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Neutrosophic Under Soft Generalized Continuous Functions: Foundations and Applications in Recruitment Analysis

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Abstract. Empirical Correlation System is an important tool that expresses the linear interrelations between two variables. Its significance lies in describing the obvious link without explicitly declaring the causality existing between the involved sets. This new framework presents an elaborate study of basic definitions and operations associated with Neutrosophic Under Soft Sets. Furthermore, the paper introduces a new notion: a topological space together with the Neutrosophic Under Soft Sets($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-sets). Additionally, Neutrosophic Under Soft Generalized Continuous Functions are introduced to extend the theoretical framework. These inclusions are expected to enable a deeper understanding and broader applications in the mathematical environment. The research not only establishes the foundational aspects of these concepts but also investigates several properties and theorems related to them. This is complemented by a variety of numerical illustrations to elucidate and enhance the understanding of the topics. To demonstrate the practical relevance of these ideas, the paper utilizes the correlation framework to present a numerical example of this relation in application.

Keywords: Neutrosophic Under Soft Set and Neutrosophic Under Soft Topological Space.

1. Introduction

Uncertainty arises in the performance of daily activities. For instance, if one tosses a coin on an uneven table top or throws a die, then some uncertainties arise. In 1965, Zadeh founded fuzzy logic by developing fuzzy sets, which is a basic concept to manage uncertainty in several fields such as artificial intelligence and information theory [26]. In 1970, Bellman and Zadeh developed the idea by introducing fuzzy sets in decision-making under uncertainty and breaking through management science [3]. Two years later, in 1978, Zadeh, based on the fuzzy set, formulated the theory of possibility and thus proceeded with further development of tools for managing uncertainty and vagueness [27]. Intuitionistic fuzzy sets (IFS), introduced by Atanassov [1], extend fuzzy sets by incorporating both membership and non-membership functions with the condition that their sum does not exceed one.

Molodtsov introduced the soft set theory, which emerged in 1999 as a flexible approach for managing indeterminate data and was applied in various computational fields [16]. In 1999, Smarandache established neutrosophic logic that integrates degrees of truth, indeterminacy, and falsity, thus offering an all-encompassing framework to manage paradoxical or incomplete information [23].

Soft set theory was further developed in 2003, where Maji, Biswas, and Roy gave new insights of the applications in computational mathematics that strengthen its role in decision-making processes [14]. In 2012, Atanassov managed to extend the fuzzy theory by developing intuitionistic fuzzy sets, which entails membership and non-membership degrees that enhance the applicability of fuzzy sets in real-life problems [2].

Broumi's work in 2013 on generalized neutrosophic soft sets extended neutrosophic set applications to a broader range of computational problems which provide an allowance for more subtle analyses of uncertainty [4]. Mondal and Pramanik suggested a neutrosophic tangent similarity measure for application in multi-attribute decision-making in 2015, especially useful for rankings and selection [17]. In the same year, Broumi and Deli proposed correlation measures of neutrosophic refined sets by applying it to a medical diagnosis for better accuracy under uncertain conditions [5]. Furthermore, Pramanik and Mondal helped in medical diagnosis by using the weighted fuzzy similarity measure, which provides a novel mechanism to process medical diagnosis data [19].

Smarrandache discussed other varieties of neutrosophic set, which include over-set, under-set, and off-set in 2016. These provided neutrosophic theory with more depth both in its logical and probabilistic aspects [25]. Smarandache and Pramanik acted as editors for a volume on emerging trends in neutrosophic theory, capturing what are developing and being applied today [24].

Forward, in 2017, Dhavaseelan and Jafari studied generalized neutrosophic closed sets to raise new insights to neutrosophic topology and its applications in complex systems [11]. In 2019, the study of correlation measures in Pythagorean neutrosophic sets, especially with dependent neutrosophic components, was done by Jansi, Mohana, and Smarandache to enhance analytical tools for decision-making through fuzzy-based systems [12].

In 2020, Broumi and his co-author proposed single and multi-valued neutrosophic hypersoft sets, along with a tangent similarity measure that makes it easier to expand applications of neutrosophic to multifarious fields [22]. Radha, Mary, and Prema together with Broumi presented neutrosophic Pythagorean sets with dependent components in the year 2021 for enhancing correlation techniques toward better uncertainty analysis [20].

In the year 2023, Kumaravel et al. have worked on fuzzy cognitive maps and neutrosophic cognitive maps to determine dengue fever; for this, they used concepts of fuzziness and neutrosophy in healthcare [13]. Furthermore, Murugesan, Parthiban, et al., discussed the comparison of cognitive maps related to COVID variant analysis, reflecting the methodology of neutrosophic approach in pandemic research [18]. As a result, Rodrigo and Maheswari proposed neutrosophic gsα*-open and closed maps in the year 2023, which helped to investigate the neutrosophic topological spaces [21]. Majumder, Paul, and Pramanik developed a hyperbolic tangent similarity measure to detect environmental risks during the COVID-19 period contributing to public health applications of neutrosophic systems.

In 2023, Devi and Parthiban provided a correlation-based decision-making process over neutrosophic Pythagorean soft sets [6]. Continuing in 2024, they came up with the topic of decision making in neutrosophic over soft topological spaces to present techniques of decision-making under vague environments [7]. Another contribution from Devi and Parthiban was the propotion of plithogenic hypersoft set approach in school selection using TOPSIS method along with some novel applications of neutrosophic sets for education, which gave significant outcomes in school selection [8]. Another 2024 work by the same authors considered neutrosophic over supra-exterior modal topological structures while exploring healthcare decision-making and thus serves as an example of neutrosophic applications in critical decision domains [9]. The work by Devi, Sowmiya, and Parthiban [10] presents a novel approach to solving assignment problems using Pythagorean Octagonal Neutrosophic Fuzzy Numbers. It emphasizes the practical application of these numbers in optimization techniques.

This paper introduces innovative concepts, including the $\mathcal{N}\mathfrak{s}^{\mathfrak{u}}$-set and the $\mathcal{N}\mathfrak{s}^{\mathfrak{u}}$-topological space, which serve as a foundation for describing basic notions, operations, and theorems within the framework of Neutrosophic Under Soft Sets. Furthermore, the study introduces Neutrosophic Under Soft Generalized Continuous Functions, enhancing the theoretical understanding of these structures and their continuity properties. To demonstrate practical relevance, the study incorporates numerical examples derived from a survey conducted with five incharges at a school during a teacher recruitment process. These examples satisfy specific conditions related to selecting suitable candidates, providing insights into real-world decision-making scenarios. By integrating theoretical advancements, illustrative numerical examples, and practical applications, this manuscript makes a significant contribution to the field, offering a robust framework for further exploration and application in mathematical and practical environments.

2. **Preliminary**

This section presents the fundamental definitions for Neutrosophic Set (NS), Neutrosophic Under Set (NUS), Neutrosophic Over Soft Set ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set), and Neutrosophic Over Soft Topological Space ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space).

Definition 2.1. [23] Let $\mathcal{H}$ be a non-empty set, and let $\mathcal{J}$ be a Neutrosophic Set (NS). Then

$$\mathcal{J} = \langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}$$

where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, 1]$ and $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$. Here, $\aleph(\mathsf{h})$, $\eth(\mathsf{h})$, and $\Upsilon(\mathsf{h})$ represent the degree of truth membership, indeterminacy, and falsity, respectively.

Definition 2.2. [25] Let $\mathcal{J}$ be an NS in $\mathcal{H}$.If $\mathcal{J}$ is said to be an NUS in an non-empty set $\mathcal{H}$ then it has at-least one neutrosophic component is < 0 and no other components are > 1 is defined as,

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\}$$

Where $\aleph, \eth, \Upsilon : \mathcal{H} \to [\psi, 1]$, $0 \leq \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \leq 3$ and ψ is said to be under-limit of NUS

Note: $\rho(\mathcal{H})$ is a set of all the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$ subset of an non-empty set $\mathcal{H}$

Definition 2.3. [7] Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameter on $\mathcal{H}$.Then $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is defined by a set valued function

$$\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}} : \mathcal{E} \to \rho(\mathcal{H})$$

where $\rho(\mathcal{H})$ is an set of all $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set on $\mathcal{H}$.$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set is an valued function from the set of parameter $\mathcal{E}$ on $\mathcal{H}$ is defined as

$$\mathcal{J} = (\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E}) = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Definition 2.4. [7] A $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\odot = e, \langle \mathsf{h}, 0, 0, \Omega \rangle : \mathsf{h} \in \mathcal{H} : e \in \mathcal{E}$ is called a Null $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set, and $\circledast = e, \langle \mathsf{h}, \Omega, \Omega, 0 \rangle : \mathsf{h} \in \mathcal{H} : e \in \mathcal{E}$ is called a Universal $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set.

Definition 2.5. [7] Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}, \mathcal{E})$ be a two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set.If $\mathcal{J}$ is said to be a subset of $\mathcal{W}$ i.e.,$\mathcal{J} \subseteq \mathcal{W}$ then

$$\aleph_{\mathcal{J}}(\mathsf{h}) \leq \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}) \leq \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \geq \Upsilon_{\mathcal{W}}(\mathsf{h})$$

In other words $\mathcal{W}$ is an super set of $\mathcal{J}$

Note: Let $\mathcal{J} \subset \mathcal{W}$ and $\mathcal{W} \subset \mathcal{J}$ then $\mathcal{J} = \mathcal{W}$

Definition 2.6. [7] Let $\mathcal{J}$ and $\mathcal{W}$ be any $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-sets, and let $\forall \mathsf{h} \in \mathcal{H}$ and $\mathsf{e} \in \mathcal{E}$. Then the union, intersection, and complement are defined as follows:

(i) Union:

$$\mathcal{J} \mho \mathcal{W} = \{\mathsf{e}, \{\langle \mathsf{h}, \max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), \max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), \min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle\}\}$$

(ii) Intersection:

$$\mathcal{J} \Omega \mathcal{W} = \{\mathsf{e}, \{\langle \mathsf{h}, \min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), \min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), \max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle\}\}$$

(iii) Complement:

$$\mathcal{J}^{☾} = \{\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle\}\}$$

Definition 2.7. [7] A neutrosophic over soft topology ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topology) $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ on a non-empty set $\mathcal{H}$ satisfies the following conditions:

(i) $\odot, \circledast \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.
(ii) The union of any arbitrary collection of sets in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is also in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.
(iii) The finite intersection of sets in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is also in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$.

Then, $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ is called a neutrosophic over soft topological space ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space). An element of $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is called a neutrosophic over soft open set ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-open set), and the complement of any element in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$ is called a neutrosophic over soft closed set ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-closedset).

Definition 2.8. [7] For an operator on a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-set $\mathcal{J} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}$, the neutrosophic over soft topological closure and interior, denoted by $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J})$ and $int\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}(\mathcal{J})$, are defined as follows:

$$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}) = \Omega \{\mathcal{G} : \mathcal{G} \text{ is a } \mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}\text{-closed set in } \mathcal{H} \text{ and } \mathcal{J} \subseteq \mathcal{G}\}.$$

$$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}) = \mho \{\mathcal{O} : \mathcal{O} \text{ is a } \mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}\text{-open set in } \mathcal{H} \text{ and } \mathcal{J} \supseteq \mathcal{O}\}.$$

Note:

(i) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}^{☾}) = (int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}))^{☾}$
(ii) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}^{☾}) = (cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J}))^{☾}$

Proposition 2.9. *[7] Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}$-topological space and $\mathcal{J}$ is a subset of $\mathcal{H}$, then*

(i) *$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{R})$ is the smallest $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}} -$ closedset containing $\mathcal{R}$.*
(ii) *$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}}}(\mathcal{J})$ is the largest $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{o}} -$ openset contained in $\mathcal{J}$.*

3. Neutrosophic Under Soft Topological Space

Definition 3.1. Let $\mathcal{H}$ be an non-empty set and $\mathcal{E}$ be a set of parameter on $\mathcal{H}$.Then $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set is defined by a set valued function

$$\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} : \mathcal{E} \to \rho(\mathcal{H})$$

where $\rho(\mathcal{H})$ is an set of all $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set on $\mathcal{H}$.$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set is an valued function from the set of parameter $\mathcal{E}$ on $\mathcal{H}$ is defined as

$$\mathcal{J} = (\lambda_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}, \mathcal{E}) = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

Definition 3.2. Let $\mathcal{J} = (\mathcal{J}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}, \mathcal{E})$ and $\mathcal{W} = (\mathcal{W}_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}, \mathcal{E})$ be a two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set.If $\mathcal{J}$ is said to be a subset of $\mathcal{W}$ i.e.,$\mathcal{J} \subseteq \mathcal{W}$ then

$$\aleph_{\mathcal{J}}(\mathsf{h}) \leq \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}) \leq \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \geq \Upsilon_{\mathcal{W}}(\mathsf{h})$$

In other words $\mathcal{W}$ is an super set of $\mathcal{J}$

Definition 3.3. Let $\mathcal{J} \subset \mathcal{W}$ and $\mathcal{W} \subset \mathcal{J}$ then $\mathcal{J} = \mathcal{W}$

Definition 3.4. Let $\mathcal{J}$ and $\mathcal{W}$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set, Then the union,intersection and compliment are defined by

(i)$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(ii)$\mathcal{J} \Omega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

(iii)$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$

Proposition 3.5. *Let $\mathcal{J}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set on $\mathcal{H}$. Then*

(i). $\odot^{☾} = \circledast$

(ii). $\circledast^{☾} = \odot$

(iii). $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

Proof. 1. $\odot^{☾} = \circledast$

$$\odot = \{e, \{\langle \mathsf{h}, 0, 0, \psi\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$$

$$\odot^{☾} = \{\langle \mathsf{h}, \psi, \psi, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \circledast$$

$$\implies \odot^{☾} = \circledast$$

2. $\circledast^{☾} = \odot$

$$\circledast = \{\langle \mathsf{h}, \psi, \psi, 0\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\circledast^{☾} = \{\langle \mathsf{h}, \psi, \psi, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \odot$$

$$\implies \circledast^{☾} = \odot$$

3. $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J}^{☾})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \psi + (\psi + \eth_{\mathcal{J}}(\mathsf{h})), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} = \mathcal{J}$$

$$\implies (\mathcal{J}^{☾})^{☾} = \mathcal{J}$$

□

Proposition 3.6. *Let $\mathcal{J}$ and $\mathcal{W}$ be an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set on $\mathcal{H}$. Then*

(i). $\mathcal{J} \mho \mathcal{J} = \mathcal{J} \Omega \mathcal{J} = \mathcal{J}$

(ii). $\mathcal{J} \mho \mathcal{W} = \mathcal{W} \mho \mathcal{J}$

(iii). $\mathcal{J} \Omega \mathcal{W} = \mathcal{W} \Omega \mathcal{J}$

(iv). $\mathcal{J} \mho \odot = \mathcal{J}$ *and* $\mathcal{J} \mho \circledast = \circledast$

(v). $\mathcal{J} \Omega \odot = \odot$ *and* $\mathcal{J} \Omega \circledast = \mathcal{J}$

Proof. The proof is obvious from the definition. □

Theorem 3.7. *Let $\mathcal{J}$ and $\mathcal{W} \in \mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set. Then*

(i). $(\mathcal{J} \mho \mathcal{W})^{☾} = \mathcal{J}^{☾} \mho \mathcal{W}^{☾}$

(ii). $(\mathcal{J} \Omega \mathcal{W})^{☾} = \mathcal{J}^{☾} \Omega \mathcal{W}^{☾}$

Proof. (i).By the union definition,

$$\mathcal{J} \mho \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J} \mho \mathcal{W})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\psi + \eth_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (1)$$

By the definition of compliment

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \psi + \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{☾} \mho \mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\psi + \eth_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (2)$$

From (1) and (2) we get,

$$(\mathcal{J} \mho \mathcal{W})^{☾} = \mathcal{J}^{☾} \mho \mathcal{W}^{☾}$$

(ii). By the union definition we know that,

$$\mathcal{J} \Omega \mathcal{W} = \{(\mathsf{e}, \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathcal{J}\Omega\mathcal{W})^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\psi+\eth_{\mathcal{J}}(\mathsf{h}), \psi+\eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (3)$$

By the definition of compliment

$$\mathcal{J}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \psi + \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \psi + \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$$

$$\mathcal{J}^{☾}\Omega\mathcal{W}^{☾} = \{(\mathsf{e}, \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\psi+\eth_{\mathcal{J}}(\mathsf{h}), \Omega-\eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\} \quad (4)$$

From (3) and (4) we get,

$(\mathcal{J}\Omega\mathcal{W})^{☾} = \mathcal{J}^{☾}\Omega\mathcal{W}^{☾}$ □

Definition 3.8. Let $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ be a neutrosophic under soft topology($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topology) in $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{J}$ is a collection of subset of an non-empty set $\mathcal{H}$ such that

(i) $\odot, \circledast \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$.

(ii) The union of an arbitrary collection $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$.

(iii) The finite intersection of subsets $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ is in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$.

Then $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ is called neutrosophic under soft topological space($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space).An element of $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ is called an neutrosophic under soft open set($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-open set) and $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}^{☾}$ is named an neutrosophic under soft closed set($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set).

Theorem 3.9. *Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space on $\mathcal{H}$, then $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space in $\mathcal{H}$.*

Proof. Let $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ and $(\mathcal{H}, \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space in $\mathcal{H}$.

$\Longrightarrow$ $\odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ and $\odot, \circledast \in \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$

$\Longrightarrow$ $\odot, \circledast \in \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ $\therefore$ $(\mathcal{H}, \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\Omega\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space in $\mathcal{H}$. □

Remark 3.10. In the theorem 2.2 instead of the intersection operation if we use union operation the claim may not be true. It can be seen following example.

Example 3.11. Let $\mathcal{H} = \{\mathsf{r}_1, \mathsf{r}_2\}$ be the two mobile phone and $\mathcal{A} = \{batterydurability(\mathsf{q}_1), workingspeed(\mathsf{q}_2)\}$.

Then $(\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A}) \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}^{☾}$ such that,

$$\mathcal{R}_1(\mathsf{q}_1) = \{\langle \mathsf{r}_1, 0.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, 0.1, 0.3, 0.5\rangle\}$$

$$\mathcal{R}_1(\mathsf{q}_2) = \{\langle \mathsf{r}_1, 0.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, 0.2, 0.5, 0.6\rangle\}$$

$$(\mathcal{R}_1, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, -0.2, 0.4, 0.6\rangle, \langle \mathsf{r}_2, -0.1, 0.3, 0.5\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, -0.3, 0.5, 0.6\rangle, \langle \mathsf{r}_2, -0.2, 0.5, 0.6\rangle\}\}\}$$

$$\mathcal{R}_2(\mathsf{q}_1) = \{\langle \mathsf{r}_1, -0.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, -0.2, 0.3, 0.3\rangle\}$$

$$\mathcal{R}_2(\mathsf{q}_2) = \{\langle \mathsf{r}_1 - 0.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2 - 0.1, 0.6, 0.6\rangle\}$$

$(\mathcal{R}_2, \mathcal{A}) = \{\{\mathsf{q}_1 = \{\langle \mathsf{r}_1, -0.3, 0.4, 0.5\rangle, \langle \mathsf{r}_2, -0.2, 0.3, 0.3\rangle\}, \{\mathsf{q}_2 = \{\langle \mathsf{r}_1, -0.4, 0.5, 0.7\rangle, \langle \mathsf{r}_2, -0.1, 0.6, 0.6\rangle\}\}\}$

Then, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A})\}$ and $\tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} = \{\odot, \circledast, (\mathcal{R}_2, \mathcal{A})\}$ are two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space on $\mathcal{W}$.

But $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} = \{\odot, \circledast, (\mathcal{R}_1, \mathcal{A}), (\mathcal{R}_2, \mathcal{A})\}$.

Because $(\mathcal{R}_1, \mathcal{A}) \Omega (\mathcal{R}_2, \mathcal{A}) \notin \tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$.So, $\tau_{1\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}} \mho \tau_{2\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$ is not $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space on $\mathcal{H}$.

Definition 3.12. An operators of $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ $\mathcal{R} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}^{\complement}$, then neutrosophic under soft topological interior and closure are $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ and $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is defined as:

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) = \mho\{\mathcal{N} : \mathcal{N} \subseteq \mathcal{H} \text{and} \mathcal{N} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\}$ and

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\}$.

Proposition 3.13. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space and $\mathcal{R}$ is a subset of $\mathcal{H}$, then*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is the largest open set contained in $\mathcal{R}$.

(ii) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is the smallest NOS closed set containing $\mathcal{R}$.

Proof. (i) By the definition of interior, $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$.Let $\mathcal{N}$ be an open set such that $\mathcal{N} \subset \mathcal{R}$. $\because \mathcal{N}$ is open and $\mathcal{N} \subset \mathcal{R}$,then

$\mathcal{N} \subset int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is the largest open set contained in $\mathcal{R}$.

(ii) By the closure definition,

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \text{and} \mathcal{O}^{\complement} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}\}$

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is the smallest closed set containing $\mathcal{R}$. □

Theorem 3.14. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}^{\complement}$.Then,*

(i) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\odot) = \odot$ and $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\circledast) = \circledast$.

(ii) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-open set iff $\mathcal{Q} = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$.

(iv) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

(vi) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \mho int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \mho \mathcal{Q})$

(vii) $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \Omega int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

(iii) If $\mathcal{Q}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-open set in $\mathcal{H}$, then $\mathcal{Q}$ is itself a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-open set in $\mathcal{H}$ which contains $\mathcal{Q}$.

So,$\mathcal{Q}$ is the largest $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ contained in $\mathcal{Q}$

$\implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) = \mathcal{Q}$.

Conversely, suppose that $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) = \mathcal{Q}$.then $\mathcal{Q} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}$.

(iv) Let $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) = \mathcal{Q}$.

Then, $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) = \mathcal{Q}$ from (iii).

$\implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$

(v) Suppose that $\mathcal{R} \subseteq \mathcal{Q}$.As $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq \mathcal{R} \subseteq \mathcal{Q}.int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$ is a Neutrosophic under soft subset of $\mathcal{Q}$

From definition (3.2) we get, $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$.

(vi) It is clear that $\mathcal{R} \subseteq \mathcal{R} \mho \mathcal{Q}$ and $\mathcal{Q} \subseteq \mathcal{R} \mho \mathcal{Q}$.

Thus,

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \mho int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$ and

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \mho int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

$\implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \mho int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \mho \mathcal{Q})$ [By (v)].

(vii) Clearly w.k.t.

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}((R))$ and $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}((Q))$[By (v)].

So, that $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \Omega int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

Also,$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq \mathcal{R}$ and $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) \subseteq \mathcal{Q}$ we have

$int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \Omega int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) \subseteq \mathcal{R} \Omega \mathcal{Q}$.

$\implies int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) = int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \Omega int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$ □

Theorem 3.15. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space on $\mathcal{H}$.Let $\mathcal{R}$ and $\mathcal{Q}$ in $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}^{☾}$.Then,*

(i) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\odot) = \odot$ and $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\circledast) = \circledast$.

(ii) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \supseteq \mathcal{R}$.

(iii) $\mathcal{Q}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set iff $\mathcal{Q} = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$.

(iv) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})) = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$

(v) $\mathcal{R} \subseteq \mathcal{Q} \implies cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \subseteq cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

(vi) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \mho cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q}) = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \mho \mathcal{Q})$

(vii) $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R} \Omega \mathcal{Q}) \subseteq cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) \Omega cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$

Proof. (i) and (ii) are obviously true.

Proof of (vi) and (vii) similar to the Theorem 2.3 (vi) and (vii)

(iii) If $\mathcal{R}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set on $\mathcal{H}$ then $\mathcal{R}$ is itself a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set in $\mathcal{H}$ which contains $\mathcal{R}$.

$\therefore \mathcal{R}$ is a smallest $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set containing $\mathcal{R}$. and $\mathcal{R} = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$.

Conversely, Suppose that $\mathcal{R} = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$. As. $\mathcal{R}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set, so $\mathcal{R}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set in $\mathcal{H}$.

(vi) $\mathcal{R}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set then by the proof (iii)

$\mathcal{R} = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R})$..

(v) Suppose $\mathcal{R} \subseteq \mathcal{Q}$.Then every neutrosophic under soft closed super-set of $\mathcal{Q}$ also contained in $\mathcal{R}$.

$\implies$ super-sets of $\mathcal{Q}$ is also a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closed set. Thus,

$cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{R}) = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{Q})$. □

4. Neutrosophic Under Soft Generalized Continuous Functions

Definition 4.1. Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. A $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{J}$ is said to be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Generalized Closed Set ($\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set) if $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}) \subseteq \mathcal{G}$ whenever $\mathcal{J} \subseteq \mathcal{G}$ and $\mathcal{G}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset. The complement of a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set is called a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Generalized Open Set ($\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set).

Definition 4.2. Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. Then for any $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{J}$, $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ generalized topological interior($int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J})$) and closure($cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J})$)operators are defined as:

$int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) = \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \supseteq \mathcal{O}\}$ and

$cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) = \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \subseteq \mathcal{G}\}$.

Proposition 4.3. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. Let $\mathcal{J}$ and $\mathcal{W}$ be any two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$. Then the $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set satisfy the following properties:*

(i) $\mathcal{J} \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J})$

(ii) $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq \mathcal{J}$

(iii) $\mathcal{J} \subseteq \mathcal{W} \implies cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

(iv) $\mathcal{J} \subseteq \mathcal{W} \implies int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W})$

(v) $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J} \mho \mathcal{W}) = cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \mho cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

(vi) $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J} \Omega \mathcal{W}) = int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \Omega int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W})$

(vii)$(cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$

(viii) $(int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$

Proof. (i) $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) = \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \subseteq \mathcal{G}\}$

Thus$\mathcal{J} \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J})$

(ii) $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) = \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \supseteq \mathcal{O}\}$

Thus $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq \mathcal{J}$

(iii) $\mathcal{J} \subseteq \mathcal{W}$

$$\begin{aligned} cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W}) &= \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{W} \subseteq \mathcal{G}\} \\ &\supseteq \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \subseteq \mathcal{G}\} \\ &\supseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \end{aligned}$$

Thus $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

(iv) $\mathcal{J} \subseteq \mathcal{W}$

$$\begin{aligned} int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W}) &= \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{W} \supseteq \mathcal{O}\} \\ &\supseteq \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \supseteq \mathcal{O}\} \\ &\supseteq int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \end{aligned}$$

Thus $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W})$

(v) $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J} \mho \mathcal{W}) = \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad (\mathcal{J} \mho \mathcal{W}) \subseteq \mathcal{G}\}$

$= (\Omega\{\mathcal{G} : \mathcal{G} \quad is \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \subseteq \mathcal{G}\}) \mho$

$(\Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{W} \subseteq \mathcal{G}\})$

$= cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \mho cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

$\therefore cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J} \mho \mathcal{W}) = cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \mho cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

(vi) $int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J} \Omega \mathcal{W}) = \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad (\mathcal{J} \Omega \mathcal{W}) \supseteq \mathcal{O}\}$

$= (\mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \supseteq \mathcal{O}\}) \Omega$

$(\mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{W} \supseteq \mathcal{O}\})$

$= int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \Omega int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W})$

$\therefore int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J} \Omega \mathcal{W}) = int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) \Omega int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{W})$

(vii)$cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) = \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \subseteq \mathcal{G}\}$

$(cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = \mho\{\mathcal{G}^{\complement} : \mathcal{G}^{\complement} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \mathcal{H} \quad \text{and} \quad \mathcal{J}^{\complement} \supseteq \mathcal{G}^{\complement}\}$

$= \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J}^{\complement} \supseteq \mathcal{O}\}$

$= int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$

$\therefore (cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$

(viii)$int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}) = \mho\{\mathcal{O} : \mathcal{O} \quad is \quad \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J} \supseteq \mathcal{O}\}$

$(int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = \Omega\{\mathcal{O}^{\complement} : \mathcal{O}^{\complement} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J}^{\complement} \subseteq \mathcal{O}^{\complement}\}$

$= \Omega\{\mathcal{G} : \mathcal{G} \quad is \quad \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}} \quad in \quad \mathcal{H} \quad \text{and} \quad \mathcal{J}^{\complement} \subseteq \mathcal{G}\}$

$= cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$

$\therefore (int_{\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}}(\mathcal{J}))^{\complement} = cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}^{\complement})$ □

Proposition 4.4. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. If $\mathcal{W}$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ and $\mathcal{W} \subseteq \mathcal{J} \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$, then $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$.*

Proof. Let $\mathcal{L}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ such that $\mathcal{J} \subseteq \mathcal{L}$

Since $\mathcal{W} \subseteq \mathcal{J}$

$\mathcal{W} \subseteq \mathcal{L}$

Now, $\mathcal{W}$ is $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set and

$cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W}) \subseteq \mathcal{L}$

But $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W})$

Since $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{W}) \subseteq \mathcal{L}$

$\implies cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq \mathcal{L}$

Hence $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set □

Proposition 4.5. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space and $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}})$. Then $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set if and only if $\mathcal{W} \subseteq int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J})$,whenever $\mathcal{W}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closedset and $\mathcal{W} \subseteq \mathcal{J}$.*

Proof. The proof is obvious. □

Proposition 4.6. *If $int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}) \subseteq \mathcal{W} \subseteq \mathcal{J}$ and if $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set then $\mathcal{W}$ is also a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set.*

Proof. Now, $\mathcal{J}^{\complement} \subseteq \mathcal{W}^{\complement} \subseteq (int_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}))^{\complement} = cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}^{\complement})$

Since $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set, then $\mathcal{J}^{\complement}$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set

By proposition (3.6)

$$\mathcal{W}^{\complement} \text{ is a } \mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}\text{-set} \implies \mathcal{W} \text{ is a } \mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}\text{-set,}$$ □

Definition 4.7. Let $\mathcal{H}$ and $\mathcal{I}$ be any two nonempty sets, and let $\mathfrak{f} : \mathcal{H} \to \mathcal{I}$ be a function. The notions of image and preimage of a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set are defined as follows:

(i) If $\mathcal{K} = \{\langle \mathsf{i}, \aleph_{\mathcal{K}}(\mathsf{i}), \eth_{\mathcal{K}}(\mathsf{i}), \Upsilon_{\mathcal{K}}(\mathsf{i})\rangle : \mathsf{i} \in \mathcal{I}\}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{I}$, then the preimage of $\mathcal{K}$ under $\mathfrak{f}$, denoted by $\mathfrak{f}^{\rightarrow}(\mathcal{K})$, is the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{H}$ defined by

$$\mathfrak{f}^{\rightarrow}(\mathcal{K}) = \{(\mathsf{e}, \{\langle \mathsf{h}, \mathfrak{f}^{\rightarrow}(\aleph_{\mathcal{K}})(\mathsf{h}), \mathfrak{f}^{\rightarrow}(\eth_{\mathcal{K}})(\mathsf{h}), \mathfrak{f}^{\rightarrow}(\Upsilon_{\mathcal{K}})(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}.$$

(ii) If $\mathcal{J} = \{(\mathsf{e}, \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}) : \mathsf{e} \in \mathcal{E}\}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{H}$, then the image of $\mathcal{J}$ under $\mathfrak{f}$, denoted by $\mathfrak{f}(\mathcal{J})$, is the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{I}$ defined by

$$\mathfrak{f}(\mathcal{J}) = \{(\mathsf{e}, \{\langle \mathsf{i}, \mathfrak{f}(\aleph_{\mathcal{J}})(\mathsf{i}), \mathfrak{f}(\eth_{\mathcal{J}})(\mathsf{i}), (1 - \mathfrak{f}(1 - \Upsilon_{\mathcal{J}})(\mathsf{i}))\rangle : \mathsf{i} \in \mathcal{I}\}) : \mathsf{e} \in \mathcal{E}\}.$$

where,

$$\mathfrak{f}(\aleph_{\mathcal{J}})(\mathsf{i}) = \begin{cases} \sup_{\mathsf{h} \in \mathfrak{f}^{\rightarrow}(\mathsf{i})} \aleph_{\mathcal{J}}(\mathsf{h}), & \text{if } \mathfrak{f}^{\rightarrow}(\mathsf{i}) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mathfrak{f}(\eth_{\mathcal{J}})(\mathsf{i}) = \begin{cases} \sup_{\mathsf{h} \in \mathfrak{f}^{\rightarrow}(\mathsf{i})} \eth_{\mathcal{J}}(\mathsf{h}), & \text{if } \mathfrak{f}^{\rightarrow}(\mathsf{i}) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mathfrak{f}(\Upsilon_{\mathcal{J}})(\mathsf{i}) = \begin{cases} \inf_{\mathsf{h} \in \mathfrak{f}^{\rightarrow}(\mathsf{i})} \Upsilon_{\mathcal{J}}(\mathsf{h}), & \text{if } \mathfrak{f}^{\rightarrow}(\mathsf{i}) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

Corollary 4.8. *Let $\mathcal{J}_{\mathfrak{n}}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{H}(\forall \mathfrak{n} = 1, 2, \dots)$, $\mathcal{K}_{\mathfrak{m}}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $\mathcal{I}(\forall \mathfrak{m} = 1, 2, \dots)$ and $\mathfrak{f} : \mathcal{H} \to \mathcal{I}$ be a function. Then*

(i) $\mathcal{J}_1 \subseteq \mathcal{J}_2 \implies \mathfrak{f}(\mathcal{J}_1) \subseteq \mathfrak{f}(\mathcal{J}_2)$

(ii) $\mathcal{K}_1 \subseteq \mathcal{K}_2 \implies \mathfrak{f}^{\rightarrow}(\mathcal{K}_1) \subseteq \mathfrak{f}^{\rightarrow}(\mathcal{K}_2)$

(iii) $\mathcal{J} \subseteq \mathfrak{f}^{\rightarrow}(\mathfrak{f}(\mathcal{J}))$ {*if $\mathfrak{f}$ is injective, then $\mathcal{J} = \mathfrak{f}^{\rightarrow}(\mathfrak{f}(\mathcal{J}))$*}

(iv) $\mathfrak{f}(\mathfrak{f}^{\rightarrow}(\mathcal{K})) \subseteq \mathcal{K}$ {*if $\mathfrak{f}$ is surjective, then $\mathfrak{f}(\mathfrak{f}^{\rightarrow}(\mathcal{K})) = \mathcal{K}$*}

(v) $\mathfrak{f}^{\rightarrow}(\mho \mathcal{K}_{\mathfrak{m}}) = \mho \mathfrak{f}^{\rightarrow}(\mathcal{K}_{\mathfrak{m}})$

(vi) $\mathfrak{f}^{\rightarrow}(\Omega \mathcal{K}_{\mathfrak{m}}) = \Omega \mathfrak{f}^{\rightarrow}(\mathcal{K}_{\mathfrak{m}})$

(viii) $\mathfrak{f}(\mho \mathcal{J}_{\mathfrak{n}}) = \mho \mathfrak{f}(\mathcal{J}_{\mathfrak{n}})$

(viii) $\mathfrak{f}(\Omega \mathcal{J}_{\mathfrak{n}}) \subseteq \Omega \mathfrak{f}(\mathcal{J}_{\mathfrak{n}})\{$ *if* $\mathfrak{f}$ *is injective, then* $\mathfrak{f}(\Omega \mathcal{J}_{\mathfrak{n}}) = \Omega \mathfrak{f}(\mathcal{J}_{\mathfrak{n}})\}$

(ix) $\mathfrak{f}^{\rightarrow}(\circledast) = \circledast$

(x) $\mathfrak{f}^{\rightarrow}(\odot) = \odot$

(xi) $\mathfrak{f}(\circledast) = \circledast$, *if* $\mathfrak{f}$ *is surjective*

(xii) $\mathfrak{f}(\odot) = \odot$

(xiii) $(\mathfrak{f}(\mathcal{J}))^{\complement} \subseteq \mathfrak{f}(\mathcal{J}^{\complement})$, *if* $\mathfrak{f}$ *is surjective*

(xiv) $\mathfrak{f}(\mathcal{J}^{\complement}) = (\mathfrak{f}(\mathcal{J}))^{\complement}$

Definition 4.9. Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ and $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ be any two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological spaces. Let $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \rightarrow (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is a function.

(i) $\mathfrak{f}$ is said to be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Generalized Continuous Function ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function) if the inverse image of every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closedset in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$

Similarly if the inverse image of every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$

(ii) $\mathfrak{f}$ is said to be a Strongly $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Continuous Function (strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function)if $\mathfrak{f}^{\rightarrow}(\mathcal{J})$ is both $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset and $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closedset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ for each $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$

(iii) $\mathfrak{f}$ is said to be a Strongly $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Generalized Continuous Function (strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function) if the inverse image of every $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$

Proposition 4.10. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ and $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ be any two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. Let $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \rightarrow (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is said to be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function. Then for every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{J}$ in $\mathcal{H}$,$\mathfrak{f}(cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J})) \subseteq cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J}))$*

Proof. Let $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$.

Since $cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J}))$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closedset and $\mathfrak{f}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function

$\implies \mathfrak{f}^{\rightarrow}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J})))$ is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set and $\mathfrak{f}^{\rightarrow}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J}))) \supseteq \mathcal{J}$

Now, $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J}) \subseteq \mathfrak{f}^{\rightarrow}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J})))$

$\therefore \mathfrak{f}(cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathcal{J})) \subseteq cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathcal{J}))$ □

Proposition 4.11. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ and $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ be any two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. Let $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \rightarrow (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is said to be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function. Then for every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{J}$ in $\mathcal{I}$,$cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathfrak{f}^{\rightarrow}(\mathcal{J})) \subseteq \mathfrak{f}^{\rightarrow}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}))$*

Proof. Let $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$.Let $\mathcal{K} = \mathfrak{f}^{\rightarrow}(\mathcal{J})$ then

$\mathfrak{f}(\mathcal{K}) = \mathfrak{f}(\mathfrak{f}^{\rightarrow}(\mathcal{J})) \subseteq \mathcal{J}$

By the proposition (4.10),

$\mathfrak{f}(cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathfrak{f}^{\rightarrow}(\mathcal{J}))) \subseteq cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathfrak{f}(\mathfrak{f}^{\rightarrow}(\mathcal{J})))$

Thus, $cl_{\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}}(\mathfrak{f}^{\rightarrow}(\mathcal{J})) \subseteq \mathfrak{f}^{\rightarrow}(cl_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}}(\mathcal{J}))$ □

Proposition 4.12. *Let $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ and $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ be any two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological space. Let $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \to (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is said to be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Continuous function($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function) then it is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function.*

Proof. Let $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$.

Since $\mathfrak{f}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function, $\mathfrak{f}^{\to}(\mathcal{J})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$.

Every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset is a $\mathcal{N}_{\mathfrak{sc}}^{\mathfrak{ug}}$-set.

Now, $\mathfrak{f}^{\to}(\mathcal{J})$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$.

Hence $\mathfrak{f}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function □

Proposition 4.13. *The converse of the proposition is not necessarily true. That is, if $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \to (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function, it does not imply that $\mathfrak{f}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$ Continuous function ($\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function).*

Proof. It is proved by the help of the example (4.14). □

Example 4.14. Let $\mathcal{H} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$, $\psi = -0.1$ and $\mathcal{E} = \mathsf{e}$. Let $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-sets $\mathcal{J}$ and $\mathcal{K}$ in $\mathcal{H}$ as follows:

$$\mathcal{J} = \{\langle \mathfrak{a}, -0.4, 0.7, 0.3\rangle, \langle \mathfrak{b}, 0.6, 0.9, -0.1\rangle, \langle \mathfrak{c}, -0.3, 0.9, 0.5\rangle\}$$

$$\mathcal{K} = \{\langle \mathfrak{a}, -0.4, 0.7, 0.2\rangle, \langle \mathfrak{b}, 0.7, 0.9, -0.1\rangle, \langle \mathfrak{c}, -0.2, 0.7, 0.4\rangle\}$$

Then two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topologies $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}} = \{\odot, \circledast, \mathcal{J}\}$ and $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}} = \{\odot, \circledast, \mathcal{K}\}$. Thus $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ and $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ are two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological spaces.

Define $\mathfrak{f}_1 : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \to (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ as $\mathfrak{f}_1(\mathfrak{a}) = \mathfrak{b}$,$\mathfrak{f}(\mathfrak{b}) = \mathfrak{a}$ and $\mathfrak{f}(\mathfrak{c}) = \mathfrak{c}$.

$$\mathfrak{f}_1^{-1}(\mathcal{K}) = \{\mathsf{e}, \{\langle \mathfrak{a}, 0.7, 0.9, -0.1\rangle, \langle \mathfrak{b}, -0.4, 0.7, 0.2\rangle, \langle \mathfrak{c}, -0.2, 0.7, 0.4\rangle\} : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathfrak{f}_1^{-1}(\mathcal{K}))^{☾} = \{\mathsf{e}, \{\langle \mathfrak{a}, -0.1, 0.8, 0.7\rangle, \langle \mathfrak{b}, 0.2, 0.6, -0.4\rangle, \langle \mathfrak{c}, 0.4, 0.7, -0.3\rangle\} : \mathsf{e} \in \mathcal{E}\}$$

$$(\mathfrak{f}_1^{-1}(\mathcal{K}))^{☾} \subseteq \mathcal{G}$$

$$\text{where,} \mathcal{G} = \{\circledast, \mathcal{J}\}$$

$$(\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})^{☾} = \{\odot, \circledast, (\mathcal{J})^{☾}\}$$

$$(\mathcal{J})^{☾} = \{\mathsf{e}, \{\langle \mathfrak{a}, 0.3, 0.6, -0.4\rangle, \langle \mathfrak{b}, -0.1, 0.8, 0.6\rangle, \langle \mathfrak{c}, 0.5, 0.8, -0.3\rangle\} : \mathsf{e} \in \mathcal{E}\}$$

$$cl_{\tau^{\mathfrak{u}}_{\mathcal{N}_{\mathfrak{s}}}}(\mathfrak{f}^{\to}(\mathcal{K})^{☾}) = \{\circledast\} \subseteq \mathcal{G}$$

$$\Longrightarrow cl_{\tau^{\mathfrak{u}}_{\mathcal{N}_{\mathfrak{s}}}}(\mathfrak{f}^{\to}(\mathcal{K})^{☾}) \subseteq \mathcal{G}$$

Then $\mathfrak{f}_1$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function.

$\therefore \mathfrak{f}_1^{-1}(\mathcal{K})$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset

But $\mathfrak{f}_1^{-1}(\mathcal{K})$ is not $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ $\forall \mathcal{K} \in \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}}$

$\Longrightarrow \mathfrak{f}_1$ is not a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function.

Proposition 4.15. *Let* $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ *and* $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ *be any two* $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$*-topological space. Let* $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \to (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ *is a Strongly Neutrosophic Under Soft Generalized Continuous function (strongly-*$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$*-function) then* $\mathfrak{f}$ *is a* $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$*-function.*

Proof. Let $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$. Every $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set

Now, $\mathcal{J}$ is a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$

Since $\mathfrak{f}$ is strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$, $\mathfrak{f}^{\rightarrow}(\mathcal{J})$ is $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$

Hence, $\mathfrak{f}$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function. □

Remark 4.16. Converse of proposition 4.15 need not be true(shown in example 4.17).

Example 4.17. Let $\mathcal{H} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ and $\mathcal{E} = \mathsf{e}$. Let $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set $\mathcal{K}$ and $\mathcal{L}$ in $\mathcal{H}$ as follows:

$$\mathcal{K} = \{\mathsf{e}, \{\langle \mathfrak{a}, -0.4, 0.7, 0.2\rangle, \langle \mathfrak{b}, -0.3, 0.7, 0.4\rangle, \langle \mathfrak{c}, 0.7, 0.9, -0.1\rangle\}$$

$$\mathcal{L} = \{\mathsf{e}, \{\langle \mathfrak{a}, -0.4, 0.7, 0.2\rangle, \langle \mathfrak{b}, 0.7, 0.9, -0.1\rangle, \langle \mathfrak{c}, -0.3, 0.7, 0.4\rangle\}$$

Then two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topologies $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}3}} = \{\odot, \circledast, \mathcal{K}\}$ and $\tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}4}} = \{\odot, \circledast, \mathcal{L}\}$. Thus $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}3}})$ and $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}4}})$ are two $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-topological spaces.

Define $\mathfrak{f}_2 : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}3}}) \to (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}4}})$ as $\mathfrak{f}_2(\mathfrak{a}) = \mathfrak{a}$,$\mathfrak{f}_2(\mathfrak{b}) = \mathfrak{c}$ and $\mathfrak{f}_2(\mathfrak{c}) = \mathfrak{b}$.

Then $\mathfrak{f}_2$ is a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function.

Let $\mathcal{M} = \{\mathsf{e}, \{\langle \mathfrak{a}, -0.4, 0.8, 0.1\rangle, \langle \mathfrak{b}, 0.8, 0.9, -0.1\rangle, \langle \mathfrak{c}, -0.3, 0.7, 0.4\rangle\}$ be a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}4}})$.

But $\mathfrak{f}_2^{\rightarrow}(M)$ is not an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}3}})$.

$\implies$ $\mathfrak{f}_2$ is not a strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function.

Proposition 4.18. *Let* $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$ *and* $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ *be two* $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$*-topological spaces. If* $\mathfrak{f} : (\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}}) \to (\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$ *is a strongly-*$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$*-function, then* $\mathfrak{f}$ *is a strongly-*$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$*-function.*

Proof. Let $\mathcal{J}$ be a $\mathcal{N}_{\mathfrak{so}}^{\mathfrak{ug}}$-set in $(\mathcal{I}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}2}})$.

Since $\mathfrak{f}$ is a strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}\mathcal{C}}$-function, it follows that $\mathfrak{f}^{\rightarrow}(\mathcal{J})$ is both an $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-openset and a $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-closedset in $(\mathcal{H}, \tau_{\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}1}})$.

Hence, $\mathfrak{f}$ is a strongly-$\mathcal{N}_{\mathfrak{s}}^{\mathfrak{ug}\mathcal{C}}$-function. □

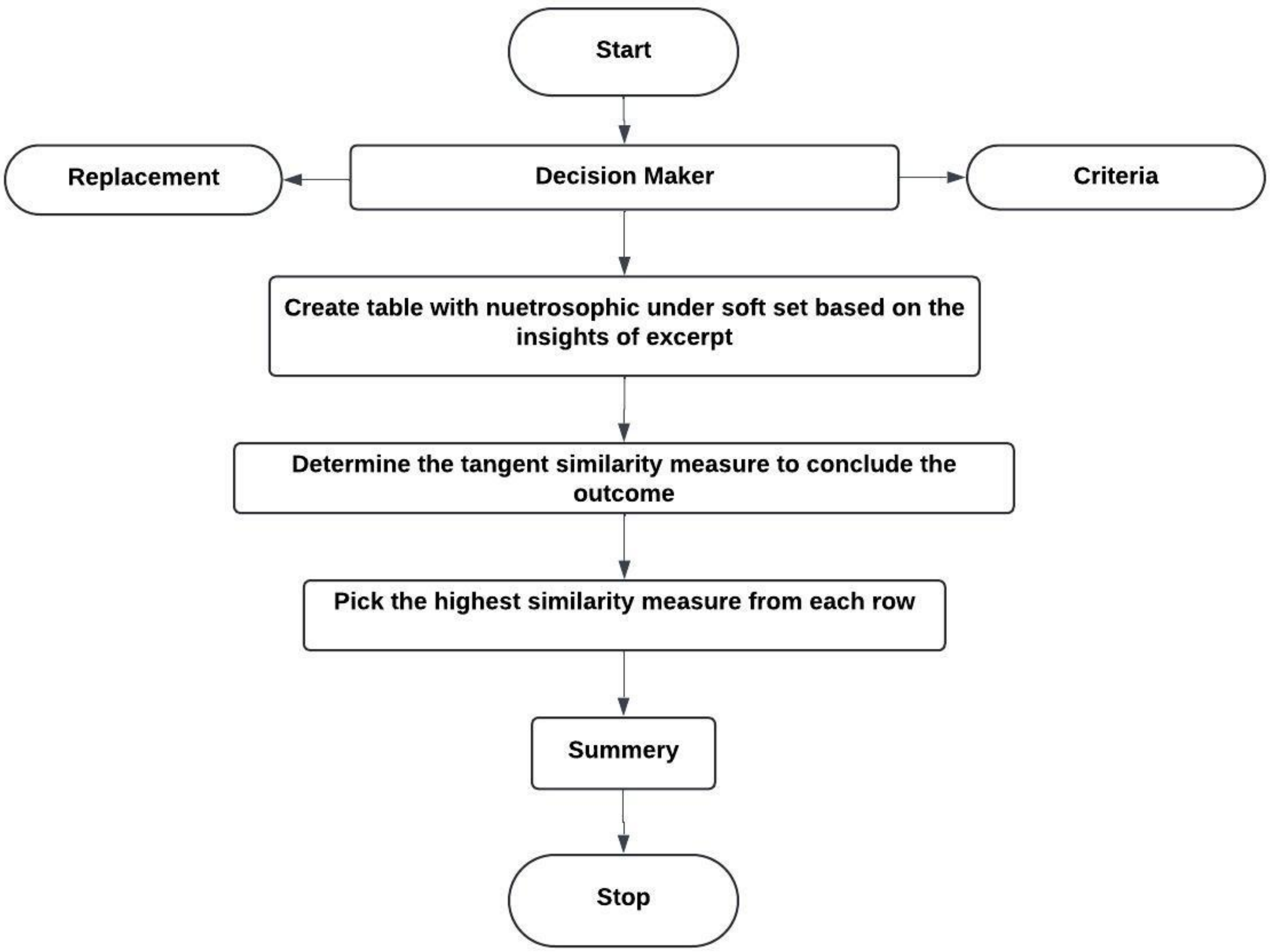
5. Similarity measure for Neutrosophic Under Soft Set

Consider neutrosophic under soft sets $\mathcal{Q}i$ and $\mathcal{S}i$ in $\mathcal{H}$. To establish a similarity metric that assesses the likeness between these sets based exclusively on their alignment, without taking into account the separation between them. This similarity measure can be articulated as follows:

Tangent Similarity Measure:

$$\rho(\mathcal{Q}i, \mathcal{S}i) = \frac{1}{n}\left(\sum_{i=1}^{n} 1 - tan\left[\frac{\pi\left[|\aleph_{\mathcal{Q}i}(\mathsf{h}) - \aleph_{\mathcal{S}i}(\mathsf{h})| + |\eth_{\mathcal{Q}i}(\mathsf{h}) - \eth_{\mathcal{S}i}(\mathsf{h})| + |\Upsilon_{\mathcal{Q}i}(\mathsf{h}) - \Upsilon_{\mathcal{S}i}(\mathsf{h})|\right]}{12}\right]\right) \tag{5}$$

6. Flow Chart To Solving $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set Using Tangent Similarity Measure



7. Numerical Illustration

Modern teaching skills integrate a variety of instructional approaches that collectively elevate student engagement and adapt to individual learning needs. Inquiry-based learning, which emphasizes exploration and critical thinking, requires teachers to guide students through open-ended questioning and foster a sense of curiosity. The flipped classroom model reimagines traditional learning by shifting content review to students' home environments, allowing classroom time to focus on applying concepts through active, hands-on activities. This approach calls for strong instructional design, as well as effective classroom management to support in-depth, interactive learning. Collaborative learning emphasizes teamwork, with teachers facilitating group problem-solving and constructive dialogue, which requires interpersonal skills and the ability to create activities that drive shared knowledge-building. Teacher-centered methods focus on delivering content directly, where clarity in communication and structured organization of information are paramount for fostering comprehension. Personalized learning, meanwhile, adapts to individual students' interests, pace, and abilities, demanding flexibility, ongoing assessment, and an understanding of varied learning styles. By developing these diverse skills, educators can create dynamic, supportive, and inclusive environments that inspire and accommodate all students.

In the context of our discussion, teacher identification problem serves as a practical demonstration of the aforementioned strategy's efficiency and relevance. This illustrative example demonstrates a systematic and comprehensive approach to identifying a teacher based on various teaching methods, with the assistance of educational professionals. Marks are assigned based on their performance, allowing for both positive and negative scoring.

For example,

Positive and Negative Marks for True, Indeterminacy, and Falsity Membership in Flipped Classroom

Positive True Membership:

- Students experience active and immersive participation in their own learning process.
- Facilitates improved classroom interaction and collaboration due to prior preparation by students.
- Promotes self-reliance and responsibility for learning, aligning with native cognitive habits.

Negative True Membership:

- Some students face difficulty adapting to the flipped model due to ingrained habits of passive learning.
- Requires significant effort and time for teachers to prepare pre-class materials, which may not suit native teaching practices.
- Overemphasis on individual preparation may lead to unequal outcomes if students lack self-discipline.

Positive Indeterminacy:

- Some students may be uncertain about how to approach pre-class materials or lack guidance.
- The effectiveness of the flipped model can vary depending on cultural or regional learning norms.

- Mixed results may occur if native teaching methods do not align with flipped classroom principles.

Negative Indeterminacy:

- Students' engagement with pre-class materials is uncertain and cannot be reliably measured.
- Teachers might be unsure of whether in-class discussions effectively address all students' doubts.
- Mixed success when integrating the flipped model with native classroom dynamics (e.g., teacher-led methods).

Positive Falsity Membership:

- Resistance to the flipped model in regions where traditional teacher-centric methods are deeply rooted.
- Students accustomed to passive learning may struggle to adapt to active self-learning.
- Lack of access to resources or technology for completing pre-class activities can hinder progress.

Negative Falsity Membership:

- Students may entirely skip pre-class preparation, resulting in ineffective or failed classroom sessions.
- Heavy reliance on technology could alienate students in areas with limited digital access or infrastructure.
- The flipped model can cause frustration or demotivation among students unfamiliar with self-directed learning.

Step 1:Collection of data

The data gathered through consultations with educational professionals, encompassing four categories of teaching types -Flipped Classroom($\mathfrak{U}_1$), Collaborative Learning($\mathfrak{U}_2$), Teacher Centered Method($\mathfrak{U}_3$), Personalized Learning ($\mathfrak{U}_4$),Inquiry Based Learning($\mathfrak{U}_5$) - known for replacement to finding teachers for an interview among three teachers where selected or not selected has been systematically organized into a table.

Let three teachers as $\mathfrak{T}_1$, $\mathfrak{T}_2$, $\mathfrak{T}_3$ and factors as Selected, Not selected .

TABLE 1. Teachers with replacement

Teachers\Replacement	$\mathfrak{U}_1$	$\mathfrak{U}_2$	$\mathfrak{U}_3$	$\mathfrak{U}_4$	$\mathfrak{U}_5$
$\mathfrak{T}_1$	(0.8,0.3,-0.1)	(-0.5,0.4,0.1)	(0.6,0.4,-0.1)	(0.6,0.2,-0.2)	(-0.2,0.3,0.2)
$\mathfrak{T}_2$	(0.7,0.3,-0.2)	(-0.1,0.4,0.0)	(0.7,-0.3,-0.2)	(0.8,0.2,-0.1)	(-0.2,0.4,0.1)
$\mathfrak{T}_3$	(-0.3,0.2,0.2)	(0.9,-0.1,-0.1)	(0.9,-0.1,-0.2)	(0.5,0.4,-0.2)	(-0.3,0.4,0.1)

TABLE 2. Replacement with Factor

Replacement\Factors	Selected	Not selected
$\mathfrak{U}_1$	(-0.1,0.1,0.3)	(0.5,0.3,-0.3)
$\mathfrak{U}_2$	(-0.6,0.3,0.3)	(0.6,-0.3,-0.2)
$\mathfrak{U}_3$	(0.7,0.2,-0.1)	(-0.2,0.3,0.1)
$\mathfrak{U}_4$	(-0.3,0.4,0.1)	(-0.3,0.7,-0.2)
$\mathfrak{U}_5$	(0.8,0.0,-0.2)	(-0.8,0.3,0.9)

TABLE 3. Teachers with factors

Replacement\Factors	Selected	Not selected
$\mathfrak{T}_1$	**0.7081**	0.6442
$\mathfrak{T}_2$	**0.6591**	0.6150
$\mathfrak{T}_3$	0.6486	**0.6535**

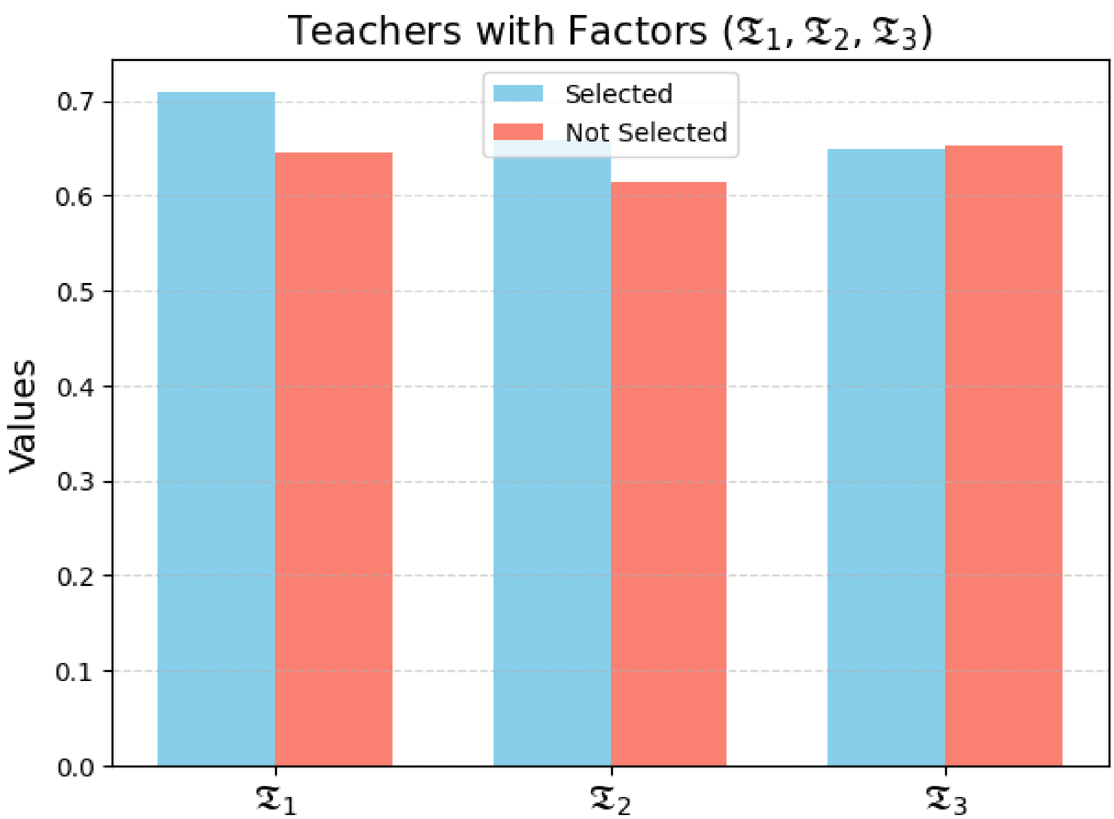


Table 4. Result

Replacement\Factors	Decision
$\mathfrak{T}_1$	Selected
$\mathfrak{T}_2$	Selected
$\mathfrak{T}_3$	Not-Selected

Therefore $\mathfrak{T}_1$ and $\mathfrak{T}_2$ are selected for teaching job.

8. Conclusion

This manuscript is the first-ever attempt at presenting a new look into Neutrosophic Under Soft Set and Neutrosophic Under Soft Topological Space and Generalized Continuous Function that will prove to be an important contribution toward theoretical development in this specialized area of mathematical science. As such, this manuscript goes deeper into operational characteristics of Neutrosophic Under Soft Sets, and it proposes a tangential similarity measure of correlation that goes beyond the classical approach, enriching the understanding of interrelations within that structure. Another aspect under the manuscript is the real application of these correlation measures using the practicality. Scenario: a $\mathfrak{T}_1$ and $\mathfrak{T}_2$ teachers have been chosen and selected for assignments in teaching. Here, there is that union of insight at the theoretical level as well as real applications towards the study of Neutrosophic Under Soft Sets, implying the practically significant nature of the proposed measures. Consequently, this research is important for the theoretical development researcher and practical searcher looking to the discipline for ever more complicated analytical tools.

Further, the $\mathcal{N}_{\mathfrak{s}}^{\mathfrak{u}}$-set correlation measure has many applications in medicine, industry, and construction.

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