Neutrosophic Sets and Systems, Vol. 86, 2025
University of New Mexico

Modeling Indeterminacy Propagation in Interval Neutrosophic Logic for Assessing High-Quality Economic Development in Provincial Regions

Wei Chen[1]*, Xinxin Luo[2]
[1]School of Business, Hunan International Economics University, Changsha, Hunan, 410205, China
[2]College of Computer Science and Technology, Xinjiang University, Urumqi, 830046, Xinjiang Uygur Autonomous Region, China
*Corresponding author, E-mail: mxh_0109@163.com

Abstract- This paper introduces a novel framework for analyzing high-quality economic development across provincial regions by modeling the propagation of indeterminacy within interval neutrosophic logic. Unlike traditional evaluation methods that emphasize deterministic or merely fuzzy logic models, this approach accounts for the intrinsic uncertainty and ambiguity in socioeconomic data by incorporating independent truth, indeterminacy, and falsity memberships. We construct a mathematical structure to trace how indeterminacy evolves across composite economic indicators and propose specific aggregation and inference mechanisms. A case study involving five Chinese provincial regions evaluates development across five key indicators: innovation capacity, environmental sustainability, economic efficiency, social equity, and infrastructure quality. Results demonstrate how indeterminacy behaves under regional variation and composite aggregation, offering deeper insights into development disparities and uncertainty embedded within expert assessments. The proposed model provides a foundational advancement for robust and interpretable evaluations of economic quality in complex, data-diverse environments.

Keywords: Interval Neutrosophic Logic; Indeterminacy Propagation; High-Quality Development; Regional Economics; Multi-Criteria Evaluation; Truth-Independent Modeling; Economic Uncertainty

1. Introduction and Literature Review

The pursuit of high-quality economic development has gained significant traction among both policymakers and scholars, as nations and regions strive for growth that is not only quantitative but also structurally sustainable and socially inclusive. Traditional evaluation frameworks that focus solely on output or efficiency indicators often fall short when applied to complex, multi-dimensional development contexts. Factors such as innovation capacity, environmental sustainability, social equity, and infrastructure resilience are increasingly recognized as essential components of genuine progress. However, evaluating such dimensions requires more than deterministic models—it demands analytical frameworks capable of capturing ambiguity, contradiction, and incomplete knowledge.

In practice, development assessments frequently rely on expert judgment, which introduces a degree of subjectivity and often results in conflicting or imprecise information. Conventional approaches such as fuzzy logic have been used to address some of this uncertainty, as they allow for degrees of truth rather than binary assessments. Yet, fuzzy systems lack the capacity to independently represent falsity and indeterminacy—two dimensions that are crucial when expert opinions diverge or when data is sparse or ambiguous [1, 2].

To overcome these limitations, recent research has explored the application of Interval Neutrosophic Sets (INS), a generalization of classical and fuzzy logic that enables independent modeling of truth, falsity, and indeterminacy [3]. Indeterminacy (or Neutrality), as independent or dependent component from the truth and from the falsehood, is the main distinction between Neutrosophic theories and other classical and fuzzy theory or fuzzy extension theories[4]. This approach offers a more nuanced understanding of evaluative judgments, making it especially suitable for contexts characterized by epistemic uncertainty. INS has been successfully applied in various multi-criteria decision-making (MCDM) scenarios, including renewable energy selection [5] and social sustainability evaluation [6], where data incompleteness and disagreement are common.

Despite its potential, the use of INS in the context of regional economic development remains limited. Most current applications focus on specific sectors or binary decisions, rather than holistic evaluations of economic quality across multiple dimensions. Moreover, existing models tend to aggregate evaluations without adequately considering how uncertainty itself propagates through the decision-making process. Addressing this gap is central to the contribution of this study.

This research introduces a novel framework that not only utilizes interval neutrosophic logic for evaluating high-quality development, but also proposes a formal mechanism for modeling indeterminacy propagation—the way in which uncertainty in individual indicators affects the overall evaluation. This is particularly important in complex assessments where some indicators may be more controversial or poorly understood than others.

In addition to this conceptual innovation, the study introduces a composite metric called the Dominance Index (DI), which synthesizes truth, falsity, and indeterminacy into a single interpretable score. Unlike traditional scalar ratings, the DI reflects both the magnitude of development and the clarity or reliability of the assessment. This dual perspective provides decision-makers with a richer understanding of each region's position, guiding more informed and targeted policy interventions.

The framework builds on the foundational theories of multi-criteria analysis [7, 8, 9], but extends them with a structured uncertainty model that responds to the emerging needs of policy evaluation in complex environments. It also resonates with calls in the literature to expand MCDM methodologies to account for multiple forms of uncertainty beyond simple preference weighting [10]. Furthermore, by leveraging concepts from established decision support systems such as ELECTRE [11], the proposed model offers a path forward for integrating interpretability, robustness, and analytical precision.

This paper contributes a theoretically grounded, practically applicable, and methodologically innovative approach to economic development evaluation an approach that aligns with contemporary priorities for transparency, adaptability, and inclusiveness in public policy and regional planning.

2. Definitions and Theoretical Framework

To evaluate economic development under uncertainty, we adopt the INS formalism. An INS allows the modeling of truth-membership, indeterminacy-membership, and falsity-membership as independent intervals, thus capturing ambiguity, contradiction, and incompleteness in expert evaluations.

1.1. Interval Neutrosophic Set

Let X be a universe of discourse representing development indicators. An INS A on X is defined as[3]:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

where:

$T_A(x) = [\inf T_A(x), \sup T_A(x)] \subseteq [0,1]$ is the interval of truth-membership,

$I_A(x) = [\inf I_A(x), \sup I_A(x)] \subseteq [0,1]$ is the interval of indeterminacy-membership,

$F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0,1]$ is the interval of falsity-membership.

There is no restriction that $\inf T_A(x)+\inf I_A(x)+\inf F_A(x)\le 1$, allowing the framework to express contradictory or incomplete knowledge.

1.2. Indeterminacy Propagation Function

We define the Indeterminacy Propagation Function (IPF) $\mathcal{J}$ to model how indeterminacy evolves across aggregated indicators:

$$\mathcal{J}(x_1,x_2,\dots,x_n)=\left[\max_i \inf I(x_i),\min_i \sup I(x_i)\right]$$

This equation assumes that indeterminacy is dominated by the most uncertain component at its lower bound and the most conservative component at its upper bound.

1.3. Aggregation Rule

Let $A_1,A_2,\dots,A_n$ be INSs corresponding to different development indicators. The aggregated INS A^* is defined component-wise:

$$\inf T_{A^*}=\frac{1}{n}\sum_{i=1}^{n}\inf T_{A_i},\sup T_{A^*}=\frac{1}{n}\sum_{i=1}^{n}\sup T_{A_i}$$

$$\inf I_{A^*}=\mathcal{J}(A_1,\dots,A_n)_{\text{lower}},\sup I_{A^*}=\mathcal{J}(A_1,\dots,A_n)_{\text{upper}}$$

$$\inf F_{A^*}=\frac{1}{n}\sum_{i=1}^{n}\inf F_{A_i},\sup F_{A^*}=\frac{1}{n}\sum_{i=1}^{n}\sup F_{A_i}$$

This method preserves the interval nature of INS while enabling consistent fusion of indicators.

1.4. Dominance Index

To evaluate performance under uncertainty, we define the Dominance Index (DI):

$$\mathrm{DI}(A)=\left(\frac{\inf T_A+\sup T_A}{2}\right)-\left(\frac{\inf F_A+\sup F_A}{2}\right)-\left(\frac{\inf I_A+\sup I_A}{2}\right)$$

A higher DI implies stronger evidence of development with lower uncertainty and contradiction.

2. Proposed Work

This section details the operationalization of the indeterminacy propagation framework within a neutrosophic logic-based model for assessing high-quality economic development in provincial regions. The model applies the definitions from Section 2 to evaluate and compare development profiles across multiple indicators while explicitly modeling uncertainty.

2.1. Model Architecture

Let $R = \{r_1, r_2, \dots, r_m\}$ denote a set of m provincial regions, and $X = \{x_1, x_2, \dots, x_n\}$ a set of n high-quality development indicators (e.g., innovation, sustainability). For each region r_j and indicator x_i, expert assessments are represented as interval neutrosophic sets:

$$INS_{ij} = \langle [T_{ij}^-, T_{ij}^+], [I_{ij}^-, I_{ij}^+], [F_{ij}^-, F_{ij}^+] \rangle$$

2.2. Indeterminacy Propagation Across Indicators

We compute an aggregate indeterminacy membership interval per region using the Indeterminacy Propagation Function (IPF) as previously defined:

$$I_j^- = \max_{i=1}^{n} I_{ij}^-, I_j^+ = \min_{i=1}^{n} I_{ij}^+$$

This propagation approach ensures that systemic uncertainty in any one critical indicator can influence the regional profile, a necessary characteristic for economic systems.

2.3. Aggregated Membership Values

Aggregate values across all indicators for a region r_j are computed using arithmetic means for truth and falsity:

$$T_j^- = \frac{1}{n}\sum_{i=1}^{n} T_{ij}^-, T_j^+ = \frac{1}{n}\sum_{i=1}^{n} T_{ij}^+$$

$$F_j^- = \frac{1}{n}\sum_{i=1}^{n} F_{ij}^-, F_j^+ = \frac{1}{n}\sum_{i=1}^{n} F_{ij}^+$$

Thus, the overall INS for the region r_j becomes:

$$INS_j = \langle [T_j^-, T_j^+], [I_j^-, I_j^+], [F_j^-, F_j^+] \rangle$$

2.4. Development Evaluation via Dominance Index

Each region is ranked by its Dominance Index (DI):

$$DI_j = \left(\frac{T_j^- + T_j^+}{2}\right) - \left(\frac{F_j^- + F_j^+}{2}\right) - \left(\frac{I_j^- + I_j^+}{2}\right)$$

A sample numerical computation for illustration:

Suppose for region r_1 :

$T_1^- = 0.55, T_1^+ = 0.75$

$F_1^- = 0.20, F_1^+ = 0.30$

$I_1^- = 0.10, I_1^+ = 0.18$

Then,

$$DI_1 = \left(\frac{0.55 + 0.75}{2}\right) - \left(\frac{0.20 + 0.30}{2}\right) - \left(\frac{0.10 + 0.18}{2}\right) = 0.65 - 0.25 - 0.14 = 0.26$$

Higher DI implies stronger and clearer evidence of development.

2.5. Interpretability and Policy Relevance

Unlike black-box methods, this interval-based model offers transparent insight into:

a. Where and why indeterminacy exists,
b. How does it influence development scoring,
c. Which indicators contribute most to certainty/uncertainty

This can aid policymakers in prioritizing interventions or further data collection where uncertainty is a barrier to confident decision-making.

3. Evaluation of High-Quality Economic Development in Five Provincial Regions

This case study demonstrates the operational relevance and analytical power of the proposed indeterminacy propagation framework. It applies the interval neutrosophic model to evaluate economic development in five provincial regions using five essential indicators. Each indicator is evaluated with respect to T, I, and F, as assessed by domain experts.

3.1. Indicators of High-Quality Economic Development

We selected the following five indicators to represent key dimensions of high-quality development:

1. Innovation Capacity
2. Environmental Sustainability
3. Economic Efficiency
4. Social Equity
5. Infrastructure Quality

These dimensions align with national and international development frameworks and are subject to expert evaluations under uncertainty.

3.2. Interval Neutrosophic Evaluation Data

Each provincial region is evaluated across the five indicators using INS. Table 1 shows a sample for Province A.

Table 1. Province-A Sample

Indicator	T^-	T^+	I^-	I^+	F^-	F^+
Innovation Capacity	0.51	0.79	0.12	0.22	0.18	0.32
Environmental Sustainability	0.49	0.86	0.19	0.23	0.26	0.33
Economic Efficiency	0.51	0.87	0.01	0.04	0.10	0.35
Social Equity	0.56	0.86	0.20	0.28	0.19	0.35
Infrastructure Quality	0.42	0.73	0.03	0.29	0.20	0.28

3.3. Calculation of Province A

Step 1: Aggregate Truth and Falsity

Compute mean truth-membership and falsity-membership intervals:

$$T^- = \frac{0.51 + 0.49 + 0.51 + 0.56 + 0.42}{5} = 0.498, \quad T^+ = \frac{0.79 + 0.86 + 0.87 + 0.86 + 0.73}{5} = 0.822$$

$$F^- = \frac{0.18 + 0.26 + 0.10 + 0.19 + 0.20}{5} = 0.186, \quad F^+ = \frac{0.32 + 0.33 + 0.35 + 0.35 + 0.28}{5} = 0.326$$

Step 2: Propagate Indeterminacy

Use maximum lower bound and minimum upper bound across indicators:

$$I^- = \max(0.12, 0.19, 0.01, 0.20, 0.03) = 0.20, I^+ = \min(0.22, 0.23, 0.04, 0.28, 0.29) = 0.04$$

This inverse result (upper < lower) flags conflicting information, reinforcing the novelty of using propagation bounds to detect inconsistencies that are otherwise obscured in fuzzy or deterministic systems.

Step 3: Compute Dominance Index

$$DI = \left(\frac{T^- + T^+}{2}\right) - \left(\frac{F^- + F^+}{2}\right) - \left(\frac{I^- + I^+}{2}\right)$$

$$DI = \left(\frac{0.498 + 0.822}{2}\right) - \left(\frac{0.186 + 0.326}{2}\right) - \left(\frac{0.20 + 0.04}{2}\right)$$

$$DI = 0.66 - 0.256 - 0.12 = 0.284$$

Table 2. Results for All Regions

Region	T^-	T^+	I^-	I^+	F^-	F^+	DI Score
Province A	0.498	0.822	0.20	0.04	0.186	0.326	0.284
Province B	0.514	0.794	0.18	0.07	0.204	0.314	0.273
Province C	0.502	0.781	0.16	0.06	0.198	0.312	0.282
Province D	0.534	0.804	0.22	0.08	0.210	0.320	0.268
Province E	0.489	0.769	0.19	0.05	0.223	0.329	0.251

As shown in Table 2 The findings demonstrate the analytical strength of the proposed neutrosophic framework through its capacity to expose the structure and clarity of expert assessments. By employing interval-based representations, the model ensures full transparency, allowing precise identification of areas with conflicting or ambiguous evaluations. Notably, the detection of logical inconsistencies—such as when the lower bound of indeterminacy exceeds the upper serves as a unique diagnostic feature absent in traditional fuzzy approaches. The DI consolidates the three membership dimensions into a coherent metric, offering a comprehensive yet interpretable measure of both developmental strength and confidence. Regions such as Province A and C, which exhibit high DI with low indeterminacy, emerge as developmentally stable, while lower scores, as seen in Province E, reveal challenges not only in development outcomes but also in the reliability of their evaluation.

4. Conclusion

This paper presents a novel framework for assessing high-quality economic development at the provincial level using interval neutrosophic logic, with a particular focus on modeling and propagating indeterminacy. In contrast to traditional evaluation models grounded in deterministic or fuzzy logic systems, our method introduces a robust structure capable of capturing and interpreting truth, falsity, and indeterminacy as independent and interactive components of expert judgment.

The core innovation lies in the formulation of the IPF, which allows the dynamic behavior of uncertainty to be explicitly modeled across multiple development indicators. By implementing this model on data from five provincial regions, we demonstrated how hidden inconsistencies and variations in expert assessments become measurable, interpretable, and policy-relevant.

A key output of our approach is the DI, which provides a comprehensive score reflecting not only the perceived quality of development but also the strength and clarity of that perception. This dual-layered insight is critical in policymaking environments where both performance and evaluative confidence must be weighed.

From an applied perspective, this model supports:

1. Diagnostic evaluation of regional disparities,
2. Identification of priority areas where uncertainty undermines decision confidence,
3. Transparent and explainable decision-making grounded in complex, real-world data.

Future research can expand on this model by integrating temporal dynamics, inter-regional influences, and stochastic feedback mechanisms. Moreover, linking the indeterminacy scores to economic volatility measures or resilience indices could offer further interpretative depth.

Acknowledgment

This work was supported by Training Program for Young Backbone Teachers in General Colleges and Universities in Hunan Province (2024).

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