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Plithogenic Sets-Based Academic Quality Appraisal Under Uncertainty in Vocational College Art Courses

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Abstract: This paper presents a rigorous application of plithogenic sets, as introduced by Smarandache (2018), to evaluate teaching quality in vocational college art courses. Plithogenic sets generalize crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets by characterizing elements with multiple attributes, each associated with appurtenance and contradiction degrees. We develop a tailored plithogenic set model for art education, incorporating attributes such as instructional clarity, student engagement, creative facilitation, and assessment fairness. The model employs plithogenic aggregation operators, inclusion relations, and refined sets to achieve precise evaluations. Through detailed numerical examples, verified equations, and multiple tables, we demonstrate the frameworks effectiveness. The paper integrates accurate references and addresses practical implications for vocational education.

Keywords: multiple attributes; plithogenic set model; vocational college art courses; teaching quality

1. Introduction

Evaluating teaching quality in vocational college art courses is a complex task due to the subjective nature of artistic instruction and the multiplicity of evaluation criteria. Traditional methods, such as crisp sets or fuzzy sets [1-3], often fail to capture the nuanced interplay of attributes like instructional clarity, student engagement, creative facilitation, and assessment fairness. Plithogenic sets, introduced by Smarandache [5], offer a robust framework that extends existing set theories by incorporating multiple attributes with varying degrees of appurtenance and contradiction. This paper adapts the plithogenic set methodology to develop a comprehensive model for assessing teaching quality, specifically tailored to vocational college art courses.

The plithogenic set model is ideal for this application because it handles multi-dimensional attributes and their interdependencies, which are critical in art education where creativity and engagement are paramount [5-7]. By defining plithogenic aggregation operators, inclusion relations, and refined sets, we provide a precise and flexible evaluation model. This paper enhances the original model with additional equations, tables, and rigorous verification to ensure accuracy and completeness of the model.

2. Formal Definition of Plithogenic Sets

A plithogenic set P is a non-empty subset of a universe of discourse U, where $P \subseteq U$, and its elements are characterized by one or more attributes [5]. Let $\mathcal{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}, m \geq 1$,
be a set of uni-dimensional attributes, and let $\alpha \in \mathcal{A}$ be an attribute with a spectrum of possible values S.

2.1 Attribute Value Spectrum

The spectrum S for an attribute α can be:

- Finite discrete: $S = \{s_1, s_2, \ldots, s_l\}, 1 \leq l < \infty$.
- Infinitely countable: $S = \{s_1, s_2, \ldots, s_\infty\}$.
- Infinitely uncountable: $S =]a, b[, a < b$.

For teaching quality evaluation, we use finite discrete sets to ensure practical applicability, e.g., $S = \{$ Clear, Moderate, Unclear $\}$ for instructional clarity.

2.2 Attribute Value Range

The attribute value range $V \subseteq S$ is the subset of values relevant to the application:

$$V = \{v_1, v_2, \ldots, v_n\}, n \geq 1 \quad (1)$$

Each element $x \in P$ is characterized by all values in V. For example, an instructor's clarity is assessed across $V = \{c_1, c_2, c_3\}$.

2.3 Dominant Attribute Value

A dominant attribute value $v_D \in V$ is the most significant value, determined by experts. If no dominant value exists, the contradiction degree is set to zero:

$$c(v_D, v_i) = 0 \text{ if } v_D \text{ is undefined.} \quad (2)$$

In art education, Clear is often the dominant value for instructional clarity.

2.4 Appurtenance Degree Function

Each attribute value $v \in V$ has a degree of appurtenance $d(x, v)$ for an element $x \in P$ to the set P :

$$\forall x \in P, d: P \times V \rightarrow \mathcal{P}([0,1]^z), \quad (3)$$

where $\mathcal{P}([0,1]^z)$ is the power set of $[0,1]^z$, and $z = 1$ (fuzzy), $z = 2$ (intuitionistic fuzzy), or $z = 3$ (neutrosophic). We use fuzzy degrees ($z = 1$), where $d(x, v) \in [0,1]$.

2.5 Contradiction Degree Function

The contradiction degree function $c: V \times V \rightarrow [0,1]$ measures dissimilarity:

$$c(v_1, v_1) = 0, c(v_1, v_2) = c(v_2, v_1) \quad (4)$$

We use a fuzzy contradiction degree c_F :

$$c_F: V \times V \rightarrow [0,1]. \quad (5)$$

For example, c (Clear, Unclear) = 1, indicating maximum dissimilarity.
Thus, a plithogenic set is (P, α, V, d, c), where P is the set of instructors, α is an attribute, V is the value range, d is the appurtenance degree, and c is the contradiction degree.

3. Plithogenic Aggregation Operators

Plithogenic aggregation operators enhance accuracy by incorporating contradiction degrees. We use fuzzy t-norm $a \wedge_F b = a \cdot b$ and t-conorm $a \vee_F b = a + b - a \cdot b$.

3.1 Intersection

For a dominant value v_D and value v_2, with $c(v_D, v_2)$:
$d_A(x, v_2) \wedge_P d_B(x, v_2) = [1 - c(v_D, v_2)] \cdot [d_A(x, v_2) \wedge_F d_B(x, v_2)] + c(v_D, v_2) \cdot [d_A(x, v_2) \vee_F d_B(x, v_2)]$.

3.2 Union

$d_A(x, v_2) \vee_P d_B(x, v_2) = [1 - c(v_D, v_2)] \cdot [d_A(x, v_2) \vee_F d_B(x, v_2)] + c(v_D, v_2) \cdot [d_A(x, v_2) \wedge_F d_B(x, v_2)]$.

3.3 Complement

The complement of an attribute value v_i with $c_i = c(v_D, v_i)$:

$$\neg_P v_i = \operatorname{anti}(v_i) = (1 - c_i) \cdot v_i, \neg_P d_A(x, (1 - c_i) v_i) = d_A(x, v_i) \quad (8)$$

3.4 Inclusion (Partial Order)

The plithogenic inclusion for fuzzy degrees:

$$d_A(x, v) \leq_P d_B(x, v) \text{ iff } d_A(x, v) \leq [1 - c(v_D, v)] \cdot d_B(x, v) \quad (9)$$

This accounts for contradiction degrees, enhancing partial ordering.

3.5 Verification

The weights in equations (6) and (7) sum to 1 :

$$[1 - c(v_D, v_2)] + c(v_D, v_2) = 1$$

This ensures a valid convex combination. For $c(v_D, v_2) = 0$, the intersection reduces to $\wedge_F$, and for $c(v_D, v_2) = 1$, it becomes $\vee_F$, consistent with the document.

4. Plithogenic Set Model for Teaching Quality

We define a plithogenic set $P \subseteq U$, where U is all instructors in a vocational college, and P is art course instructors. The attributes are:

— Instructional Clarity (α_1): $V_1 = \{$ Clear, Moderate, Unclear $\} \cong \{c_1, c_2, c_3\}$.
— Student Engagement (α_2): $V_2 = \{$ High, Medium, Low $\} \cong \{e_1, e_2, e_3\}$.
— Creative Facilitation (α_3): $V_3 = \{$ Innovative, Standard, Basic $\} \cong \{f_1, f_2, f_3\}$.
— Assessment Fairness (α_4): $V_4 = \{$ Fair, Partially Fair, Unfair $\} \cong \{a_1, a_2, a_3\}$.

The multi-attribute value set is:

$$V_4 = \{(c_i, e_j, f_k, a_l) \mid 1 \leq i, j, k, l \leq 3\}, |V_4| = 3^4 = 81$$

Dominant values: c_1, e_1, f_1, a_1. Contradiction degrees:

$$c(c_1, c_2) = 0.5, \quad c(c_1, c_3) = 1; \quad c(e_1, e_2) = 0.5,$$
$$c(f_1, f_2) = 0.5, \quad c(f_1, f_3) = 1; \quad c(e_1, e_3) = 1$$
$$(a_1, a_2) = 0.5, \quad c(a_1, a_3) = 1$$

Table 1: Attribute Definitions and Contradiction Degrees

Attribute	Values	Dominant Value	Contradiction Degrees
Clarity	{Clear, Moderate, Unclear}	Clear	$c(c_1, c_2) = 0.5, c(c_1, c_3) = 1$
Engagement	{High, Medium, Low}	High	$c(e_1, e_2) = 0.5, c(e_1, e_3) = 1$
Facilitation	{Innovative, Standard, Basic}	Innovative	$c(f_1, f_2) = 0.5, c(f_1, f_3) = 1$
Fairness	{Fair, Partially Fair, Unfair}	Fair	$c(a_1, a_2) = 0.5, c(a_1, a_3) = 1$

5. Application: Evaluating an instructor

We evaluate an instructor $x \in P$ for the 4-attribute value (c_1, e_2, f_3, a_2), assessed by two experts: A (department head) and B (student representative).

Input Data Expert A:

$$d_A(c_1) = 0.8, d_A(e_2) = 0.6, d_A(f_3) = 0.5, d_A(a_2) = 0.7$$

Expert B:

$$d_B(c_1) = 0.7, d_B(e_2) = 0.4, d_B(f_3) = 0.6, d_B(a_2) = 0.5$$

Contradiction degrees:

$$c(c_1, c_1) = 0, c(e_1, e_2) = 0.5, c(f_1, f_3) = 1, c(a_1, a_2) = 0.5$$

Plithogenic Intersection Using equation (6):

1. For c_1 :

$$d_A(c_1) \wedge_P d_B(c_1) = (1 - 0) \cdot [0.8 \cdot 0.7] + 0 \cdot [0.8 + 0.7 - 0.8 \cdot 0.7] = 0.56$$

2. For e_2 :
   $d_A(e_2) \wedge_P d_B(e_2) = (1 - 0.5) \cdot [0.6 \cdot 0.4] + 0.5 \cdot [0.6 + 0.4 - 0.6 \cdot 0.4] = 0.5 \cdot 0.24 + 0.5 \cdot 0.76 = 0.50$.
3. For f_3 :
   $d_A(f_3) \wedge_P d_B(f_3) = (1 - 1) \cdot [0.5 \cdot 0.6] + 1 \cdot [0.5 + 0.6 - 0.5 \cdot 0.6] = 0 \cdot 0.3 + 1 \cdot 0.8 = 0.80$.
4. For a_2 :
   $d_A(a_2) \wedge_P d_B(a_2) = (1 - 0.5) \cdot [0.7 \cdot 0.5] + 0.5 \cdot [0.7 + 0.5 - 0.7 \cdot 0.5] = 0.5 \cdot 0.35 + 0.5 \cdot 0.85 = 0.60$.

Result: $x_A \wedge_P x_B(c_1, e_2, f_3, a_2) = (0.56, 0.50, 0.80, 0.60)$.

Plithogenic Union Using equation (7):

1. For c_1 :

$$d_A(c_1) \vee_P d_B(c_1) = (1 - 0) \cdot [0.8 + 0.7 - 0.8 \cdot 0.7] + 0 \cdot [0.8 \cdot 0.7] = 0.94.$$

2. For e_2 :
   $d_A(e_2) \vee_P d_B(e_2) = (1 - 0.5) \cdot [0.6 + 0.4 - 0.6 \cdot 0.4] + 0.5 \cdot [0.6 \cdot 0.4] = 0.5 \cdot 0.76 + 0.5 \cdot 0.24 = 0.50$.
3. For f_3 :
   $d_A(f_3) \vee_P d_B(f_3) = (1 - 1) \cdot [0.5 + 0.6 - 0.5 \cdot 0.6] + 1 \cdot [0.5 \cdot 0.6] = 0 \cdot 0.8 + 1 \cdot 0.3 = 0.30$.
4. For a_2 :
   $d_A(a_2) \vee_P d_B(a_2) = (1 - 0.5) \cdot [0.7 + 0.5 - 0.7 \cdot 0.5] + 0.5 \cdot [0.7 \cdot 0.5] = 0.5 \cdot 0.85 + 0.5 \cdot 0.35 = 0.60$.

   Result: $x_A \vee_P x_B(c_1, e_2, f_3, a_2) = (0.94, 0.50, 0.30, 0.60)$.

   Plithogenic Complement Using equation (8):

$$\neg_P x_A = \left(d_A(c_1), d_A(e_1), d_A(f_1), d_A(a_1)\right).$$

Assuming $d_A(e_1) = 0.7, d_A(f_1) = 0.9, d_A(a_1) = 0.8$:

$$\neg_P x_A = (0.8,0.7,0.9,0.8).$$

Table 2: Evaluation Results for Instructor x

Attribute Value	Contradiction	Expert A	Expert B	Intersection	Union
Clear (c_1)	0	0.8	0.7	0.56	0.94
Medium Engagement (e_2)	0.5	0.6	0.4	0.50	0.50
Basic Facilitation (f_3)	1	0.5	0.6	0.80	0.30
Partially Fair (a_2)	0.5	0.7	0.5	0.60	0.60

5.1 Interpretation

The intersection (0.56,0.50,0.80,0.60) indicates moderate consensus on clarity and fairness, balanced engagement, and high agreement on basic facilitation due to $c(f_1, f_3) = 1$. The union (0.94,0.50,0.30,0.60) highlights optimal clarity but low creative facilitation, reflecting the t-norms dominance for f_3. The complement identifies ideal qualities for improvement.

6. Refined Plithogenic Set

We refined Creative Facilitation:

$V_3 =$ { Highly Innovative, Moderately Innovative, Standard, Basic } $\cong \{f_1, f_{1.1}, f_2, f_3\}$.

Contradiction degrees:

$$c(f_1, f_{1.1}) = 0.25, c(f_1, f_2) = 0.5, c(f_1, f_3) = 1.$$

For $f_{1.1}$, Expert A: $d_A(f_{1.1}) = 0.7$, Expert B: $d_B(f_{1.1}) = 0.6$. Intersection:

$d_A(f_{1.1}) \wedge_P d_B(f_{1.1}) = (1 - 0.25) \cdot [0.7 \cdot 0.6] + 0.25 \cdot [0.7 + 0.6 - 0.7 \cdot 0.6] = 0.75 \cdot 0.42 + 0.25 \cdot 0.88 = 0.535$.

Table 3: Refined Creative Facilitation Evaluation

Value	Contradiction	Expert A	Expert B	Intersection
Highly Innovative (f_1)	0	0.9	0.8	0.72
Moderately Innovative ($f_{1.1}$)	0.25	0.7	0.6	0.535
Standard (f_2)	0.5	0.6	0.5	0.45
Basic (f_3)	1	0.5	0.6	0.80

6.1 Explanation

The refined model distinguishes between levels of innovation, with the intersection for $f_{1.1}$ (0.535) reflecting a nuanced consensus. The table shows how contradiction degrees modulate results, enhancing precision.

7. Plithogenic Inclusion

For $v = e_2$, with $c(e_1, e_2) = 0.5$, compare Expert A and B:

$$d_A(e_2) = 0.6 \leq_P d_B(e_2) = 0.4 \text{ iff } 0.6 \leq (1 - 0.5) \cdot 0.4 = 0.2$$

Since $0.6 > 0.2, d_A(e_2) \Big/ \mathbb{Z}_P d_B(e_2)$. This indicates Expert A rates higher engagement, adjusted by contradiction.

Table 4: Inclusion Comparison for Engagement

Value	Expert A	Expert B	Inclusion Check

Medium Engagement (e_2)	0.6	0.4	$0.6 \nleq 0.5 \cdot 0.4 = 0.2$

8. Comparison with Other Sets

Plithogenic sets generalize: - Crisp Set:

$$d(x, \text{ membership }) = 1, c(\text{ membership }, \text{ nonmembership }) = 1.$$

Fuzzy Set:

$$d(x, \text{ membership }) \in [0,1], \neg_F a = 1 - a.$$

Intuitionistic Fuzzy Set:

$$d(x, \text{ membership }) + d(x, \text{ nonmembership }) \leq 1$$

Neutrosophic Set:

$$0 \leq d(x, \text{ membership }) + d(x, \text{ indeterminacy }) + d(x, \text{ nonmembership }) \leq 3.$$

Plithogenic sets handle multiple attributes, making them superior for complex evaluations [5-7].

Table 5: Comparison of Set Models

Set Type	Attributes	Contradiction Degree
Crisp	1 (membership)	Binary (0 or 1)
Fuzzy	1 (membership)	None
Intuitionistic Fuzzy	2 (membership, nonmembership)	Fixed (1)
Neutrosophic	3 (membership, indeterminacy, nonmembership)	Variable (0,0.5,1)
Plithogenic	Multiple	Flexible [0,1]

9. Discussion

The practical implementation of the plithogenic set model in this study demonstrates its strong capacity to handle the complexity and subjectivity inherent in evaluating teaching quality in vocational college art courses. Unlike traditional models that rely on binary or single-dimensional membership values, the plithogenic approach uses multiple attributes and incorporates contradiction degrees, allowing for a more detailed and accurate evaluation process. For example, the inclusion of contradiction values in Table 2 directly influenced the intersection and union results, adjusting the evaluations based on the degree of disagreement between expert opinions.

A clear advantage of the model is its ability to handle contradictory expert assessments without losing coherence. For instance, in evaluating the "Basic Facilitation" attribute where the contradiction degree was 1, the model still produced a mathematically consistent intersection and union by appropriately applying the defined aggregation operators. This flexibility allows the model to realistically reflect educational settings where evaluators often have different perspectives due to their roles such as the department head versus student representative.

The refinement of the "Creative Facilitation" attribute in Section 6 further illustrates the model's adaptability. By breaking down the attribute into four finer categories and assigning specific contradiction degrees to each, the model captured subtle variations in instructional quality that a standard three-level scale would miss. The resulting intersection values, especially for "Moderately Innovative," revealed nuanced consensus levels between experts that could be directly interpreted for program improvement.

Moreover, the plithogenic inclusion analysis in Section 7 offered a structured way to compare how different experts perceive the same attribute. For example, the comparison of "Medium Engagement" showed a measurable difference between expert A and B's assessments, adjusted through contradiction-aware ordering. This kind of insight is valuable for decision-makers seeking not only evaluation results but also an understanding of the evaluators' perspectives.

In summary, the discussion reveals that the plithogenic set model does not only provide quantitative evaluation but also preserves the interpretive depth needed in educational assessments. The combination of multi-attribute logic, contradiction modulation, and refined structuring makes it especially suitable for art education, where qualitative judgments play a central role.

10. Conclusion and Future Research

This study applied plithogenic set theory to evaluate teaching quality in vocational art programs by combining multiple teaching attributes with contradiction degrees. The model adjusted for differences in expert opinions and maintained mathematical accuracy through step-by-step calculations.

Practical examples showed that attributes with low contradiction, like clarity, led to consistent results, while higher-contradiction areas, such as creative facilitation, demonstrated the model's strength in handling disagreement. Refining attributes further improved evaluation precision.

The model also compared expert input using inclusion logic, offering useful insights for educational development. Verified equations and tables ensured the model's reliability.

Future research could explore time-based evaluations, integrate AI tools for smarter analysis, and apply the model in other fields to test its broader effectiveness.

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