NSS

Neutrosophic Sets and Systems, Vol. 86, 2025

University of New Mexico

A Robust Approach to Possibility Single-Valued Neutrosophic Dombi-Weighted Aggregation Operators for Multiple Attribute Decision-Making

Yousef Al-Qudah[1], Ayman. A Hazaymeh[2], Faisal Al-Sharqi[3,*],Mamika Ujianita Romdhini [4] and Sarah Jawad Shoja[5]

[1]Department of Mathematics, Faculty of Arts and Science, Amman Arab University, Amman, Jordan; y.alqudah@aau.edu.jo ; alquyousef82@gmail.com

[2]Department of Mathematics, Faculty of Science, Jadara University, Irbid 21110, Jordan; aymanha@jadara.edu.jo

[3]Department of Mathematics, Faculty of Education for Pure Sciences, University of Anbar,Ramadi, Anbar, Iraq; faisal.ghazi@uoanbar.edu.iq

[4]Department of Mathematics, Faculty of Mathematics and Natural Sciences, University of Mataram, Mataram 83125, Indonesia; mamika@unram.ac.id

[5]College of Dentistry, Al-Ayen Iraqi University, An Nasiriyah, Iraq ; sarah.shoja@alayen.edu.iq

[*]Correspondence: Faisal Al-Sharqi, Email: faisal.ghazi@uoanbar.edu.iq

Abstract. Smarandache hands out the neutrosophic set (NS) theory that provides a general tri-mathematical framework to avail oneself of handling uncertainties during the information analysis. In the NS-environment there are three functions (T, I, F) that give a comprehensive description of each element or active case in the universal set. In real-life applications such as decision-making, this description of the elements of the universal set represents the decision-maker's opinion, which must be objective and reliable.This decision-maker's opinion, which each of the three functions works to imagine, has no degree of acceptability, such as a degree of probability of acceptance.This bar captures our attention and pay us to proposed a new notion of possibility single value neutrosophic set (PSVNS) as a new view where every NS-membership function has a different degree of probability of congruence (product).In this paper, based on PSVNN, the tools of Dombi t-norm (TN) and Dombi t-conorm (TCN) are applied to initiate more flexible and feasible operation rules for managing a parameter in a PSVN-environment when We introduce the notion of PSVN-Dombi arithmetic average aggregation (PSVDWAA) and PSVN-Dombi ordered weighted geometric aggregation (PSVNDWGA) operators.Due to the PSVNDWAA and PSVNDWGA operators having the advantage of virtual pliability, we work to discuss their different properties in addition to proofs, and also investigate multi-attribute decision-making (MADM) techniques according to proposed aggregation operators under PSVN-environment.Finally, an illustrative numerical example about MACM that takes selecting an ideal bridge according to the standards required by the bridge construction company to demonstrate the application and feasibility of the developed approach.

Keywords: Possibility single-valued neutrosophic number; Dombi operation; Possibility single-valued neutrosophic Dombi weighted arithmetic average (PSVNDWAA) operator; Possibility single-valued neutrosophic Dombi weighted geometric average (PSVNDWGA) operator; multiple attribute decision-making (MADM).

---

1. Introduction

Multi-criteria decision-making (MCDM) is a major issue in decision-making (DM)science, as it involves choosing the optimal modern alternative from among several modern alternatives based on data related to these modern alternatives. This data is visualized accurately according to mathematical tools, as these tools depict human thinking about any alternative from a group of options into a numerical value. Accordingly, Zadeh [1] initiated the fuzzy sets (FSs) to work on visualizing the data accompanying the decision-making process into numerical values (true membership function (TMF)) ranging between 0 and 1. The FSs offer appropriate solutions in several real-life issues, such as medical image processing, protein structure analysis, and medical data classification etc. However, the TMF is sometimes not enough to handle human thinking, which is based on the idea that for every correct logic, there is an incorrect logic. Therefore, to overcome this barrier, Atanassov [2] established the model of an intuitionistic fuzzy set (IFS), which is distinguished by two membership functions, namely, TMF and FMF, between 0 and 1. Furthermore, since FSs and IFSs cannot represent indeterminate and inconsistent information in human thinking, Smarandache [3] handled this issue when he introduced a neutrosophic set (NS) as an extension of FSs and IFSs. From a philosophical point of view, NS expresses indeterminate and inconsistent information in human thinking. As a mathematical structure, it consists of NS of three functions whose domain is the universal set and whose range lies in the real standard or nonstandard interval. This makes it difficult to apply this new concept to real-life issues. To overcome this problem, Wang et al [4] efficiently implemented the notion of the concept of a single-valued neutrosophic set (SVNS) when every membership function is given in single value form. Recently, many researchers have been progressing extensions of SVNS in different mathematical environments, such as: Pranab et al. [5] proposed a new approach for MADM based on similarity to the ideal solution to SVNSs. Alqahtani et al. [6] combined SVNS with graph theory ideas and checked their applications. Ding et al. [7] showing MCDM with SVNS-graphs. Khalil et al. [8] studied balance during compiling the soft set and SVNS. Al-Sharqi [9]– [11] studied some mathematical structures on SVNS and SS. Al-Quran et al. [12]– [13] extend the SVNS-inviroment from real numbers to complex numbers, and they are working on the application of these tools in real life. Alsharari et al. [14] introduced SVN-Primal Theory and its properties. Hazaymeh et al. [15, 16] studied some notes on SVNS-topological spaces with some applications. in addition, a lot of other studies as [16]– [24].

Moreover, Alkhazaleh and Salleh [25] invented a new mathematical structure by combining the structure of fuzzy theory with a probability score ranging from 0 to 1. This idea greatly attracted researchers and prompted them to present several research studies: Garai et al. [26]

expanded a link between SVNS and probability score in 2020. Al-Sharqi et al. [27]– [28] proposed the new similarity measures (SM) of SVNS and examined the application of medical issues. Zhao et al. [29] proposed a novel PSVN-hypersoft set (psv-NHSS) for the evaluation of investment projects. Al-Hijjawi and Alkhazaleh [30] characterized PSVN-hypersoft set (psv-NHSS) and examined the application of medical points.

Previous studies on associating probability scores with fuzzy sets and their extensions work on the mechanism that each fuzzy structure has a probability score (evaluation score). This point aroused our interest as those interested in this field and prompted us to present a new model for associating probability scores with the three-function neutrosophic set, where each function has a probability score written as a product. This new structure presented in this work gives the decision maker more freedom and flexibility by assigning a probability score to the truth score, a probability score to the neutrality score, and a probability score to the non-coincidence score. All of these scores are written as the product of the value of the neutrosophic function and the probability score.

In addition to the above, the Aggregation Operators (AOs) tools play a vital role in collecting numerical data by collecting huge data and simplifying it into simple numerical values that are easy to deal with. Accordingly, many research works have been presented by applying these mathematical tools with fuzzy concepts. Ye [31] proposed some simplified neutrosophic weighted arithmetic average operators (SNWAAOs) and a simplified neutrosophic weighted geometric average operators (SNWGAOs) based on the two AOs and the cosine-SM for SNSs. Jana and Pal [32, 33] presented both AOs and Domba-AOs on SVNS with an SS environment. Chen and Ye [34] used AOs and Domba-AOs for SVNS in solving MADM. Al-Quran et al. [35, 36] developed a new AOs for q-Rung -SVNNs (q-RSVNNs) based on the extension of the weighted arithmetic average operators and weighted geometric average operators. In addition to many other research works in this area [37–43]. Following this series of research works, we will present in this article a new approach for Dombi aggregation operators (DAOs) on PSVNNs, namely, we propose the PSVN-Dombi weighted arithmetic average (PSVNDWAA) operator and a PSVN-Dombi weighted geometric average (PSVNDWGA) operator for the PSVNN information as well as their properties. Finily, to develop a DM approach based on the PSVNDWAA and PSVNDWGA operators for dealing with MADM issues with PSVNN data.

This paper is literally organized as follows: In the Section 2, the definition of SVNS is presented in addition to the new definition of PSVNS is given as well as the details and illustrative examples that illustrate the difference between the given definition and previous definitions that depend on the degree of probability. In the Section 3, we define each of the Dombi T-norm and T-conorm between two real numbers p and q and based on this definition we define the Dombi operations of PSVNNs (DOPSVNNs). Section 4 presents the two Dombi weighted aggregation operators, the PSVNDWAA and PSVNDWGA operators, based on the Dombi operations of PSVNNs in Definition 3.1, and examines their properties. Section 5, we work on presenting the importance of the tools provided in this work, i.e., PSVNDWAA operator or the PSVNDWGA operator, in helping to solve one of the problems with PSVNN information. We formulating a hypothetical problem based on PSVNN data. Finally, in Section 6, we provide a complete summary of the work.

2. **Possibility single-valued neutrosophic environment**

In this section we will provide the basic definitions of our proposed concept in addition to presenting the related properties.But first we will give the basic definition of SVN as following:

Definition 2.1. [3] The following structure defined on **U**

$$\tilde{\Theta} = \left\{ \left\langle \ddot{v}, \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

is called SVNS where the three memberships functions: true $\widehat{\mathcal{T}}_{\Theta}(\ddot{v})$:$\mathbf{U}\rightarrow[0,1]$,indeterminacy $\widehat{\mathcal{I}}_{\Theta}(\ddot{v})$:$\mathbf{U}\rightarrow[0,1]$,falsity $\widehat{\mathcal{F}}_{\Theta}(\ddot{v})$:$\mathbf{U}\rightarrow[0,1]$ all of them for component $(\ddot{v})$ in **U** with stander condition $0\leq \widehat{\mathcal{T}}_{\Theta}(\ddot{v})+ \widehat{\mathcal{I}}_{\Theta}(\ddot{v})+ \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \leq 3$.

Below we present the basic definition of PSVNS as which is the product of each nitrosophic function with a different degree of probability such that the result falls in $[0,1]$.

Definition 2.2. Assume that $\mathbf{U}=\{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, ..., \ddot{v}_m\}$. Then the PSVNS is given as follows structure:

$$\tilde{\Theta}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

where

$\wp_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}) : U \rightarrow [0,1]$ is the product of the probability function and the neutrosophic truth function. $\wp_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}) : U \rightarrow [0,1]$ is the product of the probability function and the neutrosophic non-neutrality function and $\wp_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) : U \rightarrow [0,1]$ is the product of the probability function and the neutrosophic non-truth function, such that $0 \leq \wp_{\widehat{T}_{\Theta}(\ddot{v})} \widehat{T}_{\Theta}(\ddot{v}) + \wp_{\widehat{I}_{\Theta}(\ddot{v})} \widehat{I}_{\Theta}(\ddot{v}) + \wp_{\widehat{F}_{\Theta}(\ddot{v})} \widehat{F}_{\Theta}(\ddot{v}) \leq 3$.

To clarify the above definition, we present the following numerical example:

Example 2.3. Let $\mathbf{U}=\{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3\}$. Then the PSVNS $\tilde{\Theta}_{\wp}$ on $\mathbf{U}$ given as following:

$$\tilde{\Theta}_{\wp} = \begin{Bmatrix} \langle \ddot{v}_1, (.3\,(.8)\,, .9\,(.6)\,, .2\,(.7)) \rangle \\ \langle \ddot{v}_2, (.7\,(.1)\,, .8\,(.6)\,, .3\,(.9)) \rangle \\ \langle \ddot{v}_3, (.6\,(.1)\,, .6\,(.7)\,, .5\,(.6)) \rangle \end{Bmatrix}$$

Definition 2.4. A PSVNS $\tilde{\Theta}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ on $\mathbf{U}$ is side to Null-PSVNS and given as following

$$\tilde{\Theta}^{\varnothing}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})} \widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v}), \wp_{\widehat{I}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})} \widehat{I}_{\tilde{\Theta}^{\varnothing}}(\ddot{v}), \wp_{\widehat{F}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})} \widehat{F}_{\tilde{\Theta}^{\varnothing}}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

Where

$\wp_{\widehat{\mathcal{T}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{T}}_{\Theta^{\varnothing}}(\ddot{v}) = 0,\ \wp_{\widehat{\mathcal{I}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{I}}_{\Theta^{\varnothing}}(\ddot{v}) = 0$ and $\wp_{\widehat{\mathcal{F}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{F}}_{\Theta^{\varnothing}}(\ddot{v}) = 0$

Definition 2.5. A PSVNS $\tilde{\Theta}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ on $\mathbf{U}$ is side to Null-PSVNS and given as following

$$\tilde{\Theta}^{U}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})} \widehat{T}_{\tilde{\Theta}^{U}}(\ddot{v}), \wp_{\widehat{I}_{\tilde{\Theta}^{U}}(\ddot{v})} \widehat{I}_{\tilde{\Theta}^{U}}(\ddot{v}), \wp_{\widehat{F}_{\tilde{\Theta}^{U}}(\ddot{v})} \widehat{F}_{\tilde{\Theta}^{U}}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

Where

$\wp_{\widehat{\mathcal{T}}_{\Theta^{U}}(\ddot{v})} = 1, \widehat{\mathcal{T}}_{\Theta^{U}}(\ddot{v}) = 1,\ \wp_{\widehat{\mathcal{I}}_{\Theta^{U}}(\ddot{v})} = 1, \widehat{\mathcal{I}}_{\Theta^{U}}(\ddot{v}) = 1$ and $\wp_{\widehat{\mathcal{F}}_{\Theta^{U}}(\ddot{v})} = 1, \widehat{\mathcal{F}}_{\Theta^{U}}(\ddot{v}) = 1$

Definition 2.6. We say that the $\tilde{\Theta}^{1}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ is subset of $\tilde{\Theta}^{2}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ and denotes as $\tilde{\Theta}^{1}_{\wp} \leq \tilde{\Theta}^{2}_{\wp}$ if : $\wp_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}) \leq \wp_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})$, $\wp_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}) \geq \wp_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})$, $\wp_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \geq \wp_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})$.

Definition 2.7. We say that the $\tilde{\Theta}^{1}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ is equal of $\tilde{\Theta}^{2}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ and denotes as $\tilde{\Theta}^{1}_{\wp} \leq \tilde{\Theta}^{2}_{\wp}$ if : $\wp_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}) = \wp_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})$, $\wp_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}) = \wp_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})$, $\wp_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) = \wp_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})$.

Definition 2.8. For a PSVNS $\tilde{\Theta}_{\wp} = \left\{ \left\langle \ddot{v}, \wp_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), \wp_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$. Then the score function (SF) $\breve{E}\left(\tilde{\Theta}_{\wp}\right)$ and accuracy function (AF) $\breve{A}\left(\tilde{\Theta}_{\wp}\right)$ given as following:

$$\breve{E}\left(\tilde{\Theta}_{\wp}\right)=\frac{\left(2+\left(\wp_{\widehat{\mathcal{T}}_{\Theta(\ddot{v})}}\widehat{\mathcal{T}}_{\Theta}(\ddot{v})\right)-\left(\wp_{\widehat{\mathcal{I}}_{\Theta(\ddot{v})}}\widehat{\mathcal{I}}_{\Theta}(\ddot{v})\right)-\left(\wp_{\widehat{\mathcal{F}}_{\Theta(\ddot{v})}}\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\right)\right)}{3}. \tag{1}$$

Where the value of $\breve{E}\left(\tilde{\Theta}_{\wp}\right)\in[0,1]$.

$$\breve{A}\left(\tilde{\Theta}_{\wp}\right)=\left(\wp_{\widehat{\mathcal{T}}_{\Theta(\ddot{v})}}\widehat{\mathcal{T}}_{\Theta}\left(\ddot{v}\right)\right)-\left(\wp_{\widehat{\mathcal{F}}_{\Theta(\ddot{v})}}\widehat{\mathcal{F}}_{\Theta}\left(\ddot{v}\right)\right). \tag{2}$$

Where the value of $\breve{A}\left(\tilde{\Theta}_{\wp}\right)\in[-1,1]$.

Definition 2.9. The ranking method between two PSVNSs
$\tilde{\Theta}^1_{\wp}=\left\{\left\langle\ddot{v},\wp_{\widehat{\mathcal{T}}_{\Theta^1(\ddot{v})}}\widehat{\mathcal{T}}_{\Theta^1}\left(\ddot{v}\right),\wp_{\widehat{\mathcal{I}}_{\Theta^1(\ddot{v})}}\widehat{\mathcal{I}}_{\Theta^1}\left(\ddot{v}\right),\wp_{\widehat{\mathcal{F}}_{\Theta^1(\ddot{v})}}\widehat{\mathcal{F}}_{\Theta^1}\left(\ddot{v}\right)\right\rangle|\ddot{v}\in U\right\}$ and
$\tilde{\Theta}^2_{\wp}=\left\{\left\langle\ddot{v},\wp_{\widehat{\mathcal{T}}_{\Theta^2(\ddot{v})}}\widehat{\mathcal{T}}_{\Theta^2}\left(\ddot{v}\right),\wp_{\widehat{\mathcal{I}}_{\Theta^2(\ddot{v})}}\widehat{\mathcal{I}}_{\Theta^2}\left(\ddot{v}\right),\wp_{\widehat{\mathcal{F}}_{\Theta^2(\ddot{v})}}\widehat{\mathcal{F}}_{\Theta^2}\left(\ddot{v}\right)\right\rangle|\ddot{v}\in U\right\}$ is defined as follows:
(i.) If $\breve{E}\left(\tilde{\Theta}^1_{\wp}\right)>\breve{E}\left(\tilde{\Theta}^2_{\wp}\right)$ then $\tilde{\Theta}^1_{\wp}>\tilde{\Theta}^2_{\wp}$
(ii.) If $\breve{E}\left(\tilde{\Theta}^1_{\wp}\right)=\breve{E}\left(\tilde{\Theta}^2_{\wp}\right)$ and $\breve{A}\left(\tilde{\Theta}^1_{\wp}\right)>\breve{A}\left(\tilde{\Theta}^2_{\wp}\right)$ then $\tilde{\Theta}^1_{\wp}>\tilde{\Theta}^2_{\wp}$.
(iii.) If $\breve{E}\left(\tilde{\Theta}^1_{\wp}\right)=\breve{E}\left(\tilde{\Theta}^2_{\wp}\right)$ and $\breve{A}\left(\tilde{\Theta}^1_{\wp}\right)=\breve{A}\left(\tilde{\Theta}^2_{\wp}\right)$ then $\tilde{\Theta}^1_{\wp}=\tilde{\Theta}^2_{\wp}$.

3. **Possibility Single-Valued Neutrosophic Dombi Operations**

In the following section we provide a definition of each of Dombi T-norm and T-conorm between two real numbers p and q and based on this definition we define the Dombi operations of PSVNNs (DOPSVNNs).

Definition 3.1. [34] The Dombi T-norm and T-conorm between two real numbers p and q given as follows:

$$D_O\left(p,q\right)=\frac{1}{1+\left\{\left(\frac{1-p}{p}\right)^{\rho}+\left(\frac{1-q}{q}\right)^{\rho}\right\}^{1/\rho}} \tag{1}$$

$$D_O^C\left(p,q\right)=1-\frac{1}{1+\left\{\left(\frac{1-p}{p}\right)^{\rho}+\left(\frac{1-q}{q}\right)^{\rho}\right\}^{1/\rho}} \tag{2}$$

Where $\rho\geq1$ and $(p,q)\in[0,1]\times[0,1]$.

Now based on the Dombi T-norm and T-conorm in the above definition, we define the Dombi operations of PSVNNs (DOPSVNNs) as following.

Definition 3.2. Assume that $\tilde{\Theta}^1_{\wp}=\left\{\left\langle\left(\wp^{\widehat{\mathcal{T}}}_{\Theta^1}\widehat{\mathcal{T}}_{\Theta^1}\right),\left(\wp^{\widehat{\mathcal{I}}}_{\Theta^1}\widehat{\mathcal{I}}_{\Theta^1}\right),\left(\wp^{\widehat{\mathcal{F}}}_{\Theta^1}\widehat{\mathcal{F}}_{\Theta^1}\right)\right\rangle\right\}$ and $\tilde{\Theta}^2_{\wp}=\left\{\left\langle\left(\wp^{\widehat{\mathcal{T}}}_{\Theta^2}\widehat{\mathcal{T}}_{\Theta^2}\right),\left(\wp^{\widehat{\mathcal{I}}}_{\Theta^2}\widehat{\mathcal{I}}_{\Theta^2}\right),\left(\wp^{\widehat{\mathcal{F}}}_{\Theta^2}\widehat{\mathcal{F}}_{\Theta^2}\right)\right\rangle\right\}$ be two PSVNNs and both $\rho\geq1,\delta\geq1$. Then all Dombi T-norm and T-conorm operations of PSVNNs are given as follows:

(i.) $$\tilde{\Theta}^1_{\wp} \oplus \tilde{\Theta}^2_{\wp} = \left\langle 1 - \frac{1}{1+\left\{\left(\frac{\left(\wp_{\widehat{\mathcal{T}}_{\Theta_1}}\widehat{\mathcal{T}}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{\mathcal{T}}_{\Theta_1}}\widehat{\mathcal{T}}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{\left(\wp_{\widehat{\mathcal{T}}_{\Theta_2}}\widehat{\mathcal{T}}_{\Theta_2}\right)}{1-\left(\wp_{\widehat{\mathcal{T}}_{\Theta_2}}\widehat{\mathcal{T}}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}, \right.$$

$$\left. \frac{1}{1+\left\{\left(\frac{1-\left(\wp_{\widehat{\mathcal{I}}_{\Theta_1}}\widehat{\mathcal{I}}_{\Theta_1}\right)}{\left(\wp_{\widehat{\mathcal{I}}_{\Theta_1}}\widehat{\mathcal{I}}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{1-\left(\wp_{\widehat{\mathcal{I}}_{\Theta_2}}\widehat{\mathcal{I}}_{\Theta_2}\right)}{\left(\wp_{\widehat{\mathcal{I}}_{\Theta_2}}\widehat{\mathcal{I}}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-\left(\wp_{\widehat{\mathcal{F}}_{\Theta_1}}\widehat{\mathcal{F}}_{\Theta_1}\right)}{\left(\wp_{\widehat{\mathcal{F}}_{\Theta_1}}\widehat{\mathcal{F}}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{\left(\wp_{\widehat{\mathcal{F}}_{\Theta_2}}\widehat{\mathcal{F}}_{\Theta_2}\right)}{\left(\wp_{\widehat{\mathcal{F}}_{\Theta_2}}\widehat{\mathcal{F}}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

(ii.) $$\tilde{\Theta}^1_{\wp} \otimes \tilde{\Theta}^2_{\wp} = \left\langle \frac{1}{1+\left\{\left(\frac{1-\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}{\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{1-\left(\wp_{\widehat{T}_{\Theta_2}}\widehat{T}_{\Theta_2}\right)}{\left(\wp_{\widehat{T}_{\Theta_2}}\widehat{T}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}, \right.$$

$$\left. 1-\frac{1}{1+\left\{\left(\frac{\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{\left(\wp_{\widehat{I}_{\Theta_2}}\widehat{I}_{\Theta_2}\right)}{1-\left(\wp_{\widehat{I}_{\Theta_2}}\widehat{I}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\left(\frac{\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}\right)^{\rho}+\left(\frac{\left(\wp_{\widehat{F}_{\Theta_2}}\widehat{F}_{\Theta_2}\right)}{1-\left(\wp_{\widehat{F}_{\Theta_2}}\widehat{F}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

(iii.) $$\delta\tilde{\Theta}^1_{\wp} = \left\langle \frac{1}{1+\left\{\delta\left(\frac{\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}},\ 1-\frac{1}{1+\left\{\delta\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}{\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\delta\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}{\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

(iv) $$\left(\tilde{\Theta}^1_{\wp}\right)^{\delta} = \left\langle \frac{1}{1+\left\{\delta\left(\frac{1-\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}{\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}},\ 1-\frac{1}{1+\left\{\delta\left(\frac{\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}}, 1-\frac{1}{1+\left\{\delta\left(\frac{\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

4. The Two Novel Dombi Weighted Aggregation Operators of PSVNNs

In this part, we present the two Dombi weighted aggregation operators, the PSVNDWAA and PSVNDWGA operators, based on the Dombi operations of PSVNNs in Definition 4, and inquire into their properties.

Definition 4.1. Assume that $\tilde{\Theta}^i_{\wp} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle$ for i=1,2,3,...,n be a group of PSVNNs and $w = (w_1, w_2, w_3, ..., w_n)$ be a group of weight vector for $\tilde{\Theta}^i_{\wp}$ such that $w_k \in [0, 1]$ and $w_1+w_2+w_3+...+w_n = 1$.Then the PSVNDWAA and PSVNDWGA operators, respectively, are given as follows:

$$\text{PSVNDWAA}\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=\overset{n}{\underset{k=1}{\oplus}} w_k\tilde{\Theta}_{\wp}^{k}. \tag{3}$$

$$\text{PSVNDWGA}\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=\overset{n}{\underset{k=1}{\otimes}}\left(\tilde{\Theta}_{\wp}^{k}\right)^{w_k}. \tag{4}$$

Theorem 4.2. *Assume that* $\tilde{\Theta}_{\wp}^{i}=\left\langle\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)\right\rangle$ *for i=1,2,3,...,n be a group of PSVNNs and* $w=(w_1,w_2,w_3,...,w_n)$ *be a group of weight vector for* $\tilde{\Theta}_{\wp}^{i}$ *such that* $w_k\in[0,1]$ *and* $w_1+w_2+w_3+...+w_n=1$*.Then the aggregated value of the PSVNDWAA operator is still a PSVNN, which is determined by the next formula:*

$$PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=$$
$$\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right)}{1-\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{1-\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right)}{\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{1-\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)}{\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}}\right\rangle.$$

Proof. To prove this theory satisfactorily, it is necessary to use mathematical induction as following:

If n=2 then from the Dombi operations of PSVNNs given in Definition 3.2, we can come by the following outcome:

$$PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2}\right)=\tilde{\Theta}_{\wp}^{1}\oplus\tilde{\Theta}_{\wp}^{2}=$$
$$\left\langle 1-\frac{1}{1+\left\{w_1\left(\frac{\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_1}}\widehat{T}_{\Theta_1}\right)}\right)^{\rho}+w_2\left(\frac{\left(\wp_{\widehat{T}_{\Theta_2}}\widehat{T}_{\Theta_2}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_2}}\widehat{T}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{w_1\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}{\left(\wp_{\widehat{I}_{\Theta_1}}\widehat{I}_{\Theta_1}\right)}\right)^{\rho}+w_2\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_2}}\widehat{I}_{\Theta_2}\right)}{\left(\wp_{\widehat{I}_{\Theta_2}}\widehat{I}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}\right.$$
$$\left.,\frac{1}{1+\left\{w_1\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}{\left(\wp_{\widehat{F}_{\Theta_1}}\widehat{F}_{\Theta_1}\right)}\right)^{\rho}+w_2\left(\frac{\left(\wp_{\widehat{F}_{\Theta_2}}\widehat{F}_{\Theta_2}\right)}{\left(\wp_{\widehat{F}_{\Theta_2}}\widehat{F}_{\Theta_2}\right)}\right)^{\rho}\right\}^{1/\rho}}\right\rangle.$$
$$=$$
$$\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{2}w_i\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{2}w_i\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{2}w_i\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}\right\rangle$$

If n = k, then from Equation in theorem 4.2, we got the following equation:

$$PSVNDWAA\left(\tilde{\Theta}_\wp^1,\tilde{\Theta}_\wp^2,\tilde{\Theta}_\wp^3,...,\tilde{\Theta}_\wp^k\right) = \tilde{\Theta}_\wp^1 \oplus \tilde{\Theta}_\wp^2 \oplus \tilde{\Theta}_\wp^3 \oplus ... \oplus \tilde{\Theta}_\wp^k = \left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

Now, if n = k + 1, then we get the following equation:

$$PSVNDWAA\left(\tilde{\Theta}_\wp^1,\tilde{\Theta}_\wp^2,\tilde{\Theta}_\wp^3,...,\tilde{\Theta}_\wp^k,\tilde{\Theta}_\wp^{k+1}\right) = \tilde{\Theta}_\wp^1 \oplus \tilde{\Theta}_\wp^2 \oplus \tilde{\Theta}_\wp^3 \oplus ... \oplus \tilde{\Theta}_\wp^k \oplus \tilde{\Theta}_\wp^{k+1}$$

$$= \left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle \oplus w_{k+1}\tilde{\Theta}_\wp^{k+1}$$

$$= \left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k+1} w_i\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k+1} w_i\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k+1} w_i\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

Hence, we can conclude that Theorem 4.2 is true for n = k + 1. Thus, Equation (******)in the above theorem holds for all n. □

In the next steps of this work, we explore more properties that are associated with the PSVNDWAA operator, as follows:

Proposition 4.3. ***(Reducibility):*** *If* $w = \left(\frac{1}{n},\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)$. *Then,*

$$PSVNDWAA\left(\tilde{\Theta}_\wp^1,\tilde{\Theta}_\wp^2,\tilde{\Theta}_\wp^3,...,\tilde{\Theta}_\wp^n\right) = \tilde{\Theta}_\wp^1 \oplus \tilde{\Theta}_\wp^2 \oplus \tilde{\Theta}_\wp^3 \oplus ... \oplus \tilde{\Theta}_\wp^n$$

$$= \left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{n} \frac{1}{n}\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

Proof. From Equation in theorem 4.2, the property is clear. □

Proposition 4.4. ***(Idempotency):*** *Assume that all PSVNNs* $\tilde{\Theta}_{\wp}^{i} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle$ $= \tilde{\Theta}_{\wp}^{1}$ *for i=1,2,3,...,n.Then* $PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right) = \tilde{\Theta}_{\wp}^{1}$.

Proof. Since $\tilde{\Theta}_{\wp}^{i} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle = \tilde{\Theta}_{\wp}$ for i=1,2,3,...,n. Then, based on Equation stated in Theorem 4.2, we can get the the following outcome:

$$PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{k}\right) = \tilde{\Theta}_{\wp}^{1} \oplus \tilde{\Theta}_{\wp}^{2} \oplus \tilde{\Theta}_{\wp}^{3} \oplus ... \oplus \tilde{\Theta}_{\wp}^{n} =$$

$$\left\langle 1 - \frac{1}{1+\left\{\sum_{i=1}^{k} w_i \left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i \left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{i=1}^{k} w_i \left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left\{\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle =$$

$$\left\langle 1 - \frac{1}{1+\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}}, \frac{1}{1+\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}}, \frac{1}{1+\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}} \right\rangle$$

$$= \left\langle \left(\wp_{\widehat{T}_{\Theta}}\widehat{T}_{\Theta}\right), \left(\wp_{\widehat{I}_{\Theta}}\widehat{I}_{\Theta}\right), \left(\wp_{\widehat{F}_{\Theta}}\widehat{F}_{\Theta}\right) \right\rangle$$

$$= \Theta_{\wp}$$

Hence $\tilde{\Theta}_{\wp}^{i} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle = \tilde{\Theta}_{\wp}$ is hold. □

Proposition 4.5. ***(Commutativity):*** *Assume that PSVNS* $\left(\tilde{\Theta}_{\wp}^{1'}, \tilde{\Theta}_{\wp}^{2'}, \tilde{\Theta}_{\wp}^{3'}, ..., \tilde{\Theta}_{\wp}^{n'}\right)$ *be any permutation of* $\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right)$*.Then there is* $PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1'}, \tilde{\Theta}_{\wp}^{2'}, \tilde{\Theta}_{\wp}^{3'}, ..., \tilde{\Theta}_{\wp}^{n'}\right) = PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right)$.

Proof. The property is clear. □

Proposition 4.6. ***(Boundedness):*** *Assume that* $\tilde{\Theta}_{\wp}^{\min} = \min\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right)$ *and* $\tilde{\Theta}_{\wp}^{\max} = \max\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right)$*. Then* $\tilde{\Theta}_{\wp}^{\min} \leq PSVNDWAA\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right) \leq \tilde{\Theta}_{\wp}^{\max}$.

Proof. Assume that $\tilde{\Theta}_{\wp}^{\min} = \min\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right) = \left\langle \left(\wp_{\Theta^-}\widehat{T}_{\Theta^-}\right), \left(\wp_{\Theta^-}\widehat{I}_{\Theta^-}\right), \left(\wp_{\Theta^-}\widehat{F}_{\Theta^-}\right) \right\rangle$ and $\tilde{\Theta}_{\wp}^{\max} = \max\left(\tilde{\Theta}_{\wp}^{1}, \tilde{\Theta}_{\wp}^{2}, \tilde{\Theta}_{\wp}^{3}, ..., \tilde{\Theta}_{\wp}^{n}\right) = \left\langle \left(\wp_{\Theta^+}\widehat{T}_{\Theta^+}\right), \left(\wp_{\Theta^+}\widehat{I}_{\Theta^+}\right), \left(\wp_{\Theta^+}\widehat{F}_{\Theta^+}\right) \right\rangle$

Then, we have, $\left(\wp_{\Theta^-}\widehat{T}_{\Theta^-}\right) = \min\left(\wp_{\Theta}^{i}\widehat{T}_{\Theta}^{i}\right)$, $\left(\wp_{\Theta^-}\widehat{I}_{\Theta^-}\right) = \max\left(\wp_{\Theta}^{i}\widehat{I}_{\Theta}^{i}\right)$, $\left(\wp_{\Theta^-}\widehat{F}_{\Theta^-}\right) = \max\left(\wp_{\Theta}^{i}\widehat{F}_{\Theta}^{i}\right)$

and

$$\left(\wp_{\Theta^+}\widehat{T}_{\Theta^-}\right)=\max\left(\wp^i_{\Theta}\widehat{T}^i_{\Theta}\right),\ \left(\wp_{\Theta^+}\widehat{I}_{\Theta^+}\right)=\min\left(\wp^i_{\Theta}\widehat{I}^i_{\Theta}\right),\ \left(\wp_{\Theta^+}\widehat{F}_{\Theta^+}\right)=\min\left(\wp^i_{\Theta}\widehat{F}^i_{\Theta}\right)$$

Thus, there are the following inequalities:

$$1-\frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{T}^-_\Theta\right)}{1-\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{T}^-_\Theta\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ 1-\frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\ \widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\ \widehat{T}_{\Theta_i}\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ 1-\frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{T}^+_\Theta\right)}{1-\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{T}^+_\Theta\right)}\right)^\rho\right\}^{1/\rho}}$$

$$\frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{I}^-_\Theta\right)}{\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{I}^-_\Theta\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ \frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\ \widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\ \widehat{I}_{\Theta_i}\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ \frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{I}^+_\Theta\right)}{\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{I}^+_\Theta\right)}\right)^\rho\right\}^{1/\rho}}$$

$$\frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{F}^-_\Theta\right)}{\left(\wp^-_{\widehat{T}_\Theta}\ \widehat{F}^-_\Theta\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ \frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\ \widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\ \widehat{F}_{\Theta_i}\right)}\right)^\rho\right\}^{1/\rho}}\ \leq\ \frac{1}{1+\left\{\sum_{i=1}^{k}w_i\left(\frac{1-\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{F}^+_\Theta\right)}{\left(\wp^+_{\widehat{T}_\Theta}\ \widehat{F}^+_\Theta\right)}\right)^\rho\right\}^{1/\rho}}$$

Hence $\tilde{\Theta}^{\min}_{\wp}\leq P\text{SVNDWAA}\left(\tilde{\Theta}^1_{\wp},\tilde{\Theta}^2_{\wp},\tilde{\Theta}^3_{\wp},...,\tilde{\Theta}^n_{\wp}\right)\leq\tilde{\Theta}^{\max}_{\wp}$. is hold. □

Theorem 4.7. *Assume that* $\tilde{\Theta}^i_{\wp}=\left\langle\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)\right\rangle$ *for i=1,2,3,...,n be a group of PSVNNs and* $w=(w_1,w_2,w_3,...,w_n)$ *be a group of weight vector for* $\tilde{\Theta}^i_{\wp}$ *such that* $w_k\in[0,1]$ *and* $w_1+w_2+w_3+...+w_n=1$.*Then the aggregated value of the PSVNDWGA operator is still a PSVNN, which is determined by the next formula:*

$$PSVNDWGA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=$$
$$\left\langle \frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{1-\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right)}{\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}},\ 1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right)}{1-\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}},1-\frac{1}{1+\left\{\sum_{i=1}^{n}w_n\left(\frac{\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)}{1-\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)}\right)^{\rho}\right\}^{1/\rho}}\right\rangle.$$

Proof. The proof of Theorem 4.2 is also by using mathematical induction, similar to that of Theorem 4.1. Thus, it is not taken out here. □

In the next steps of this work, we explore more properties that are associated with the PSVNDWGA operator, as follows:

Proposition 4.8. ***(Reducibility):*** *If* $w=\left(\frac{1}{n},\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)$. *Then,*
$PSVNDWGA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=\tilde{\Theta}_{\wp}^{1}\oplus\tilde{\Theta}_{\wp}^{2}\oplus\tilde{\Theta}_{\wp}^{3}\oplus...\oplus\tilde{\Theta}_{\wp}^{n}$
$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{n}\frac{1}{n}\left(\frac{\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}{1-\left(\wp_{\widehat{T}_{\Theta_i}}\widehat{T}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{n}\frac{1}{n}\left(\frac{1-\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}{\left(\wp_{\widehat{I}_{\Theta_i}}\widehat{I}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}},\frac{1}{1+\left\{\sum_{i=1}^{n}\frac{1}{n}\left(\frac{1-\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}{\left(\wp_{\widehat{F}_{\Theta_i}}\widehat{F}_{\Theta_i}\right)}\right)^{\rho}\right\}^{1/\rho}}\right\rangle.$$

Proof. The proof is similar to the method of the property 4.3,proofs. □

Proposition 4.9. ***(Idempotency):*** *Assume that all PSVNNs* $\tilde{\Theta}_{\wp}^{i}=\left\langle\left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right),\left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right)\right\rangle$
$=\tilde{\Theta}_{\wp}^{1}$ *for i=1,2,3,...,n.Then* $PSVNDWGA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)=\tilde{\Theta}_{\wp}^{1}$.

Proof. The proof is similar to the method of the property 4.4,proofs. □

Proposition 4.10. ***(Commutativity):*** *Assume that PSVNS*$\left(\tilde{\Theta}_{\wp}^{1'},\tilde{\Theta}_{\wp}^{2'},\tilde{\Theta}_{\wp}^{3'},...,\tilde{\Theta}_{\wp}^{n'}\right)$ *be any permutation of* $\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)$.*Then there is PSVNDWGA* $\left(\tilde{\Theta}_{\wp}^{1'},\tilde{\Theta}_{\wp}^{2'},\tilde{\Theta}_{\wp}^{3'},...,\tilde{\Theta}_{\wp}^{n'}\right)=$ *PSVNDWAA* $\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)$.

Proof. The proof is similar to the method of the property 4.5,proofs. □

Proposition 4.11. ***(Boundedness):*** *Assume that* $\tilde{\Theta}_{\wp}^{\min}=\min\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)$ *and* $\tilde{\Theta}_{\wp}^{\max}=\max\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)$. *Then* $\tilde{\Theta}_{\wp}^{\min}\leq PSVNDWGA\left(\tilde{\Theta}_{\wp}^{1},\tilde{\Theta}_{\wp}^{2},\tilde{\Theta}_{\wp}^{3},...,\tilde{\Theta}_{\wp}^{n}\right)\leq\tilde{\Theta}_{\wp}^{\max}$.

Proof. The proof is similar to the method of the property 4.6,proofs. □

5. Employ both PSVNDWAA Operator or the PSVNDWGA Operator in handling approach for MADM issues

The MADM method is a critical tool widely employed to analyze data on the available options to help choose the best alternative (choice) from among several alternatives (options) presented. In this section, we work on presenting the importance of the tools provided in this work, i.e. (PSVNDWAA operator or the PSVNDWGA operator) in helping to solve one of the problems with PSVNN information. Where we formulating a hypothetical problem based on PSVNN data.

Let $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ and $\mathcal{Q} = \{\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_m\}$ be a discrete set of alternatives and a discrete set of attributes, respectively.Let the weight vector value of certain be given as following $w = (w_1, w_2, w_3, ..., w_n)$ where $w_1, w_2, w_3, ..., w_n \in [0, 1]$ such that $w_1 + w_2 + w_3 + ... + w_n = 1$. To facilitate the process of evaluating alternatives by decision makers, we can resort to creating a matrix $\widehat{D}\left(\tilde{\Theta}^{ij}_{\wp}\right)_{m\times n}$ containing values of a PSVNNs $\tilde{\Theta}^{ij}_{\wp} = \left\langle \left(\wp_{\Theta^{ij}} \widehat{T}_{\Theta^{ij}}\right), \left(\wp_{\Theta^{ij}} \widehat{I}_{\Theta^{ij}}\right), \left(\wp_{\Theta^{ij}} \widehat{F}_{\Theta^{ij}}\right)\right\rangle$ for i=1,2,3,...,n,j=1,2,3,...,m that represent evaluation about the alternative $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ under the attribute$\mathcal{Q} = \{\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_m\}$. Thus, in the following, we are developing a new algorithm that depends on PSVNDWAA operator or the PSVNDWGA operator for PSVNNs to manage a new approach for MADM issues with PSVNN data.

Algorithm

Step1. Compute the group PSVNN $\tilde{\Theta}^{i}_{\wp} = \left\langle \left(\wp_{\Theta^{i}} \widehat{T}_{\Theta^{i}}\right), \left(\wp_{\Theta^{i}} \widehat{I}_{\Theta^{i}}\right), \left(\wp_{\Theta^{i}} \widehat{F}_{\Theta^{i}}\right)\right\rangle$ for i=1,2,3,...,n for the certain alternative $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ by employing the following PSVNDWAA operator:

$$\tilde{\Theta}^{i}_{\wp} = PSVNDWAA\left(\tilde{\Theta}^{i1}_{\wp}, \tilde{\Theta}^{i2}_{\wp}, \tilde{\Theta}^{i3}_{\wp}, ..., \tilde{\Theta}^{in}_{\wp}\right) =$$

$$\left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{\left(\wp_{\Theta^{ij}} \widehat{T}_{\Theta^{ij}}\right)}{1-\left(\wp_{\Theta^{ij}} \widehat{T}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{1-\left(\wp_{\Theta^{ij}} \widehat{I}_{\Theta^{ij}}\right)}{\left(\wp_{\Theta^{ij}} \widehat{I}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}}, \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{1-\left(\wp_{\Theta^{ij}} \widehat{F}_{\Theta^{ij}}\right)}{\left(\wp_{\Theta^{ij}} \widehat{F}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

or by employing the following PSVNDWGA operator:

$$\tilde{\Theta}^{i}_{\wp} = PSVNDWGA\left(\tilde{\Theta}^{i1}_{\wp}, \tilde{\Theta}^{i2}_{\wp}, \tilde{\Theta}^{i3}_{\wp}, ..., \tilde{\Theta}^{in}_{\wp}\right) = \left\langle \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{1-\left(\wp_{\Theta^{ij}}\widehat{T}_{\Theta^{ij}}\right)}{\left(\wp_{\Theta^{ij}}\widehat{T}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{\left(\wp_{\Theta^{ij}}\widehat{I}_{\Theta^{ij}}\right)}{1-\left(\wp_{\Theta^{ij}}\widehat{I}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{n} w_j \left(\frac{\left(\wp_{\Theta^{ij}}\widehat{F}_{\Theta^{ij}}\right)}{1-\left(\wp_{\Theta^{ij}}\widehat{F}_{\Theta^{ij}}\right)}\right)^{\rho}\right\}^{1/\rho}} \right\rangle.$$

Here it is worth noting that $w = (w_1, w_2, w_3, ..., w_n)$ where $w_1, w_2, w_3, ..., w_n \in [0, 1]$ such that $w_1 + w_2 + w_3 + ... + w_n = 1$.

Step2. Compute the SF-values $\breve{E}\left(\tilde{\Theta}_{\wp}\right)$ and, if necessary AF-value $\breve{A}\left(\tilde{\Theta}_{\wp}\right)$ of all collective PSVNN $\tilde{\Theta}^{i}_{\wp} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle$ for i=1,2,3,...,n.

Step3. Rank the certain alternatives and pick the optimal one(s).

Step4. End proposed algorithm.

5.1 Illustrative Numerical Example

In the following, we will organize a MADM problem adapted from engineering field where a civil engineer is required to design a bridge crossing a river. This engineer has three possible designs: $\mathcal{A}_1$ =Suspension bridge, $\mathcal{A}_2$ =Arch bridge, and $\mathcal{A}_3$ =Cantilever bridge. Here, the investment company contracting for this project, in which this engineer works, must make the appropriate decision according to the following four attributes: (1)$\mathcal{Q}_1$ = Cost: (Total cost to build the bridge (in dollars))., (2)$\mathcal{Q}_2$ = Durability: (The expected life of the bridge (in years))., (3)$\mathcal{Q}_3$ = Aesthetic appearance and (4)$\mathcal{Q}_4$ = Aesthetic Construction time (The expected time period for building the bridge (in months)). Here, in this case, the suitability assessment of the alternative $\mathcal{A}_i (i = 1, 2, 3)$ is identical to the four features of $\mathcal{G}_j (j = 1, 2, 3, 4)$. Accordingly, a matrix $D\left(\tilde{\Theta}^{ij}_{\wp}\right)_{3\times4}$ can be created that contains the opinions of experts (decision makers) in the field of bridge construction, where these opinions are in the form of PSVNNs as following:

$$D\left(\tilde{\Theta}^{ij}_{\wp}\right)_{3\times4} = \begin{bmatrix} \langle(.3\,(.8)\,, .9\,(.6)\,, .2\,(.7))\rangle & \langle(.2\,(.5)\,, .6\,(.2)\,, .4\,(.2))\rangle & \langle(.3\,(.4)\,, .2\,(.6)\,, .1\,(.1))\rangle & \langle(.6\,(.4)\,, .9\,(.8)\,, .2\,(.8))\rangle \\ \langle(.7\,(.1)\,, .8\,(.6)\,, .3\,(.9))\rangle & \langle(.5\,(.5)\,, .3\,(.7)\,, .2\,(.4))\rangle & \langle(.3\,(.5)\,, .8\,(.2)\,, .6\,(.1))\rangle & \langle(.4\,(.9)\,, .4\,(.3)\,, .6\,(.7))\rangle \\ \langle(.6\,(.1)\,, .6\,(.7)\,, .5\,(.6))\rangle & \langle(.6\,(.3)\,, .5\,(.6)\,, .4\,(.3))\rangle & \langle(.2\,(.5)\,, .6\,(.2)\,, .4\,(.2))\rangle & \langle(.6\,(.4)\,, .9\,(.8)\,, .2\,(.8))\rangle \end{bmatrix}$$

With weight vector $w_j, j = 1, 2, 3, 4$ of the four features is given as $w = (.15, .30, .30, .25)$. Thus, we are using the formalism of the PSVNDWAA operator, as shown above, or using the formalism of the PSVNDWGA operator, as shown above, to pick up the MADM issue with PSVNN information.

Now we go to implement the above algorithm, specifically based on the PSVNDWAA operator can be outlined as follows:

Step1. Compute the group PSVNN $\tilde{\Theta}^i_{\wp} = \left\langle \left(\wp_{\Theta^i}\widehat{T}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{I}_{\Theta^i}\right), \left(\wp_{\Theta^i}\widehat{F}_{\Theta^i}\right) \right\rangle$ for i=1,2,3 for the certain alternative and for $\rho = 1$.

$\mathcal{A}_1 = (.1662, .1778, .0274)$,
$\mathcal{A}_2 = (.2336, .1754, .1532)$,
$\mathcal{A}_3 = (.2542, .2643, .0836)$

Step2. Compute the SF-values $\breve{E}\left(\tilde{\Theta}^j_{\wp}\right)$ j=1,2,3. Such that:
$\breve{E}\left(\tilde{\Theta}^1_{\wp}\right) = .6537$,
$\breve{E}\left(\tilde{\Theta}^2_{\wp}\right) = .6350$,
$\breve{E}\left(\tilde{\Theta}^3_{\wp}\right) = .6354$.

Step3. Based on the pick-up score values, we get the ranking order of the alternatives as following:
$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$ So the best selection is $\mathcal{A}_1$.

In order to demonstrate the extent of the impact of the alternatives on the change in the data resulting from the proposed tools $\rho \in [0, 1]$, we resort to changing the parameter values as shown in Tables 1 and 2.

Based on Tables 1 and 2, we can judge the MADM issues based on the PSVNDWAA and PSVNDWGA operators indicate the upper hand of its flexibility in verified applications that deal with real-life issues. Therefore, the developed MADM method provides a new effective way for decision-makers to handle PSVN-MADM problems.

Table 1. Ranking outcome of different prepared parameters of the PSVNN-Dombi weighted arithmetic average (PSVNDWAA) operator.

ρ	$\breve{E}\left(\tilde{\Theta}^1_{\wp}\right)$	$\breve{E}\left(\tilde{\Theta}^2_{\wp}\right)$	$\breve{E}\left(\tilde{\Theta}^3_{\wp}\right)$	Order of alternatives
$\rho = 1$	.6537	.6350	.6354	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 4$	.4103	.4048	.4037	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 7$	.4194	.4086	.4079	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 10$	.4274	.4183	.4175	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 12$	.4307	.4208	.4194	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 15$	.4572	.4297	.4274	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$

Table 2. Ranking outcome of different prepared parameters of the PSVNN-Dombi weighted geometric average (PSVNDWGA) operator.

ρ	$\breve{E}\left(\tilde{\Theta}^1_{\wp}\right)$	$\breve{E}\left(\tilde{\Theta}^2_{\wp}\right)$	$\breve{E}\left(\tilde{\Theta}^3_{\wp}\right)$	Order of alternatives
$\rho = 1$	.4873	.4264	.4358	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 4$	.5036	.4462	.4501	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 7$	.5262	.4732	.4866	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 10$	.5682	.5026	.5116	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 12$	.5835	.5241	.5371	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$
$\rho = 15$	.6027	.5532	.5684	$\mathcal{A}_1 \geq \mathcal{A}_3 \geq \mathcal{A}_2$

6. **Conclusion**

As an innovative idea and a new framework that differs from previous concepts that were presented in the literature, we presented in this work a new concept called PSVNS that depends on the product of each of the probability degrees between 0 and 1 and each of the degrees of truth, neutrality and untruth that are present in the mathematical structure of NS. According to this concept, we presented Dombi operations of PVNNs based on the Dombi T-norm and T-conorm operations, and then put forward the PSVNDWAA and PSVNDWGA operators, and study their important properties. as a new view of PSVNS-enviroments. We utilized both of PSVNDWAA operator and the PSVNDWGA operator with the score function (accuracy) to rank the alternatives and to act on the prime one(s) as stated by the score function values in the different number of operational parameters.In addition to the above, some properties related to PSVNDWAA operator and the PSVNDWGA operator the mechanism of operation of each were addressed and worked on proving them mathematically. Finally, we presented an example of a multi-attribute decision-making problem in the engineering field to choose the best bridge connecting the two banks of a river based on a number of criteria related to the human thinking mentality. As for future studies that can be built on this topic, these tools can be applied and linked with other vague concepts, as shown in the following research works [44]– [47].

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L. A. Fuzzy sets. Information and Control, 1965, 8(3): 338-353.
2. Atanassov,K. T. Intuitionistic fuzzy sets. Fuzzy Sets and Systems,1986, 20(1), 87-96.
3. Smarandache, F. Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis and synthetic analysis, 1998.
4. Wang, H.; Smarandache, F.; Zhang, Y.; & Sunderraman, R. Single valued neutrosophic sets. Infinite study,2010, 4 , 410-413.
5. Biswas, P.; Pramanik, S.; & Giri, B.C. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Comput & Applic,2016 27, 727-737.
6. Alqahtani, M.; Kaviyarasu, M.; Al-Masarwah, A.; & Rajeshwari, M. Application of Complex Neutrosophic Graphs in Hospital Infrastructure Design. Mathematics,2024, 12(5), 719.
7. Ding, J.; Bai, W.; & Zhang, C. A new multi-attribute decision making method with single-valued neutrosophic graphs. International Journal of Neutrosophic Science,2020, 11(2), 76–86.
8. Khalil, A. M.; Cao, D., Azzam, A.; Smarandache, F.; & Alharbi, W. R. Combination of the single-valued neutrosophic fuzzy set and the soft set with applications in decision-making. Symmetry,2020, 12(8), 1361.
9. Ismail, J. N.; Rodzi, Z.; Hashim, H.; Sulaiman, N. H.; Al-Sharqi, F.; Al-Quran, A., & Ahmad, A. G. Enhancing Decision Accuracy in DEMATEL using Bonferroni Mean Aggregation under Pythagorean Neutrosophic Environment. Journal of Fuzzy Extension & Applications (JFEA),2023, 4(4), 281–298.
10. Al-Sharqi, F.; Al-Quran, A.; Ahmad, A. G. Mapping on Interval Complex Neutrosophic Soft Sets. International Journal of Neutrosophic Science, 2022, 19(4), 77-85.
11. Ismail, J. N.; Rodzi, Z.; Al-Sharqi, F.; Hashim, H.; & Sulaiman, N. H. (2023). The Integrated Novel Framework: Linguistic Variables in Pythagorean Neutrosophic Set with DEMATEL for Enhanced Decision Support. International Journal of Neutrosophic Science (IJNS), 2023, 21(2), 129-141.

12. Al-Qudah, Y.; O., A.; Abdalla, N.; Al-Sharqi, F. A new generalization of interval-valued Q-neutrosophic soft matrix and its applications. International Journal of Neutrosophic Science,2025, 25(3), 242-257. https://doi.org/10.54216/IJNS.250322
13. Romdhini, M. U.; Al-Quran, A.; Al-Sharqi, F.; Tahat, M. K.; & Lutfi, A. Exploring the Algebraic Structures of Q-Complex Neutrosophic Soft Fields. International Journal of Neutrosophic Science,2023, 22(04), 93-105.
14. Alsharari, F.; Alohali, H.; Saber, Y.; & Smarandache, F. An Introduction to Single-Valued Neutrosophic Primal Theory. Symmetry,2024, 16(4), 402.
15. Hazaymeh,A. Time Effective Fuzzy Soft Set and Its Some Applications with and Without a Neutrosophic, International Journal of Neutrosophic Science, 2024, 23(2), 129–149.
16. Bataihah,A.; Hazaymeh, A. Quasi Contractions and Fixed Point Theorems in the Context of Neutrosophic Fuzzy Metric Spaces. European Journal of Pure and Applied Mathematics,2025, 18(1), 5785–5785.
17. Alqahtani, M.; Kaviyarasu, M.; Al-Masarwah, A.; and Rajeshwari,M. Application of Complex Neutrosophic Graphs in Hospital Infrastructure Design, Mathematics,2024, 12(5),719.
18. Hazaymeh, A.; Abdullah, I.B.; Balkhi, Z.; and Ibrahim, R. Fuzzy Parameterized Fuzzy Soft Expert Set. Applied Mathematical Sciences,2012, 6(12), 5547 - 5564.
19. Alsubie, A.; Al-Masarwah, A. MBJ-neutrosophic hyper BCK-ideals in hyper BCK-algebras, AIMS Mathematics,2021, 6(6), 6107-6121.
20. Hazaymeh, A.A.; Abdullah, I.B.; Balkhi, Z.T.; and Ibrahim, R.I. Generalized fuzzy soft expert set. Journal of Applied Mathematics,2012, 2012(1), p.328195.
21. Ganie, A. H.; Gheith, N. E. M.; Al-Qudah, Y.; Ganie, A. H.; Sharma, S. K.; Aqlan, A. M.; & Khalaf, M. M. An innovative Fermatean fuzzy distance metric with its application in classification and bidirectional approximate reasoning. IEEE Access,2024, 12, 4780-4791. https://doi.org/10.1109/ACCESS.2023.3348780
22. Al-Qudah, Y.; Jaradat, A.; Sharma, S. K.; & Bhat, V. K. Mathematical analysis of the structure of one-heptagonal carbon nanocone in terms of its basis and dimension. Physica Scripta, 2024, 99(5), 055252. https://doi.org/10.1088/1402-4896/ad3add
23. Alqudah, Y.; Alaroud, M.; Qoqazeh, H.; Alhazmi, S.E.; Alomari, S. Approximate AnalyticNumeric Fuzzy Solutions of Fuzzy Fractional Equations Using a Residual Power Series Approach. Symmetry, 2022, 14(4), 804 https://doi.org/10.3390/sym14040804
24. Al-Qudah Y. A robust framework for the decision-making based on single-valued neutrosophic fuzzy soft expert setting. International Journal of Neutrosophic Science, 2024,23(2):195-95.https://doi.org/10.54216/IJNS.230216
25. Alkhazaleh, S.; Salleh, AR.; Hassan ,N. Possibility fuzzy soft set. Advances in Decision Sciences. 2011 Sep 8;2011.
26. Garai, T.; Garg, H.; & Roy, T. K. A ranking method based on possibility mean for multi-attribute decision making with single valued neutrosophic numbers. Journal of Ambient Intelligence and Humanized Computing,2020, 11, 5245–5258.
27. Al-Sharqi, F.; Al-Quran, A.; Romdhini MU. Decision-making techniques based on similarity measures of possibility interval fuzzy soft environment. Iraqi Journal for Computer Science and Mathematics,2023, 4(4),18-29.
28. Al-Qudah, Y.; Al-Sharqi, F. Algorithm for decision-making based on similarity measures of possibility interval-valued neutrosophic soft setting settings. International Journal of Neutrosophic Science, 2023, 22(3), 69-83.
29. Zhao, J.; Li, B.; Rahman, A. U.; & Saeed, M. An intelligent multiple-criteria decision-making approach based on sv-neutrosophic hypersoft set with possibility degree setting for investment selection. Management Decision,2023, 61(2), 472-485.

30. Al-Hijjawi, S.; & Alkhazaleh, S. Possibility Neutrosophic Hypersoft Set (PNHSS). Neutrosophic Sets and Systems, vol. 53/2023: An International Journal in Information Science and Engineering,2023, 117.
31. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of intelligent & fuzzy systems, 2014, 26(5), 2459-2466.
32. Jana, C.,; & Pal, M. A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making. Symmetry, 2019, 11(1), 110.
33. Jana, C., & Pal, M. Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators. Soft Computing, 2021, 25, 5055-5072.
34. Chen, J.; & Ye, J. Some single-valued neutrosophic Dombi weighted aggregation operators for multiple attribute decision-making. Symmetry,2017, 9(6), 82.
35. Al-Quran, A.; Al-Sharqi, F.; Djaouti, AM. $\mathfrak{q}$-Rung simplified neutrosophic set: a generalization of intuitionistic, Pythagorean and Fermatean neutrosophic sets, AIMS Math.,2025, 10(4), 8615-8646.
36. Al-Quran A.; Al-Sharqi,F.; Rahman A. U.; and Rodzi, Z. M. The q-rung orthopair fuzzy-valued neutrosophic sets: Axiomatic properties, aggregation operators and applications. AIMS Mathematics,2024, 9(2), 5038–5070.
37. Riaz, M.; Farid, H. M. A.; Ashraf, S.; & Kamac, H. Single-valued neutrosophic fairly aggregation operators with multi-criteria decision-making. Computational and applied mathematics,2023, 42(3), 104.
38. Al-Qudah, Y.; Alhazaymeh, K.; Hassan, N.; Almousa, M.; Alaroud, M. Transitive Closure of Vague Soft Set Relations and its Operators. International Journal of Fuzzy Logic and Intelligent Systems, 2022, 22(1), pp. 59-68. https://doi.org/10.5391/IJFIS.2022.22.1.59
39. Riaz, M.; Farid, H. M. A.; Ashraf, S.; & Kamac, H. Single-valued neutrosophic fairly aggregation operators with multi-criteria decision-making. Computational and applied mathematics,2023, 42(3), 104.
40. Fathi H.; Myvizhi M.; Abdelhafeez A.; Abdellah MR.; Eassa M.; Sawah MS.; Elbehiery H. Single-Valued Neutrosophic Graph with Heptapartitioend Structure. Neutrosophic Sets and Systems. 2025, vol.80, 728-48.
41. Hazaymeh, A. A.; & Bataihah, A. Neutrosophic fuzzy metric spaces and fixed points for contractions of nonlinear type. Neutrosophic Sets and Systems,2025, 77, 96-112.
42. Al-Qudah, Y.; Al-Sharqi, F.; Mishlish, M.; & Rasheed, M. M. Hybrid integrated decision-making algorithm based on AO of possibility interval-valued neutrosophic soft settings. International Journal of Neutrosophic Science,2023, 22(3), 84-98. https://doi.org/10.54216/IJNS.220306
43. Romdhini, M. U.; & Nawawi, A. On the spectral radius and sombor energy of the non-commuting graph for dihedral groups. Malaysian Journal of Fundamental and Applied Sciences,2024, 20(1), 65-73.
44. Qoqazeh, H.; Al-Qudah, Y.; Almousa, M.; Jaradat, A. On D-Compact topological spaces. Journal of Applied Mathematics and Informatics, 2021, 39(5-6), pp. 883-894. https://doi.org/10.14317/jami.2021.883
45. Abed MM.; Al-Sharqi FG.; Mhassin AA. Study fractional ideals over some domains. InAIP conference Proceedings 2019 Aug 21 (Vol. 2138, No. 1, p. 030001). AIP Publishing LLC.
46. Elbehiery H.; Fathi H.; Eassa M.; Abdelhafeez A.; Abdellah MR.; Mahmoud H. Advanced Machine Learning Approaches for Breast Cancer Detection with Neutrosophic Sets. Neutrosophic Sets and Systems, 2025 Apr, Vol.81,273-84.
47. Romdhini, M. U.; Al-Sharqi, F.; Al-Obaidi, R. H.; & Rodzi, Z. M. Modeling uncertainties associated with decision-making algorithms based on similarity measures of possibility belief interval-valued fuzzy hypersoft setting. Neutrosophic Sets and Systems,2025, 77, 282-309.

Received: Nov. 27, 2024. Accepted: May 23, 2025