NSS

Neutrosophic Sets and Systems, Vol. 86, 2025

University of New Mexico

Optimizing Defective Product Distribution for Sustainability and Profitability in Manufacturing Model under Neutrosophic Environment

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Abstract. This study investigates the challenges associated with defective product distribution in manufacturing industries, contributing to environmental degradation, resource wastage, and financial losses. A mathematical model is developed to address these issues, integrating a screening mechanism to identify and rework defective items, transforming them into usable products. Additionally, the model incorporates supplier discounts on defective items as an incentive for sustainable practices. Preservation technology is employed to mitigate product deterioration and enhance overall profitability. The proposed framework accounts for multiple demand-influencing factors, including price, stock levels, product quality, advertising, shortages, and backlogging. By employing both crisp and fuzzy mathematical approaches, the optimal solution is derived through an advanced defuzzification technique within a generalized triangular neutrosophic environment. A comprehensive numerical example and sensitivity analysis validate the model's effectiveness under fuzzy conditions, highlighting its potential to enhance economic and environmental sustainability in practical manufacturing scenarios.

Keywords: Inventory; Quality dependent demand; Preservation technology; Defective items; Deterioration.

1. Introduction

Managing defective items in inventory is a significant challenge in production and commercial sectors, particularly in industries where product imperfections or damages are common. Traditional approaches, such as discarding defective items or selling them at reduced prices, often lead to compromised profitability, increased waste, and environmental degradation. In response, reworking defective products—repairing or restoring them to their original functionality has emerged as a sustainable and strategic alternative. These innovations streamline the identification, classification, and reprocessing of defective items, enabling businesses to optimize resource utilization, reduce costs, and align with sustainability goals. By enhancing product recovery rates and extending product life-cycles, rework strategies contribute to the delivery of high-quality outputs and resonate with circular economy principles. Furthermore, the integration of predictive analytics and real-time monitoring strengthens the rework framework by providing actionable insights for proactive inventory management. This study builds on these advancements, proposing a comprehensive model that integrates rework processes and preservation technologies to address the economic and environmental challenges associated with defective item management.

This study aims to develop an economic order quantity (EOQ) model emphasizing production recovery by reworking defective items while minimizing waste. Preservation technology is incorporated into the model to reduce product deterioration, extending the product's lifespan and enhancing profitability. The deterioration rate is assumed to remain constant throughout the inventory cycle. The model considers demand factors as critical determinants of stock levels and warehouse performance, incorporating elements such as price, stock availability, quality, and advertising [1–3].

The optimal solution is computed within a fuzzy environment, effectively addressing system parameter uncertainties. This study addresses key research questions related to the management of defective items, as outlined below:

- How can preservation technology be leveraged to reduce the deterioration of items?
- How can high profitability be achieved through the combined use of preservation technology and the reprocessing of defective items?
- What impact does uncertainty, particularly within a fuzzy environment, have on the optimal solution of inventory models?

To address these questions, this study proposes an inventory model with defective items; it integrates strategies such as reduction of decay through preservation technology, minimization of waste, defect reduction via rework processes, and mitigation of uncertainties using neutrosophic fuzzy to derive an optimal solution. This study's main goal aims to optimize overall profit while minimizing defects, deterioration, and waste.

2. Literature review

This section presents supporting literature to identify this research advancement and gaps in the field of inventory management systems. Relevant articles for this study were gathered from various databases using keywords such as demand, deteriorating items, preservation technology, inventory models, defective items, rework process, and fuzzy environment [4, 5]. The first subsection examines various types of demand regulation in inventory models [6], the second focuses on deteriorating items, the third discusses preservation technology, and the fourth and fifth sections cover the rework process and fuzzy environment, respectively.

Inventory demand forecasting involves predicting customer demand for a particular item over a specified time period to maintain ideal stock levels and ensure optimal inventory management. Namdeo et al. [7] and Priyamvada et al. [8] developed a model for deteriorating items where demand is based on price and stock availability, while Hatibaruah and Saha [9] examined accounting in shortages with a cost- and stock-dependent demand inventory model. Rini et al. [10] presented a price-driven demand production model. Liu et al. [11] constructed an inventory model for perishable food items driven by price and quality demand. while Feng and Wang [12] focused on price demand for fresh items. Rahman et al. [13] demonstrated optimal strategies for managing perishable goods under combined demand that is influenced by both price and stock. Kundu et al. [14] presented an inventory system with price discounts with fuzzy requirements.

In real-world inventory systems, deterioration is a significant factor that cannot be overlooked, typically defined as the decline in quality, damage, evaporation, spoilage, obsolescence, loss of utility, theft, or reduction in the value of a product, rendering it unsuitable for its intended use. Jani et al. [15] presented an inventory model assuming a constant rate of deterioration. Rapolu and Kandpal [16] analyzed an inventory model for items with non-instantaneous deterioration, incorporating reduction technology to mitigate decay rates. which was similarly examined by Li et al. [17] in their model for items deteriorating gradually rather than instantaneously. Yadav and Swami [18] introduced an inventory model for two warehouses, accounting for gradual deterioration, while Indrajitsingh and Sahu [19] developed a system where deterioration is calculated based on instantaneous decay.

In an inventory system, incorporating fuzzy provides more reliable and acceptable results when compared to traditional crisp inventory models, especially when handling uncertain data [4]. Islam et al. [20] developed an inventory model with cost parameters represented as generalized trapezoidal fuzzy numbers, using a neutrosophic hesitant fuzzy programming approach to solve it. Rani et al. [21] created an imprecise inventory system for perishable products with triangular fuzzy number-based cost parameters. Saarumathi et al. [22] proposed a production strategy for utilizing fragile commodities in the medical treatment of juvenile diabetes, employing trapezoidal neutrosophic fuzzy numbers. Bhavani et al. [23] introduced a two-warehouse inventory system for controlling carbon emissions applying green technology under fuzziness. Sindhuja et al. [24] analyzed energy consumption in inventory management using a Neutrosophic Trapezoidal Number. Tripathy and Bag [25] developed an economic order quantity model for deteriorating items, expressing demand and cost as trapezoidal fuzzy numbers. Furthermore, several researchers have applied neutrosophic concepts, as noted by [26–30].

In an inventory system, preservation technology is crucial in reducing item deterioration [32], extending their lifespan, minimizing losses, and decreasing waste [31]. The inventory system can enhance profits by integrating preservation processes [33] while minimizing waste. Shaikh et al. [34] created an inventory model to mitigate item deterioration by employing preservation technology.

2.1. *Research gap and contributions*

The literature review reveals that significant research has been carried out on defective items within the rework process. However, a clear gap exists in developing an integrated EOQ model designed to minimize defects and waste while accounting for realistic demand in a fuzzy environment [35]. This innovative model incorporates factors such as price, stock, quality, advertising demand, preservation technology investment, fuzzy logic, and more, with the goal of promoting sustainable development.

TABLE 1. Contribution-based comparative analysis of this work with earlier models.

Author	**Demand**	**Deterioration**	**Preservation technology**	**Fuzzy Variables**	**Nature of Fuzziness**
Singh et al. [36]	Constant	✓	✓	×	×
Shastri et al. [37]	Price	✓	✓	×	×
Pal et al. [38]	Ramp type	✓	✓	Fuzzy	Fuzzy
Tayal et al. [39]	Constant	✓	✓	×	×
Karmakar et al. [40]	Constant	×	×	Cost parameters	Triangular dense fuzzy
Li [41]	Fuzzy	×	×	Demand	Type-2
Singh [42]	Price, stock and time	✓	✓	×	×
Poswal et al. [43]	Price & stock	×	×	Cost parameters	Trapezoidal fuzzy number
Ruidas et al. [44]	Price	✓	✓	Cost parameters	Interval number
Aka & Akyüz [45]	Time	×	×	Demand, purchase cost	Fuzzy
Mohanta et al. [46]	Market	✓	✓	Demand, costs	Triangular neutrosophic fuzzy
Gulia et al. [47]	Average yearly demand	×	×	×	Triangular fuzzy number
This paper	Price, stock, quality and advertaisement	✓	✓	Cost parameters	Generalized triangular neutrosophic

The research gap identified in previous studies and the contributions of this paper are summarized in the literature review presented in Table 1. The contributions of the proposed model are outlined below:

- To create an EOQ model for defective items that focuses on production recovery via reworking and reduces waste.
- Employ preservation techniques to mitigate the degradation of products.
- Incorporates uncertainty to determine the optimal solution.
- Implementing the demand rate, influenced by factors such as price, quality, stock, and advertising, contributes to increasing total profit.

3. Development of inventory system

Notations:

The following notations are used in this work for model formulation and solutions.

Parameters	
p	Selling price of unit item per unit time.
A_0	Ordering expanse for every replenish.
A_1	Per-unit holding cost over a specific duration.
A_2	Waste management cost per item per time unit.
A_3	Purchasing expanse of unit item.
A_4	Item shortage expanse per unit.
A_5	Item reworking expanse per unit.
A_6	Screening cost of unit time.
θ_o	deterioration rate without preservation.
$\theta(\theta_o, \xi)$	Deterioration rate under preservation.
T	Cycle time in each replenishment.
ξ	Preservation investment cost per unit time.
p	Selling price per unit item per unit time.
q_s	Supplier's raw material quality.
q_r	Retailer's product quality.
α	Defect ratio per unit$(0 < \alpha < 1)$.
γ	Discount rate on defective item purchases.
Decision variable	
t_1	The time at the reaching point of zero inventory level.

Assumptions:

(1) The demand rate of this model depends on concerns with the quality of retailers products, i.e., $q_r = (1 - e^{-a_1 q_s})$. The quality of the retailer's products is contingent upon the raw materials provided by the supplier.

(2) The demand for this model is influenced by inventory levels, pricing, and advertisement frequency and usage percentage of herbal and chemical ingredients of the product that is $D(J(t), A, p, q_r) = (mp^{-\delta} + bJ(t) + q_r)A^{\nu}$, where $m, b, \gamma > 0$, $A > 1$ and $0 \leq \nu, \delta \leq 1$. Here, A represents the number of advertisements per cycle, and ν denotes the impact of each advertisement.

(3) The rate of item deterioration in the inventory model is stable and can be controlled using preservation technology, i.e., $\theta = \theta_o e^{-a_1\xi}$where $0 < \theta_0 < 1$ and $a_1 > 0$.

(4) Shortages are allowed and completely backlogged.

(5) The cost parameters imprecision has been represented using GTNNs and a novel deneutrosophication method to convert neutrosophic to a crisp environment.

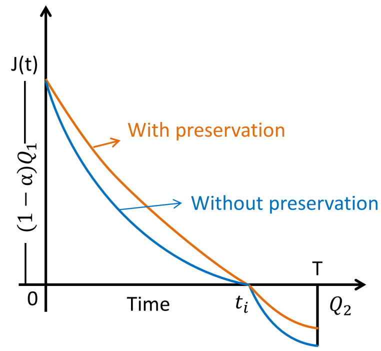

Figure 1. Proposed inventory model in graphical representation

Let $J(t)$ represent the inventory level at a given time t. The inventory system's stock levels are governed by differential equations that account for reductions due to demand and deterioration over the time interval $(0, t_i)$. Q_1 denotes the initial inventory at time $(0, t_i)$. During the time interval (t_1, T), the system experiences complete backlogging. Q_2 represents the inventory shortages that accumulate over the time period (t_1, T).

$$\frac{dJ(t)}{dt} + \theta(\theta_o, \xi)J(t) = -(mp^{-\delta} + bJ(t) + cq_r)A^{\nu}, \quad 0 \le t \le t_i. \tag{1}$$

$$\frac{dJ(t)}{dt} = -(mp^{-\delta} + c^{'}q_r)A^{\nu}, \quad t_i \le t \le T. \tag{2}$$

with boundary conditions $J(0) = (1-\alpha)Q_1$, $J(t_i) = 0$ and $J(T) = -Q_2$.

The boundary conditions Eqn.(3) and (4) are used to derive the solution for these equations (1) and (2) is given by

$$J(t) = \frac{(mp^{-\delta} + cq_r)A^{\nu}}{\theta + bA^{\nu}}\left(e^{(\theta+bA^{\nu})(t_i-t)} - 1\right), \qquad 0 \le t \le t_i. \tag{3}$$

$$J(t) = (mp^{-\delta} + c^{'}q_r)A^{\nu}(t_i - t), \quad t_i \le t \le T. \tag{4}$$

Putting $t = 0$ in Eqn. (3), we get

$$(1-\alpha)Q_1 = \frac{(mp^{-\delta} + cq_r)A^{\nu}}{\theta + bA^{\nu}}\left(e^{(\theta+bA^{\nu})t_i} - 1\right). \tag{5}$$

$$Q_2 = (mp^{-\delta} + c^{'}q_r)A^{\nu}(T - t_i) \tag{6}$$

Since the ordering quantity is $Q = (1-\alpha)Q_1 + \alpha Q_1 + Q_2$, therefore

$$Q = \frac{(mp^{-\delta} + cq_r)A^{\nu}}{(\theta + bA^{\nu})(1-\alpha)}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) + (mp^{-\delta} + c^{'}q_r)A^{\nu}(T - t_i). \tag{7}$$

3.1. *Mathematical Analysis for Suggested Inventory Framework*

The described inventory model ordering cost ($\mathcal{OC}$) Eqn. (8), which includes transportation costs, is represented by

$$\mathcal{OC} = A_0. \tag{8}$$

Holding costs ($\mathcal{HC}$) Eqn. (9) are added to maintaining the cost of inventory. The inventory model experiences under-stocking during the specified time period $(0, t_i)$ is

$$\begin{aligned} \mathcal{HC} &= A_1 \int_0^{t_i} J(t)dt \qquad (9) \\ &= \frac{A_1(mp^{-\delta} + cq_r)A^{\nu}}{(\theta + bA^{\nu})^2}\left(e^{(\theta+bA^{\nu})t_i} - 1 - (\theta + bA^{\nu})t_i\right). \end{aligned}$$

The cost of deterioration arises from the decay or degradation of stock. The period time $(0, t_i)$ the total inventory in the model is Q_1. The stock Q_1's quantity does not increase because of demand; the remaining stock declines further deterioration in inventory over $(0, t_i)$ time. Therefore, the deterioration cost ($\mathcal{DC}$) for the items in the inventory system, as given by Eqn. (10), is expressed as follows:

$$\begin{aligned} \mathcal{DC} &= A_2\left[(1-\alpha)Q_1 - \int_0^{t_i} \mathcal{D}(J(t), A, p, q_r)dt\right] \qquad (10) \\ &= \frac{A_2\theta(mp^{-\delta} + cq_r)A^{\nu}}{(\theta + bA^{\nu})^2}\left(e^{(\theta+bA^{\nu})t_i} - 1 - (\theta + bA^{\nu})t_i\right). \end{aligned}$$

The purchasing cost of items in the inventory system is defined in Eqn. (11):

$$\begin{aligned} \mathcal{PC} &= A_3(1-\alpha)Q_1 + A_3\alpha Q_1 + A_3 Q_2 \\ &= \frac{A_3(mp^{-\delta} + cq_r)A^{\nu}}{\theta + bA^{\nu}}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) + A_3(mp^{-\delta} + c^{'}q_r)A^{\nu}(T - t_i) \\ &\quad + \frac{A_3\gamma\alpha(mp^{-\delta} + cq_r)A^{\nu}}{\theta + bA^{\nu}(1-\alpha)}\left(e^{(\theta+bA^{\nu})t_i} - 1\right). \qquad (11) \end{aligned}$$

During the time period (t_1, T), the inventory was in excess, leading to shortages caused by the unavailability of stock within this interval. The stock cost ($\mathcal{SC}$) incurred as a consequence of insufficient stock (Eqn. (12)) is represented below.

$$\mathcal{SC} = -A_4 \int_{t_i}^{T} J(t)dt = A_4(mp^{-\delta} + c^{'}q_r)A^{\nu}\frac{(T - t_i)^2}{2}. \tag{12}$$

Eqn. (12) provides the sales revenue ($\mathcal{SR}$) for items sold within the time frame $(0, T)$.

$$\begin{aligned}\mathcal{SR} &= p\int_0^T \mathcal{D}(J(t), A, p, q_r)dt \qquad (13)\\ &= p\int_0^{t_i} \mathcal{D}(J(t), A, p, q_r)dt + p\int_{t_i}^T \mathcal{D}(J(t), A, p, q_r)dt\\ &= p\Bigg[(c-c^{'})q_r A^{\nu} t_i + \frac{b(mp^{-\delta}+cq_r)A^{2\nu}}{(\theta+bA^{\nu})^2}\left(e^{(\theta+bA^{\nu})t_i} - 1 - (\theta+bA^{\nu})t_i\right) + (mp^{-\delta}+c^{'}q_r)A^{\nu}T\Bigg]\end{aligned}$$

The rework cost ($\mathcal{RC}$) for the retailer's defective items, as defined in Eqn. (14), is calculated as follows:

$$\mathcal{RC} = \frac{A_5\alpha(mp^{-\delta}+cq_r)A^{\nu}}{(\theta+bA^{\nu})(1-\alpha)}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) \qquad (14)$$

Screening cost ($\mathcal{SCR}$) of the inventory model is defined by Eqn. (15) as follows:

$$\mathcal{SCR} = A_6 Q_1 = \frac{A_6(mp^{-\delta}+cq_r)A^{\nu}}{(1-\alpha)(\theta+bA^{\nu})}\left(e^{(\theta+bA^{\nu})t_i} - 1\right). \qquad (15)$$

The inventory system's advertisement cost ($\mathcal{AC}$) equals the cost per advertisement (g) multiplied by the number of advertisements; Eqn. (16) reflects the same.

$$\mathcal{AC} = gA. \qquad (16)$$

The preservation technology investment ($\mathcal{PTI}$) per cycle is detailed in Eqn. (17).

$$\mathcal{PTI} = \xi T. \qquad (17)$$

The total profit of the inventory system per unit of time is determined using Eqn. (18), as follows:

$$\mathcal{TP} = \frac{1}{T}\Big[\mathcal{SR} - \mathcal{OC} - \mathcal{HC} - \mathcal{DC} - \mathcal{PC} - \mathcal{SC} - \mathcal{RC} - \mathcal{SCR} - \mathcal{AC} - \mathcal{PTI}\Big]$$

$$\begin{aligned}\mathcal{TP} &= \frac{1}{T}\Bigg[p\Bigg((c-c^{'})q_r A^{\nu} t_i + \frac{b(mp^{-\delta}+cq_r)A^{2\nu}}{(\theta+bA^{\nu})^2}\left(e^{(\theta+bA^{\nu})t_i} - 1 - (\theta+bA^{\nu})t_i\right) + (mp^{-\delta}+c^{'}q_r)A^{\nu}T\Bigg)\\ &\quad -\frac{(A_1+A_2\theta)(mp^{-\delta}+cq_r)A^{\nu}}{(\theta+bA^{\nu})^2}\left(e^{(\theta+bA^{\nu})t_i} - 1 - (\theta+bA^{\nu})t_i\right) - A_4(mp^{-\delta}+c^{'}q_r)A^{\nu}\frac{(T-t_i)^2}{2}\\ &\quad -\frac{(A_5\alpha+A_6)(mp^{-\delta}+cq_r)A^{\nu}}{(\theta+bA^{\nu})(1-\alpha)}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) - \frac{A_3(mp^{-\delta}+cq_r)A^{\nu}}{\theta+bA^{\nu}}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) \qquad (18)\\ &\quad -A_3(mp^{-\delta}+c^{'}q_r)A^{\nu}(T-t_i) - \frac{A_3\gamma\alpha(mp^{-\delta}+cq_r)A^{\nu}}{\theta+bA^{\nu}(1-\alpha)}\left(e^{(\theta+bA^{\nu})t_i} - 1\right) - A_0 - gA - \xi T\Bigg]\end{aligned}$$

4. De-neutrosophication of Generalized Triangular Neutrosophic Numbers

In real situations, most of the incident parameters are imprecise, which can be represented using fuzzy to address this uncertainty. Generalized triangular Neutrosophic Numbers (GTNNs) can analyze and determine the given parameters' truth, hesitation, and falsity.

Definition 4.1. A GTNN $\tilde{M} = \langle (u_1, u_2, u_3; \Psi), (v_1, v_2, v_3; \Pi), (w_1, w_2, w_3; \Phi)\rangle$, where $\Psi, \Pi, \Phi \in [0, 1]$. Here $\pi_{\tilde{N}} : \mathbf{R} \to [0, \Psi]$ is the truth membership function, $\theta_{\tilde{N}} : \mathbf{R} \to [\Pi, 1]$ is the hesitation membership function, and the falsity membership function is $\eta_{\tilde{N}} : \mathbf{R} \to [\Phi, 1]$, where the membership functions are mathematically defined as follows:

$$\pi_{\tilde{N}}(x) = \begin{cases} \frac{(x-u_1)}{(u_2-u_1)}\Psi, & \text{for } u_1 \leq x < u_2 \\ \Psi, & \text{for } x = u_2 \\ \frac{(u_3-x)}{(u_3-u_2)}\Psi, & \text{for } u_2 < x \leq u_3 \\ 0, & \text{otherwise} \end{cases} \qquad \theta_{\tilde{N}}(x) = \begin{cases} \frac{(v_2-x)+\Pi(x-v_1)}{(v_2-v_1)}, & \text{for } v_1 \leq x < v_2 \\ \Pi, & \text{for } x = v_2 \\ \frac{(x-v_2)+\Pi(v_3-x)}{(v_3-v_2)}, & \text{for } v_2 < x \leq v_3 \\ 1, & \text{otherwise} \end{cases}$$

and

$$\eta_{\tilde{N}}(x) = \begin{cases} \frac{(w_2-x)+\Phi(x-w_1)}{(w_2-w_1)}, & \text{for } w_1 \leq x < w_2 \\ \Phi, & \text{for } x = u_2 \\ \frac{(x-w_2)+\Phi(w_3-x)}{(w_3-w_2)}, & \text{for } w_2 < x \leq w_3 \\ 1, & \text{otherwise} \end{cases}$$

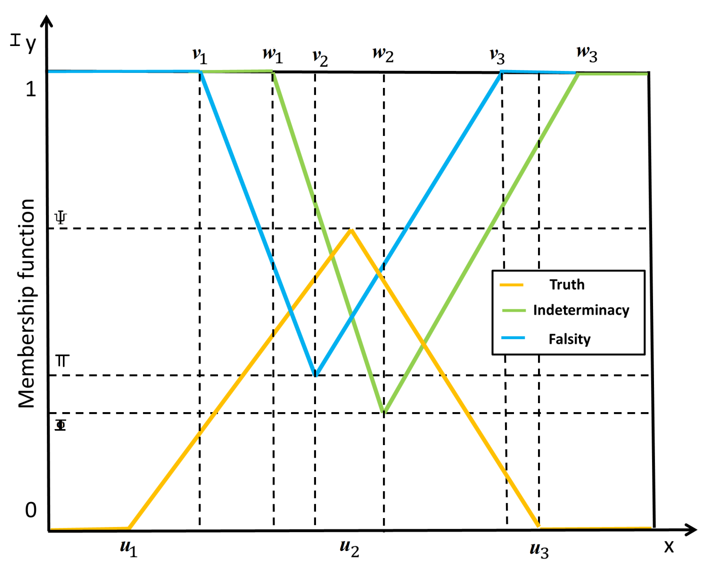

FIGURE 2. Graphical representation of a Generalized Triangular Neutosophic Number

Existing methods in the literature for the defuzzification of fuzzy numbers are compared in this paper, which also proposes a new defuzzification technique for GTNN.

4.1. *Rouben's De-neutrosophication*

The existing Rouben's function for any fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}$ is as follows:

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 \big(inf\tilde{A}_\alpha + sup\tilde{A}_\alpha\big)d\alpha \tag{19}$$

Definition 4.2. The defuzzification of neutrosophic numbers $\tilde{N}$, which involve the truth membership function $\pi_{\tilde{N}}$, the indeterminacy membership function $\theta_{\tilde{N}}$, and the falsity membership function $\eta_{\tilde{N}}$, is guided by Rouben's ranking function (Eqn. (19)) presented in Eqn. (20). When $\tilde{N} = \langle(u_1, u_2, u_3; \Psi), (v_1, v_2, v_3; \Pi), (w_1, w_2, w_3; \Phi)\rangle$ is expressed as a GTNN, the defuzzification is carried out as follows:

$$\begin{aligned} R(\tilde{N}) \;=\; & \frac{1}{3}\Bigg(\frac{1}{4\Psi}\big(2u_2 + (2\Psi-1)(u_1+u_3)\big) + \frac{1}{4(1-\Pi)}\big(2v_2 + (1-2\Pi)(v_1+v_3)\big) \\ & + \frac{1}{4(1-\Phi)}\big(2w_2 + (1-2\Phi)(w_1+w_3)\big)\Bigg). \end{aligned} \tag{20}$$

4.2. *Proposed De-neutrosophication*

The center of area (or centroid) for the α-cut of a triangular fuzzy number $\tilde{A}$ can be calculated as the midpoint of the interval $[l_\alpha, u_\alpha]$, which is: $C_\alpha = \frac{l_\alpha + u_\alpha}{2}$. A more generalized approach is to compute the average centroid over the entire range of α:

$$\mathcal{R}(\tilde{N})_{New} = \frac{1}{\alpha_{\max}}\int_0^{\alpha_{\max}} C_\alpha \, d\alpha \tag{21}$$

Where $\alpha_{\max}$ is the maximum value of α (usually $\alpha_{\max} = 1$).

The process of defuzzifying a neutrosophic number $\tilde{N}$, characterized by the truth membership function $\pi_{\tilde{N}}$, the indeterminacy membership function $\theta_{\tilde{N}}$, and the falsity membership function $\eta_{\tilde{N}}$, is defined as follows:

$$\mathcal{R}(\tilde{N})_{New} = \frac{1}{3}[\mathcal{R}_r(\pi_{\tilde{N}}) + \mathcal{R}_r(\theta_{\tilde{N}}) + \mathcal{R}_r(\eta_{\tilde{N}})], \tag{22}$$

$$\mathcal{R}_r(\pi_{\tilde{N}}) = \frac{u_1 + u_3 + \Psi(2u_2 - u_1 - u_3)}{2}, \mathcal{R}_r(\theta_{\tilde{N}}) = \frac{v_1 + v_3 + \frac{2v_2 - v_1 - v_3}{\Pi}}{2},$$
$$\mathcal{R}_r(\eta_{\tilde{N}}) = \frac{w_1 + w_3 + \frac{2w_2 - w_1 - w_3}{\Phi}}{2}$$

The proposed de-neutrosophication in Eq.(23) is defined using Eq.(21)and (22):

$$\begin{aligned} \mathcal{R}(\tilde{N})_{New} = \frac{1}{3}\Bigg(& \frac{u_1+u_3}{2} + \frac{\Psi(2u_2 - u_1 - u_3)}{2} + \frac{v_1+v_3}{2} + \frac{2v_2 - v_1 - v_3}{2\Pi} \\ & + \frac{w_1+w_3}{2} + \frac{2w_2 - w_1 - w_3}{2\Phi}\Bigg) \end{aligned} \tag{23}$$

TABLE 2. Comparison of Old ($\mathcal{R}(\tilde{N})$) and New ($\mathcal{R}(\tilde{N})_{New}$) De-neutrosophic Values

Example	Methods	Values of Ψ, Π and Φ									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
Example 1	$\mathcal{R}(\tilde{N})$	—	6	6	6	6	6	6	6	6	6
	$\mathcal{R}(\tilde{N})_{New}$	6	6	6	6	6	6	6	6	6	6
Example 2	$\mathcal{R}(\tilde{N})$	—	6.3333	5.9167	5.7778	5.7083	5.6667	5.6389	5.6190	5.6042	5.5926
	$\mathcal{R}(\tilde{N})_{New}$	5.6667	5.6852	5.7083	5.7381	5.7778	5.8333	5.9167	6.0556	6.3333	7.1667

The comparison between Rouben's de-neutrosophication method ($\mathcal{R}(\tilde{N})$) and the proposed approach ($\mathcal{R}(\tilde{N})_{New}$) is further illustrated in Figure 3, which highlights their numerical behavior across different examples. In Example 1 (Figure 3(a)), both methods yield identical results, emphasizing equivalence under symmetric membership scenarios. However, in Example 2 (Figure 3(b)), the proposed method ($\mathcal{R}(\tilde{N})_{New}$) demonstrates dynamic variations in response to changes in the membership parameters (Ψ, Π, Φ), showcasing its sensitivity and adaptability. These visualizations reinforce the proposed method's superiority in capturing nuanced parameter trends, making it more effective for complex applications under uncertainty.

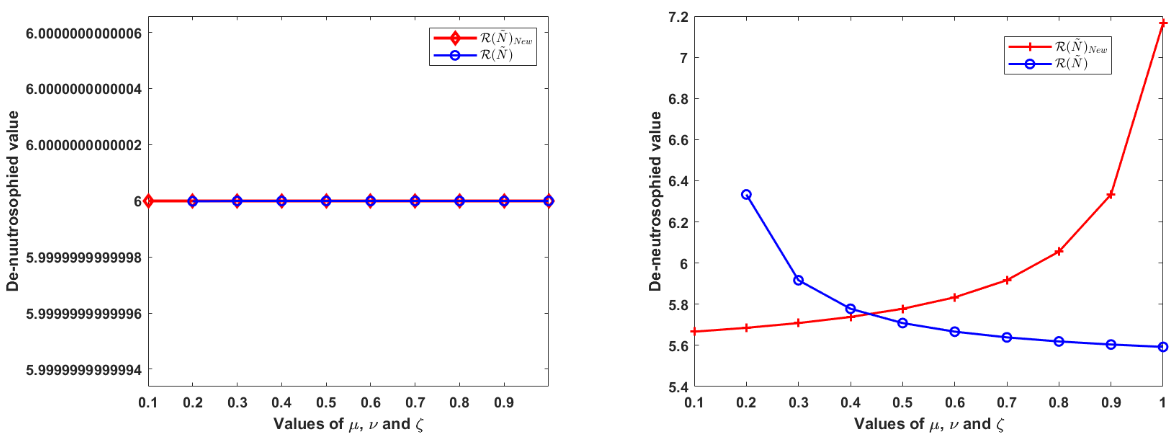

FIGURE 3. Comparison of de-neutrosophication methods for Example 1 in Fig 3(a) and Example 2 in Fig 3(b).

The proposed de-neutrosophication introduces greater flexibility in handling the parameters (Ψ, Π, Φ) through explicit dependence on these values. This contrasts with Rouben's approach, which applies uniform averaging irrespective of parameter variations. Table 2 compares the computed de-neutrosophic values for two examples. The trends reveal key differences between the methods:

Example 1: Both methods yield identical values (6) across all parameter variations, indicating equivalence in this specific scenario. This suggests that both methods may perform similarly for uniformly distributed membership functions or symmetric inputs.

Example 2: Distinct trends are observed: $\mathcal{R}(\tilde{N})$ values generally decrease as Ψ, Π, and Φ approach their upper limits (1.0). In contrast, it $(\mathcal{R}(\tilde{N})_{New})$ shows a dynamic increase with increasing parameter values, reflecting its sensitivity to changes in the underlying membership functions.

This distinction suggests that the proposed method is more responsive to parameter shifts, making it better suited for scenarios with significant variation in truth, indeterminacy, and falsity degrees.

- **Rouben's Method:** Suitable for simpler systems where uniform averaging suffices and computational simplicity are prioritized.
- **Proposed Method:** More appropriate for complex systems where the variability of membership parameters (Ψ, Π, Φ) plays a critical role in decision-making or ranking.

The proposed method $(\mathcal{R}(\tilde{N})_{New})$ provides a refined perspective by capturing subtle variations in membership functions, which could be crucial in applications requiring nuanced interpretations of neutrosophic data, such as optimization processes or uncertainty modeling. The comparative analysis highlights the adaptability and precision of the proposed deneutrosophication method over Rouben's method. While both methods demonstrate utility in different scenarios, the proposed method stands out for its ability to handle parameter variations more effectively, making it a promising tool for advanced applications in neutrosophic environments.

5. Numerical example

Numerical solutions are presented to support the suggested model using the optimization tool of MATLAB 23b version using a computer with an Intel i5 processor, 8GB RAM, and 256GB SSD. The parameter values are listed as follows: the scaling parameters of the demand are $m = 20$, $b = 0.5$, $c = 4$, $c' = 4.5$ and δ is 0.3. The purchasing cost per unit time $\tilde{A_3}$ is \$30, ordering cost per order $\tilde{A_0}$ is 500, cost of selling unit item is $\tilde{p}$ is \$60, holding cost per unit of time $\tilde{A_1}$ is \$6.5, shortage cost per unit time $\tilde{A_4}$ is \$12 and deterioration cost per decaying goods per unit of time $\tilde{A_2}$ is \$6, rework cost of defective item per unit time $\tilde{A_5}$ is \$8 and screening cost per unit time $\tilde{A_6}$ is \$3. The preservation investment of the inventory system per unit of time is $\xi = 5$, and the initial decay rate θ_0 is 0.1. The quality of the product values are q_s is 0.9 and a_1 is 0.5. demand depends advertisement the values are A is 30, ν is 0.4 and g is 10. the defective item rate is α is 0.2 and suppliers give the discount for defective items γ is 0.5. Substitute these values in Eqn. (18) obtained the optimal values and optimal profit.

The generalized triangular neutrosophic fuzzy cost parameters are $\tilde{A_0} =< (350, 500, 650 : 0.5), (400, 550, 700 : 0.7), (500, 550, 600 : 0.8) >$, $\tilde{A_1} =< (5.5, 6, 6.5 : 0.2), (5.5, 6.5, 7.5 :$

0.4), (6, 7, 8 : 0.6) >, $\tilde{A}_2$ =< (4, 5, 6 : 0.7), (5, 6, 7 : 0.4), (6, 7, 8 : 0.3) >, $\tilde{A}_3$ =< (25, 28, 31 : 0.6), (28, 30, 32 : 0.8), (30, 32, 34 : 1) >, $\tilde{A}_4$ =< (8, 10, 12 : 0.7), (10, 12, 14 : 0.3), (12 : 14 : 16 : 0.3) >, $\tilde{A}_5$ =< (6, 7, 8 : 0.7), (7, 8, 9 : 0.2), (8, 9, 10 : 0.2) >, $\tilde{A}_6$ =< (1, 2, 3 : 0.7), (2, 3, 4 : 0.2), (3, 4, 5 : 0.2) >, and the de-neutrosophication values are $\tilde{A}_{0(D)} = \$500$, $\tilde{A}_{1(D)} = \$6.5$, $\tilde{A}_{2(D)} = \$6$, $\tilde{A}_{3(D)} = \$30$, $\tilde{A}_{4(D)} = \$12$, $\tilde{A}_{5(D)} = \$8$ and $\tilde{A}_{6(D)} = \$3$.

TABLE 3. Optimal average profit for inventory model

Preservation Technology	$t_1 = t_1^*$(year)	$Q = Q^*$	$\widetilde{TP}^*$
With	0.8664	55.7028	1516
Without	0.8259	52.0498	1316

In Table 3, with preservation technology, the total optimal profit $\widetilde{TP}^*$ is \$1516, the optimal order quantity Q^* is 55.7028 and the optimal replenishment time t_i^* is 0.8664. These values increase with the investment in preservation technology. Conversely, in the without of such investment, the total profit decreases to $\widetilde{TP}^*$ is \$1316, replenishment time drops to t_i^* is 0.8259, and quantity of order reduces to Q^* is 52.0498. Preservation technology enhances product lifespan, optimizes replenishment cycles, reduces lost sales, and minimizes waste. It is observed that investments in preservation technology by retailers lead to higher profits.

6. **Sensitivity analysis**

Sensitivity analysis plays a crucial role in examining parameter behavior and evaluating the practicality of theoretical models, particularly in the context of remanufacturing processes for businesses or industries managing defective products. Implementing a rework process for defective items, along with an inventory model that considers factors like price, stock, quality, and advertising demand, aims to increase profit by reducing costs and minimize the number of decaying items through preservation technology.

TABLE 4. Costs and other parameters influence the overall system performance evaluation.

Parameters	% variation	$t_i = t_i^*$ (year)	% variation in t_i^*	$\widetilde{TP}^*$	% variation in $\widetilde{TP}^*$
A_0	-20	0.8664	0	1616	6.60
	-10	0.8664	0	1566	2.90
	10	0.8664	0	1466	-3.30
	20	0.8664	0	1416	-6.60
$A1$	-20	0.8725	0.70	1543	1.78
	-10	0.8694	0.35	1530	0.92
	10	0.8634	-0.35	1503	-0.86
	20	0.8604	-0.69	1490	-1.72
A_2	-20	0.8664	0	1517	0.07
	-10	0.8664	0	1516	0
	10	0.8664	0	1516	0
	20	0.8663	-0.01	1516	0
A_3	-20	0.9352	7.94	2007	38.38
	-10	0.8991	3.77	1754	15.70
	10	0.8364	-3.46	1293	-14.71
	20	0.8090	-6.63	1081	-28.70
A_4	-20	0.8657	-0.08	1517	0.07
	-10	0.8660	-0.05	1517	0.07
	10	0.8667	0.03	1516	0
	20	0.8671	0.08	1516	0
A_5	-20	0.8708	0.51	1542	1.72
	-10	0.8686	0.25	1529	0.86
	10	0.8642	-0.25	1503	-0.86
	20	0.8619	-0.52	1491	-1.65
A_6	-20	0.8748	0.97	1565	3.23
	-10	0.8706	0.48	1541	1.65
	10	0.8622	-0.48	1492	-1.59
	20	0.8581	-0.96	1468	-3.17

- Raising the retailer's ordering cost shows in Table 4, A_0 results in a reduction in profit. Conversely, lowering the retailer's ordering costs leads to an increase in profit, allowing the store to enhance profit by reducing these costs.
- The holding cost parameter A_1 exhibits moderate sensitivity. An increase in A_1 leads to a decrease in the retailer's total profit and also reduces the optimal replenishment time t_i^*. Therefore, the retailer can enhance profit by lowering the holding cost.
- The sensitivity analysis of the purchasing cost A_3 shows high sensitivity. An increase leads to a decrease in total optimal profit and a reduction in replenishment time t_i^*. Therefore, the retailer can boost total profit by reducing the amount of purchased products, as shows in Fig 4.

- The sensitivity of the deterioration cost A_2 and shortage cost A_4 is relatively low. Increasing the deterioration cost has no significant effect on total profit, and the replenishment time increases due to the use of preservation technology. Both deterioration and shortage costs have minimal impact on the total profit.
- Raising the rework cost A_5 and screening cost A_6 for defective items will negatively affect the overall profit and reduce the replenishment time. On the other hand, lowering the rework and screening costs will have a positive effect on total profit and lead to an increase in replenishment time, as shows in Fig 5.

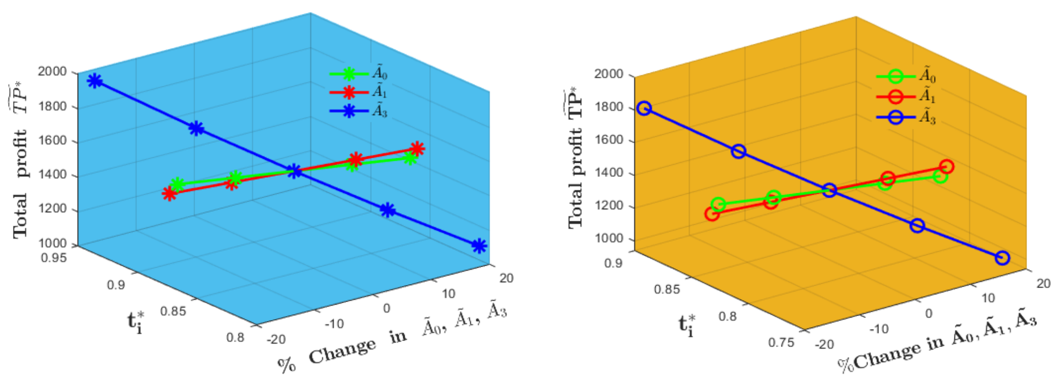

FIGURE 4. Graph of sensitivity analysis of ordering cost, holding cost and purchase cost shown in Fig 4(a) with preservation technology scenario, while Fig 4(b) shows the scenario without it.

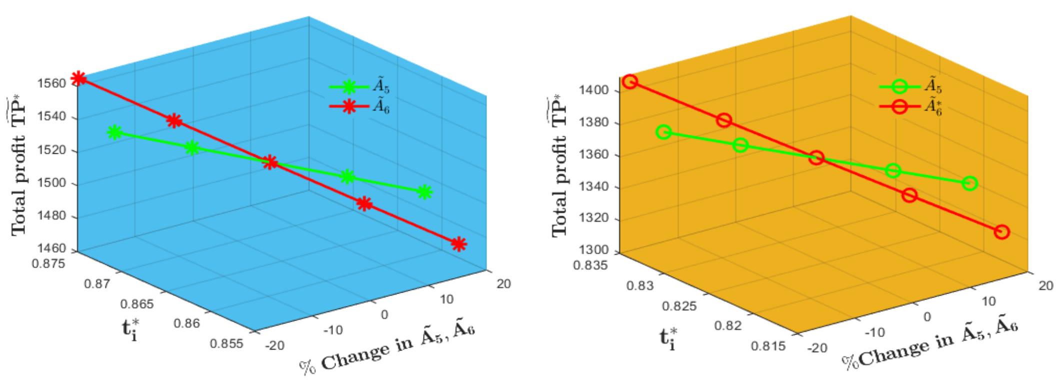

FIGURE 5. Representation of sensitivity of rework cost for defective items and screening cost in Fig 5(a) with preservation technology for profit $\widetilde{TP}^*$. Similarly in Fig 5(b) without preservation technology for profit $\widetilde{TP}^*$.

TABLE 5. Present model parameters impact in performance analysis.

Parameters	% variation	$t_i = t_i^*$ (year)	% variation in t_i^*	$\widetilde{TP}^*$	% variation in $\widetilde{TP}^*$
p	-20	0.7798	-9.10	627	-58.64
	-10	0.8251	-4.77	1069	-29.49
	10	0.9043	4.37	1968	29.82
	20	0.9393	8.41	2423	59.83
ν	-20	1.0137	17.00	825	-45.58
	-10	0.9317	7.54	1078	-28.89
	10	0.8155	-5.88	2259	49.01
	20	0.7774	-10.27	3526	132.59
δ	-20	0.8729	0.75	2086	37.60
	-10	0.8697	0.38	1784	17.68
	10	0.8627	-0.43	1280	-15.57
	20	0.8586	-0.88	1071	-29.35
ξ	-20	0.8639	-0.29	1507	-0.59
	-10	0.8652	-0.14	1512	-0.26
	10	0.8672	0.09	1519	0.20
	20	0.8679	0.17	1522	0.40
θ_0	-20	0.8672	0.09	1520	0.26
	-10	0.8668	0.05	1518	0.13
	10	0.8660	-0.05	1515	-0.07
	20	0.8656	-0.09	1513	-0.20

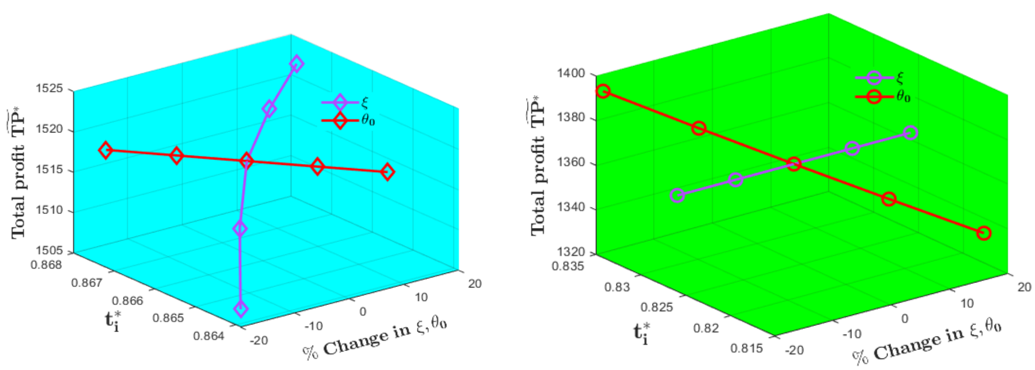

FIGURE 6. Graphical representation of sensitivity analysis of preservation investment cost and initial decay rate shown in Fig 6 (a) with preservation technology, while Fig 6(b) shows the scenario without it.

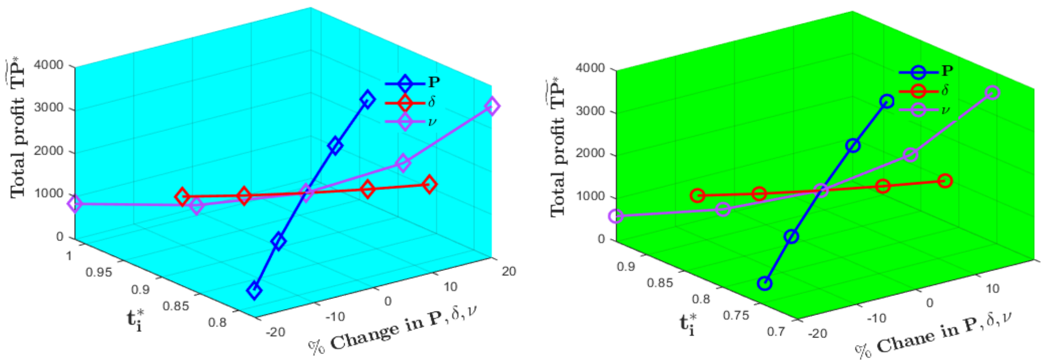

FIGURE 7. Graphical representation of sensitivity analysis of price, price sensitive and advertisement parameters are shown in Fig 7(a) and Fig 7(b) for profit $\widetilde{TP}^*$ with and without preservation technology, respectively.

- The price parameter, represented by p,is highly responsive. When the value of p increases, both the total profit and replenishment time t_1 increase. On the other hand, lowering the price results in a decrease in total profit and replenishment time t_i. Therefore, managers and decision-makers should carefully consider the selling price, as increasing it can drive up demand and lead to higher overall profit. It is shown in Fig 7.
- The quality parameter c is highly sensitive. When the value of c increases, both the total profit and the replenishment time t_i also increase. The retailer's profit is influenced by the product's quality, as they receive high-quality products from the supplier. Therefore, demand is based on the quality of the product.
- In this model, where demand depends on advertising, the parameter ν demonstrates high sensitivity. The parameter ν represents the impact of advertising on the demand rate. Interestingly, the optimal total profit is the optimal total profit despite a decrease in advertisement-induced demand rates at time period t_i^*.
- The overall profit is impacted by an increase in the price sensitivity parameter δ. A Because a high value of the product δ reduces the product's demand, prompting the retailer to sell at a lower price, ultimately leading to higher profit. As a result, at the time, t_i^* increases; meanwhile, the optimal total profit decreases. The sensitivity of the parameter δ is high.
- The parameter b represents the sensitivity related to stock. When the value of b increases, the total profit rises, and the replenishment time t_1 decreases. On the other hand, when b decreases, the total profit declines, and the replenishment time t_i increases. Additionally, customers consider the stock levels, so retailers must maintain appropriate inventory to influence demand effectively and increase profit.

- The sensitivity of the preservation technology investment ξ , is relatively low . Investing in deterioration reduction technology (ξ) increases total profit, while the replenishment cycle time t_1 also rises. Conversely, decreasing the value of ξ reduces overall profit and replenishment time t_i^*. It is shown in Fig 6.
- Based on the sensitivity results, parameter θ_0 is shown in Table 5, which represents the preservation technology's initial decay rate, has less sensitivity. In this scenario, θ_0 at time t_i^* are decreased, leading to a decrease in the overall optimal profit.

TABLE 6. Sensitivity analysis for the different inventory relate parameters

Parameter	% change	$t_i = t_i^*$ (year)	% change in t_i^*	$\widetilde{TP}^*$	% change in $\widetilde{TP}^*$
b	-20	0.9821	13.35	1327	-12.47
	-10	0.9156	5.68	1376	-9.23
	10	0.8294	-4.27	1740	14.78
	20	0.8013	-7.51	2045	34.89
γ	-20	0.8713	0.57	1470	3.03
	-10	0.8706	0.48	1541	1.65
	10	0.8622	-0.48	1492	-1.58
	20	0.8581	-0.96	1468	-3.17
α	-20	0.8839	2.02	1619	6.80
	-10	0.8753	1.03	1568	3.48
	10	0.8572	-1.06	1463	-3.50
	20	0.8478	-2.14	1408	-7.12
r	-20	0.8687	0.27	1494	-1.45
	-10	0.8675	0.13	1505	-0.73
	10	0.8652	-0.14	1527	0.73
	20	0.8642	-0.25	1538	1.45

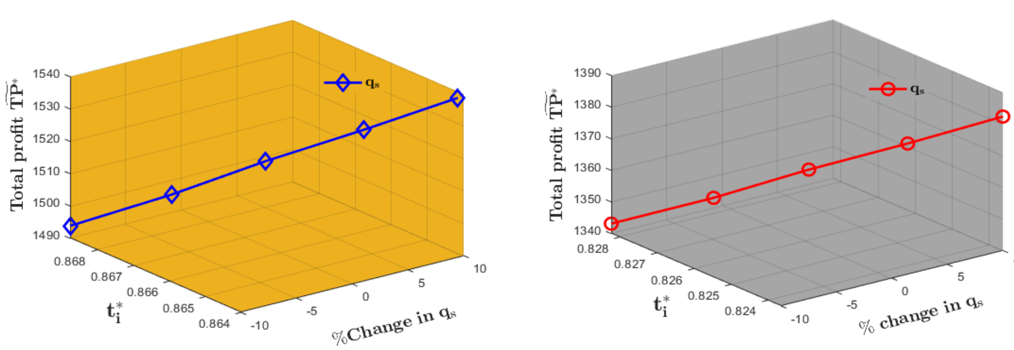

FIGURE 8. The graphical representation presents a sensitivity analysis of suppliers quality products shown in Fig 8 (a) depicts the scenario with preservation technology, while Fig 8(b) shows the scenario without it.

- The parameter b represents the sensitivity related to stock. When the value of b increases, the total profit rises, and the replenishment time t_i^* decreases. On the other hand, when b decreases, the total profit declines, and the replenishment time t_i^* increases. Additionally, customers take the stock levels into account, so retailers must maintain appropriate inventory to influence demand effectively and increase their profit.
- The defective items rate parameter shown in Table 6 is moderately sensitive. An increase in the defect rate will negatively affect the total profit, while a decrease in defective items will positively impact the total profit. Therefore, the retailer should carefully inspect the products and ensure they receive non-defective items from the supplier.
- The sensitivity analysis of the quality parameter q_s shows that it is relatively insensitive. When the supplier's product quality increases, the retailer automatically maintains higher-quality products, positively impacting total profit. Conversely, a decrease in the supplier's product quality results in the retailer maintaining lower-quality products, negatively affecting total profit as illustrated in Fig 8.

6.1. *Managerial insights*

- Sustainable Inventory Management: The EOQ model integrates a rework mechanism, reducing waste, minimizing emissions, and enhancing profitability in remanufacturing industries.
- Preservation Technology Benefits: Implementing preservation technology helps mitigate deterioration, ensuring higher product quality and reducing financial losses.
- Strategic Demand Management: Sales and profitability are influenced by key factors such as price, stock levels, quality, and advertising efforts, highlighting the need for optimized inventory strategies.
- Impact of Advertising and Information Dissemination: Effective marketing strategies and timely dissemination of product information significantly enhance customer engagement and boost sales.
- Practical Decision-Making: Managers can use numerical insights to refine replenishment strategies, select optimal technologies, and enhance operational efficiency.

7. **Conclusion and Future direction**

This study introduces an EOQ model tailored for defective items, which, if unmanaged, contribute to waste and environmental degradation. Designed for application in remanufacturing industries, the model integrates a rework mechanism supported by an efficient screening process, emphasizing its role in reducing defects, minimizing waste, lowering emissions, and enhancing profitability. As product deterioration progresses, businesses must adjust their replenishment and pricing strategies to mitigate potential losses. To address this, the model incorporates preservation technology to slow the decay rate, assuming an initially constant rate of deterioration.

Customer demand is influenced by multiple factors, including price, stock availability, product quality, and advertising efforts. Retailers prioritize high-quality offerings, and the study examines the impact of quality, stock levels, and advertising on sales performance. The results indicate that the timely dissemination of product information significantly boosts sales. Table 3 presents the optimal profit, replenishment time, and order quantity, both with and without preservation technology. Numerical experiments highlight the complexities involved in selecting effective retail strategies and technologies. Additionally, sensitivity analysis, conducted under shortages and full backlogging conditions, identifies key demand parameters b, ν, c, and p as the most influential factors affecting optimal profitability. The detailed findings of this analysis, summarized in Table 5, offer valuable insights into how variations in these parameters impact business performance.

Future research can extend the proposed model to incorporate additional features such as delayed payments, partial backlogging, and continuous decay rates. The use of GTNNs in addressing uncertain data values will be further explored to refine decision-making processes under unpredictable market conditions. The model could also be adapted to accommodate various demand patterns, such as stochastic or ramp-type demands, and integrated with production models for enhanced inventory management. These extensions broaden the model's applicability to more diverse and dynamic industrial settings.

CRediT authorship contribution statement:

B. Priskilla: Conceptualization, Methodology, Formal analysis, Writing-original draft, Visualization. **B. Baranidharan:** Methodology, investigation, Writing-review & editing, Visualization, Data curation. **G. S. Mahapatra:** Methodology, Visualization, Project administration, Supervision, Data curation, Writing - review & editing.

Data availability statements: "The data that support the findings of this study are available from the corresponding author upon reasonable request."

Funding: "This research received no external funding".

Acknowledgment: "The present work has not received any financial support."

Conflicts of Interest: "The authors declare that they have no conflict of interest."

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Received: Nov. 27, 2024. Accepted: May 23, 2025