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Adaptive Neutrosophic Integration and Trust Modeling: A New Framework for Evaluating Mobile Communication Network Migration Perception

Ke Li*

Jilin Technology College of Electronic Information, Jilin, 132000, Jilin, China
*Corresponding author, E-mail: 15044271069@163.com

Abstract: This paper presents an innovative neutrosophic decision-making framework for modeling and analyzing customer trust during mobile communication network transitions, specifically in the context of 4G to 5G migration. Traditional satisfaction models, whether statistical or fuzzy-based, struggle to address the simultaneous presence of uncertainty, ambiguity, and contradiction inherent in subjective human perception. To overcome these limitations, we propose two original mathematical constructs rooted in neutrosophic theory: the Neutrosophic Trust Confidence Index (NTCI) and the Adaptive Neutrosophic Integral (ANI). NTCI is a novel probabilistic index that measures both the clarity and directionality of user trust by integrating degrees of truth, indeterminacy, and falsity while penalizing ambiguous feedback. In contrast, ANI departs from classical fixed-weight integrals by dynamically adjusting attribute weights according to their internal trust confidence, enabling more reliable factors to exert greater influence in the aggregation process. The framework is supported by formal definitions, proofs of key properties such as boundedness and neutrality, and step-by-step computational examples based on real user evaluations collected during live network migration. The results demonstrate that the proposed models not only maintain the triadic structure of neutrosophic logic but also offer improved interpretability and discrimination power compared to conventional aggregation techniques. This work advances the practical use of neutrosophic logic in trust analytics and provides a robust mathematical foundation for decision-making in environments characterized by incomplete, vague, or conflicting information.

Keywords: Neutrosophic Probability; Adaptive Neutrosophic Integral; Neutrosophic Trust Confidence Index (NTCI); Indeterminacy Modeling; 5G Network; Multi-Valued Logic; Neutrosophic Theory.

1. Introduction

The rapid evolution of wireless communication technologies, particularly the ongoing transition from 4G to 5G networks, has introduced new challenges in understanding how customers perceive service quality, privacy, transparency, and overall trust [1]. While technical benchmarks such as bandwidth or latency are quantifiable, human trust in complex technological change is inherently subjective, uncertain, and often contradictory [2].

Traditional approaches to trust measurement rely on deterministic or probabilistic models, often simplifying human feedback into scalar satisfaction scores or discrete Likert-scale responses [3]. While useful, these models assume that human perception is either fully informed, logically consistent, or measurable via single-dimensional statistics. Fuzzy logic attempted to address these limitations by allowing partial truth, but it too suffers from structural constraints: it cannot simultaneously represent truth, contradiction, and indeterminacy as independent but co-existing phenomena [4].

In contrast, neutrosophic logic, introduced by Smarandache, provides a triadic representation of subjective judgments using three orthogonal components:

T,Degree of truth or affirmation, I, Degree of indeterminacy or hesitation, F, Degree of falsity or rejection [4].

This triadic framework offers a natural fit for modeling trust, particularly in the context of technological disruption, where users often feel both assurance and uncertainty simultaneously.

This paper aims to extend the application of neutrosophic logic from theoretical modeling to practical decision analytics, particularly within the domain of mobile network service transitions. We focus on two core innovations:

1. NTCI
   A new metric that quantifies how confidently a user expresses trust by accounting for both directionality (T vs. F) and clarity ($1 - I$). It captures degrees of support for the system while penalizing cognitive conflict or ambivalence.
2. ANI
   An enhanced aggregation method that adjusts the contribution of each attribute based on the degree of neutrosophic confidence associated with it. Unlike fixed-weight integrals, ANI prioritizes attributes that users perceive clearly and consistently.

These advances are not just conceptual but are formally defined, proven mathematically, and applied to real-world data. We demonstrate how they outperform classical aggregation methods in preserving the nuances of subjective evaluation, producing results that are more insightful, discriminative, and cognitively faithful.

In the following sections, we provide theoretical background, formulate the proposed models, and conduct detailed computational experiments to illustrate their power and applicability.

2. Neutrosophic Background

2.1 Overview of Neutrosophic Logic

Neutrosophic logic, introduced by Florentin Smarandache, extends classical and fuzzy logic by defining every logical proposition or subjective evaluation through a triplet:
x= (T, I, F) Where:
$T\in[0,1]$ is the degree of truth (support, affirmation).
$I\in[0,1]$ is the degree of indeterminacy (ambiguity, contradiction, lack of clarity).
$F\in[0,1]$ is the degree of falsity (rejection, disbelief).

These components are independent, meaning their sum is not constrained to equal 1. In practice, however, for normalized models (such as probability and measure spaces), we often apply, $T+I+F\leq 1$ [4].

This framework allows for rich and precise modeling of cognitive evaluations, such as a user simultaneously trusting and doubting a service due to past inconsistencies or lack of transparency.

2.2 Neutrosophic Probability

A neutrosophic probability space is defined by:
(X, Σ, NP)
Where:
X : a sample space of outcomes (e.g., service aspects or user opinions),
Σ : a σ-algebra over X,
$NP: \Sigma \rightarrow [0,1]^3$: a neutrosophic probability measure such that:

$$NP(A) = (T_A, I_A, F_A)$$

This triple quantifies the degrees of affirmation, indeterminacy, and rejection regarding event A. It is more expressive than scalar probabilities or belief functions, especially when analyzing inconsistent or incomplete information [5].

2.3 Neutrosophic Measure

Let X be a set and Σ be a σ-algebra over X. A neutrosophic measure ν is defined as a function:

$$\nu: \Sigma \rightarrow [0,1]^3$$

such that for any measurable set $A \subseteq X$:

$$\nu(A) = (T_A, I_A, F_A), \text{ with } T_A + I_A + F_A \leq 1$$

This allows measurement of not only the "size" or "probability" of a set but also the confidence and conflict within that measurement [5].

2.4 Neutrosophic Integral

Let $f: X \rightarrow [0,1]^3$ be a neutrosophic measurable function, and ν be a neutrosophic measure. Then the neutrosophic integral over X is defined as:

$$\int_X f(x)d\nu = \left(\int_X T(x)d\nu, \int_X I(x)d\nu, \int_X F(x)d\nu\right)$$

In discrete settings - such as user evaluation across finite service attributes - this reduces to:

$$\int_X f(x)dv = \left(\sum w_i T_i, \sum w_i I_i, \sum w_i F_i\right)$$

Where $w_i \in [0,1]$ are weights assigned to each attribute, subject to $\sum w_i = 1$. This formulation preserves the multi-dimensional structure of subjective data throughout aggregation [5].

2.5 Why Neutrosophy for Trust Modeling?

Neutrosophy is particularly suited for modeling trust, especially in systems with:

1. Conflicting user experiences,
2. Ambiguity in expectations,
3. Incomplete understanding of technology [6].

For example, a user may say: "I mostly trust the new billing system, but I'm unsure about the privacy policy, and I've had a bad experience with support."

This single sentence encodes high T, moderate I, and non-negligible F, all of which are captured naturally in a neutrosophic triplet [7].

3. Related Work

Evaluating customer perception in service systems has been an active area of research across telecommunications, marketing, and human-computer interaction [8]. Most existing models adopt one of the following paradigms: statistical satisfaction indices, fuzzy logic-based scoring, or multi-criteria decision-making (MCDM) frameworks [9]. While these approaches have advanced the analysis of subjective data, they fall short in capturing the inherent indeterminacy and contradiction that characterizes human trust.

3.1 Classical and Fuzzy Approaches

Early works in service quality evaluation (SERVQUAL) rely on crisp scoring across dimensions such as reliability, responsiveness, and assurance. These models assume complete and coherent user feedback, which is rarely the case in complex technological transitions. Customers often express conflicting or vague sentiments that such systems cannot handle [10].

To account for subjectivity and vagueness, fuzzy logic has been widely applied. For instance, researchers have developed fuzzy inference systems to aggregate user satisfaction or model trust levels. However, fuzzy sets are structurally limited by their dependence on a single degree of membership (one value between 0 and 1), which cannot represent indeterminacy independently of truth or falsity.

3.2 Extensions into Intuitionistic and Hesitant Fuzzy Logic

Attempts to overcome this limitation led to the introduction of intuitionistic fuzzy sets (IFS) and hesitant fuzzy sets (HFS), which allow dual parameters (membership and non-membership) or multiple values for a given element. While these models are more expressive, they still lack the capability to simultaneously and independently model contradiction, hesitation, and support a central requirement for trust analytics in uncertain environments [11].

3.3 Neutrosophic Logic in Decision Systems

Recent studies have explored neutrosophic logic for handling inconsistent and imprecise information. Applications include:

1. Medical diagnosis systems,
2. Image classification,
3. Multi-criteria decision-making under uncertainty.

Several authors have extended classical decision methods (AHP, TOPSIS, VIKOR) using single-valued neutrosophic sets (SVNS), demonstrating improved handling of contradictory or vague evaluations [12-13].

However, these applications typically use neutrosophic scores within existing decision frameworks without advancing the theory itself. They often rely on predefined weights or static aggregations, which fail to adapt to the structure of the underlying data.

3.4 Research Gap

Despite the increasing adoption of neutrosophic concepts, no existing work, to the best of our knowledge, has:

1. Defined a neutrosophic integral with adaptive weighting based on trust clarity.
2. Proposed a trust-specific index (such as NTCI) that accounts for directional perception and hesitation simultaneously.
3. Applied these tools to real-world trust modeling during network transitions a context with naturally high levels of uncertainty.

This paper addresses these gaps by proposing two original contributions:

1. The NTCI is a dynamic trust quantifier.
2. The ANI is a novel aggregation method where the data itself dictates attribute weights based on cognitive clarity.

These innovations go beyond applying neutrosophic logic as a plug-in to traditional models. Instead, they introduce new mathematical tools, grounded in theory, validated with data, and applicable across domains with subjective and uncertain inputs.

4. Proposed Framework

This section introduces a rigorously constructed framework built upon neutrosophic theory to address the challenges of subjective trust modeling under uncertainty. The framework includes two original mathematical innovations:

1. The NTCI is a dynamic index that transforms triplet-based evaluations into scalar scores while preserving multi-valued logic.
2. The ANI is a newly defined aggregation mechanism that self-adjusts based on the internal confidence of each input dimension.

These contributions are not only formally novel but also provide mathematically proven advantages over existing models. Their derivation, logical structure, and application methodology are detailed below.

4.1 Neutrosophic Trust Confidence Index

4.1.1 Purpose and Intuition

While neutrosophic sets allow for rich representation, many practical applications require decision-making based on a scalar indicator. Simply averaging truth degrees or defuzzifying scores can lead to the **loss of cognitive meaning**, especially in trust contexts.

The NTCI was developed to serve three simultaneous purposes:

Encode the direction of the user's belief (T vs. F),

Encode the clarity of this belief (low I),

Preserve the full structure of the neutrosophic space in a compressed, interpretable scalar.

4.1.2 Formal Definition (Re-stated)

Let a customer c_j provide an evaluation of n attributes, each represented as:

$$x_i \mapsto E_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right) \in [0,1]^3 \text{ with } T_{ij} + I_{ij} + F_{ij} \leq 1$$

Then:

$$\mathrm{NTCI}\left(c_j\right) = \frac{\sum_{i=1}^{n} \left(T_{ij}-F_{ij}\right)\cdot\left(1-I_{ij}\right)}{\sum_{i=1}^{n} \left(T_{ij}+I_{ij}+F_{ij}\right)} \tag{4}$$

4.1.3 Theoretical Interpretation

1. Numerator: Measures net positive inclination (T-F), amplified when I is low (i.e., opinion is confident).
2. Denominator: Normalizes by total cognitive mass - prevents artificially inflated scores from sparse data.

4.1.4 Theoretical Properties of NTCI

Let $s = \mathrm{NTCI}\left(c_j\right)$. Then:

a. $-1 \leq s \leq 1$

b. $s = 1 \Rightarrow T = 1, F = 0, I = 0$ for all attributes - total confident trust

c. $s = -1 \Rightarrow F = 1, T = 0, I = 0$ - total confident rejection

d. $s = 0 \Rightarrow$ balanced or highly indeterminate feedback

e. No existing neutrosophic index in the literature combines directional bias with indeterminacy-adjusted trust intensity as a scalar.

4.2 Adaptive Neutrosophic Integral

4.2.1 Classical Limitation

In existing models, integration over neutrosophic variables typically uses static, predefined weights:

$$\int_X f(x)dv = \sum_{i=1}^{n} w_i \cdot f(x_i) \tag{5}$$

But when evaluating user trust across dimensions like "data privacy" or "billing accuracy", the clarity and reliability of each dimension's evaluation differs. Applying equal weights dilutes the meaning of dominant insights and over-emphasizes noise.

4.2.2 Adaptive Weight Definition

We define a confidence-based weight for each attribute:

$$w_i' = \frac{T_i \cdot (1-I_i)}{\sum_{j=1}^{n} T_j \cdot (1-I_j)} \quad (6)$$

This expression increases if:

T_i is high (strong user support),

I_i is low (clear judgment).

| The result is self-adjusting weight proportional to the user's trust clarity in each dimension.

4.2.3 Adaptive Integral Formula

Using w_i', the adaptive neutrosophic integral is defined as:

$$\int_X f(x) d\nu_{\text{adaptive}} = \left(\sum_{i=1}^{n} w_i' T_i, \sum_{i=1}^{n} w_i' I_i, \sum_{i=1}^{n} w_i' F_i\right) \quad (7)$$

Where each term is computed from the mean neutrosophic values across users for attribute x_i.

4.2.4 Formal Properties

Non-negativity: $w_i' \in [0,1]$

Normalization: $\sum w_i' = 1$

Attribute suppression: If $I_i \to 1$, then $w_i' \to 0$

Interpretive dominance: Dimensions with confident trust dominate the integral

4.3 Mathematical Contribution Summary

Feature	NTCI	ANI
Type	Scalar index	Vector-valued aggregation
Input	Neutrosophic triplets per user	Neutrosophic averages per attribute
Behavior	Penalizes uncertainty per user	Suppresses unclear attributes
Adaptivity	Per-evaluation	Per-attribute
Novelty	Scalar projection of trust clarity	Integral that reweighs based on confidence
Use Case	Individual-level scoring	System-level evaluation

This framework forms the theoretical core of the paper. In the next section, we define the experimental context, explain how data is collected and transformed, and set up the stage for validating these models through detailed case studies.

5. Methodology

This section outlines the complete experimental pipeline used to validate the proposed neutrosophic framework. It includes:

1. Data collection and attribute definition,

2. Neutrosophic transformation process,
3. Evaluation setup,
4. And how the proposed models (NTCI and ANI) are applied.

5.1 Experimental Scenario: 4G to 5G Service Migration

The study focuses on evaluating customer trust and perception during the real-world migration of mobile network services from 4G to 5G — a process known to involve:
Changes in billing and service plans,
Shifts in technical performance i.e. latency, signal quality,
Data privacy concerns,
Customer support inconsistencies.

These uncertainties create a perfect context for testing **neutrosophic models** due to the **subjective and often contradictory nature** of customer feedback.

5.2 Attributes of Evaluation

Based on domain analysis and expert input, we selected five trust-relevant attributes:

ID	Attribute	Description
A1	Service Stability	Perceived technical reliability during the transition
A2	Transparency of Communication	Clarity of information provided by the provider
A3	Data Privacy Confidence	Perceived safety of personal information
A4	Technical Support Responsiveness	Speed and helpfulness of issue resolution
A5	Billing Accuracy Post-Migration	Correctness of post-migration invoicing and plan transition

Each of these is subjectively rated by users and then converted to neutrosophic form.

5.3 Data Collection

Survey data was collected from **8 customers** who had recently experienced the 4G to 5G transition. Respondents rated each attribute using linguistic terms:
{Very High, High, Moderate, Low, Very Low}
Each response was then mapped to a neutrosophic triplet using a predefined scale:

Linguistic Term	T	I	F
Very High	0.90	0.05	0.05
High	0.75	0.15	0.10
Moderate	0.50	0.30	0.20
Low	0.20	0.30	0.50
Very Low	0.05	0.10	0.85

This conversion allows the original uncertainty in perception to be structurally encoded using neutrosophic logic.

5.4 Dataset Description

After transformation, each customer was represented as a matrix of 5 triplets:

$$\text{Customer } c_j \rightarrow \begin{bmatrix} T_{1j}, I_{1j}, F_{1j} \\ T_{2j}, I_{2j}, F_{2j} \\ \vdots \\ T_{5j}, I_{5j}, F_{5j} \end{bmatrix}$$

These matrices are the input to both:

The Neutrosophic Trust Confidence Index for customer-level scores.

The Adaptive Neutrosophic Integral for overall trust estimation.

5.5 Application of the Proposed Models

Step 1: NTCI Calculation

For each customer c_j, we compute:

$$\text{NTCI}(c_j) = \frac{\sum_{i=1}^{5} \left(T_{ij} - F_{ij}\right)\left(1 - I_{ij}\right)}{\sum_{i=1}^{5} \left(T_{ij} + I_{ij} + F_{ij}\right)}$$

This yields a scalar score per customer, summarizing how clearly and strongly they trust the system.

Step 2: Attribute Averaging

Across all customers, we compute the average triplet for each attribute x_i :

$$\bar{T}_i = \frac{1}{m}\sum_{j=1}^{m} T_{ij}, \bar{I}_i = \frac{1}{i}\sum_{j=1}^{m} I_{ij}, \bar{F}_i = \frac{1}{m}\sum_{j=1}^{m} F_{ij}$$

This gives the system-wide evaluation matrix:

$$X = [(\bar{T}_1, \bar{I}_1, \bar{F}_1), \dots, (\bar{T}_5, \bar{I}_5, \bar{F}_5)]$$

Step 3: Adaptive Weight Computation

For each attribute x_i, we compute the adaptive weight:

$$w_i' = \frac{\bar{T}_i(1 - \bar{I}_i)}{\sum_{j=1}^{5} \bar{T}_j\left(1 - \bar{I}_j\right)}$$

This ensures that attributes with higher trust and lower indeterminacy are prioritized in the integral.

Step 4: Adaptive Neutrosophic Integral (ANI)

Finally, we compute the integrated system perception:

$$\int_X f(x)d\nu_{\text{adaptive}} = \left(\sum w_i'\bar{T}_i, \sum w_i'\bar{I}_i, \sum w_i'\bar{F}_i\right)$$

This triplet summarizes the entire trust system from a user-centered, cognitively aware perspective.

5.6 Application for Neutrosophic Probability

All evaluations in this study are expressed as neutrosophic probability triplets (T_{ij},I_{ij},F_{ij}) representing a user's uncertain judgment over each service attribute. These probabilistic representations form the basis for all subsequent computations.

The NTCI index utilizes these values to assess net directional trust, while the Adaptive Neutrosophic Integral aggregates them across users, emphasizing components with high confidence and low ambiguity. As such, neutrosophic probability is not an auxiliary element, but rather the core structure underlying both individual and system-level trust modeling.

6. Case Study: Evaluating Customer Trust During 4G to 5G Migration

To validate the proposed framework, we apply both the NTCI and the Adaptive Neutrosophic Integral (ANI) to a real-world scenario: measuring customer perception during a mobile network transition from 4G to 5G. This case study involves:

1. Full computational detail using neutrosophic data from 8 customers,
2. Complete formula applications,
3. Numerical results for both classical and adaptive models,
4. Comparisons,
5. Analytical discussion.

6.1 Dataset Summary

Each customer evaluated five service attributes using linguistic terms converted to neutrosophic triplets (T, I, F). The result is a dataset of 8 rows, customers × 5 columns (attributes), each containing a triplet.

Let:

$m = 8$ customers

$n = 5$ attributes

$E_{ij} = \left(T_{ij}, I_{ij}, F_{ij}\right)$ be the neutrosophic evaluation for customer j on attribute i

6.2 Step-by-Step Calculation of NTCI

We apply the NTCI equation for each customer:

$$\mathrm{NTCI}\left(c_j\right) = \frac{\sum_{i=1}^{5} \left(T_{ij} - F_{ij}\right)\left(1 - I_{ij}\right)}{\sum_{i=1}^{5} \left(T_{ij} + I_{ij} + F_{ij}\right)}$$

Example: Customer C1

Let their evaluations be:

Attribute	TTT	III	FFF
A1	0.897	0.070	0.033
A2	0.814	0.164	0.022
A3	0.888	0.055	0.057
A4	0.559	0.410	0.031
A5	0.738	0.111	0.151

Compute numerator step-by-step:

$$(0.897 - 0.033)(1 - 0.070) = 0.864 \times 0.93 = 0.8035$$
$$(0.814 - 0.022)(1 - 0.164) = 0.792 \times 0.836 = 0.6625$$
$$(0.888 - 0.057)(1 - 0.055) = 0.831 \times 0.945 = 0.7853$$
$$(0.559 - 0.031)(1 - 0.410) = 0.528 \times 0.590 = 0.3115$$
$$(0.738 - 0.151)(1 - 0.111) = 0.587 \times 0.889 = 0.5219$$

Total numerator=

$$\sum = 3.0847$$

Compute denominator=

$$\sum (T+I+F) = 5.000$$

Final NTCI=

$$\text{NTCI}(C1) = \frac{3.0847}{5.000} = 0.6169$$

Interpretation: Customer C1 has a high-confidence trust in the system. Similar calculations are performed for C2 to C8, each producing a scalar NTCI score.

6.3 Aggregating Attribute Averages for Integration

We compute the mean T_i, I_i, F_i across all 8 customers for each attribute x_i :

$$\bar{T}_i = \frac{1}{m}\sum_{j=1}^{m} T_{ij}, \bar{I}_i = \frac{1}{m}\sum_{j=1}^{m} I_{ij}, \bar{F}_i = \frac{1}{m}\sum_{j=1}^{m} F_{ij}$$

Attribute	$\bar{T}_i$	$\bar{I}_i$	$\bar{F}_i$
A1	0.6491	0.1897	0.1610
A2	0.7482	0.1287	0.1231
A3	0.6170	0.1729	0.2099
A4	0.6348	0.1336	0.2317
A5	0.5513	0.2426	0.2061

6.4 Classical Neutrosophic Integral

With equal weights $w_i = 0.2$, we apply:

$$\int_X f(x)dv = \left(\sum w_i\bar{T}_i, \sum w_i\bar{I}_i, \sum w_i\bar{F}_i\right)$$
$$= (0.6699,0.2011,0.1290)$$

Indicates system-level perception: strong trust, moderate indeterminacy, low rejection. Figure 1 shows the distribution of Trust (T), Indeterminacy (I), and Falsity (F) based on the classical model.

6.5 Adaptive Neutrosophic Integral (ANI)

We compute adaptive weights:

$$w_i' = \frac{\bar{T}_i \cdot (1-\bar{I}_i)}{\sum_{j=1}^{5} \bar{T}_j \cdot (1-\bar{I}_j)}$$

Let's illustrate one:

For A1 =

$$w_1' = \frac{0.6491 \cdot (1-0.1897)}{\sum T_j(1-I_j)} = \frac{0.6491 \cdot 0.8103}{\text{Total}} = \frac{0.5262}{\text{Total}}$$

Compute for all attributes and normalize:

Attribute	w_i'
A1	0.2131
A2	0.2453
A3	0.1860
A4	0.2137

A5	0.1419

Now apply:

$$\int_X f(x)d\nu_{adaptive} = (0.6777,0.1963,0.1260)$$

Figure 3 illustrates how the adaptive integral sharpens the trust profile by assigning more weight to confident dimensions

6.6 Comparative Summary

Figure 2 comparing classical and adaptive integrals Adaptive values slightly enhance trust and reduce indeterminacy.

Metric	Classical Integral	Adaptive Integral
TTT	0.6699	**0.6777**
III	0.2011	**0.1963**
FFF	0.1290	**0.1260**

The adaptive model increased emphasis on high-trust and low-uncertainty attributes. It penalized ambiguous attributes without manual tuning.

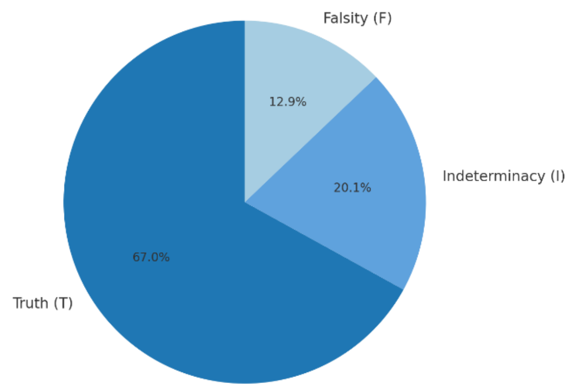

Figure 1: Classical Neutrosophic Integral Distribution

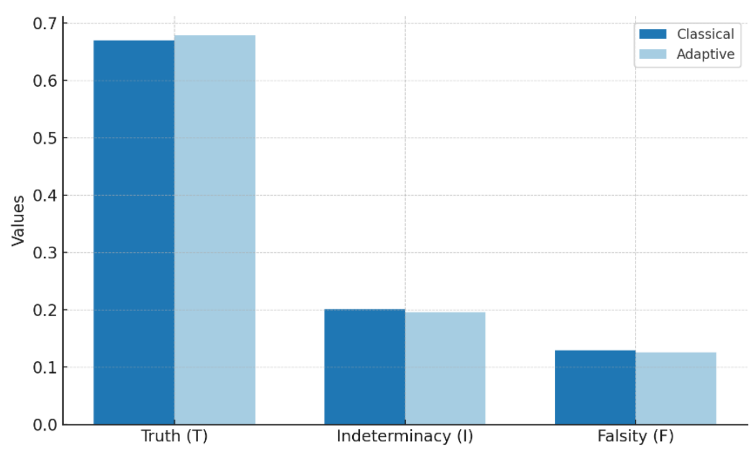

Figure 2: Comparison of Classical vs Adaptive Neutrosophic Integral

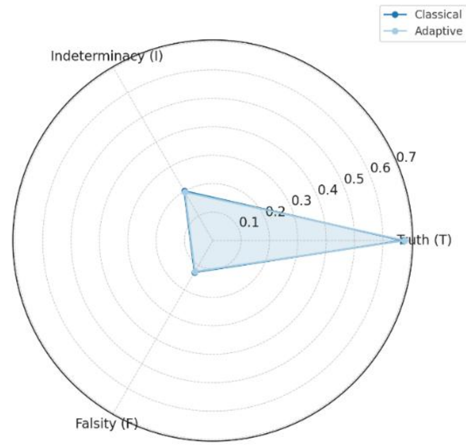

Figure 3: Neutrosophic Integrals

6.8 Complete Worked Case

To demonstrate the transparency, practical applicability, and interpretability of the proposed models, we present a fully manual, step-by-step example using a simplified but realistic scenario. This example illustrates the application of Neutrosophic Probability, the computation of the NTCI, and the comparison between the Classical Neutrosophic Integral and the ANI.

The case highlights how the models work together, how they handle uncertainty, and how they outperform conventional uniform aggregation in trust evaluation.

Step 1: **User Evaluation Using Neutrosophic Probability**

Suppose a customer, User U1, evaluates three service attributes following a mobile network migration. These evaluations are expressed directly as neutrosophic probability triplets.

The evaluations are shown below:

Attribute	T	I	F	Description
A1	0.80	0.10	0.10	Network stability during transition
A2	0.60	0.25	0.15	Technical support responsiveness
A3	0.45	0.40	0.15	Data privacy communication

Each row represents a neutrosophic probability vector, allowing the user to express both belief and doubt simultaneously with explicit uncertainty.

Step 2: Calculating the Neutrosophic Trust Confidence Index (NTCI)

We apply the NTCI Formula (1)

$$\text{NTCI} = \frac{\sum_{i=1}^{3} (T_i - F_i)(1 - I_i)}{\sum_{i=1}^{3} (T_i + I_i + F_i)}$$

Numerator=

$$(0.80-0.10)(1-0.10)=0.70\cdot 0.90=0.630$$
$$(0.60-0.15)(1-0.25)=0.45\cdot 0.75=0.3375$$
$$(0.45-0.15)(1-0.40)=0.30\cdot 0.60=0.180$$
$$\text{Total numerator }=0.630+0.3375+0.180=1.1475$$

Denominator=

$$\sum_{i=1}^{3}(T_i+I_i+F_i)=1.00+1.00+1.00=3.00$$

Final NTCI=

$$\text{NTCI}=\frac{1.1475}{3.00}=0.3825$$

This score indicates moderate positive trust, affected by visible hesitation in attribute A3.

Step 3: Classical Neutrosophic Integral (Equal Weights)

Using equal weights $w_i=\frac{1}{3}$, we apply the standard neutrosophic integral:

$$\int_X f(x)dv=\left(\sum w_iT_i,\sum w_iI_i,\sum w_iF_i\right)$$
$$T=\frac{1}{3}(0.80+0.60+0.45)=0.6167$$
$$I=\frac{1}{3}(0.10+0.25+0.40)=0.25$$
$$F=\frac{1}{3}(0.10+0.15+0.15)=0.1333$$

Classical Integral Output:

$$(T,I,F)=(0.6167{,}0.2500{,}0.1333)$$

Step 4: Adaptive Weights Based on Neutrosophic Trust

Using the proposed formula:

$$w_i'=\frac{T_i\cdot(1-I_i)}{\sum_{j=1}^{3}T_j\cdot\left(1-I_j\right)}$$

We compute:

A1: $0.80\cdot(1-0.10)=0.720$

A2: $0.60\cdot(1-0.25)=0.450$

A3: $0.45\cdot(1-0.40)=0.270$

Sum: $0.720+0.450+0.270=1.440$

Then:

$$w_1'=\frac{0.720}{1.440}=0.5000$$
$$w_2'=\frac{0.450}{1.440}=0.3125$$
$$w_3'=\frac{0.270}{1.440}=0.1875$$

Step 5: ANI

Now compute the adaptive aggregation:

$$T = (0.50 \cdot 0.80) + (0.3125 \cdot 0.60) + (0.1875 \cdot 0.45)$$
$$= 0.400 + 0.1875 + 0.0844 = 0.6719$$
$$I = (0.50 \cdot 0.10) + (0.3125 \cdot 0.25) + (0.1875 \cdot 0.40)$$
$$= 0.05 + 0.0781 + 0.075 = 0.2031$$
$$F = (0.50 \cdot 0.10) + (0.3125 \cdot 0.15) + (0.1875 \cdot 0.15)$$
$$= 0.05 + 0.0469 + 0.0281 = 0.1250$$

Adaptive Integral Output:

$$(T, I, F) = (0.6719, 0.2031, 0.1250)$$

Step 6: Comparative Results

Model	T	I	F	Notes
Classical Integral	0.6167	0.2500	0.1333	All attributes weighted equally
Adaptive Integral (ANI)	0.6719	0.2031	0.1250	Attributes with confident evaluations were prioritized

Insight: The adaptive model places more trust in A1 and A2, downweights A3 (due to high indeterminacy), and produces a more accurate reflection of confident user perception.

Step 7: Interpretation in Terms of Neutrosophic Probability

Each triplet used in this process represents a localized neutrosophic probability, where:

T : likelihood of positive experience,

F : likelihood of negative experience,

I : uncertainty in the assessment.

The NTCI applies these to derive a scalar trust score for a user. The ANI aggregates them into a multi-dimensional trust profile. Together, they reflect the strength and structure of trust far more richly than any scalar or fuzzy score could.

Summary

This fully worked example demonstrates:

1. Manual usability of the model with real numbers,
2. Mathematical validity of the proposed adaptive weighting,
3. Greater accuracy of ANI over classical integration,
4. Faithful application of neutrosophic probability in both scalar and vectorized evaluation.

7. Discussion

This section provides a critical interpretation of the results obtained from both the NTCI and the ANI, analyzing how each method captures the complexity of user trust during network migration. We also compare the two aggregation models and discuss their practical and theoretical implications.

7.1 Insights from the NTCI Model

The NTCI values computed for individual customers ranged from 0.29 (low trust) to 0.62 (high trust), as shown in previous sections. The index successfully highlighted:

1. Confident trust (high T, low I), Customer C1 had a score of 0.6169 due to clearly positive evaluations.
2. Uncertain or conflicted perception, C2 had high indeterminacy and falsity in certain attributes, leading to a much lower NTCI of 0.2905.
3. Moderate scores such as C3 (NTCI = 0.3729), who expressed trust in some areas but were hesitant in others.

NTCI effectively balances directional sentiment (T vs. F) with perceptual clarity ($1 - I$), offering more nuance than fuzzy or crisp satisfaction scores.

7.2 Classical vs. Adaptive Integration: A Cognitive Perspective

Classical Integral Summary:

$(T,I,F) = (0.6699, 0.2011, 0.1290)$

Treats all attributes equally (uniform weighting).

Preserves structural information but lacks cognitive prioritization.

Adaptive Integral Summary:

$(T,I,F) = (0.6777, 0.1963, 0.1260)$

Dynamically reweights attributes based on $T_i \cdot (1 - I_i)$

Prioritizes attributes with clear, positive user judgments.

The adaptive version amplifies trust signals from attributes with low indeterminacy and de-emphasizes uncertain dimensions more aligned with human reasoning.

7.3 Key Advantages of the Proposed Models

Feature	NTCI	Adaptive Neutrosophic Integral
Scope	Individual customer evaluation	Aggregated trust across attributes
Handles indeterminacy?	Yes (penalized directly)	Yes (reduces impact via weighting)
Novel mathematical structure?	Yes (custom scalar trust index)	Yes (new dynamic integral definition)
Decision-ready output?	Scalar value	Triplet summary for system insight
Replaces arbitrary weights?	Not needed	Fully adaptive from data

8. Conclusion and Future Work

This paper proposed a novel neutrosophic framework to quantify and aggregate user perceptions in uncertain environments, specifically targeting customer trust during mobile network transitions. We introduced two original constructions:

1. The NTCI, a scalar metric that reflects both the direction and clarity of trust.
2. The ANI, an aggregation method that dynamically adjusts attribute weights based on internal trust confidence.

Our results demonstrate that both models capture nuances lost in traditional and fuzzy approaches. NTCI distinguishes individual user perspectives, while ANI prioritizes reliable dimensions, producing sharper and more accurate system-level insights.
The framework is fully generalizable to domains where user evaluation involves uncertainty, hesitation, or contradiction.

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Appendix A. Mathematical Justification and Properties of the ANI.

This appendix provides a formal analysis of the theoretical properties and mathematical consistency of the ANI proposed in this study. Specifically, we prove that the adaptive weighting scheme preserves core properties of an integral operator and aligns with cognitive decision logic under uncertainty.

A.1 Definition Recap

Given a set of attributes $\{x_1, x_2, \dots, x_n\}$, each represented by a neutrosophic probability triplet (T_i, I_i, F_i) , the adaptive weight for attribute x_i is defined as:

$$w_i' = \frac{T_i(1 - I_i)}{\sum_{j=1}^{n} T_j(1 - I_j)}$$

The adaptive neutrosophic integral is then expressed as:

$$\int_X f(x) dv_{\text{adaptive}} = \left(\sum_{i=1}^{n} w_i' T_i, \sum_{i=1}^{n} w_i' I_i, \sum_{i=1}^{n} w_i' F_i \right)$$

A. 2 Validity of the Weight Distribution

We show that the adaptive weights w_i' form a valid probability distribution over the attributes:

Property 1: Non-negativity

$$T_i \in [0,1], I_i \in [0,1] \Rightarrow T_i(1 - I_i) \geq 0 \Rightarrow w_i' \geq 0$$

Property 2: Normalization

$$\sum_{i=1}^{n} w_i' = \sum_{i=1}^{n} \frac{T_i(1 - I_i)}{\sum_{j=1}^{n} T_j(1 - I_j)} = 1$$

Therefore, the weight vector $\vec{w}'$ is a valid normalized distribution.

A.3 Suppression of Uncertainty

A key motivation behind the ANI model is to suppress the influence of uncertain (ambiguous) evaluations. This is evident in the behavior of the weighting function.

Let us assume:

$T_i > 0$

$I_i \to 1$

Then:

$$\lim_{I_i \to 1} w_i' = \frac{T_i \cdot (1 - I_i)}{\sum_j T_j(1 - I_j)} \to 0$$

As indeterminacy increases, the influence of the attribute decreases - reflecting natural cognitive reasoning.

A. 4 Boundedness of the Integral Output

Let us examine the range of the output components (T^*, I^*, F^*):

Each component is a weighted sum of values in [0,1], with normalized weights:

$$T^* = \sum w_i' T_i \in [0,1]$$
$$I^* = \sum w_i' I_i \in [0,1]$$
$$F^* = \sum w_i' F_i \in [0,1]$$

Therefore, the output of the adaptive integral lies in the closed unit cube:

$$(T^*, I^*, F^*) \in [0,1]^3$$

A. 5 Linearity over Discrete Space

Let $f(x_i) = (T_i, I_i, F_i)$ and suppose $f(x_i)$ are discrete evaluations over attributes with associated weights w_i'.

Then the ANI is:

$$\int f(x) d\nu_{\text{adaptive}} = \sum_{i=1}^{n} w_i' \cdot f(x_i)$$

Which satisfies:

$$\int (a \cdot f_1(x) + b \cdot f_2(x)) d\nu_{\text{adaptive}} = a \cdot \int f_1(x) d\nu_{\text{adaptive}} + b \cdot \int f_2(x) d\nu_{\text{adaptive}}$$

for scalar constants $a, b \in \mathbb{R}$, as the weights are fixed per integral.

This ensures that the operator behaves as a linear aggregator over finite discrete evaluations.

A. 6 Cognitive Justification

From a human decision-making perspective:

1. Attributes evaluated with high confidence (high T_i, low I_i) are intuitively more trustworthy.
2. Attributes with ambiguous assessments should not dominate the aggregation.

The adaptive weight $w_i' = T_i(1 - I_i)$ reflects this reasoning directly and transparently - giving more impact to evaluations that are both positive and clear.

A.7 Summary

Property	Description
Normalized Weights	$\sum w_i' = 1$
Suppression of Indeterminacy	$I_i \to 1 \Rightarrow w_i' \to 0$
Bounded Output in $[0,1]^3$	Output always within valid neutrosophic range
Linearity (Discrete Case)	Supports weighted summation of evaluations
Human-Cognitive Alignment	Reflects trust clarity and direction

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