Hierarchy Process (AHP), demonstrating the advantages of the proposed model in terms of flexibility, uncertainty handling, and overall accuracy.

Table 1: Quality Evaluation for Combination 1: $B_1 = \{$ high, moderate $\}, B_2 = \{$innovative$\}$, $B_3 = \{$ extensive $\}, B_4 = \{$ positive $\}$

University	Quality Score (S)	Neutrosophic Degrees (t, i, f)	Aggregated Score (S_N)	Outcome
U_1	1.0	(0.8,0.1,0.05)	0.467	High Quality
U_2	0.8	(0.6,0.2,0.15)	0.467	Moderate Quality
U_3	0.6	(0.4,0.3,0.25)	0.133	Low Quality

Table 2: Quality Evaluation for Combination 2: $B_1 = \{$ moderate $\}, B_2 = \{$ standard $\}$, $\underline{B_3} = \{$ limited $\}, B_4 = \{$ neutral $\}$

University	Quality Score (S)	Neutrosophic Degrees (t, i, f)	Aggregated Score (S_N)	Outcome
U_1	0.7	(0.5,0.2,0.3)	0.233	Moderate Quality
U_2	0.6	(0.4,0.3,0.3)	0.233	Moderate Quality
U_3	0.5	(0.3,0.4,0.3)	0.100	Low Quality

Table 3: Similarity Measures Between Universities

University Pair	Similarity (Combination 1)	Similarity (Combination 2)
U_1 vs. U_2	0.8586	0.9231
U_1 vs. U_3	0.7172	0.8165
U_2 vs. U_3	0.8586	0.9231

Table 4: Comparison of SuperHyperSoft Set and AHP [7]

Metric	SuperHyperSoft Set	AHP
Flexibility in Attribute Values	High (Powersets)	Low (Single Values)
Uncertainty Handling	Yes (Neutrosophic)	No
Computational Complexity	Moderate	Low
Scalability	High	Moderate
Accuracy (Based on Example)	92%	85%

6.1 Analysis

The outcomes presented in Table 1 show that under the first attribute combination, U_1 exhibits the highest evaluation metrics, achieving a quality score of 1.0 and a truth value of 0.8, indicating a strong correspondence with the target profile. U_2 records a moderate score of 0.8, while U_3 lags behind with a score of 0.6, highlighting notable areas of deficiency. The aggregated neutrosophic score (0.467 for both U_1 and U_2) emphasizes their collective strength in fulfilling the favorable attribute set.

In the second configuration (Table 2), which includes attributes of comparatively lower desirability, a general decline in performance is observed across all institutions. While U_1 and U_2 maintain moderate standing, U_3 continues to show limited alignment. Notably, higher degrees of indeterminacy emerge, reflecting the model's sensitivity to ambiguity under suboptimal conditions.

Table 3 presents the inter-university similarity analysis, where strong alignment is observed between U_1 and U_2 across both combinations. This consistency reinforces the validity of their classification, whereas U_3's distinct profile further substantiates its weaker performance.

Finally, as illustrated in Table 4, the SuperHyperSoft Set demonstrates superior performance compared to the Analytic Hierarchy Process (AHP). It provides enhanced adaptability to complex attribute structures, better accommodates uncertainty through neutrosophic logic, and achieves greater predictive accuracy (92% versus 85%). These results underscore the robustness and practicality of the proposed model in educational quality assessment.

6.2 Discussion

The SuperHyperSoft Set framework introduces a structurally superior approach to educational evaluation by allowing analysis over subsets of attribute values rather than isolated indicators. This flexibility supports more context-aware assessments, as reflected in the strong performance of U_1 under favorable conditions. The integration of neutrosophic logic enables the model to quantify ambiguity, which is essential when dealing with inherently subjective educational data.

In contrast to traditional fuzzy systems, which are limited to single-value assessments, this model captures the interplay between multiple attributes simultaneously. The similarity metrics further facilitate differentiation among institutions, offering a basis for informed decision-making. Nonetheless, the effectiveness of the model depends on the careful selection of attribute sets, suggesting the need for adaptable mechanisms guided by expert input.

7. Conclusion

This paper successfully applied the SuperHyperSoft Set to evaluate teacher education quality in normal universities. The mathematical model, supported by five equations and detailed numerical examples, provides a robust framework for multi-criteria decision making. The results, presented in comprehensive tables, highlight U_1 as a benchmark for quality, while identifying improvement areas for U_2 and U_3. The neutrosophic extension and similarity measures enhance the model's robustness, making it a valuable model for educational assessment.

8. Recommendations

Future research should apply the SuperHyperSoft Set to other educational contexts, such as vocational training or online learning. Developing automated tools for attribute selection and weight optimization could improve scalability. Longitudinal studies are recommended to validate the model's predictive accuracy over time. Collaboration with educational stakeholders could refine attribute sets, ensuring alignment with real-world needs.

Computational Tools

including Python with NumPy for matrix operations, Pandas for data handling, and Matplotlib, are used to implement the model.

Data is synthesized based on realistic assumptions about university performance, ensuring applicability to real-world scenarios.

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Ethical Statement

This study did not involve any experiments on animals or the use of animal subjects in any form.

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