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A Neutrosophic Paradox-Based Assessment Framework with Over/Under/Off-Indeterminacy for Green Transformation in the Industrial Economy of Resource-Based Regions

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Abstract-Evaluating green transformation in resource-intensive industrial regions demands more than conventional logic or binary classifications. Projects in these environments often contain conflicting, incomplete, or uncertain information conditions inadequately handled by existing models. This paper introduces an advanced neutrosophic evaluation framework built upon a novel concept: Compensatory Indeterminacy Regulation. The framework incorporates over-, under-, and off indeterminacy conditions and provides mathematical mechanisms for their redistribution within a multi-project system. A new formulation for effective indeterminacy is derived, supported by rigorous equations and real-world-inspired scenarios. Additionally, we develop a generalized weighted model to reflect project interrelations, enabling context-sensitive evaluation. The proposed approach expands the analytical reach of neutrosophic logic, allowing it to model dynamic, uncertain, and multi-agent environments more faithfully.

Keywords: Neutrosophic Logic; Green Transformation; Paradox Modeling; Indeterminacy; Industrial Economy; Resource-Based Regions; Sustainability Evaluation; GPI; NEV.

1.Introduction

Industries rooted in resource extraction and high-impact manufacturing are now under increasing pressure to adopt sustainable practices. The transition to green technologies and operations in these sectors, however, is rarely straightforward. Projects meant to drive environmental improvement often introduce their own contradictions-such as clean energy plants relying on pollutive materials, or recycling processes with unclear health risks. Standard evaluation models fail to fully capture such complexity, especially when the available data is vague, disputed, or incomplete

Neutrosophic theory, with its triadic structure of truth (T), indeterminacy (I), and falsehood (F), offers a flexible basis for dealing with ambiguity. Yet, existing applications typically confine these values within a normalized range, discarding extreme values as anomalies. This limits their ability to reflect true system uncertainty [1-11].

In 2007, Florentin Smarandache extended the concept of the "uncertain set" which includes fuzzy sets and their various extensions such as intuitionistic fuzzy, neutrosophic, spherical fuzzy, and plithogenic sets by introducing three new types: OverSet, UnderSet, and OffSet. This expansion was motivated by real-life observations where traditional membership degrees, limited to the interval [0,1], were not sufficient to model certain situations accurately.

An OverSet allows for membership degrees greater than 1. For instance, an employee who works overtime and exceeds normal expectations could be assigned a membership degree higher than 1, as a way to reflect their extra contribution compared to a regular full-time employee whose membership degree is 1.

On the other hand, an UnderSet permits membership degrees that are less than 0. This is useful in situations where an entity has a negative impact. For example, an employee who causes more harm than benefit to a company could receive a degree of membership below 0, to indicate their negative contribution in contrast to a productive employee with a positive membership value.

The OffSet is a broader concept that includes both extremes components that are greater than 1 and others that are less than 0 thus allowing for degrees of membership that fall outside the standard [0,1] interval entirely.

This generalization was not limited to sets alone; it was further extended to other fields such as logic, probability, measures, and statistics. As a result, we now have Over-/Under-/Off- Logics, Probabilities, Measures, and Statistics, which provide more flexible and realistic tools for modeling uncertainty in complex systems [3-9].

2. Neutrosophic Mathematical Foundations

Let each project or industrial action x be evaluated using a triplet:

$$E(x) = (T_x, I_x, F_x)$$

Where:

$T_x \in [0,1]$: Degree of truth (contribution to green goals).

$F_x \in [0,1]$: Degree of falsehood (contradiction or harm).

$I_x \in [0,1]$: Degree of indeterminacy (uncertainty, conflict, or vagueness).

Assumption: T_x, F_x, I_x are independent; hence, $T_x + F_x + I_x$ can exceed 1 (paraconsistent logic).

Example 1: Battery Recycling Plant
Let a plant be evaluated as follows:

Benefits: Reduces landfill waste → $T = 0.8$

Harms: Emits heavy-metal byproducts → $F = 0.5$

Indeterminate: Inconclusive health impact data → $I = 0.6$

Thus:

$$E_{\text{Battery}} = (0.8, 0.6, 0.5)$$

3. Green Paradox Index (GPI)

To quantify paradox intensity:

$$\text{GPI}_x = \frac{T_x \cdot F_x \cdot I_x}{T_x + F_x + I_x + \epsilon}$$

Where:

ϵ is a small stabilizer to prevent division by zero, e.g., $\epsilon = 10^{-6}$.

Example 2: Hydrogen Production Using Fossil Fuels

$$T = 0.6, F = 0.4, I = 0.8$$
$$\text{GPI}_{\text{Hydrogen}} = \frac{0.6 \cdot 0.4 \cdot 0.8}{0.6 + 0.4 + 0.8 + 10^{-6}} \approx \frac{0.192}{1.8} \approx 0.1067$$

Interpretation: Moderate paradox exists. High indeterminacy prevents clear classification.

4. Neutrosophic Evaluation Vector (NEV)

We introduce:

$$\text{NEV}_x = (w_T \cdot T_x, w_I \cdot I_x, w_F \cdot F_x)$$

Where:

w_T, w_I, w_F are weights for truth, indeterminacy, and falsehood, respectively.

Example 3:
Let $w_T = 0.5, w_I = 0.3, w_F = 0.2$ and use values from the battery plant:

$$\text{NEV}_{\text{Battery}} = (0.5 \cdot 0.8, 0.3 \cdot 0.6, 0.2 \cdot 0.5) = (0.4, 0.18, 0.1)$$

This vector reflects a decision profile where benefits dominate but indeterminacy is not negligible.

5. Neutrosophic Paradox Decision Matrix (NPDM)

Construct a matrix for multiple projects:

$$\mathrm{NPDM} = \begin{bmatrix} T_1 & I_1 & F_1 & \mathrm{GPI}_1 \\ T_2 & I_2 & F_2 & \mathrm{GPI}_2 \\ \vdots & \vdots & \vdots & \vdots \\ T_n & I_n & F_n & \mathrm{GPI}_n \end{bmatrix}$$

This matrix allows clustering or principal component analysis to identify high-paradox sectors.

6. Proposed Novel Metric: Contradiction Sharpness Index (CSI)

Let:

$$\mathrm{CSI}_x = \frac{|T_x - F_x|}{I_x + \epsilon}$$

a. Measures how sharp the contradiction is relative to the uncertainty buffer.

b. High CSI = clearly opposing trends under weak uncertainty.
Example:

$$T = 0.9, F = 0.7, I = 0.1 \Rightarrow \mathrm{CSI} = \frac{|0.9 - 0.7|}{0.1} = 2.0$$

→ Clear paradox with insufficient uncertainty to resolve conflict.
7. § Proposed Framework Summary

Step-by-Step Model:

1. Data Collection: For each project, extract T, I, F via expert elicitation or data proxies.
2. Compute GPI and CSI for each project.
3. Build NPDM and rank projects.
4. NEV Vectorization: Normalize evaluations for comparative plotting.
5. Rule Decision: Flag projects with:

High GPI → paradox-rich projects.
High CSI → dangerous ambiguity.

8. Case Application: Resource-Based Region Transition Plan

This section shows how the proposed neutrosophic paradox framework can be used in real-world situations. We focus on green transformation projects in regions that rely heavily on natural resources, such as coal, oil, and mining. These areas often face challenges in shifting to cleaner industries, where projects may have both positive and negative effects at the same time.

To evaluate these projects, we use the neutrosophic values T, I, and F, which help us understand their benefits, drawbacks, and uncertainties. Based on these values, we calculate two key measures: the Green Paradox Index (GPI) and the Contradiction Sharpness Index (CSI). These indicators show how much contradiction and uncertainty each project contains.

Table 2: Neutrosophic Evaluation of Transition Projects in a Resource-Based Region

Project	T	I	F	GPI	CSI
Battery Recycling Plant	0.8	0.6	0.5	0.1067	0.5
Hydrogen from Methane Reforming	0.6	0.8	0.4	0.1067	0.25
CCS Facility (Coal Plant)	0.7	0.4	0.7	0.1377	0.0

9. Compensatory Neutrosophic Regulation of Over/Under/OffIndeterminacy

9.1. Motivation

In complex neutrosophic evaluations-such as those involved in green transformation projectsindeterminacy values I frequently fall outside the classical range [0,1]. This results in:

Over-indeterminacy: $I > 1$, caused by extreme uncertainty or contradictory inputs.

Under-indeterminacy: $I < 0$, which may result from biased, incomplete, or overly confident sources.

Off-indeterminacy: any instance where $I \notin [0,1]$.

Traditional models ignore or "clip" these values to force-fit them within [0,1], which loses important evaluative signal. Instead, we propose a Compensatory Neutrosophic Balance Model (CNBM) that redistributes the indeterminacy burden across multiple projects in a system based on their neutrosophic surplus or deficit.

9.2. Mathematical Formulation

Let there be n green transformation projects, where each project P_i is evaluated with the triplet:

$$E(P_i) = (T_i, I_i, F_i)$$

Let:

$T_i \in [0,1]$: the degree of truth or benefit

$F_i \in [0,1]$: the degree of falsehood or harm

$I_i \in \mathbb{R}$: the raw indeterminacy (possibly off-bound)

Step 1: Residual Indeterminacy Potential
We define the Residual Indeterminacy Potential for each project as:

$$R_i = 1 - T_i - F_i$$

This represents the "available" or "excess" space for indeterminacy if the sum of truth and falsehood is less than 1.

a. If $R_i > 0$, the project has capacity to absorb more indeterminacy.
b. If $R_i < 0$, the project is over-determined, implying conflicting or redundant information.

Step 2: Systemic Compensation Mechanism
To regulate abnormal I_i values, we introduce a compensated indeterminacy function:

$$I_i^{\text{eff}} = I_i + \lambda \cdot \sum_{j \neq i} R_j$$

Where:

$I_i^{\text{eff.}}$ the effective indeterminacy after system-wide compensation.

$\lambda \in \mathbb{R}$: a scaling factor (typically between 0 anc ↓ that controls how strongly external residuals affect

The summation aggregates the residual potential from all other projects in the system.

9.3. Application to Green Projects

To demonstrate the compensatory indeterminacy model in a real-world context, we apply it to three typical green transformation projects. The results, shown in Table 2, illustrate how over- and under-indeterminacy values are recalibrated through internal system compensation.

Table 1: Effective Indeterminacy Adjustment for Green Projects

Project	Description
A	Biomass Conversion Plant
B	Hydrogen Production (Steam Methane Reforming)
C	Battery Recycling Facility

Their neutrosophic evaluations are:

$$E(P_A) = (0.6, 1.2, 0.3), (\text{ Over-indeterminacy })$$
$$E(P_B) = (0.7, -0.3, 0.5), (\text{ Under-indeterminacy })$$
$$E(P_C) = (0.5, 0.0, 0.4), (\text{ Normal })$$

Step-by-step Calculation:
Step 1 - Compute Residuals R_i :

$$R_A = 1 - 0.6 - 0.3 = 0.1$$
$$R_B = 1 - 0.7 - 0.5 = -0.2$$
$$R_C = 1 - 0.5 - 0.4 = 0.1$$

Step 2 - Apply Compensation Rule:
Let $\lambda = 0.6$. We compute:

$$I_A^{\text{eff}} = 1.2 + 0.6 \cdot (R_B + R_C) = 1.2 + 0.6 \cdot (-0.2 + 0.1) = 1.2 + 0.6 \cdot (-0.1) = 1.2 - 0.06 = 1.14$$
$$I_B^{\text{eff}} = -0.3 + 0.6 \cdot (R_A + R_C) = -0.3 + 0.6 \cdot (0.1 + 0.1) = -0.3 + 0.6 \cdot 0.2 = -0.3 + 0.12 = -0.18$$
$$I_C^{\text{eff}} = 0.0 + 0.6 \cdot (R_A + R_B) = 0.0 + 0.6 \cdot (0.1 - 0.2) = 0.0 + 0.6 \cdot (-0.1) = -0.06$$

9.4. Clarification

Project A initially had a dangerously high indeterminacy ($I = 1.2$). The system redistributed some of the deficit from Project B, slightly reducing it to 1.14.

Project B started with negative indeterminacy. The surplus from Projects A and C helped raise its effective value to -0.18, softening its under-confidence.

Project C, though balanced, slightly shifted due to compensatory interactions.

This demonstrates that extreme indeterminacies are not simply errors; they can be absorbed, diluted, or reinforced by neighboring evaluations, echoing real-world dynamics in interrelated systems.

9.5. Generalization

This model can be generalized with weightings:

$$I_i^{\text{eff}} = I_i + \lambda \cdot \sum_{j \neq i} w_{ij} \cdot R_j$$

Where w_{ij} is a weight matrix encoding spatial distance, project similarity, priority, or stakeholder alignment.

This is the first mathematically defined model designed to regulate Over-, Under-, and Off-Indeterminacy through a systemic compensation approach. It introduces a non-linear, distributed correction mechanism that preserves the interpretive significance of extreme indeterminacy values rather than suppressing them. The model establishes a dynamic logical framework that is well-suited for handling fuzzy, conflicting, and evolving data conditions commonly found in complex systems such as industrial green transition planning.

10. Conclusion

This research presents a comprehensive extension to neutrosophic evaluation frameworks by introducing a mathematically defined method to handle extreme and off-range indeterminacy. Through the Compensatory Neutrosophic Balance Model, we have shown that indeterminacy is not merely a byproduct of poor data quality but a systemic signal that can be redistributed across interrelated projects. The model accounts for the shared uncertainty space in complex systems and introduces a logic structure capable of dynamically regulating it.

By preserving the full spectrum of indeterminacy including its exaggerated or negative forms we avoid information loss and gain deeper insight into the inconsistencies and interactions that shape green transition outcomes. The generalized weighted formulation allows the framework to reflect context, scale, and stakeholder dynamics. As green transformation efforts continue to grow in scale and complexity, models that embrace not erase uncertainty will be crucial in guiding sustainable, balanced, and context-aware decisions.

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